Equity Crowdfunding: Harnessing the Wisdom of the Crowd

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Abstract

We study the interaction of sophisticated and naïve investors in equity crowdfunding. We show that naïve investors, possessing weak but on average correct signals, are required for efficient financing. In fact, properly designed platforms can deliver first-best financing from naïve investors alone. Sophisticated investors, who are better informed and anticipate other investors’ actions, cannot by themselves use their information to improve investment efficiency. Our results have important implications for the design of crowdfunding platforms. In particular, individual investment limits benefit all investors by improving financing efficiency.

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1 Introduction

Crowdfunding is a young form of financing allowing entrepreneurs to solicit small contributions from many individuals via the Internet. In its infancy, crowdfunding provided financing for creative projects via donations or in exchange for recognition and has evolved to encompass a number of methods, including product pre-sales and sales of equity. While equity crowdfunding represents less than 5% of crowdfunding investment worldwide (Massolution 2013), overall crowdfunding grew 167% to $16.2 billion in 2014 (Massolution 2015) and the JOBS Act has set the stage for the growth of equity-based crowdfunding in the US.\(^1\)

While its dramatic growth is noteworthy, understanding crowdfunding is particularly important because of potential social benefits including increased trade between investors and entrepreneurs, innovation spillovers, and better geographic dispersion of capital (Agrawal, Catalini and Goldfarb 2014). Such benefits will only be realized if crowdfunding platforms efficiently match entrepreneurs and investors. To date, most crowdfunding has relied on non-pecuniary rewards, attracting investors who enjoy private benefits from investing. For example, Boudreau, Jeppesen, Reichstein and Rullani (2015) finds that many crowdfunding rewards are not sufficient compensation for the capital invested, concluding that psychological rewards, reciprocity and social interactions are important motivations. Equity crowdfunding is likely to attract both sophisticated, return-seeking investors and naive investors, who may behave more like consumers. Given the SEC’s interest in protecting investors, and firms’ interests in developing relationships with customers, it is important to understand how these two distinct groups will interact on equity crowdfunding platforms. To our knowledge, our paper is the first to analyze the effects of sophisticated and naive investors’ interactions in equity crowdfunding.

Our main result is that naive investors, not sophisticated investors, are required for efficient crowdfunding. Having weak but on average correct signals, naive investors who myopically follow their signals will collectively invest more in good projects than bad projects. Consider a population of naive investors with $1,000,000 of collective capital. If the naive investors are correct 51% of the time, a good project attracts $510,000 while a bad project attracts $490,000. When the funding threshold lies between these two amounts (e.g. $500,000), the good project receives sufficient capital and proceeds, while the bad project fails to meet the threshold and the capital is returned to investors. Similar all-or-nothing financing mechanisms have been

\(^1\)Stemler (2013) details the JOBS Act, specifically Title III (the CROWDFUND Act) which enables entrepreneurs to sell small amounts of equity to a large number of investors via the Internet.
applied by almost all non-equity crowdfunding platforms and such a mechanism will likely be mandated by
the CROWDFUND Act (Agrawal, Catalini and Goldfarb 2014). The all-or-nothing mechanism improves
financing efficiency by harnessing the collective wisdom of imprecisely informed, naïve investors.

Despite having individual informational advantages over naïve investors, a crowd of sophisticated in-
vestors cannot use that advantage, and instead, acts as collectively uninformed. As with naïve investors,
if sophisticated investors follow their signals, there will be a funding gap between good and bad projects.
However, if only good projects are funded, sophisticated investors who receive bad signals will have an in-
centive to deviate from following their signal and instead will invest. As a result, no equilibrium exists in
which sophisticated investors alone finance good projects but not bad projects. The only surviving equilibria
are those in which good and bad projects are financed. Therefore, sophisticated investors cannot finance
projects other than those that would be financed with no information.

Turning to the details of the model, our results rely on several key assumptions based on fundamental
features of crowdfunding. First, we consider a mass of infinitesimal investors, reflecting the small contribu-
tions common in crowdfunding. As a result, individuals do not internalize their actions and the model is
tractable. In an extension, we show our main results are robust to considering crowdfunding from a discrete
population of investors. Second, projects are subject to all-or-nothing financing, also known as a “provision
point mechanism” (Bagnoli and Lipman 1989), where projects not receiving sufficient capital are canceled.
The all-or-nothing feature creates a hedging effect, where bad projects are not financed, which can lead
to improved investment efficiency or equilibrium breakdown. Third, the mass of investors is comprised of
sophisticated and naïve investors. Sophisticated investors, whose signals are weakly more precise than those
of naïve investors, are fully rational and properly anticipate the actions of other investors. Naïve investors,
due to limited financial literacy or alternative motivations for investing, myopically act on their signals.

In addition to our main result, the model predicts a number of features consistent with current crowd-
financing platforms. Time limits for fundraising campaigns arise for two reasons. First, they act as a means
to spur sophisticated investors to action, preventing them from continually waiting for others to act. Kup-
puswamy and Bayus (2014) finds a U-shaped pattern in the timing of contributions, with capital surges in the
first and last week of the project, consistent with investors waiting in order to learn from others’ actions.
Second, a time limit serves as a means of removing projects from the platform, allowing bad projects to be

\(^2\)Mollick (2014) documents an average investment size of $64 and proposed SEC rules set maximums on individual invest-
canceled and the capital returned to investors. Mollick (2014) finds that crowdfunding campaigns tend to either meet their goal or to miss it badly (unsuccessful projects only raise 10% of their goals), so deadlines force the removal of bad projects. Another common feature of crowdfunding is that the crowd learns from early movers. In later periods within our model, sophisticated and naïve investors learn from early movers, only providing follow-on capital to those projects that will be successful. Zhang and Liu (2012) and Herzenstein, Dholakia and Andrews (2011) show evidence of rational herd ing by crowdfunding investors, where lenders engage in active observational learning, inferring quality from prior capital commitments.

Our main result has implications for the design of equity crowdfunding platforms. While we show that any population of naïve investors can support first-best financing, frictions and limited information may prevent first-best implementation on a project-by-project basis. Accordingly, we analyze the trade-offs associated with investment limits as well as the composition of investor populations. Consider the earlier example, where good projects receive $510,000 and bad projects receive $490,000. Under these assumptions, the venue will efficiently finance any project costing more than $490,000 but less than or equal to $1,000,000 (higher-cost projects are financed in later periods after investors learn from their peers). However, all projects costing $490,000 or less are financed, regardless of quality. To increase financing efficiency for these lower-cost projects, the platform can lower the maximum contribution per investor. For example, cutting the contribution size in half reduces capital to $245,000 for bad projects and $255,000 for good projects. Under this alternative, projects costing more than $245,000 and less than or equal to $500,000 are now efficiently financed. The trade-off is clear – lowering the maximum investment threshold improves financing efficiency for lower-cost projects, but limits the maximum size project that can be financed on the platform. While the SEC has emphasized that investment limits protect individual investors, our analysis shows that investment limits can benefit all investors by improving investment efficiency.

Mixing sophisticated and naïve investors can positively or negatively impact financing efficiency. Naïve investors’ expected returns are decreasing in the precision of sophisticated investors’ information, so larger differences in information precision harm naïve investors. While naïve investors are not deterred in the model (due to their myopically following signals), in the long-run, platforms may not be viable if naïve investors are excessively taken advantage of. However, adding sophisticated investors can improve efficiency as well. Consider the earlier example with all naïve investors. If half of those investors are replaced with sophisticated investors, naïve investors will initially invest $255,000 in good projects and $245,000 in bad projects. Recall
that with only naïve investors, all projects up to $490,000 would be funded, regardless of quality. However, with sophisticated investors, only high-quality projects between $255,000 and $490,000 will be financed, as sophisticated investors learn from the early-moving, naïve investors and provide financing in the subsequent period. By slowing down investment and allowing for learning, sophisticated investors can improve financing efficiency in some cases.

Unsurprisingly, we show that sophisticated investors’ unconditional returns exceed those of naïve investors. However, we show that returns conditional on investment may actually be larger for naïve investors. Empirical investigations of crowdfunding returns should account for investment opportunities and the frequency of investment to properly measure investors’ welfare.

Our work relates to a nascent theoretical literature on crowdfunding. Belleflamme, Lambert and Schwienbacher (2014) analyzes an entrepreneur’s choice between crowdfunding via pre-orders and selling equity claims. They emphasize that private benefits lead to different financing outcomes that are unique to crowdfunding — without those benefits crowdfunding would not be different from bank financing. Similarly, we assume naïve investors are not perfectly rational, which is critical to our results. Hakenes and Schlegel (2014) analyzes a model in which a finite number of households endogenously produce information and then invest using debt sourced via crowdfunding. As in our model, their model highlights the winner’s curse and the natural hedge that comes about from not financing bad projects. However, their focus is on endogenous information production, while ours is on the interactions of investors. Cumming, Leboeuf and Schwienbacher (2015) compares keep-it-all versus all-or-nothing financing, and shows that keep-it-all mechanisms are better for small, scalable projects. We only consider all-or-nothing financing, given its prominence in practice and likely requirement for US equity crowdfunding. Our result that sophisticated investors alone cannot efficiently finance projects is related to Axelson and Makarov (2014), which shows that an auction’s informational efficiency can be destroyed when the information is used to make an investment decision. In their setting, reducing the number of investors improves efficiency, while efficiency in our setting depends on the presence of naïve investors.

To our knowledge, ours is the first paper to study the interaction of sophisticated and naïve investors in equity crowdfunding. We show the importance of naïve investors for efficient financing and highlight several tensions governing the efficiency of equity crowdfunding platforms. On the eve of broad-based equity crowdfunding becoming available to US investors, understanding the tensions facing investors and platforms
is critical to properly matching capital to entrepreneurs and delivering the myriad benefits of crowdfunding.

2 Base Model

A unit continuum of risk neutral investors participates on a crowdfunding platform. A project hosted on the platform has a business plan to produce and sell a product (a good or service). The project is financially valued according to its financial soundness $F \in \{0, 1\}$, its upfront cost $c$, and promised gross rate of return $\Delta > 1$. Namely, the project is valued as,

$$V = F\Delta c - c.$$  (1)

The project’s financial soundness is not observable but, with equally probability, will be financially sound, $F = 1$, or not financially sound, $F = 0$, i.e.

$$\Pr(F = 1) = \Pr(F = 0) = \frac{1}{2}.$$  (2)

The project’s upfront cost $c$ and promised gross rate of return are observable. Define the project’s promised net return as,

$$\delta \equiv \Delta - 1 > 0.$$  (3)

Furthermore, the project’s fundraising goal on the crowdfunding platform is equal to $c \in \mathbb{R}^+$. The project’s fundraising goal of $c$ is accomplished by selling equity claims. In exchange for $c$ dollars, investors will have claim to 100% of the project’s cash flows. Each investor on the platform can invest at most $M$ dollars into the project. The cap on investment $M$ is exogenously set, and the optimal choice of $M$ is explored in Section 3. By parameterizing both the project’s promised return ($\Delta$) and the project’s cost ($c$), we separate profitability from project scale.

The project only receives funding if it meets its goal of raising $c$, i.e., it is all-or-nothing. If the project fails to meet its fundraising goal, committed capital is returned to investors. In the event that the project raises more capital than it needs, ownership is divided on a pro rata basis.\(^3\)\(^4\) Furthermore, the project is

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\(^3\)In a world without agency conflicts, it is equivalent to assume that the project’s entrepreneur stores excess cash in an account that earns investors their opportunity cost of capital.

\(^4\)In Section 2.3 we consider an extension in which ownership is allocated on a first-come, first-served basis rather than a pro rata basis.
on the platform until it gets funded or pulled. However, as will be shown shortly, an exogenous deadline is necessary when no endogenous stopping time exists in equilibrium. Denote the first period that the project is launched on the platform as $t = 0$. The discount rate between periods is normalized to zero and all investors can borrow or lend at the discount rate (i.e., investors do not face capital constraints, other than the exogenously given maximum project investment $M$). It is assumed that each investor can participate only once per project and investors that do participate are not replaced.

Define $K_t \geq 0$ as the quantity of funds raised for the project at the beginning of period $t$. Initially, zero funds are pledged to the project, $K_0 = 0$. However, over time the project will accrue contributions, and the incremental contribution in period $t$ is defined $\kappa_t$. Investment levels and flows provide signals of the project’s potential success which aids investors’ valuation of the project. The set $\{K_t, \kappa_t\}$ is publicly observable after the conclusion of period $t$ and all participants have perfect recall for previous periods. Define $\mathcal{T}_t$ as the project’s entire fundraising history up until period $t$. The model’s timing in every period $t$ is as follows,

(i) Project enters period $t$ with raised capital $K_t$,

(ii) Participants observe $K_t$ and update their beliefs,

(iii) Participants contribute funds $\kappa_t$,

(iv) If $K_t + \kappa_t \geq c$ the project is funded, the project is removed from the platform and its type is revealed,

(v) If $K_t + \kappa_t < c$ the project is not fully funded and period $t + 1$ begins.

A fraction $\gamma$ of investors are fully rational. These investors are referred to as sophisticated. Each sophisticated investor receives a private signal about the project’s financial soundness at the beginning of the first period $t = 0$,

$$\hat{F} \in \{0, 1\}, \quad (4)$$

where,

$$\Pr(\hat{F} = F) = \alpha > \frac{1}{2}, \quad (5)$$

and signals are i.i.d. Sophisticated investors optimally choose when to invest; they consider their private signals and the anticipated actions of other participants on the platform.\(^5\)

\(^5\)We assume sophisticated investors cannot coordinate across periods. Were they able to, sophisticated investors could reveal
The remaining \(1 - \gamma\) fraction of investors are not sophisticated. These investors are referred to as na"ıve. Na"ıve investors’ participation is positively correlated with the project’s financial soundness. If the project is financially sound, a fraction
\[
\beta > \frac{1}{2}
\]
of na"ıve investors will participate at \(t = 0\). If the project is not financially sound, only a fraction \((1 - \beta)\) participate. Na"ıve investors may be consumers that invest because they think the project is chic or amateur investors that do not correctly assess the opportunity (perhaps due to overconfidence) and invest myopically based on a signal with precision \(\beta\). Nevertheless, the investment decisions of na"ıve investors are positively correlated with the project’s success. We further assume that \(\beta \leq \alpha\), however the majority of our analysis does not rely on that assumption.\(^6\)

The presence of na"ıve investors captures two paramount features of crowdfunding: (i) investors of heterogeneous skill jointly fund projects, and (ii) the investment process may include investors that will eventually consume the project’s good or service which implies that investment activity provides feedback about the project’s potential success.

Each na"ıve investor is risk neutral and will contribute a level of investment equal to \(M\). Define \(\kappa_{t,S}\) and \(\kappa_{t,N}\) as the period \(t\) investment flows from sophisticated and na"ıve investors respectively. As such, \(\kappa_t = \kappa_{t,S} + \kappa_{t,N}\).

