

CALCULUS II SAMPLE PLACEMENT EXAMINATION

1. The arc length of $y = 8x^{3/2}$, $0 \leq x \leq 1$ is

- a) 7π b) $\pi(\sqrt{145} - 1)$ c) $\frac{1}{216}(145^{3/2} - 1)$ d) $\frac{3}{4}$ e) none of a) - d)

2. The surface area generated by the rotation of $y = x^2$, $0 \leq x \leq 4$, about the y-axis is

- a) $\frac{15\pi}{2}$ b) $\frac{\pi}{2}(65^{3/2} - 1)$ c) $\frac{\pi}{6}(65^{3/2} - 1)$ d) 2π
e) none of a) - d)

3. The area, bounded by $y = x^3$, $y = 0$, and $x = 1$, is rotated about the x-axis. The volume generated is

- a) $\frac{1}{7}$ b) $\frac{\pi}{7}$ c) π d) $\frac{2\pi}{5}$ e) none of a) - d)

4. What is the area of the region bounded by the curve $y = 3x^2$ and the line $y = 12$?

- a) 12 b) $\frac{7}{3}$ c) $\frac{1}{2}$ d) 32 e) 24π

5. $\int \tan^{-1} x dx =$

- a) $\ln|\tan x| + C$ b) $x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2) + C$ c) $\ln|\sin x| + C$
d) $\frac{1}{1+x^2} + C$ e) $-\ln|\cos x| + C$

6. How many **terms** should appear in the partial-fraction decomposition of $f(x) = \frac{2x^6 + 7x^4 + 5x^2 + 2}{x^3(x^2 + 1)^2}$

NOTE: You do not need to find the numerical values of the coefficients in order to answer this question.

- a) 2 b) 3 c) 4 d) 5 e) 6

7. The form of the decomposition in partial fractions of $\frac{x^8 + 2x^2 + 1}{(x - 2)^3(x^2 + 1)^2}$ is:

a) $3\frac{C}{x-2} + 2\frac{Fx+G}{x^2+1}$

b) $\frac{C}{x-2} + \frac{D}{(x-2)^2} + \frac{E}{(x-2)^3} + \frac{F}{x^2+1} + \frac{H}{(x^2+1)^2}$

c) $Ax+B + \frac{C}{x-2} + \frac{D}{(x-2)^2} + \frac{E}{(x-2)^3} + \frac{Fx+G}{x^2+1} + \frac{Hx+K}{(x^2+1)^2}$

d) $\frac{C}{x-2} + \frac{D}{(x-2)^2} + \frac{E}{(x-2)^3} + \frac{Fx+G}{x^2+1} + \frac{Hx+K}{(x^2+1)^2}$

e) none of the above

8. $\int \frac{1}{x^2 + 4x - 5} dx =$

a) $\frac{1}{6} \ln \left| \frac{x-1}{x+5} \right| + C$

b) $\ln |x^2 + 4x - 5| + C$

c) $\ln x^2 + 4 \ln x - \ln 5 + C$

d) $\tan^{-1}(x+2) + C$

e) $\frac{1}{3}x^3 + 2x^2 - 5x + C$

9. Which of the following integrals is convergent?

a) $\int_1^{\infty} \frac{dx}{(x+7)^{0.5}}$

b) $\int_{-\infty}^{\infty} \frac{x^3}{x^4+1} dx$

c) $\int_{-1}^3 \frac{dx}{x^2+x-2}$

d) $\int_1^{\infty} \frac{dx}{(x-7)^{2.5}}$

e) $\int_{-\infty}^{\infty} \frac{x^3}{x^6+1} dx$

10. The interval of convergence, excluding possible end points, for the series $\sum_{n=0}^{\infty} \frac{(2x+3)^n}{7^n(n+4)}$ is

a) $-1 < x < 1$

b) The entire real axis

c) $-5 < x < 2$

d) $-7 < x < 7$

e) none of the above

11. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{6n}}{(2n)!}$ is the power series in x for

- a) $\frac{1}{1-2x^3}$ b) $\cos(2x^3)$ c) $\sin(6x)$ d) $\exp(2x^3)$ e) none of the above

12. $\int \frac{1}{x \ln x} dx$ can be evaluated

- a) with the substitution $u = \ln x$
 b) with the substitution $u = \frac{1}{x}$
 c) with integration by parts: $u = x$ and $dv = \ln x dx$
 d) by evaluating $\int x \ln x dx$, then taking the reciprocal
 e) none of the above

13. $\int_1^e \ln x dx =$

- a) 1 b) $\frac{1}{e}$ c) $e + 1$ d) e e) $\frac{1}{e} - 1$

14. $\int_1^{\infty} \frac{1}{x^3} dx$

- a) diverges b) converges to 2
 c) converges to 1/2 d) converges to 1 e) can only be approximated

15. The interval of convergence, excluding possible end points, of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{3^n}$ is

- a) The entire real axis b) $-3 < x < 3$ c) $\{0\}$ d) $-1/3 < x < 1/3$ e) $-9 < x < 9$

16. The series $\sum_{n=0}^{\infty} 2x^n$ converges, for $|x| < 1$, to

- a) $\frac{1}{2+x}$ b) $\frac{1}{1-2x}$ c) $\frac{1}{1+2x}$ d) $\frac{2}{1-x}$ e) $\frac{2}{1+x}$

17. Consider the following series: (I) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2.5^n}$ and (II) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{.25^n}$
- a) both series converge b) both series diverge c) only I converges d) only II converges e) cannot be determined
18. Consider the following series: (I) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ and (II) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$
- a) both series converge absolutely b) only I converges absolutely c) only II converges absolutely d) neither series converges absolutely e) can not be determined
19. The first two non-zero terms of the power series in x , about $x = 0$, for $\ln x$ is
- a) $1 + x$ b) $1 - x$ c) x d) $-x$ e) none of the above
20. The first two non-zero terms of the Taylor series, about $x = 1$, for x^3 is
- a) $(x - 1)^3$ b) $1 + (x - 1)$ c) $1 + 3(x - 1)$ d) $1 + x$ e) $1 + x^2$
21. $\int \sin^2 x dx =$
- a) $\frac{\sin^3 x}{3} + C$ b) $\cos^2 x + C$ c) $\frac{x}{2} - \frac{1}{4} \sin(2x) + C$
- d) $\frac{x}{2} - \frac{1}{2} \sin(2x) + C$ e) $1 - \frac{\cos^3 x}{3} + C$
22. $\int \cos^3 x \sin^2 x dx =$
- a) $\frac{\cos^4 x \sin^3 x}{12} + C$ b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$ c) $\sin^3 x - \sin^5 x + C$
- d) $-\frac{\cos^4 x}{4} + C$ e) $\frac{\cos^4 x}{4} + \frac{\sin^3 x}{3} + C$

Answer key:

1. C 2. C 3. B 4. D 5. B 6. D 7. C 8. A 9. E 10. C 11. B 12. A
13. A 14. C 15. B 16. D 17. C 18. B 19. E 20. C 21. C 22. B

Passing score: 17 correct answers.