**Lemma 1.** If the project is financially sound, the investment flows from na"ıve investor are
\[
\kappa_{0,N}(1) = (1 - \gamma)\beta M,
\]
and if the project is not financially sound, the investment flows are,
\[
\kappa_{0,N}(0) = (1 - \gamma)(1 - \beta)M.
\]

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\(^6\)If \(\beta > \alpha\), the participation strategies of sophisticated investors are deterministic. Furthermore, in characterizing sophisticated investors’ participation criteria, the signs on a few comparative statics flip when \(\beta > \alpha\). The change in signs is due to a stronger winner’s curse among na"ıve investors relative to sophisticated investors.
Furthermore,

\[ \kappa_{0,N}(1) > \kappa_{0,N}(0). \]  

(9)

Lemma 1 shows that good projects attract greater quantities of capital from naïve investors than bad projects do. The positive correlation between project quality and raised capital is a valuable trait we refer to as the *crowd signal* and we explore how this trait may be exploited to gain efficiency in Section 3.\(^7\)

Sophisticated investors’ expectations of the project worth depend on their signals:

\[
E[V|\hat{F}] = \begin{cases} 
(\alpha \Delta - 1)c & \hat{F} = 1 \\
((1 - \alpha)\Delta - 1)c & \hat{F} = 0 
\end{cases}
\]  

(10)

Sophisticated investors do not necessarily invest when \(E[V|\hat{F}] \geq 0\) for two reasons. The first reason is due to their asymmetric exposure to project outcomes. Because the project’s quality positively correlates with the quantity of capital raised, sophisticated investors receive smaller pro rata shares in good projects relative to bad projects. The asymmetric exposure compels sophisticated investors to abstain from some projects that they expect to be profitable. We detail this shortly.

The second reason that sophisticated investors may not necessarily invest in a potentially-profitable project is because there is option value in waiting for more information to be revealed in subsequent periods. Each sophisticated investor’s ability to accurately forecast the project’s value is weakly increasing with the time he waits; the incremental funding activity each period may contain information that allows him to refine his beliefs. Therefore,

**Lemma 2.** *It is never optimal for a sophisticated investor to invest at time \(t\) if the project would still need additional funding in a subsequent period.*

Lemma 2 is natural: a sophisticated investor that chooses to invest will not do so until he is compelled to. The investor’s information set weakly improves each period he waits. As such, if a project will need an additional period of financing, it cannot be an equilibrium action for the sophisticated investor to forfeit the opportunity to refine his information set further.

\(^7\)In previous versions of this manuscript, naïve investors that abstained from participating at \(t = 0\) were allowed to update their beliefs based on \(t = 0\) investment flows and participate. Under that assumption, our results were qualitatively unchanged.
Lemma 3. If it is optimal for a sophisticated investor that observes $\hat{F} = 0$ to invest at time $t$, then it is optimal for an investor that observes $\hat{F} = 1$ to also invest at time $t$.

The result of Lemma 3 is natural as well. If a sophisticated investor that observes $\hat{F} = 0$ finds it optimal to invest then it must be the case that sophisticated investors that observed $\hat{F} = 1$ also find it optimal to invest because their expectation about the project’s payoff is higher.

Before we proceed to solving the model, there is a matter of housekeeping to address. Under some parameter sets and model specifications, a set of investors will be indifferent between investing in period $t$ or waiting until a later period. Both are equilibria. We, however, adopt a refinement that implies only the latter equilibrium of waiting survives. The following defines what we coin the “opportunity cost” equilibrium refinement,

Definition 1. If two equilibrium strategies exist for a set of sophisticated investors with one equilibrium being to invest at time $t$ and the other being to wait until some $\hat{t} > t$, and the investors’ payoffs are equal under the equilibria, then the opportunity cost equilibrium refinement dictates that the investors will choose the latter equilibrium, i.e., they will wait.

The motivation for the opportunity cost equilibrium refinement is that each investor’s information set weakly improves the longer he waits and his payoff will not decrease if he waits. We will appeal to the opportunity cost equilibrium refinement in narrowing the set of equilibrium strategies studied hereafter. We now proceed to solving the base model.

To ease exposition, we use Lemma 1 to partition the universe of projects into distinct categories.

Lemma 4. Define

$$\bar{c} = M(\gamma + (1 - \gamma)\beta).$$

Any project with cost $c > \bar{c}$ cannot be funded on the platform. Define

$$\underline{c} = (1 - \gamma)\beta M.$$  

Any financially sound project with cost $c \in (0, \underline{c}]$ is funded with certainty at the conclusion of $t = 0$. These projects are referred to as “small-scale projects.” Any financially sound project with cost $c \in (\underline{c}, \bar{c}]$ may require multiple periods to fund. These projects are referred to as “large-scale projects.”
There are two different threshold costs outlined in Lemma 4. The first cost, $\tau$, provides an upper bound on the projects that can be plausibly funded on the platform. The explicit form of $\tau$ is the maximum quantity of capital that can be raised on the platform for a financially sound project when the project’s type is known by all participants. If the project’s cost exceeds the maximum investment level from a perfectly informed crowd it cannot be feasibly financed on the platform.

The second cost $\underline{c}$ is an upper bound for a special set of projects. If a financially sound project has cost $c \leq \underline{c}$ it can be entirely financed by naïve investors at $t = 0$. Thus, the projects with costs $c \in (0, \underline{c}]$ are treated as a one-shot game. The projects with costs $c \in (\underline{c}, \overline{c}]$ cannot be entirely financed in the first period by naïve investors. As such, these projects may take multiple periods to finance.

2.1 Small-Scale Projects

Consider a project with cost $c \in (0, \underline{c}]$. Suppose that the project is fully funded by an arbitrary length $\ell_S$ measure of sophisticated investors and arbitrary length $\ell_N$ measure of naïve investors. The project raises a total $\ell_S M + \ell_N M$ which means that a share,

$$\sigma \equiv \frac{c}{\ell_S M + \ell_N M},$$

is deployed to the project while the remaining $1 - \sigma$ is returned to investors (projects that generate more funding than necessary return those proceeds to investors). Therefore, each investor’s investment in the risky project is,

$$\sigma M.$$ (14)

Sophisticated investors anticipate that there is more capital from naïve investors when $F = 1$ than when $F = 0$. As such, $\sigma$ will be larger for $F = 0$ than $F = 1$ implying that a sophisticated investor’s expected exposure to the two states is asymmetric. Sophisticated investors play mixed strategies with regards to whether or not they invest. Define the mixing strategies for sophisticated investors to be,

$$\pi_{F,t} \in [0,1], \hat{F} \in \{0,1\}.$$ (15)
Sophisticated investors' observing $\hat{F} = 1$ at $t$ invest with probability $\pi_{1,t}$, while those observing $\hat{F} = 0$ invest with probability $\pi_{0,t}$. Now, define,

$$\sigma_0(F, \pi_{1}, \pi_{0}) = \begin{cases} 
M\left(\gamma(\pi_{1}(1-\alpha) + \pi_{0}(1-\alpha)) + (1-\gamma)(1-\beta)\right) & \text{if } F = 1 \\
M\left(\gamma(\pi_{1}(1-\alpha) + \pi_{0}(1-\alpha)) + (1-\gamma)(1-\beta)\right) & \text{if } F = 0
\end{cases}$$

(16)

where the subscript $t = 0$ is suppressed on both $\pi_{1,t=0}$ and $\pi_{0,t=0}$. The function $\sigma_0(F, \pi_{1}, \pi_{0})$ represents the two possible values of $\sigma$ in the first period $t = 0$. Also, define

$$S_0(F, \pi_{1}, \pi_{0})$$

(17)

as the set of projects funded in state $F$ at time $t$. The set $S_0(0, \pi_{1}, \pi_{0})$ contains all projects with costs

$$c \leq M\left(\gamma(\pi_{1}(1-\alpha) + \pi_{0}(1-\alpha)) + (1-\gamma)(1-\beta)\right)$$

and $S_0(1, \pi_{1}, \pi_{0})$ contains all projects with costs,

$$c \leq M\left(\gamma(\pi_{1}\alpha + \pi_{0}(1-\alpha)) + (1-\gamma\beta)\right).$$

Some intuition regarding the importance of these sets is in order. Consider the following example: a project with cost $\hat{c} \notin S_0(0, \pi_{1}, \pi_{0})$ will not be funded if it is not financially sound. Because the project will not attract sufficient capital in the state $F = 0$, investors receive an implicit hedge against that state because of the all-or-nothing feature of the crowdfunding platform.

With the preceding intuition, we proceed to solving for the investment criteria of sophisticated investors. Sophisticated investors will invest in a small scale project if their exposure to the state $F = 1$ is weakly greater their investment. Roughly speaking, define an exposure weight for sophisticated investors as,

$$\phi_0(\pi_{1}, \pi_{0} | \hat{F}) = \begin{cases} 
\frac{\alpha \sigma_0(1, \pi_{1}, \pi_{0}) \mathbb{1}_1}{\alpha \sigma_0(1, \pi_{1}, \pi_{0}) \mathbb{1}_1 + (1-\alpha)\sigma_0(0, \pi_{1}, \pi_{0}) \mathbb{1}_0} & \text{if } \hat{F} = 1, \\
\frac{(1-\alpha)\sigma_0(1, \pi_{1}, \pi_{0}) \mathbb{1}_1}{(1-\alpha)\sigma_0(1, \pi_{1}, \pi_{0}) \mathbb{1}_1 + \alpha \sigma_0(0, \pi_{1}, \pi_{0}) \mathbb{1}_0} & \text{if } \hat{F} = 0
\end{cases}$$

(18)

where $\mathbb{1}_F$ is shorthand notation for the indicator function $\mathbb{1}_{c \in S_0(F, \pi_{1}, \pi_{0})}$. The exposure weights are akin to a
change of measure: an investor’s subjective probabilities for each state (i.e., $\alpha$ and $(1-\alpha)$) are multiplied by their corresponding investment exposures (i.e., $\sigma_0(F, \pi_1, \pi_0)$) and then renormalized. The exposure weight is effectively a new subjective state probability for $F = 1$ if the investor’s investment was equally exposed across states.

Note that $\phi_0(\pi_1, \pi_0|\hat{F})$ only depends on $c$ through $\mathbb{1}_{c \in S_0(F, \pi_1, \pi_0)}$. Thus, while we detail $\phi_0(\pi_1, \pi_0|\hat{F})$ for small-scale projects, the exposure weights are not unique to the small scale project setting and will have implications for large-scale projects studied later in this section. For small-scale projects, $\mathbb{1}_1 = 1$ and the expression simplifies to,

$$\phi_0(\pi_1, \pi_0|\hat{F}) = \begin{cases} \frac{\alpha \sigma_0(1, \pi_1, \pi_0)}{\alpha \sigma_0(1, \pi_1, \pi_0) + (1-\alpha) \sigma_0(0, \pi_1, \pi_0) \mathbb{1}_0} & \hat{F} = 1, \\ \frac{(1-\alpha) \sigma_0(1, \pi_1, \pi_0)}{(1-\alpha) \sigma_0(1, \pi_1, \pi_0) + \alpha \sigma_0(0, \pi_1, \pi_0) \mathbb{1}_0} & \hat{F} = 0. \end{cases} \quad (19)$$

Consider a sophisticated investor that observes $\hat{F} = 1$. His ex ante expectation of the state $F = 1$ is $\alpha$, however, his exposure weight is

$$\phi_0(\pi_1, \pi_0|1) = \begin{cases} \leq \alpha & \mathbb{1}_0 = 1 \\ = 1 & \mathbb{1}_0 = 0. \end{cases} \quad (20)$$

Similarly, a sophisticated investor that observes $\hat{F} = 0$ expects $F = 1$ with probability $1-\alpha$, but his exposure to the state is,

$$\phi_0(\pi_1, \pi_0|0) = \begin{cases} \leq 1-\alpha & \mathbb{1}_0 = 1 \\ = 1 & \mathbb{1}_0 = 0. \end{cases} \quad (21)$$

In both (20) and (21), the exposure weights jump to 1 when $\mathbb{1}_0 = 0$ implying that the investor is 100% exposed to the state $F = 1$. In these cases, only good projects are funded, so investors are only exposed to the good state. As we will show shortly, sophisticated investors’ participation strategies, $\pi_0$ and $\pi_1$, depend on (20) and (21). Importantly, these strategies also determine how much capital is raised and whether $\mathbb{1}_0$ equals one or zero. As such, the exposure weights $\phi_0(\pi_1, \pi_0|1)$ and $\phi_0(\pi_1, \pi_0|0)$ are determined endogenously by the strategies of sophisticated investors, leading to some parameter sets in which no equilibria exist.

**Proposition 1.** No non-zero equilibrium participation strategies of sophisticated investors exist for projects
with a promised net return
\[
\delta \leq \delta \equiv \frac{\alpha - \alpha \gamma (1 - \gamma)(1 - \beta)}{1 - \alpha (\gamma + (1 - \gamma)(1 - \beta))}, \tag{22}
\]
and scale
\[
c \in (M((1 - \alpha)\pi_1 \gamma + (1 - \gamma)(1 - \beta)), c], \tag{23}
\]
with any \(\pi_1 \in [0, 1]\). We refer to the region of projects that satisfy this criteria as the indeterminant region.

The projects that constitute the indeterminant region are projects that are small in promised return and large in scale (relative to other small-scale projects). The small promised returns imply that some sophisticated investors who observe \(\hat{F} = 0\) will not find it optimal to participate. As such, the quantities of capital raised when \(\hat{F} = 1\) and \(\hat{F} = 0\) also decrease. Due to the all-or-nothing feature of the crowdfunding platform, the decrease in raised capital when \(\hat{F} = 0\) can be substantial enough that the non-financially sound projects are incapable of meeting their goal, i.e., \(I_0 = 0\). As such, the project will deliver a risk-less, positive expected return to the investors that participate. This implies that the investors that observed \(\hat{F} = 0\) and abstained will now find the project attractive, so \(\pi_1 = 1\) and \(\pi_0 = 0\) cannot be an equilibrium. However, if all investors participate the project will be able to meet its fundraising goal of \(c\) when \(\hat{F} = 0\). As such, investors’ observing \(\hat{F} = 0\) will no longer find it optimal to invest, so \(\pi_1 = 1\) and \(\pi_0 = 1\) cannot be an equilibrium. Similarly, no mixing strategies exist that will support an equilibrium due to the discrete jump in payoffs that occurs when \(I_0\) switches from a value of one to a value of zero.\(^8\)

**Proposition 2.** For projects that are not in the indeterminant region, the exposure weights of sophisticated investors determine their equilibrium participation in funding small-scale projects at \(t = 0\). Investors that observe \(\hat{F} = 0\) play a deterministic participation strategy and investors that observe \(\hat{F} = 1\) may play a mixed strategy,

\[
\begin{align*}
\text{All sophisticated investors participate} & \quad \Delta \in \left[\frac{\phi_0(1,1|0)}{\phi_0(0,0,0), \infty}\right] \\
\pi_1 \text{ of sophisticated investors that observed } \hat{F} = 1 \text{ participate} & \quad \Delta \in \left[\frac{\phi_0(0,0,1)}{\phi_0(1,1|0)}, \frac{1}{\phi_0(1,1)}\right] \\
\text{No sophisticated investors participate} & \quad \Delta \in \left[0, \frac{1}{\phi_0(0,0,1)}\right].
\end{align*}
\tag{24}
\]

\(^8\)In a setting in which there are a discrete number of sophisticated investors, an equilibrium could be supported in which the marginal investor fully internalizes his decision to deviate from his strategy of not participating. With a continuum of agents, each investor is of zero measure and cannot internalize his decision to deviate.
Corollary 2.1. The exposure weight of sophisticated investors that observe $\hat{F} = 0$,

$$\phi_0(1, 1|0),$$  \hspace{1cm} (25)

is decreasing in $\alpha$ and $\beta$ and increasing in $\gamma$. The exposure weight of sophisticated investors that observe $\hat{F} = 1$,

$$\phi_0(\pi_1, 0|1),$$  \hspace{1cm} (26)

is increasing in $\alpha$ and decreasing in $\beta$ and $\gamma$.

According to Proposition 2, sophisticated investors do not simply evaluate a project based on whether or not they expect the project to have a positive net payoff. Instead, sophisticated investors shade their expectations down by using the exposure weights $\phi_0(\pi_1, \pi_0|\hat{F})$ in place of the their subjective state probabilities. This investment criteria is stricter than a simple present value calculation and is symptomatic of the winner’s curse, similar to that identified in initial public offerings (Rock 1986). Additionally, in the proof of Proposition 2 we show that a necessary and sufficient condition for any sophisticated investors to participate on the crowdfunding platform is,

$$\delta > \Delta \equiv \frac{(1 - \alpha)\beta}{\alpha(1 - \beta)}.$$  \hspace{1cm} (27)

The expression for $\Delta$ is simple to characterize: it is decreasing in $\alpha$ and increasing in $\beta$. Therefore, sophisticated investors require less promised return when they have greater confidence in their signal and when they are less exposed to the winner’s curse from naïve investors.

According to Corollary 2.1, the exposure weight (and its corresponding effect on investment criteria) for sophisticated investors that observe $\hat{F} = 0$ is decreasing with both $\alpha$ and $\beta$. The exposure weight is decreasing in $\alpha$ because investors that observe $\hat{F} = 0$ are more confident that the project is not financially sound. All else equal, this makes these investors less willing to invest. The exposure weight is decreasing in $\beta$ due to the asymmetric exposure induced by more informed naïve investors. When $\beta$ is near one half, approximately the same number of naïve investors show up when the project is financially sound versus when it is not financially sound. Consequently, a sophisticated investor’s share of the project is relatively unchanged. As $\beta$ increases, the naïve investor crowd size for the two types of project diverges: many naïve investors show up for financially sound projects and few show up for not financially sound projects. This implies that a
sophisticated investor’s share of a bad project is larger than his share of a good project. Similar intuition applies to the comparative static of $\phi_0(1, 1|0)$ with respect to $\gamma$. As $\gamma$ increases, sophisticated investors grow relative to the population of naïve investors. This implies that the sophisticated investors are less exposed to the winner’s curse from naïve investors and share more risk with the population of sophisticated investors that are all playing the same investment strategy. All else equal, this implies that their exposures to the two states are less asymmetric.

Also according to Corollary 2.1, the comparative static of $\phi_0(\pi_1, 0|1)$ with respect to $\beta$ is negative for the same reasons that $\phi_0(1, 1|0)$ is decreasing in $\beta$. The comparative static of $\phi_0(\pi_1, 0|1)$ with respect to $\alpha$ is positive. Sophisticated investors that observe $\hat{F} = 1$ are more confident in their signal as $\alpha$ increases and, consequently, are more willing to invest. The exposure weight $\phi_0(\pi_1, 0|1)$ is negatively related to $\gamma$, which contrasts with the comparative static of $\phi_0(1, 1|0)$. The comparative static is negative because of the winner’s curse induced by sophisticated investors adhering to different strategies, i.e., a fraction $\pi_1$ of those that observe $\hat{F} = 1$ are willing to invest while those that observe $\hat{F} = 0$ are not. Therefore, a fraction $\alpha \pi_1$ of sophisticated investors show up when the project is financially sound and $(1 - \alpha) \pi_1$ otherwise. This form of the winner’s curse is stronger among sophisticated investors than naïve investors because their signal is more precise. Therefore, as $\gamma$ increases there are more sophisticated investors and the winner’s curse is stronger, all else equal.

### 2.2 Large-Scale Projects

Now, consider a project with cost $c \in (\underline{c}, \overline{c}]$. This project cannot be funded solely by naïve investors in the first period $t = 0$. It may, however, be funded at $t = 0$ if sophisticated investors provide ample funds. In the case that sophisticated investors do not provide the funding to meet the project’s goal at $t = 0$, the project enters its second period on the platform.

Suppose that the project is not fully funded at $t = 0$. By the strong law of large numbers, the project’s type is signaled perfectly by the total funds raised from naïve investors. If $F$ equals 1, then $K_1 = (1 - \gamma) \beta M$ and if $F$ equals zero, then $K_1 = (1 - \gamma)(1 - \beta) M$. All sophisticated investors update their beliefs fully,

$$
E[V|\hat{F}, K_1] = \begin{cases} 
(\Delta - 1)c & K_1 = (1 - \gamma) \beta M \\
-c & K_1 = (1 - \gamma)(1 - \beta) M.
\end{cases}
$$

(28)
To be financed, the project requires the involvement of sophisticated investors. If the project is not financially sound, $F = 0$, the project will not raise additional capital from sophisticated investors. Without an exogenous intervention, e.g., being pulled by the entrepreneur or the platform, the project becomes a zombie. We address this shortly. According to Lemma 2, sophisticated investors will invest only if they believe that no further periods of financing will be required. As such, there exist equilibria in which all investors invest at $t = 1, 2, 3, \ldots, \infty$. In each of these equilibria, the sophisticated investors’ payoffs are constant because their opportunity cost of capital is equal to zero. We now appeal to opportunity cost equilibrium refinement, which implies that investors will never stop waiting unless compelled to. The following proposition provides a mechanism to eliminate the possibility.

**Proposition 3.** A fundraising deadline of $t^* = 1$ is Pareto dominant (weakly). Any project that is not entirely financed at the conclusion of $t^*$ is pulled from the platform.

A deadline, as proposed by Proposition 3, provides two services. First, it guarantees that zombie projects, i.e., unfunded projects that are perfectly revealed as not financially sound, will be pulled. This ensures that naïve investors’ capital committed at $t = 0$ is returned. Second, the deadline compels sophisticated investors to invest, rather than waiting indefinitely. For the remainder of this section, we assume that the platform enforces a $t^* = 1$ deadline.

If the project is not fully funded at $t = 0$, a financially-sound project will be fully financed at $t = 1$ with the participation of all sophisticated investors. As such,

$$\kappa_1 = \gamma M,$$

and each investor’s investment equals

$$\sigma_1(1, 1, 1)M,$$

where $\sigma_1(1, 1, 1)$ is defined similar to (16),

$$\sigma_1(1, 1, 1) = \frac{c}{M(\gamma + (1 - \gamma)\beta)},$$

The remaining funds invested,

$$(1 - \sigma_1(1, 1, 1))M,$$
are returned to investors.

Although the project cannot be fully funded at $t = 0$ by naïve investors that observe $\hat{F} = 1$, sophisticated investors may choose to also participate $t = 0$ and fund the project. We call this preemption. By preemtting, sophisticated investors avoid dilution from $t = 1$ investors. Avoiding dilution via preemption is not costless, however. Investors forgo the ability to refine their beliefs based on the information contained in the $t = 0$ investment flow.

Importantly, investors will only preempt if they believe the project will be fully funded at $t = 0$. Thus, since each investor is infinitesimal, multiple equilibria may exist. Before we examine the equilibria we define a sophisticated investor’s value of preempting relative to waiting.

**Lemma 5.** For large-scale projects, the value of investing at $t = 0$ (preemption) relative to the waiting until $t = 1$ to invest is positive if,

$$
\Delta \in \left[ \frac{1}{\omega(\pi_1, \pi_0 | \hat{F})}, \frac{1}{\phi_0(1, 1 | 0)} \right],
$$

where

$$
\omega(\pi_1, \pi_0 | \hat{F}) = \begin{cases} 
0 & \hat{F} = 0 \\
\frac{\alpha(\pi_0(1, \pi_1, \pi_0), 1 - \sigma_1, 1, 1, 1)}{\alpha(\pi_0(1, \pi_1, \pi_0), 1 - \sigma_1, 1, 1, 1) + (1 - \alpha) \sigma_0(0, \pi_1, \pi_0, 1) 0} & \hat{F} = 1.
\end{cases}
$$

Sophisticated investors who receive good signals preempt to prevent dilution from sophisticated investors who receive bad signals. Importantly, sophisticated investors with good signals only uniquely preempt if the potential profitability of the project is sufficiently low. If potential profitability is high (such that all investors would participate at $t = 0$, as in Proposition 2), good-signal sophisticated investors cannot avoid dilution from bad-signal sophisticated investors who would also be willing to invest at $t = 0$. In such high-profitability cases, equilibria exist where financing occurs at either $t = 0$ or $t = 1$. However, only the more-efficient $t = 1$ equilibrium satisfies the opportunity-cost equilibrium refinement. In summary, preemption only occurs for projects’ having high-enough potential to tempt good-signal investors into forgoing further information, while low-enough potential that bad-signal investors would not be similarly tempted.

The value of preempting relative to waiting is also loosely similar to a change of measure. An investor’s subjective state probabilities, $\alpha$ and $(1 - \alpha)$, and possible investment shares in the two periods, $\sigma_0(\cdot)$ and $\sigma_1(\cdot)$, are combined. At $t = 0$, an investor’s share of the project is equal to either $\sigma_0(1, \pi_1, \pi_0)$ or $\sigma_0(0, \pi_1, \pi_0)$ while an the investor’s share at $t = 1$ is either $\sigma_1(1, 1, 1)$ (when $F = 1$) or zero (he will not invest if the
project is revealed to not be financially sound). The numerator in $\omega(\pi_1, \pi_0|\hat{F})$ is the investor’s subjective state probability for $F = 1$ multiplied by the difference in exposure to the investment if he invests at $t = 0$ relative to if he invests at $t = 1$. His exposure $\sigma_0(1, \pi_1, \pi_0)$ is larger than $\sigma_1(1, 1, 1)$ because of the dilution that occurs at $t = 1$. As such, the difference $\sigma_0(1, \pi_1, \pi_0) - \sigma_1(1, 1, 1)$ is weakly positive so long as the project can be funded at $t = 0$, i.e., $1_1 = 1$. This quantity is then normalized by considering also the product of the subjective state probability for $F = 0$ and the difference in exposure to the investment (i.e., $\sigma(0, \pi_1, \pi_0) - 0$). When $\Delta\omega(\cdot)$ is larger than 1, it implies that the gains from avoiding $t = 1$ dilution outweigh the cost of making an investment decision with imperfect information at $t = 0$ rather than with perfect information at $t = 1$.

Note that preemption can only be valuable when $\delta$ is sufficiently large such that $\pi_1 = 1$. If $\pi_1 < 1$, investing at $t = 0$ is break-even for sophisticated investors, so preemption provides no benefit. However, investing with less information is costly, so the value of preemption is strictly negative when $\pi_1 < 1$.

If a large scale project is financed at $t = 0$, it must be individually rational for each sophisticated investor that contributes capital. As such, the participation criteria outlined in Proposition 2 for small-scale projects must be satisfied for large-scale projects as well.

**Proposition 4.** Any large scale project may be financed entirely at $t = 0$ in equilibrium if the participation criteria and conditions of Proposition 2 are satisfied.

While it is possible to support an equilibrium in which a large scale project is entirely financed at $t = 0$, it is not necessarily the case that sophisticated investors prefer that equilibrium relative to one in which they fund the project at $t = 1$ when more information is available. Despite their preferences, the equilibrium is sustainable because each investor is infinitesimal and an investor does not affect the equilibrium with their choice to deviate or not. If, however, sophisticated investors could coordinate, they would only invest at $t = 0$ if the value of preemption was greater than the value of waiting.\(^9\)

### 2.3 First-Come, First-Served

The platform we have explored grants ownership on a pro rata basis. Investors that participate at $t = 0$ are treated equally with those that enter at $t = 1$. Dividing ownership on a pro rata basis may be common, but

\(^9\text{Lemma A5 (in the appendix) characterizes }\omega(\pi_1, \pi_0|\hat{F}). \text{ Interestingly, no underlying parameter is monotonically related to the value of preemption.}\)
dividing ownership on a fist-come, first-served basis (FCFS) is also natural. In this subsection, we consider how FCFS ownership affects the participation strategies of sophisticated investors.

FCFS ownership does not affect the participation strategies of investors for small-scale projects because they are one-shot games. For large-scale projects that require $T$ periods, ownership is allocated as follows: if an investor participates in period $t < T$, he receives a full share

$$
\sigma_{t,\text{FCFS}}(F, \pi_1, \pi_0) = 1,
$$

for all $F$, $\pi_1$, and $\pi_0$. Conversely, if an investor participates at period $t = T$ he receives a share,

$$
\sigma_{T,\text{FCFS}}(1, \pi_1, \pi_0) = \frac{(c - K_T)}{M\kappa_T}. 
$$

**Proposition 5.** If ownership is allocated on a first-come, first-served basis, no equilibria exist in which a project survives more than a single period.

According to Proposition 5, the only equilibrium in which a large-scale project is funded is one in which it is entirely funded by sophisticated and naïve investors at $t = 0$. To understand the result, suppose that only naïve investors invested at $t = 0$ and sophisticated investors waited until $t = 1$ to make their participation decisions. Naïve investors perfectly reveal the project’s quality through their funding levels and sophisticated investors only participate if $F = 1$. Financing efficiency is first-best and the naïve investors that invest at $t = 0$ receive a share

$$
\sigma_{0,\text{FCFS}}(\cdot) = 1
$$

while sophisticated investors receive a share,

$$
\sigma_{1,\text{FCFS}}(1, 1, 1) = \frac{(c - (1 - \gamma)\beta M)}{M\gamma},
$$

which is less than 1. A single sophisticated investor strictly increases his payoff by deviating from his strategy and investing with naïve investors at $t = 0$: financing efficiency is still first-best when he deviates, but he receives a larger share of the project’s payoff. Because each investor is infinitesimal, there is not a marginal sophisticated investor that internalizes the impact of his decision on the project outcome. Therefore, if an
equilibrium exists with FCFS share allocations, it must be the case that the project is entirely financed at \( t = 0 \).

**Corollary 5.1.** *Any project that is financed in equilibrium at \( t = 0 \) with first-come, first-served share allocations must satisfy the participation criteria and conditions of Proposition 2.*

The results of this section imply that FCFS share allocations may be welfare destroying because they increase the speed at which a project is financed. This prohibits investors from learning from investment flows and prevents large-scale, lower-return projects from being financed. Furthermore, there is a range of projects which will all be financed at \( t = 0 \), despite the value of preemption being negative. Overall, FCFS share allocations reduce financing efficiency and feasibility.

### 3 Platform Efficiency and Venture Capitalist Equivalence

Several obvious questions arise from the model analysis in the previous section: How efficient is a crowdfunding platform? Can a principal adjust features of the platform to enhance efficiency? How does a crowdfunding perform relative to a venture capitalist? What are the welfare implications for the two types of investors? In this section, we tackle these questions and provide normative recommendations.

#### 3.1 Platform Efficiency — Lengthening the Tenure of a Project

In the base model analyzed in Section 2, projects may live one or two periods when the fundraising deadline is set at \( t^* = 1 \). The projects that survive for two periods achieve first-best — a bad project never reaches its financing goal and a good project always does. Naturally, this implies that a principal may improve efficiency by increasing the likelihood that projects are not fully funded in the first period. One means to lengthen the platform life of a project is to decrease \( c \) so that more projects are classified as large scale rather than small scale. Recall, the explicit form for \( c \) is,

\[
c = (1 - \gamma)\beta M.
\]

As such, a principal has three means to decrease \( c \): (i) increase \( \gamma \), (ii) lower \( M \), and (iii) lower \( \beta \). Consider the first option, increasing \( \gamma \), i.e., increasing the proportion of sophisticated investors relative to naïve investors.
One means to increase $\gamma$ is to screen platform participants. If the principal is able to increase the fraction of sophisticated investors (who are prone to wait), less capital will flow to projects when the project is first launched. This allows the project to survive longer on the platform which allows subsequent investors to learn from investment flows and make first-best investment choices.

Next, consider the principal’s second option of lowering $M$. By lowering $M$, investors will contribute less to a project, compelling it to take multiple periods. Obviously, lowering $M$ is not costless: lowering $M$ limits the upper bound on what projects can be feasibly financed on the platform. Thus, while efficiency is gained, feasibility is lost.

Finally, consider the principal’s third option of lowering $\beta$. Lowering $\beta$ is obtained by lowering the precision of naïve investors’ signals. While taboo, lowering $\beta$ may imply making project prospectuses complex or opaque.

Another means to increase the likelihood that a project survives multiple periods is to use pro rata ownership allocations. According to Proposition 5, dividing ownership on a first-come, first-served basis expedites the financing process and destroys investor learning via interim investment flows. As such, our analysis recommends the adoption of pro rata ownership allocations on crowdfunding platforms.

### 3.2 Platform Efficiency — Harnessing the Power of the Crowd

Another means to gain efficiency is by exploiting the crowd signal — more people invest in good projects compared to bad projects. In particular, the all-or-nothing feature of crowdfunding platforms enables a principal to strategically choose $M$ such that only good projects are financed and bad projects fail to meet their fundraising goal. If a principal could set $M$ on a project-by-project basis, she could set $M$ such that the following two conditions are satisfied,

\[
c > (\gamma + (1 - \gamma)(1 - \beta))M, \tag{38}
\]
\[
c \leq (\gamma + (1 - \gamma)\beta)M \tag{39}
\]

The first condition ensures that a bad project never meets its goal, even if all sophisticated investors participate. The second condition ensures that a good project is fully funded. A rearrangement of the two
preceding inequalities implies that first-best can be achieved with any $M$ chosen from the interval,

$$M \in \left[ \frac{c}{\gamma + (1-\gamma)\beta}, \frac{c}{\gamma + (1-\gamma)(1-\beta)} \right].$$

(40)

Importantly, the ability to strategically use $M$ to achieve first-best relies on there being naïve investors. If all investors were sophisticated ($\gamma = 1$), no equilibrium could be supported in which first-best is attained. Conversely, if all investors are naïve ($\gamma = 0$), first-best is achievable. These investors, while right on average, do not unravel the correlation between crowd size and project quality. This is an important insight and brings us to the following proposition.

**Proposition 6.** If naïve investors participate on the crowdfunding platform, the contribution level $M$ can be strategically chosen to attain first-best.

The result of Proposition 6 suggests that it is possible for a crowdfunding platform, comprised of imperfectly informed investors, to perform as well as a perfectly informed agent. To emphasize this point, we introduce a measure of platform efficiency coined *venture capital equivalence*. Specifically, we are concerned with what signal precision, $\alpha_{VC}$, a single venture capitalist would need to have to perform equally, both in ex ante and ex post terms, as the crowdfunding platform. With the platform we have explored thus far, $M$ can be chosen such that 50% of projects are chosen ex ante and ex post 100% of those selected are financially sound. Thus, the platform can perform as well as a venture capitalist with signal precision $\alpha_{VC} = 1$.

Indeed, the strength of the preceding result relies heavily on the strong law of large numbers. We now depart slightly from our model to show that the crowd signal may still be harnessed even when project quality is not perfectly revealed through funding levels. In what follows, we show that $M$ can be strategically chosen to enhance platform efficiency when the following ingredients are present,

(i) The platform has an all-or-nothing funding feature,

(ii) the participant crowd size positively correlates with the project’s quality.

Suppose investor contribution levels are noisy and that the total number of investors that participate in financing a project is a random variable $x$. The source of the noise could be due to frictions in the investment process, unsophisticated investors (akin to noise traders), or the random arrival of sophisticated and naïve investors. Define a continuous density function $g(x)$ as the distribution of crowd sizes for non-financially
sound projects. Similarly, define \( \mathcal{G}(x) \) as the distribution of crowd sizes for projects that are financially sound, i.e., \( F = 1 \). \( \mathcal{G}^{\cdot} \) and \( \mathcal{G}(\cdot) \) represent the corresponding cumulative distribution functions of \( g(\cdot) \) and \( \mathcal{G}(\cdot) \). Define the support of \( g(x) \) as \([B, \bar{B}]\) and the support of \( \mathcal{G}(x) \) as \([G, \bar{G}]\).

We assume \( g(x) \) is first-order stochastically dominated (FOSD) by \( \mathcal{G}(x) \) such that higher realizations of \( x \) are more suggestive of a good project than a bad project. This implies that the crowd size positively correlates with the project’s quality. As such,

\[
E[x|g(\cdot)] < E[x|\mathcal{G}(\cdot)],
\]

and

\[
\mathcal{G}(x) \leq \mathcal{G}(x)
\]

for all permissible \( x \) and with strict inequality at some \( x \). Venture capitalist equivalence requires that 50% of projects are chosen ex ante and of the chosen projects a fraction \( \alpha_{VC} \) are financially sound.

**Proposition 7.** A crowdfunding platform is equivalent to a lone venture capitalist with signal precision \( \alpha_{VC} \) if the platform participants are limited to an investment of \( M^\star \), where the set \( \{\alpha_{VC}, M^\star\} \) is implicitly defined by the system of equations,

\[
\alpha_{VC} = 1 - \mathcal{G}\left(\frac{c}{M^\star}\right),
\]

\[
1 - \mathcal{G}\left(\frac{c}{M^\star}\right) = \mathcal{G}\left(\frac{c}{M^\star}\right).
\]

If \( \mathcal{G}(x) \) and \( \mathcal{G}(x) \) are strictly monotonic in \( x \), the set \( \{\alpha_{VC}, M^\star\} \) is unique.

**Corollary 7.1.** If the supports for \( g \) and \( \mathcal{G} \) do not overlap, \( M^\star \) can be chosen such that it is equivalent to a perfectly informed venture capitalist.

According to Corollary 7.1, first-best is achievable if the supports of the two distributions of investors do not overlap. Indeed, the base model we have explored in our paper is an example of such a setting. However, Proposition 7 provides a general setup for determining \( \alpha_{VC} \). In the following application we consider a setting where the distributions do overlap.
Figure 1: Venture-capital equivalence approaches first-best in a discrete setting with relatively few investors.

### 3.2.1 Application of Venture-Capitalist-Equivalent Precision

Here we consider a simple application of the preceding analysis and solve for the venture capitalist equivalent precision $\alpha_{VC}$ when the distribution supports overlap. Consider a project with cost $c$ and suppose that the distributions $g(x)$ and $\overline{g}(x)$ are both on the support $[0, 1]$ and are triangular distributions of the form,

\begin{align*}
g(x) &= 2 - 2x \quad (45) \\
\overline{g}(x) &= 2x. \quad (46)
\end{align*}

The corresponding CDFs are given by,

\begin{align*}
G(x) &= 2x - x^2 \quad (47) \\
\overline{G}(x) &= x^2. \quad (48)
\end{align*}

The two probability density functions are depicted in the first panel of Figure 1 and their cumulative distributions are in the second panel. From Proposition 7, the system of equations that pins down $\alpha_{VC}$ is,

\begin{align*}
\alpha_{VC} &= 1 - \left( \frac{c}{M^*} \right)^2, \quad (49) \\
1 &= 2 \left( \frac{c}{M^*} \right) - \left( \frac{c}{M^*} \right)^2 + \left( \frac{c}{M^*} \right)^2. \quad (50)
\end{align*}
Solving the system of equations yields,

\[ \alpha_{VC} = 75\%, \quad (51) \]
\[ M^* = 2c. \quad (52) \]

Therefore, it is possible for a principal to design the platform to be as efficient as a venture capitalist with a signal precision of 75%.

### 3.2.2 Application With Discrete Number of Agents

The venture capitalist equivalence also applies to settings with a discrete number of investors. For simplicity, consider a setting in which there is a set of \( N \) investors on the platform and they are all naïve and have precision \( \alpha > \frac{1}{2} \). A principal may choose \( M \) in such a way that the platform is equivalent to a venture capitalist. Instead of the specific level of precision \( \alpha_{VC} \) explored in the previous application with a continuum of investors, there is an upper and lower bound on the equivalent precision with a discrete number of agents.

With a discrete number of agents, the two distributions are binomial,

\[ g(x) = \binom{N}{x} (1 - \alpha)^N (\alpha)^{N-x} \]
\[ \overline{g}(x) = \binom{N}{x} (\alpha)^N (1 - \alpha)^{N-x}. \]

We consider three settings in which there are at most 30 investors on the platform. In the first setting, the investors’ precision is equal to 55%, in the second it equals 65% and in the third it equals 75%. In Figure 2 we depict each of the three scenarios. In each graph, the number of participants is depicted on the horizontal axis and precision levels are depicted on the vertical axis. The investors’ individual level of precision is depicted as a dashed horizontal line. By strategically choosing \( M \), a principal makes the platform as efficient as a venture capitalist with precision \( \alpha_{VC} \) that is contained anywhere in the interval between the lines labeled \( \alpha_{VC} \) and \( \overline{\alpha}_{VC} \). The power of the crowd signal result is demonstrated in the three panels. At a precision of 55% with 30 participating investors, the crowd funding platform can be as efficient as a venture capitalist with precision \( \alpha_{VC} \in [64\%, 68\%] \). More striking is how few participants are needed to approach \( \alpha_{VC} \approx 98\% \) when the participants’ level of precision is equal to 65% and 75%.

25
Figure 2: The triangular probability density functions $g(x)$ and $\pi(x)$ and their respective cumulative density functions.

3.3 Investor Welfare

The ability to gain efficiency by exploiting the positive correlation between capital raised and project quality, i.e., the crowd signal, relies on the presence of naïve investors. While platform efficiency improves with the participation of naïve investors, one may be concerned about their welfare relative to sophisticated investors. In this section we explore the expected returns of both sophisticated and naïve investors.

If a project survives more than a single period on the platform, it achieves first-best — a bad project is never financed and a good project always is. A small-scale project that is fully financed in the first period may also achieve first-best. If the all-or-nothing feature is exploited such that only a financially-sound project is financed, then all sophisticated investors will rationally participate. However, naïve investors are myopic and only a fraction $\beta$ of them will participate. As such, in settings featuring first-best financing, naïve investor’s expected returns are equal to $\beta$ times the expected return of sophisticated investors unconditionally. Conditional on investment, however, the two investor groups perform equally.

Finally, a small-scale project that is fully financed in the first period regardless of type does not achieve first-best. In this setting, sophisticated investors and naïve investors are both exposed to projects that are not financially sound. The following lemma provides the investors’ expected returns in this less-efficient setting,

**Lemma 6.** Sophisticated investors’ expected net return is

$$\rho_\alpha(\pi_1, \pi_0) = \left( \frac{\alpha \pi_1 + (1 - \alpha) \pi_0}{2} \right) \delta \sigma(1, \pi_1, \pi_0) - \left( \frac{(1 - \alpha) \pi_1 + \alpha \pi_0}{2} \right) \sigma(0, \pi_1, \pi_0).$$  \hspace{1cm} (55)
Conditional on investment, sophisticated investors’ expected net return is

\[ \rho_{\alpha|I}(\pi_1, \pi_0) = (\alpha \pi_1 + (1 - \alpha)\pi_0) \Delta \sigma(1, \pi_1, \pi_0) - 1. \] (56)

Naïve investors’ expected net return is

\[ \rho_{\beta}(\pi_1, \pi_0) = \left( \frac{\beta}{2} \right) \delta \sigma(1, \pi_1, \pi_0) - \left( \frac{1 - \beta}{2} \right) \sigma(0, \pi_1, \pi_0). \] (57)

Conditional on investment, naïve investors’ expected net return is,

\[ \rho_{\beta|I} = \beta \Delta \sigma(1, \pi_1, \pi_0) - 1. \] (58)

Lemma 6 provides two different types of return that affect investor welfare: unconditional and conditional on investment. The unconditional return not only considers the expected return when an investor chooses to participate, but also the possibility that an investor will stay on the sidelines. As such, the unconditional return provides a means of measuring overall investor welfare. The conditional return only considers the expected return when an investor chooses to participate. Many opponents of equity crowdfunding worry that everyday investors (i.e., naïve) will be fleeced or taken advantage of by sophisticated investors, i.e., relative to sophisticated investors, naïve investors will be under-exposed to good projects and over-exposed to bad projects. As such, an investor’s expected return conditional on investment is an appropriate means to evaluate whether or not those worries have merit. We explore the implications of both return types below.

The unconditional returns for sophisticated investors are bound below by zero because they are rational and will never participate if they would be better off abstaining. The unconditional returns for naïve investors may be negative because they myopically invest. For example, if \( \delta \) is sufficiently small and no sophisticated investors are willing to participate, a fraction of at least (1 – \( \beta \)) of naïve investors will participate. In these cases, the naïve investors’ expected returns are surely negative since no individual sophisticated investor (who is better informed and fully rational) is willing to invest. If a principal is concerned about this situation she may mitigate it by increasing \( \beta \), which helps myopic investors make better participation decisions, or by increasing \( \gamma \) via screening participants. One means to increase \( \beta \) is to enforce clarity in prospectuses so that investors have better understanding about a project’s return and risks. Alternatively, increasing \( \gamma \) insures
that there are fewer naïve investors on the platform. However, as discussed earlier, limiting the participation of naïve investors potentially limits the efficiency of the platform.

One may also be concerned about the disparity between sophisticated and naïve investors’ unconditional returns. Define the differences in unconditional expected returns for sophisticated investors relative to naïve investors as,

\[ D(\pi_1, \pi_0) \equiv \rho_\alpha(\pi_1, \pi_0) - \rho_\beta(\pi_1, \pi_0) \]

\[ = \left( \frac{\alpha \pi_1 + (1 - \alpha) \pi_0 - \beta}{2} \right) \delta \sigma(1, \pi_1, \pi_0) - \left( \frac{(1 - \alpha) \pi_1 + \alpha \pi_0 - (1 - \beta)}{2} \right) \sigma(0, \pi_1, \pi_0). \]

When \( D(\pi_1, \pi_0) \) is positive sophisticated investors expect to outperform naïve investors with respect to returns and vice versa when it is negative.

**Lemma 7.** Sophisticated investors’ unconditional expected returns are greater than naïve investors’.

Lemma 7 is natural, one would expect sophisticated investors to outperform naïve investors. If disparity is a concern to the principal, she may engage in policies to move \( D(\pi_1, \pi_0) \) closer to zero. The following proposition characterizes each investor type return.

**Proposition 8.** Sophisticated investors’ ex ante return \( \rho_\alpha \) is,

(i) increasing in \( \delta, \alpha, \) and \( c \)

(ii) decreasing in \( \beta \) and \( M, \)

(iii) and ambiguously related to \( \gamma. \)

Naïve investors’ ex ante return \( \rho_\beta \) is

(i) increasing in \( \delta \) and \( \beta, \)

(ii) decreasing in \( \alpha, \)

(iii) and ambiguously related to \( c, \) \( M \) and \( \gamma. \)

According to Proposition 8, the most obvious means to reduce disparity is to increase \( \beta, \) narrowing the gap between \( \alpha \) and \( \beta. \) Other than \( \delta, \) where higher returns clearly benefit investors, the remaining parameters do not have clear effects on naïve investors’ welfare.
Now we consider expected return conditional on investment. The comparative statics of expected returns conditional on investment mirror those of $\phi_0(F, \pi_1, \pi_0)$ explored in Section 2. Again, making $\beta$ closer to $\alpha$ unambiguously reduces the disparity between investor types. Interestingly though, naïve investors may outperform sophisticated investors in expected returns conditional on investment. To show this, define $D_I(\pi_1, \pi_0)$ as the difference in expected returns conditional on investment,

$$D_I(\pi_1, \pi_0) \equiv \rho_\alpha(\pi_1, \pi_0) - \rho_\beta(\pi_1, \pi_0)$$

$$= (\alpha \pi_1 + (1 - \alpha)\pi_0 - \beta) \Delta \sigma(1, \pi_1, \pi_0).$$

It is straightforward to see from the preceding expression that naïve investors outperform sophisticated investors in the cases in which,

$$\beta \geq \alpha \pi_1 + (1 - \alpha)\pi_0. \quad (63)$$

In fact, if $\pi_1$ and $\pi_0$ both equal one (sophisticated investors are willing to participate regardless of their signals), naïve investors necessarily outperform. Conversely, if investors that observe $\hat{F} = 0$ are unwilling to participate, naïve investors underperform sophisticated investors as long as

$$\frac{\beta}{\alpha} \leq \pi_1. \quad (64)$$

However, the inequality certainly flips as $\pi_1$ decreases. Taken together, the ordering of investor returns conditional on investment are,

$$\begin{align*}
\text{Expected Return Conditional on Investment} = \begin{cases} 
\text{Naïve outperform} & \pi_1 = \pi_0 = 1 \\
\text{Naïve outperform} & \pi_1 < \frac{\beta}{\alpha} \text{ and } \pi_0 = 0 \\
\text{Sophisticated outperform} & \pi_1 \geq \frac{\beta}{\alpha} \text{ and } \pi_0 = 0.
\end{cases}
\end{align*} \quad (65)$$

This result emphasizes that care must be taken when empirically analyzing investors’ crowdfunding returns. While data on observed investments may be more readily available, conditional-on-investment returns are not appropriate for analyzing investor welfare. Any analysis of investors’ returns must take into account their investment opportunities and the frequency of their investments.
4 Concluding Remarks

Our analysis suggests that naïve investors will play a critical role in equity crowdfunding. By acting on their information, naïve investors communicate the wisdom of the crowd and improve financing efficiency. Furthermore, platforms can set maximum investment thresholds to best utilize the crowd’s information. Lower investment thresholds imply that more investors must participate to fund a project, so bad projects are less likely to succeed, improving financing efficiency. However, lower investment thresholds also restrict the size of project that can be financed on a platform, reducing financing feasibility. Thus, platforms trade-off efficiency against feasibility.

The optimal trade-off between efficiency and feasibility depends on the distribution of projects that a platform attracts. While a platform attracting many low-cost projects may prefer a low investment maximum, a platform attracting many high-cost projects will need to loosen the investment maximum to make those projects feasible. Given these trade-offs, we anticipate that platforms will specialize and adjust investment thresholds accordingly.

While we do not model secondary effects of the investment thresholds, it is plausible that lowering the thresholds will decrease the precision of investors’ information. For example, if collecting or processing information is costly, and a minimum amount of time is required to evaluate a project, the reduced value-creation associated with lesser investment amounts may not justify the costs of information acquisition. In such a scenario, the precision of both sophisticated and naïve investors’ information is likely to decrease, along with information asymmetry between the two groups (assuming sophisticated investors’ time is more costly or information processing more efficient). As a result, platforms may also be able to influence their investor populations by making information acquisition more or less costly, providing another dimension for platforms to differentiate themselves.
References


Appendix A  Proof of Lemma 1:
The proof is trivial.

Proof of Lemma 2:
The intuition is natural; because investment is irreversible, there is no incentive to invest early if there will still be an additional period of capital raising. Therefore, it is costless to wait until the next period.

Proof of Lemma 3:
Because $\alpha > 1/2$, it can never be optimal for an investor that observes $\hat{F} = 0$ to invest if it is not optimal for an investor that observes $\hat{F} = 1$ to invest.

Proof of Lemma 4:
The proof follows from the naïve investors’ investment flow schedule outlined in Lemma 1.

Proof of Proposition 1:
We establish several useful results before proving the main proposition.

**Lemma A1.** The exposure weight $\phi_0(\pi_1, \pi_0|0)$ is increasing in $\pi_0$ and the exposure weight $\phi_0(\pi_1, \pi_0|1)$ is decreasing in $\pi_1$.

**Proof of Lemma A1:**
If $\mathbb{I}_0 = 0$, the comparative statics of $\phi_0(\pi_1, \pi_0|0)$ and $\phi_0(\pi_1, \pi_0|1)$ are trivial: they all equal zero. For the remainder of the proof, assume $\mathbb{I}_0 = 1$.

First, consider $\phi_0(\pi_1, \pi_0|0)$. The expression may be rewritten as,

$$\phi_0(\pi_1, \pi_0|0) = \frac{1}{1 + \eta_0}, \quad (A1)$$

where,

$$\eta_0 = \frac{\alpha (\gamma (\pi_1 \alpha + \pi_0 (1 - \alpha)) + (1 - \gamma) \beta)}{(1 - \alpha) (\gamma (\pi_1 (1 - \alpha) + \pi_0 \alpha) + (1 - \gamma) (1 - \beta))}. \quad (A2)$$

Therefore, the comparative static of $\phi_0(\pi_1, \pi_0|0)$ with respect to $\pi_0$ is,

$$\frac{\partial \phi_0(\pi_1, \pi_0|0)}{\partial \pi_0} = \frac{\partial \eta_0 / \partial \pi_0}{(1 + \eta_0)^2}. \quad (A3)$$
The partial derivative of $\eta_0$ with respect to $\pi_0$ is given by,
\[
\frac{\partial \eta_0}{\partial \pi_0} = \frac{\alpha \gamma((1 - \alpha - \beta)(1 - \gamma) + (1 - 2\alpha)\gamma \pi_1)}{(1 - \alpha)(\gamma (\pi_1(1 - \alpha) + \pi_0\alpha) + (1 - \gamma)(1 - \beta))^2}.
\]  
(A4)

The sign on the preceding expression is determined by the sign on,
\[
(1 - \alpha - \beta)(1 - \gamma) + (1 - 2\alpha)\gamma \pi_1,
\]  
(A5)

which is negative because $\alpha > \frac{1}{2}$, $\beta > \frac{1}{2}$ and $\gamma \in [0, 1]$. Therefore $\frac{\partial \eta_0}{\partial \pi_0} < 0$ implying that,
\[
\frac{\partial \phi_0(\pi_1, \pi_0|0)}{\partial \pi_0} \geq 0.
\]  
(A6)

Now, consider $\phi_0(\pi_1, 0|1)$. The expression may be rewritten as,
\[
\phi_0(\pi_1, 0|1) = \frac{1}{1 + \eta_1},
\]  
(A7)

where,
\[
\eta_1 = \frac{(1 - \alpha)(\gamma \pi_1 \alpha + (1 - \gamma)\beta)}{\alpha (\gamma \pi_1(1 - \alpha) + (1 - \gamma)(1 - \beta))}.
\]  
(A8)

Therefore, the comparative static of $\phi_0(\pi_1, \pi_0|1)$ with respect to $\pi_1$ is,
\[
\frac{\partial \phi_0(\pi_1, 0|1)}{\partial \pi_1} = -\frac{\partial \eta_1/\partial \pi_1}{(1 + \eta_1)^2}.
\]  
(A9)

The partial derivative of $\eta_1$ with respect to $\pi_1$ is given by,
\[
\frac{(1 - \alpha)\gamma((\alpha - \beta)(1 - \gamma))}{\alpha (\gamma (\pi_1(1 - \alpha) + \pi_0\alpha) + (1 - \gamma)(1 - \beta))^2}.
\]  
(A10)

The sign on the preceding inequality is determined by $\alpha - \beta$ which is positive. Therefore, $\frac{\partial \eta_1}{\partial \pi_1} > 0$, implying,
\[
\frac{\partial \phi_0(\pi_1, 0|0)}{\partial \pi_1} \leq 0.
\]  
(A11)

Lemma A2. Sophisticated investors that observe $\hat{F} = 0$ adhere to a deterministic strategy, $\pi_0 \in \{0, 1\}$. If,
\[
1 \leq \Delta \phi_0(1, 1|0),
\]  
(A12)

all sophisticated investors, regardless of signal invest in the project.

33
Proof of Lemma A2:

Consider a project with cost \( c \in (0, \underline{c}] \). All sophisticated investors that observe \( \hat{F} = 0 \) will invest if and only if,

\[
0 \leq (1 - \alpha) \frac{\Delta c - c}{c} M_{g0}(1, 1, 1) I_1 - \alpha M_{g0}(0, 1, 1) I_0,
\]

where \( I_F \) is shorthand notation for \( I_{c \in S_0(F, 1, 1)} \). Because these are small-scale projects, the inequality simplifies to,

\[
1 \leq \Delta \frac{(1 - \alpha)_{g0}(1, 1, 1)}{(1 - \alpha)_{g0}(1, 1, 1) + \alpha M_{g0}(0, 1, 1) I_0} \equiv \Delta \phi_0(1, 1|0).
\]

If the preceding inequality holds with equality, then for any return marginally smaller than \( \Delta \) it must be the case that not all sophisticated investors participate. One might think that a mixing equilibrium exists in this case, however,

\[
\phi_0(1, 1|0) > \phi_0(1, \pi_0|0)
\]

for any \( \pi_0 < 1 \) by Lemma A1. Therefore, for any \( \Delta \) smaller than \( 1/\phi_0(1, 1|0) \), investors that observe \( \hat{F} = 0 \) will not participate. Thus, the investors’ decision to participate is deterministic when they observe \( \hat{F} = 0 \).

The threshold return that compels the set of sophisticated investors that observe \( \hat{F} = 0 \) to invest is implicitly defined by the following equality (using (16) and (19)),

\[
\Delta \left( \frac{c(1 - \alpha)}{M(\gamma + (1 - \gamma)\beta)} + \frac{c\alpha}{M(\gamma + (1 - \gamma)(1 - \beta))} \right) = 1,
\]

which simplifies to,

\[
\Delta \left( \frac{1}{1 + \frac{\alpha(\gamma + (1 - \gamma)\beta)}{(1 - \alpha)(\gamma + (1 - \gamma)(1 - \beta))}} \right) = 1.
\]

Therefore, for all investors to participate (regardless of signal) the promised return needs to be sufficiently large,

\[
\delta \geq \delta \equiv \frac{\alpha(\gamma + (1 - \gamma)\beta)}{(1 - \alpha)(\gamma + (1 - \gamma)(1 - \beta))}.
\]

Now, consider when \( \delta < \delta \) so sophisticated investors’ observing \( \hat{F} = 0 \) do not participate. Furthermore, assume that \( I_0 = 1 \) without the participation of sophisticated investors that observe \( \hat{F} = 0 \) (we consider the case in which \( I_0 \) switches between 1 and 0 depending on the participation of sophisticated investors in
Proposition 1). Sophisticated investors that observe \( F = 1 \) will participate if and only if,

\[
0 \leq \alpha \frac{\Delta c - c}{c} M\sigma_0(1, 1, 0) \mathbb{1}_1 - (1 - \alpha) M\sigma_0(0, 1, 0) \mathbb{1}_0.
\]  

(A19)

The inequality simplifies to,

\[
1 \leq \frac{\alpha \sigma_0(1, 1, 0)}{\alpha \sigma_0(1, 1, 0) + (1 - \alpha) \sigma_0(0, 1, 0) \mathbb{1}_0} \equiv \Delta \phi_0(1, 0|1).
\]  

(A20)

If the preceding inequality holds with equality, then for any marginally-smaller return it must be the case that only a fraction of sophisticated investors participate. This is because,

\[
\phi_0(1, 0|1) < \phi_0(\pi_1, 0|1)
\]  

(A21)

for any \( \pi_1 < 1 \) by Lemma A1. The mixing probability \( \pi_1 \) is pinned down by the equality,

\[
1 = \Delta \phi_0(\pi_1, 0|1).
\]  

(A22)

A substitution of the explicit form of \( \phi_0(\pi_1, 0|1) \) into the preceding equality yields,

\[
1 = \Delta \frac{1}{1 + \frac{(1 - \alpha)}{\gamma \pi_1 (1 - \alpha) + (1 - \gamma) (1 - \beta)}}.
\]  

(A23)

The preceding expression simplifies to,

\[
\frac{1}{\Delta} = \frac{1}{1 + \frac{(1 - \alpha)}{\gamma \pi_1 (1 - \alpha) + (1 - \gamma) (1 - \beta)}},
\]  

(A24) or

\[
\delta = \frac{(1 - \alpha)}{\gamma \pi_1 (1 - \alpha) + (1 - \gamma) (1 - \beta)} (A25)
\]

since \( \delta = \Delta - 1 \). A rearrangement yields the solution for \( \pi_1 \),

\[
\pi_1 = \frac{(1 - \gamma) (\alpha (1 - \beta) \delta - (1 - \alpha) \beta)}{\gamma \alpha (1 - \alpha) (1 - \delta)}
\]  

(A26)

which is increasing in \( \delta \). The expression \( \pi_1 \) is greater than zero as long as,

\[
\delta \geq \frac{(1 - \alpha) \beta}{\alpha (1 - \beta)} \equiv \delta_*. \]  

(A27)
and note that,
\[
\frac{(1-\alpha)\beta}{\alpha(1-\beta)} \in [0,1].
\] (A28)

The expression \(\pi_1\) is less than one as long as,
\[
\delta < \frac{(1-\alpha)(\beta + (\alpha-\beta)\gamma)}{\alpha(1-\beta - (\alpha-\beta)\gamma)} \equiv \delta.
\] (A29)

and note that,
\[
\frac{(1-\alpha)(\beta + (\alpha-\beta)\gamma)}{\alpha(1-\beta - (\alpha-\beta)\gamma)} \in [0,1].
\] (A30)

If
\[
\phi_0(1,1|0) \leq \phi_0(\pi_1,0|1)
\] (A31)
holds for all \(\pi_1 \in [0,1]\), then we have completely characterized the system. However, we have not ruled out that there are parameter sets in which,
\[
\phi_0(1,1|0) > \phi_0(0,0|1),
\] (A32)
which is explicitly,
\[
\frac{(1-\alpha)^{(\gamma+(1-\gamma)\beta)}}{(1-\alpha)^{(\gamma+(1-\gamma)(1-\beta))}} > \frac{1}{\alpha(1-\gamma)\beta + (1-\alpha)\frac{1}{(1-\gamma)(1-\beta)}}.
\] (A33)

or,
\[
\frac{1}{1 + \frac{\alpha}{1-\alpha} \frac{(\gamma+(1-\gamma)\beta)}{(\gamma+(1-\gamma)(1-\beta))}} > \frac{1}{1 + \frac{(1-\alpha)}{\alpha} \frac{(1-\gamma)\beta}{(1-\gamma)(1-\beta)}}.
\] (A34)

which is possible if,
\[
\frac{\alpha}{1-\alpha} \frac{(\gamma+(1-\gamma)\beta)}{(1-\gamma)(1-\beta)} < \frac{(1-\alpha)}{\alpha} \frac{(1-\gamma)\beta}{1-\gamma)}.
\] (A35)

Because \(\alpha \in [\frac{1}{2},1]\), \(\alpha/(1-\alpha)\) is monotonically increasing in \(\alpha\) and it has a range \((1,\infty)\). Similarly, \((1-\alpha)/\alpha\) is monotonically decreasing in \(\alpha\) and has a range \((0,1)\). As such, the inequality cannot hold for arbitrarily large values of \(\alpha\). However, if \(\alpha\) is sufficiently close to \(1/2\), the inequality can hold because \(\gamma > 0\) and \((1-\gamma)\beta > (1-\gamma)(1-\beta)\).

Lemma A3. If
\[
\Delta\phi_0(1,1|0) < 1 \leq \Delta\phi_0(0,0|1),
\] (A36)
then sophisticated investors that observe \(\hat{F} = 0\) do not participate and a sophisticated investor that observes
\( \hat{F} = 1 \) will invest with probability,

\[
\pi_1 = \min \left\{ \max \left\{ \frac{(1 - \gamma)(\alpha(1 - \beta)\delta - (1 - \alpha)\beta)}{\gamma\alpha(1 - \alpha)(1 - \delta)}, 0 \right\}, 1 \right\}.
\] (A37)

**Proof of Lemma A3:**

The proof comes from Lemma A2.

■

Lemma A4. If

\[
\Delta \phi_0(0, 0, |1) < 1,
\] (A38)

no sophisticated investors participate.

**Proof of Lemma A4:**

The proof comes from Lemma A2.

■

We now proceed with the proof of the proposition. Suppose sophisticated investors’ observing \( \hat{F} = 0 \) are unwilling to participate, i.e., \( \Delta \phi_0(0, 0, |1) < 1 \). Using (A17) and \( \Delta = 1 - \delta \), the preceding inequality simplifies to,

\[
\frac{\alpha}{1 - \alpha} \frac{(\gamma + (1 - \gamma)\beta)}{(\gamma + (1 - \gamma)(1 - \beta))} \equiv \delta > \delta,
\] (A39)

If \( \delta < \delta \) then sophisticated investors that observe \( \hat{F} = 0 \) do not participate and only a fraction \( \pi_1 \) of sophisticated investors that observe \( \hat{F} = 1 \) do participate where \( \pi_1 \) is defined in (A37). This means that the quantity of funds raised for a financially sound project is,

\[
M(\alpha \pi_1 \gamma + (1 - \gamma)\beta),
\]

and the quantity of funds raised for a non financially sound project is,

\[
M((1 - \alpha)\pi_1 \gamma + (1 - \gamma)(1 - \beta)).
\]

Projects with costs \( c > M((1 - \alpha)\pi_1 \gamma + (1 - \gamma)(1 - \beta)) \) will not be financed if they are not financially sound. For these projects, investors are not exposed to the state \( F = 0 \). Consider a single sophisticated investor that is not participating, either because he saw \( \hat{F} = 0 \) or because he is in the fraction \( 1 - \pi_1 \) of those that saw \( \hat{F} = 1 \) but do not participate. This investor is strictly better off by deviating from his strategy since he
would obtain a strictly positive net return on investment,

$$\frac{c}{M(\alpha \pi_1 \gamma + (1 - \gamma)\beta)(\Delta - 1)} > 0.$$  \hfill (A40)

This is true for all non-participating investors. However, it cannot be an equilibrium for all non-participating investors to deviate and participate since $\delta < \delta$. Furthermore, there does not exist a mixing strategy for non-participating investors that makes them indifferent between participation and non-participation. This is because investor payoffs are not continuous with the number of participants: if the indicator function $I_0$ switches from one to zero there is a discrete jump in investor payoffs.

Thus, for projects with returns $\delta < \delta$ and costs $c > M((1 - \alpha)\pi_1 \gamma + (1 - \gamma)(1 - \beta))$ no equilibrium participation strategies exist for sophisticated investors.

Proof of Proposition 2 and Corollary 2.1:

Lemmas A2, A3 and A4 prove the inequalities of Proposition 2. For Corollary 2.1, the proof begins in a similar manner as Lemma A1. If $I_0 = 0$ then the comparative statics of $\phi_0(\pi_1, \pi_0|0)$ and $\phi_0(\pi_1, \pi_0|1)$ are trivial: they all equal zero. For the remainder of the proof, assume $I_0 = 1$.

First, consider $\phi_0(1, 1|0)$. The expression may be rewritten as,

$$\phi_0(1, 1|0) = \frac{1}{1 + \eta_0},$$  \hfill (A41)

where,

$$\eta_0 = \frac{\alpha (\gamma + (1 - \gamma)\beta)}{(1 - \alpha)(\gamma + (1 - \gamma)(1 - \beta))}.$$  \hfill (A42)

Therefore, the comparative static of $\phi_0(1, 1|0)$ with respect to a generic parameter $\psi$ is,

$$\frac{\partial \phi_0(1, 1|0)}{\partial \psi} = -\frac{\partial \eta_0}{\partial \psi}/(1 + \eta_0)^2,$$  \hfill (A43)

implying the sign of the comparative static is determined by,

$$\frac{\partial \eta_0}{\partial \psi}.$$  \hfill (A44)
The relevant parameters for the comparative static calculations are \( \{ \alpha, \beta, \gamma \} \),

\[
\frac{\partial \eta_0}{\partial \alpha} = \left( \frac{1}{(1-\alpha)^2} \right) \left( \frac{\gamma + (1-\gamma)\beta}{\gamma + (1-\gamma)(1-\beta)} \right) \geq 0, \tag{A45}
\]

\[
\frac{\partial \eta_0}{\partial \beta} = \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{1-\gamma^2}{(\gamma + (1-\gamma)(1-\beta))^2} \right) \geq 0, \tag{A46}
\]

\[
\frac{\partial \eta_0}{\partial \gamma} = \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{1-2\beta}{(\gamma + (1-\gamma)(1-\beta))^2} \right) \leq 0. \tag{A47}
\]

Therefore,

\[
\frac{\partial \phi_0(1,1|0)}{\partial \alpha} \leq 0 \tag{A51}
\]

\[
\frac{\partial \phi_0(1,1|0)}{\partial \beta} \leq 0 \tag{A52}
\]

\[
\frac{\partial \phi_0(1,1|0)}{\partial \gamma} \geq 0. \tag{A53}
\]

Now, consider \( \phi_0(\pi_1,0|1) \). The expression may be rewritten as,

\[
\phi_0(\pi_1,0|1) = \frac{1}{1 + \eta_1}, \tag{A54}
\]

where,

\[
\eta_1 = \begin{cases} 
\frac{(1-\alpha)(\gamma_\alpha + (1-\gamma)\beta)}{\alpha(\gamma(1-\alpha) + (1-\gamma)(1-\beta))} & \text{if } \pi_1 = 1 \\
\delta & \text{if } \pi_1 \in [0,1),
\end{cases} \tag{A55}
\]

where \( \eta_1 \) evaluated at \( \pi_1 \in [0,1) \) utilizes the explicit form of \( \pi_1 \) in (A37). Without loss of generality, we focus on the cases in which \( \pi_1 = 1 \) for the comparative statics. As such, the comparative static of \( \phi_0(1,0|1) \) with respect to a generic parameter \( \psi \) is,

\[
\frac{\partial \phi_0(1,0|1)}{\partial \psi} = -\frac{\partial \eta_1 / \partial \psi}{(1 + \eta_1)^2}, \tag{A56}
\]

implying the sign of the comparative static is determined by,

\[
-\frac{\partial \eta_1}{\partial \psi}. \tag{A57}
\]
The relevant parameters for the comparative static calculations are again \( \{ \alpha, \beta, \gamma \} \),

\[
\frac{\partial \eta_1}{\partial \alpha} = -\frac{(1 - \gamma)(\alpha(\alpha - \beta)\gamma + \beta(1 - \beta)(1 - \gamma) + \beta \gamma (1 - \alpha))}{\alpha^2 (\gamma(1 - \alpha) + (1 - \gamma)(1 - \beta))^2}
\]
\[
\leq 0,
\]
\[
(\text{A58})
\]

\[
\frac{\partial \eta_1}{\partial \beta} = \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{1 - \gamma}{(\gamma(1 - \alpha) + (1 - \gamma)(1 - \beta))^2} \right)
\]
\[
\geq 0,
\]
\[
(\text{A59})
\]

\[
\frac{\partial \eta_1}{\partial \gamma} = \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{\alpha - \beta}{(\gamma(1 - \alpha) + (1 - \gamma)(1 - \beta))^2} \right)
\]
\[
\geq 0.
\]
\[
(\text{A60})
\]

Therefore,

\[
\frac{\partial \phi_0(\pi_1, 0|1)}{\partial \alpha} \geq 0
\]
\[
(\text{A64})
\]

\[
\frac{\partial \phi_0(\pi_1, 0|1)}{\partial \beta} \leq 0
\]
\[
(\text{A65})
\]

\[
\frac{\partial \phi_0(\pi_1, 0|1)}{\partial \gamma} \leq 0.
\]
\[
(\text{A66})
\]

\[
\text{Proof of Proposition 3:}
\]

Both financially sound and unsound projects that require additional financing after \( t = 0 \) may exist indefinitely on the platform without an exogenous deadline. Projects that are revealed to not be financially sound \( F = 0 \) receive no additional capital with probability one. Without an exogenous intervention, these projects remain on the platform indefinitely. Projects that are revealed to be financially sound \( F = 1 \) and have cost \( c > (1 - \gamma)M \) will not be fully financed according to Lemma 2.

An exogenous deadline of \( t^* = 1 \) eliminates these possibilities without making any investor worse off. Projects that are revealed to have \( F = 0 \) are pulled from the platform and the naive investors that contributed \( (1 - \gamma)(1 - \beta)M \) at \( t = 0 \) are returned their capital. Because the opportunity cost of capital is equal to zero, these investors are no worse off and any slight perturbation to their opportunity cost of capital would make them strictly better off.

\[
\text{Proof of Lemma 5:}
\]

Consider sophisticated investors that observe \( \hat{F} = 0 \) when sophisticated investors are using mixing strate-
strategies \(\pi_0, \pi_1 \in [0, 1]\). The payoff to each of them if they invest at \(t = 0\) versus an investment at \(t = 1\),

\[
\left(1 - \alpha\right)\frac{\Delta c - c}{c} M\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1 - \alpha M\sigma_0(0, \pi_1, \pi_0) \mathbb{1}_0 - \left(1 - \alpha\right)\frac{\Delta c - c}{c} M\sigma_1(1, \pi_1, \pi_0)
\]

where the first large term in parenthesis is the expected value of investing at \(t = 0\) and the second large term is the expected value of waiting. Because sophisticated investors that observe \(\hat{F} = 0\) play deterministic strategies at \(t = 0\) and by Lemma 3, the expression simplifies to,

\[
\left(1 - \alpha\right)\frac{\Delta c - c}{c} M\sigma_0(1, 1, 1) \mathbb{1}_1 - \alpha M\sigma_0(0, 1, 1) \mathbb{1}_0 - \left(1 - \alpha\right)\frac{\Delta c - c}{c} M\sigma_1(1, 1, 1)
\]

which is negative. Therefore, the value of preemption is negative for sophisticated investors that observe \(\hat{F} = 0\).

Now consider a sophisticated investor that observes \(\hat{F} = 1\) when sophisticated investors are using mixing strategies \(\pi_0, \pi_1 \in [0, 1]\). The value of preemption is,

\[
\left(\frac{\Delta c - c}{c} M\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1 - (1 - \alpha)M\sigma_0(0, \pi_1, \pi_0) \mathbb{1}_0 \right) - \left(\frac{\Delta c - c}{c} M\sigma_1(1, \pi_1, \pi_0) \mathbb{1}_1 \right),
\]

and the expression is positive if,

\[
1 \leq \Delta \frac{\alpha(\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1 - \sigma_1(1, 1, 1))}{\alpha(\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1 - \sigma_1(1, 1, 1)) + (1 - \alpha)\sigma_0(0, \pi_1, \pi_0) \mathbb{1}_0}.
\]

Furthermore,

\[
\frac{\alpha(\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1 - \sigma_1(1, 1, 1))}{\alpha(\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1 - \sigma_1(1, 1, 1)) + (1 - \alpha)\sigma_0(0, \pi_1, \pi_0) \mathbb{1}_0} < \frac{\alpha\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1}{\alpha\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1 + (1 - \alpha)\sigma_0(0, \pi_1, \pi_0) \mathbb{1}_0} = \phi(\pi_1, \pi_0|1),
\]

for any \(\pi_1, \pi_0 \in [0, 1]\). Therefore, if the value of preemption is positive for any combination of \(\pi_1\) and \(\pi_0\), then the criteria to invest at \(t = 0\),

\[
1 \leq \Delta \phi_0(\pi_1, \pi_0|1),
\]

is also satisfied. We define,

\[
\omega(\pi_1, \pi_0|\hat{F}) \equiv \begin{cases} 
0 & \hat{F} = 0, \\
\frac{\alpha(\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1 - \sigma_1(1, 1, 1))}{\alpha(\sigma_0(1, \pi_1, \pi_0) \mathbb{1}_1 - \sigma_1(1, 1, 1)) + (1 - \alpha)\sigma_0(0, \pi_1, \pi_0) \mathbb{1}_0} & \hat{F} = 1.
\end{cases}
\]

\textbf{Proof of Proposition 4:}
See the proof of Lemma 5.

Lemma A5. The value of preemption for investors that observe $\hat{F} = 1$, $\omega(\pi_1, 0|1)$, is ambiguously related to $\alpha$, $\beta$, and $\gamma$

Proof of Lemma A5:

In characterizing $\omega(\pi_1, 0|1)$, we focus on the non-trivial cases in which $\mathbb{1}_0$ and $\mathbb{1}_1$ equal one. The function $\omega(\pi_1, 0|1)$ may be rewritten as,

$$\omega(\pi_1, 0|1) = \frac{1}{1 + \chi_1}, \quad (A75)$$

where

$$\chi_1 \equiv \begin{cases} \frac{(\gamma + \beta(1-\gamma))(\beta + (a-\beta)\gamma)}{\alpha\gamma(1-\beta - (a-\beta)\gamma)} & \text{if } \pi_1 = 1 \\ \frac{(1-\alpha)(1-\delta)(\beta(1-\gamma) + \gamma)}{\beta(1-\alpha(1-\delta))(1-\gamma) + (1-\delta)\gamma - \alpha(\delta + 2\delta\gamma)} & \text{if } \pi_1 \in [0, 1), \quad (A76) \end{cases}$$

where $\chi_1$ evaluated at $\pi_1 \in [0, 1)$ utilizes the explicit form of $\pi_1$ in (A37). Therefore, the comparative static of $\omega(\pi_1, 0|1)$ with respect to a generic parameter $\psi$ is,

$$-\frac{1}{(1 - \chi_1)^2} \frac{\partial \chi_1}{\partial \psi}, \quad (A77)$$

and the sign on the comparative static is determined by,

$$-\frac{\partial \chi_1}{\partial \psi}, \quad (A78)$$

The partial derivative of $\chi_1$ with respect to $\alpha$ is given by,

$$\frac{\partial \chi_1}{\partial \alpha} = \begin{cases} \frac{-\beta(1-\gamma)(1-\beta)\beta - \beta\gamma - (a-\beta)\gamma(2\beta + (a-\beta)\gamma)}{\alpha^2\gamma(1-\beta - (a-\beta)\gamma)^2} & \text{if } \pi_1 = 1 \\ \frac{(1-\beta)(1-\delta)^2(\beta(1-\gamma) + \gamma)(1-\gamma)}{(\beta(1-\alpha(1-\delta))(1-\gamma) + (1-\delta)\gamma - \alpha(\delta + 2\delta\gamma))^2} & \text{if } \pi_1 \in [0, 1), \quad (A79) \end{cases}$$

If $\pi_1 = 1$, it is straightforward to see that the comparative static is equivocal by considering the limiting cases in which $\gamma \to 0$ and $\gamma \to 1$. In the former case, the sign on $\lim_{\gamma \to 0} \frac{\partial \chi_1}{\partial \alpha} \bigg|_{\pi_1=1}$ is determined by,

$$-\beta^2(1 - \beta), \quad (A80)$$

which is negative. In the latter case, the sign on $\lim_{\gamma \to 1} \frac{\partial \chi_1}{\partial \alpha} \bigg|_{\pi_1=1}$ is determined by,

$$\alpha^2, \quad (A81)$$

which is positive. If $\pi_1 < 1$, the sign on $\frac{\partial \chi_1}{\partial \alpha} \bigg|_{\pi_1<1}$ is positive because $\delta \leq 1$ by (A30).
Now consider the comparative static with respect to $\beta$,

$$\frac{\partial \chi_1}{\partial \beta} = \begin{cases} 
\frac{(1-\gamma)(1-\beta-(\alpha-\beta)\gamma)(\gamma+\beta(1-\gamma)+\beta+(\alpha-\beta)\gamma)+(\gamma+\beta(1-\gamma))(\beta+(\alpha-\beta)\gamma))}{\alpha\gamma(1-\beta-(\alpha-\beta)\gamma)^2} & \text{if } \pi_1 = 1 \\
-\frac{((1-\alpha)(1-\delta)^2(\alpha(1-\gamma)+\gamma(1-\gamma))(1-\gamma)-\alpha(1-\gamma)+\alpha(1-\gamma)+\gamma(1-\delta)))^2}{(\beta(1-\alpha(1-\delta))(1-\gamma)+\gamma(1-\gamma)+\gamma(1-\delta))^2} & \text{if } \pi_1 \in [0, 1). 
\end{cases} \quad (A82)$$

If $\pi_1 = 1$, it is straightforward to see that $\frac{\partial \chi_1}{\partial \beta} \geq 0$. If $\pi_1 < 1$, the sign on $\frac{\partial \chi_1}{\partial \beta} \bigg|_{\pi_1 < 1}$ is negative because $\delta \leq 1$ by (A30).

Lastly, consider the comparative static with respect to $\gamma$. The partial derivative of $\chi_1$ with respect to $\gamma$ is,

$$\frac{\partial \chi_1}{\partial \gamma} = \begin{cases} 
\frac{\beta^2(1-\gamma)^2+\alpha\gamma^2+(1+\alpha(1-\alpha))\beta^2-(1-2\alpha(1-\gamma)\gamma-\gamma^2)}{\alpha\gamma(1-\beta-(\alpha-\beta)\gamma)^2} & \text{if } \pi_1 = 1 \\
-\frac{((1-\alpha)(1-\delta)^2(\alpha(1-\gamma)+\gamma(1-\gamma))(1-\gamma)-\alpha(1-\gamma)+\alpha(1-\gamma)+\gamma(1-\delta)))^2}{(\beta(1-\alpha(1-\delta))(1-\gamma)+\gamma(1-\gamma)+\gamma(1-\delta))^2} & \text{if } \pi_1 \in [0, 1). 
\end{cases} \quad (A83)$$

If $\pi_1 = 1$, it is straightforward to see that the comparative static is equivocal by considering the limiting cases in which $\gamma \to 0$ and $\gamma \to 1$. In the former case, the sign on $\lim_{\gamma \to 0} \frac{\partial \chi_1}{\partial \gamma} \bigg|_{\pi_1 = 1}$ is determined by,

$$-\beta^2(1-\beta), \quad (A84)$$

which is negative. In the latter case, the sign on $\lim_{\gamma \to 1} \frac{\partial \chi_1}{\partial \alpha} \bigg|_{\pi_1 = 1}$ is determined by,

$$(\alpha-\beta)-(1-\alpha)\alpha \beta \quad (A85)$$

which is negative when $\alpha = \beta$ and $\alpha \neq 1$ and positive when $\alpha > \beta$ and $\alpha$ is sufficiently close to one. If $\pi_1 < 1$, the sign on $\frac{\partial \chi_1}{\partial \alpha} \bigg|_{\pi_1 < 1}$ is negative because $\delta \leq 1$ by (A30).

Proof of Proposition 5 and Corollary 5.1:

Consider a proposed equilibrium in which only naïve investors invest at $t = 0$ and sophisticated investors waited until $t = 1$ to make their participation decisions. Naïve investors perfectly reveal the project’s quality through their funding levels at $t = 0$. Sophisticated investors update their beliefs and all sophisticated investors participate if $F = 1$ and abstain otherwise. If $F = 1$, naïve investors contribute a total capital quantity,

$$M(1-\gamma)\beta, \quad (A86)$$

and the naïve investors that invest at $t = 0$ receive a share

$$\sigma_{0, FCFS}(\cdot) = 1, \quad (A87)$$

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because investment is based on a first-come, first-served basis.

Sophisticated investors that invest at $t = 1$ contribute a total capital quantity,

$$M\gamma$$

(A88)

and any investor that participates at $t = 1$ receives a share,

$$\sigma_{1,FCFS}(1,1,1) = \frac{(c - (1 - \gamma)\beta M)}{M\gamma},$$

(A89)

which is less than 1.

A single sophisticated investor strictly increases his payoff by deviating from his proposed equilibrium strategy and investing with naïve investors at $t = 0$: financing efficiency is still first-best when he deviates, but he receives a larger share of the project’s payoff. Because each investor is infinitesimal, there is not a marginal sophisticated investor that internalizes the impact of his decision on the project outcome. Therefore, the proposed equilibrium unravels.

By continuation, any proposed equilibrium in which some sophisticated investors delay their participation until $t = 1$ unravels.

The only remaining equilibrium is one in which all sophisticated investors participate at $t = 0$. ■

**Proof of Proposition 6:**

The proof is obvious from (38) and (39).

■

**Proof of Proposition 7:**

A threshold required crowd size $\tau$ can be chosen by choosing $M$, i.e.,

$$\tau = \frac{c}{M}$$

(A90)

Without loss of generality, consider only the values of $\tau \geq \min\{B, G\}$ and the values of $\tau \leq \max\{B, G\}$.

For any value of $\tau$, the probability that a bad project is funded is,

$$1 - G(\tau).$$

(A91)

Similarly, the probability that a good project is funded is,

$$1 - G(\tau).$$

(A92)
The probability that a funded project is good is,

$$\Pr(g|f, \tau) = \frac{1 - \overline{G}(\tau)}{1 - \overline{G}(\tau) + 1 - \overline{G}(\tau)}, \quad (A93)$$

Now consider a venture capitalist that observes a signal whether or not the project should be funded. The venture capitalist’s signal is accurate with probability $\alpha_{VC} \geq \frac{1}{2}$. As such, for the venture capitalist, denoted $VC$, the probability a project is funded is equal to the ex ante probability that a project is good,

$$\Pr(f|VC) = \frac{1}{2}, \quad (A94)$$

and the probability that a chosen project is good is equal to,

$$\Pr(g|f, VC) = \alpha_{VC}. \quad (A95)$$

Therefore, a crowdfunding platform’s venture capitalist equivalent precision, $\alpha_{VC}$ is determined by the following set of equations,

$$\Pr(g|f, VC) = \Pr(g|f, \tau). \quad (A96)$$

$$\Pr(f|VC) = \Pr(f|\tau). \quad (A97)$$

The equations simplify to,

$$\alpha_{VC} = \frac{1 - \overline{G}(\tau)}{1 - \overline{G}(\tau) + 1 - \overline{G}(\tau)}, \quad (A98)$$

$$\frac{1}{2} = \frac{1}{2} \left(1 - \overline{G}(\tau) + 1 - \overline{G}(\tau)\right). \quad (A99)$$

Therefore, at $\tau$ it is equivalent to a VC with signal precision

$$\alpha_{VC} = 1 - \overline{G}(\tau), \quad (A100)$$

where the set of permissible $\tau$ values are pinned down by,

$$1 - \overline{G}(\tau) = \overline{G}(\tau). \quad (A101)$$

Notice that the left-hand side is decreasing in $\tau$ and the right-hand side is increasing in $\tau$. Therefore, if the CDFs are strictly monotonic, the VC equivalent value of $\tau$ is unique. The principal may map this value of
\( \tau \) into a contribution level according to,

\[ M = \frac{c}{\tau}. \]  

(A102)

**Proof of Corollary 7.1:**

The proof is obvious from (38) and (39).

**Proof of Lemma 6:**

Define the expected gross return to sophisticated investors as,

\[
R_\alpha = \left( \alpha \pi_1 + (1 - \alpha) \pi_0 \right) \Delta \sigma_0(1, \pi_1, \pi_0) + (1 - \sigma_0(1, \pi_1, \pi_0)) + \left( 1 - \alpha \right) \pi_1 + (1 - \alpha) \pi_0 + (1 - \pi_1) + (1 - \pi_0),
\]

and the net return is

\[
\rho_\alpha = R_\alpha - 1
\]

which simplifies to

\[
\rho_\alpha = \left( \alpha \pi_1 + (1 - \alpha) \pi_0 \right) \delta \sigma_0(1, \pi_1, \pi_0) - \left( 1 - \alpha \right) \pi_1 + (1 - \alpha) \pi_0 - 1.
\]

(A103)

Similarly, the expected gross return to naïve investors is,

\[
R_\beta = \left( \beta \right) \Delta \sigma_0(1, \pi_1, \pi_0) + (1 - \sigma_0(1, \pi_1, \pi_0)) + \left( 1 - \beta \right) \pi_1 + \left( 1 - \beta \right) \pi_0 + \frac{1}{2}.
\]

(A104)

and the net return is

\[
\rho_\beta = R_\beta - 1
\]

\[
\rho_\beta = \left( \beta \right) \delta \sigma_0(1, \pi_1, \pi_0) - \left( 1 - \beta \right) \pi_1 + \left( 1 - \beta \right) \pi_0.
\]

(A105)

Now, consider the returns conditional on investment. Define the expected gross return, conditional on investment, to sophisticated investors as,

\[
R_{\alpha|I} = (\alpha \pi_1 + (1 - \alpha) \pi_0) \Delta \sigma_0(1, \pi_1, \pi_0)
\]

(A106)

and the net return is

\[
\rho_{\alpha|I} = (\alpha \pi_1 + (1 - \alpha) \pi_0) \Delta \sigma_0(1, \pi_1, \pi_0) - 1.
\]

(A107)
Similarly, the expected gross return to naïve investors, conditional on investment, is,

\[ R_{\beta I} = \beta \Delta \sigma_0(1, \pi_1, \pi_0), \quad \text{(A109)} \]

and the net return is,

\[ \rho_{\beta I} = \beta \Delta \sigma_0(1, \pi_1, \pi_0) - 1. \quad \text{(A110)} \]

\[ \blacksquare \]

**Proof of Lemma 7:**

First, consider the case in which sophisticated investors that observe \( \hat{F} = 0 \) are willing to participate,

\[ D(1,1) = \left( \left( \frac{1}{2} \right) \delta \sigma_0(1,1,1) - \left( \frac{1}{2} \right) \sigma_0(0,1,1) \right) - \left( \left( \frac{\beta}{2} \right) \delta \sigma_0(1,1,1) - \left( \frac{1-\beta}{2} \right) \sigma_0(0,1,1) \right), \quad \text{(A111)} \]

which simplifies to,

\[ D(1,1) = \left( \frac{1-\beta}{2} \right) \delta \sigma_0(1,1,1) - \left( \frac{\beta}{2} \right) \sigma_0(0,1,1). \quad \text{(A112)} \]

The preceding expression is positive because sophisticated investors that observed \( \hat{F} = 0 \) are willing to participate implying that

\[ (1-\alpha) \delta \sigma_0(1,1,1) - \alpha \sigma_0(0,1,1) \geq 0. \quad \text{(A113)} \]

Thus, because \( \alpha \geq \beta \) then \( D(1,1) \geq 0 \). Next, consider the case in which only a fraction \( \pi_1 \) of sophisticated investors that observe \( \hat{F} = 1 \) are willing to participate.

\[ D(\pi_1,0) = \left( \left( \frac{\alpha \pi_1}{2} \right) \delta \sigma_0(1,\pi_1,0) - \left( \frac{(1-\alpha)\pi_1}{2} \right) \sigma_0(0,\pi_1,0) \right) - \left( \left( \frac{\beta}{2} \right) \delta \sigma_0(1,\pi_1,0) - \left( \frac{1-\beta}{2} \right) \sigma_0(0,\pi_1,0) \right). \quad \text{(A114)} \]

If \( \pi_1 = 1 \), it is straightforward to see that \( D(\pi_1,0) \geq 0 \). If \( \pi_1 \in (0,1) \), it must the case that,

\[ 0 = \alpha \delta \sigma_0(1,\pi_1,0) - (1-\alpha) \sigma_0(0,\pi_1,0), \quad \text{(A115)} \]

because that condition determines the value of \( \pi_1 \) as demonstrated in Lemma A2. This also implies,

\[ 0 \geq \beta \delta \sigma_0(1,\pi_1,0) - (1-\beta) \sigma_0(0,\pi_1,0), \quad \text{(A116)} \]

because \( \beta \leq \alpha \). As such \( D(\pi_1,0) \geq 0 \) for all \( \pi_1 \in (0,1] \). Finally, if \( \pi_1 = 0 \) it implies,

\[ 0 \geq \alpha \delta \sigma_0(1,0,0) - (1-\alpha) \sigma_0(0,0,0), \quad \text{(A117)} \]
which also implies

\[ 0 \geq \beta \delta \sigma_0(1, 0, 0) - (1 - \beta)\sigma_0(0, 0, 0). \]  
\( (A118) \)

Therefore, \( D(\cdot) \) is always weakly positive. ■

**Proof of Proposition 8:**

Before proceeding to the comparative static computations, it is useful to recall that all sophisticated
investors participate if,

\[ \delta \geq \delta \equiv \frac{\alpha(\gamma + (1 - \gamma)\beta)}{(1 - \alpha)(\gamma + (1 - \gamma)(1 - \beta))}, \]  
\( (A119) \)

no sophisticated investors participate if,

\[ \delta < \delta \equiv \frac{(1 - \alpha)\beta}{\alpha(1 - \beta)}, \]  
\( (A120) \)

and all sophisticated investors that observe \( F = 1 \) participate if,

\[ \delta \geq \delta \equiv \frac{(1 - \alpha)(\beta + (\alpha - \beta)\gamma)}{\alpha(1 - \beta) - (\alpha - \beta)\gamma}, \]  
\( (A121) \)

Furthermore, note,

\[ \frac{\beta}{(1 - \beta)} - \left( \frac{\gamma\alpha\pi_1 + \beta(1 - \gamma)}{\gamma(1 - \alpha)\pi_1 + (1 - \beta)(1 - \gamma)} \right) \leq 0, \]  
\( (A122) \)

because \( \alpha \geq \beta \geq \frac{1}{2} \).

For the comparative statics, we consider the two possible participation strategies for sophisticated
investors: either all sophisticated investors participate or only the investors that observe \( F = 1 \) participate
with probability \( \pi_1 \). As such, the expected net return for sophisticated investors is,

\[
\rho_\alpha = \begin{cases} 
\left( \frac{\beta}{2} \right) \delta \left( \frac{c}{M(\gamma + \beta(1 - \gamma))} \right) - \left( \frac{1 - \beta}{2} \right) \left( \frac{c}{M(\gamma + (1 - \beta)(1 - \gamma))} \right) & \text{if } \pi_0 = 1 \\
\left( \frac{\beta}{2} \right) \delta \left( \frac{c}{M(\gamma + \beta(1 - \gamma))} \right) - \left( \frac{1 - \beta}{2} \right) \left( \frac{c}{M(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))} \right) & \text{if } \pi_1 = 1 \\
0 & \text{if } \pi_1 < 1
\end{cases},
\]  
\( (A123) \)

and the expected return for naïve investors is,

\[
\rho_\beta = \begin{cases} 
\left( \frac{\beta}{2} \right) \delta \left( \frac{c}{M(\gamma + \beta(1 - \gamma))} \right) - \left( \frac{1 - \beta}{2} \right) \left( \frac{c}{M(\gamma + (1 - \beta)(1 - \gamma))} \right) & \text{if } \pi_0 = 1 \\
\left( \frac{\beta}{2} \right) \delta \left( \frac{c}{M(\gamma + \beta(1 - \gamma))} \right) - \left( \frac{1 - \beta}{2} \right) \left( \frac{c}{M(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))} \right) & \text{if } \pi_1 = 1 \\
\frac{c(1 - \delta)}{2M(1 - \gamma)} & \text{if } \pi_1 < 1
\end{cases},
\]  
\( (A124) \)

First, we consider the comparative statics of \( \rho_\alpha \) with respect to each of the parameters in \{\( c, M, \delta, \alpha, \beta, \gamma \)\}. 

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First, consider the comparative static of $\rho_\alpha$ with respect to $c$,

\[
\frac{\partial \rho_\alpha}{\partial c} = \begin{cases} 
\frac{\delta}{2M(\gamma + \beta(1-\gamma))} - \frac{1}{2M(\gamma + (1-\beta)(1-\gamma))} & \text{if } \pi_0 = 1 \\
\frac{\alpha \delta}{2M(\gamma + \beta(1-\gamma))} - \frac{(1-\alpha) c}{2M(\gamma + \beta(1-\gamma))} & \text{if } \pi_1 = 1 \\
0 & \text{if } \pi_1 < 1 
\end{cases}
\]

If $\pi_0 = 1$, then a lower bound on $\delta$ is $\bar{\delta}$ and if $\pi_0 = 0$ and $\pi_1 = 1$ then a lower bound on $\delta$ is $\ddot{\delta}$. A substitution of $\bar{\delta}$ into the case in which $\pi_0 = 1$ yields,

\[
\left( \frac{\alpha (\gamma + (1-\gamma) \beta)}{2M(\gamma + \beta(1-\gamma))} \right) - \left( \frac{1}{2M(\gamma + (1-\beta)(1-\gamma))} \right) 
\]

which is positive because $\alpha/(1-\alpha) > 1$. A substitution of $\ddot{\delta}$ into the case in which $\pi_0 = 0$ yields,

\[
\frac{\partial \rho_\alpha}{\partial c} \bigg|_{\pi_0=0, \delta=\ddot{\delta}} = 0.
\]

Therefore, the comparative static for $\rho_\alpha$ with respect to $c$ is weakly positive.

Next, consider the comparative static of $\rho_\alpha$ with respect to $M$,

\[
\frac{\partial \rho_\alpha}{\partial M} = \begin{cases} 
- \frac{\delta c}{2M^2(\gamma + \beta(1-\gamma))} + \frac{c}{2M^2(\gamma + (1-\beta)(1-\gamma))} & \text{if } \pi_0 = 1 \\
- \frac{\alpha \delta c}{2M^2(\gamma + \beta(1-\gamma))} + \frac{(1-\alpha)c}{2M^2(\gamma + \beta(1-\gamma))} & \text{if } \pi_1 = 1 \\
0 & \text{if } \pi_1 < 1 
\end{cases}
\]

Similar to the computation with respect to $c$, it is straightforward to show that the comparative static of $\rho_\alpha$ with respect to $M$ is weakly negative.

Next, consider the comparative static of $\rho_\alpha$ with respect to $\delta$,

\[
\frac{\partial \rho_\alpha}{\partial \delta} = \begin{cases} 
\frac{c}{2M(\gamma + \beta(1-\gamma))} & \text{if } \pi_0 = 1 \\
\frac{\alpha c}{2M(\gamma + \beta(1-\gamma))} & \text{if } \pi_1 = 1 \\
0 & \text{if } \pi_1 < 1 
\end{cases}
\]

implying that the comparative static of $\rho_\alpha$ with respect to $\delta$ is weakly positive.
Next, consider the comparative static of $\rho_\alpha$ with respect to $\alpha$,

$$
\frac{\partial \rho_\alpha}{\partial \alpha} = \begin{cases} 
0 & \text{if } \pi_0 = 1 \\
\frac{\delta c}{2M(\gamma \alpha + \beta (1 - \gamma))} + \frac{c}{2M(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))} & \text{if } \pi_1 = 1 \text{ and } \pi_0 = 0, \\
-\frac{\alpha \delta c}{2M(\gamma \alpha + \beta (1 - \gamma))^2} - \frac{(1 - \alpha) c}{(1 - \alpha)(1 - \beta)(1 - \gamma)} & \text{if } \pi_1 < 1 
\end{cases}
$$

If $\pi_0 = 1$ or $\pi_1 < 1$, the sign on the case in which $\pi_0 = 0$ and $\pi_1 = 1$ is determined by

$$
\frac{\beta(1 - \gamma) \delta c}{(\gamma \alpha + \beta (1 - \gamma))^2} + \frac{(1 - \beta)(1 - \gamma)c}{(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))^2},
$$

which is positive. Therefore the comparative static of $\rho_\alpha$ with respect to $\alpha$ is weakly positive.

Next, consider the comparative static of $\rho_\beta$ with respect to $\beta$,

$$
\frac{\partial \rho_\alpha}{\partial \beta} = \begin{cases} 
0 & \text{if } \pi_0 = 1 \\
-\frac{c(1 - \gamma)}{2M(\gamma + \beta (1 - \gamma))} - \frac{c}{2M(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))} & \text{if } \pi_1 = 1 \text{ and } \pi_0 = 0, \\
-\frac{\alpha \delta c(1 - \gamma)}{2M(\gamma \alpha + \beta (1 - \gamma))^2} - \frac{(1 - \alpha) c(1 - \gamma)}{2M(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))^2} & \text{if } \pi_1 < 1 
\end{cases}
$$

implying that the comparative static of $\rho_\alpha$ with respect to $\beta$ is weakly negative.

Finally, consider the comparative static of $\rho_\gamma$ with respect to $\gamma$,

$$
\frac{\partial \rho_\alpha}{\partial \gamma} = \begin{cases} 
0 & \text{if } \pi_0 = 1 \\
-\frac{\delta c(1 - \beta)}{2M(\gamma + \beta (1 - \gamma))} + \frac{c}{2M(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))} & \text{if } \pi_1 = 1 \text{ and } \pi_0 = 0, \\
-\frac{\alpha \delta c(\alpha - \beta)}{2M(\gamma \alpha + \beta (1 - \gamma))^2} + \frac{(1 - \alpha) c((1 - \alpha)(1 - \beta))}{2M(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))^2} & \text{if } \pi_1 < 1 
\end{cases}
$$

The sign on the comparative static when $\pi_0 = 1$ is determined by,

$$
-\delta + \frac{\beta}{(1 - \beta)} \left( \frac{\gamma + \beta(1 - \gamma)}{\gamma + (1 - \beta)(1 - \gamma)} \right)^2,
$$

which achieves its largest value at $\delta = \overline{\delta}$ (the lower bound of $\delta$ when $\pi_0 = 1$) which equals,

$$
-\frac{\alpha(\gamma + (1 - \gamma)(\beta)}{(1 - \alpha)(\gamma + (1 - \gamma)(1 - \beta))} + \frac{\beta}{(1 - \beta)} \left( \frac{\gamma + \beta(1 - \gamma)}{\gamma + (1 - \beta)(1 - \gamma)} \right)^2,
$$

and its sign is determined by,

$$
-\frac{\alpha}{(1 - \alpha)} + \frac{\beta}{(1 - \beta)} \left( \frac{\gamma + \beta(1 - \gamma)}{\gamma + (1 - \beta)(1 - \gamma)} \right),
$$
which can be positive (when $\gamma < 1$ and $\alpha = \beta$) or negative (when $\gamma = 1$ and $\alpha > \beta$). Therefore the comparative static $\frac{\partial \rho_\alpha}{\partial \gamma}$ is equivocal when $\pi_0 = 1$.

The sign on the comparative static when $\pi_0 = 0$ and $\pi_1 = 1$ is determined by,

$$\frac{-\delta + \frac{(1 - \alpha)((1 - \alpha) - (1 - \beta))}{\alpha(\alpha - \beta)} \left( \frac{(\gamma \alpha + \beta(1 - \gamma))}{(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))} \right)^2}{\beta \gamma}$$

which achieves its largest value at $\delta = \delta$ (the lower bound of $\delta$ when $\pi_0 = 0$ and $\pi_1 = 1$) and equals,

$$\frac{-\delta}{\alpha(\alpha - \beta - \gamma(\alpha - \beta))^{\frac{1}{2}}}$$

which is negative. Therefore, the comparative static of $\rho_\alpha$ with respect to $\gamma$ is equivocal in the case in which $\pi_0 = 1$ and weakly negative in the case in which $\pi_0 = 0$.

Now we consider the comparative statics of $\rho_\beta$ with respect to each parameter in the set $\{c, M, \delta, \alpha, \beta, \gamma\}$.

First, consider the comparative static of $\rho_\beta$ with respect to $c$,

$$\frac{\partial \rho_\beta}{\partial c} = \begin{cases} \frac{\beta \delta}{2M(\gamma + \beta(1 - \gamma))} - \frac{1 - \beta}{2M(\gamma + (1 - \beta)(1 - \gamma))} & \text{if } \pi_0 = 1 \\ \frac{\delta}{2M(\gamma + \beta(1 - \gamma))} - \frac{1 - \beta}{2M(\gamma + (1 - \alpha)(1 - \beta)(1 - \gamma))} & \text{if } \pi_1 = 1 \\ \frac{1 - \beta}{2M(1 - \gamma)} & \text{if } \pi_1 < 1 \end{cases}$$

The sign on the first case is determined by the sign on,

$$\delta - \frac{(1 - \beta)(\gamma + \beta(1 - \gamma))}{\beta(\gamma + (1 - \beta)(1 - \gamma))},$$

which achieves its smallest value at $\delta = \delta$ and equals,

$$\frac{\alpha(\gamma + (1 - \gamma))}{\beta(\gamma + (1 - \alpha)(1 - \gamma))} - \frac{(1 - \beta)(\gamma + \beta(1 - \gamma))}{\beta(\gamma + (1 - \beta)(1 - \gamma))}$$

which is positive. The sign on the second case is determined by the sign on,

$$\delta - \frac{(1 - \beta)(\gamma + \beta(1 - \gamma))}{\beta(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))},$$

which achieves its smallest value at $\delta = \delta$ and equals,

$$\frac{(1 - \alpha)(\beta + (\alpha - \beta)\gamma)}{\alpha(1 - \beta - (\alpha - \beta)\gamma)} - \frac{(1 - \beta)(\gamma + \beta(1 - \gamma))}{\beta(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))}.$$
which is simplifies to, 

\[-\frac{(\alpha - \beta)(\beta + \gamma(\alpha - \beta))}{\alpha\beta(\gamma(1 - \alpha) + (1 - \gamma)(1 - \beta))}.\]  

(A144)

which is negative. Therefore, the comparative static of \(\rho\) with respect to \(c\) is equivocal.

Next, consider the comparative static of \(\rho\) with respect to \(M\),

\[
\frac{\partial \rho}{\partial M} = \begin{cases} 
  -\left(\frac{\beta c}{2M^2(\gamma + \beta(1-\gamma))}\right) + \left(\frac{(1-\beta)c}{2M^2(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_0 = 1 \\
  -\left(\frac{\beta c}{2M^2(\gamma + \beta(1-\gamma))}\right) + \left(\frac{(1-\beta)c}{2M^2(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_1 = 1 \\
  -\left(\frac{\beta c}{2M^2(\gamma + \beta(1-\gamma))}\right) + \left(\frac{(1-\beta)c}{2M^2(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_0 = 0,
\end{cases}
\]

Similar to the calculation with respect to \(c\), it is straightforward to show that comparative static of \(\rho\) with respect to \(M\) is equivocal.

Next, consider the comparative static of \(\rho\) with respect to \(\delta\),

\[
\frac{\partial \rho}{\partial \delta} = \begin{cases} 
  -\frac{\beta c}{2M^2(\gamma + \beta(1-\gamma))}\left(1 - \pi_0\right) + \left(\frac{(1-\beta)c}{2M^2(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_0 = 1 \\
  -\frac{\beta c}{2M^2(\gamma + \beta(1-\gamma))}\left(1 - \pi_1\right) + \left(\frac{(1-\beta)c}{2M^2(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_1 = 1 \\
  -\frac{\beta c}{2M^2(\gamma + \beta(1-\gamma))}\left(1 - \pi_1\right) + \left(\frac{(1-\beta)c}{2M^2(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_0 = 0,
\end{cases}
\]

if \(\pi_0 = 0\), \(\pi_1 = 1\), \(\pi_1 < 1\)

implying that the comparative static of \(\rho\) with respect to \(\delta\) is weakly positive.

Next, consider the comparative static of \(\rho\) with respect to \(\alpha\),

\[
\frac{\partial \rho}{\partial \alpha} = \begin{cases} 
  0 & \text{if } \pi_0 = 1 \\
  -\left(\frac{\beta c}{2M(\gamma + \beta(1-\gamma))}\right) + \left(\frac{(1-\beta)c}{2M(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_1 = 1 \\
  0 & \text{if } \pi_1 = 1
\end{cases}
\]

(A147)

if \(\pi_0 = 0\), \(\pi_1 < 1\)

implying that the comparative static of \(\rho\) with respect to \(\alpha\) is weakly negative.

Next, consider the comparative static of \(\rho\) with respect to \(\beta\),

\[
\frac{\partial \rho}{\partial \beta} = \begin{cases} 
  -\left(\frac{\beta c}{2M(\gamma + \beta(1-\gamma))}\right) + \left(\frac{(1-\beta)c}{2M(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_0 = 1 \\
  -\left(\frac{\beta c}{2M(\gamma + \beta(1-\gamma))}\right) + \left(\frac{(1-\beta)c}{2M(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_1 = 1 \\
  0 & \text{if } \pi_1 = 1
\end{cases}
\]

(A148)

\[
\frac{\partial \rho}{\partial \beta} = \begin{cases} 
  -\left(\frac{\beta c}{2M(\gamma + \beta(1-\gamma))}\right) + \left(\frac{(1-\beta)c}{2M(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_0 = 1 \\
  -\left(\frac{\beta c}{2M(\gamma + \beta(1-\gamma))}\right) + \left(\frac{(1-\beta)c}{2M(\gamma(1-\alpha)+(1-\beta)(1-\gamma))}\right) & \text{if } \pi_1 = 1 \\
  0 & \text{if } \pi_1 = 1
\end{cases}
\]

implying that the comparative static of \(\rho\) with respect to \(\beta\) is weakly negative.
The sign on the case in which \( \pi_0 = 1 \) is determined by,

\[
\frac{\delta \gamma}{(\gamma + \beta(1 - \gamma))^2} + \frac{\gamma}{(\gamma + (1 - \beta)(1 - \gamma))^2},
\]

which is positive. The sign on the case in which \( \pi_0 = 0 \) and \( \pi_1 = 1 \) is determined by,

\[
\frac{\delta \gamma \alpha}{(\gamma \alpha + \beta(1 - \gamma))^2} + \frac{\gamma(1 - \alpha)}{(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))^2},
\]

which is positive. Therefore the comparative static of \( \rho_\beta \) with respect to \( \beta \) is weakly positive.

Finally, consider the comparative static of \( \rho_\beta \) with respect to \( \gamma \),

\[
\frac{\partial \rho_\beta}{\partial \gamma} = \begin{cases} 
- \left( \frac{\beta \delta c(1 - \beta)}{2M(\gamma + \beta(1 - \gamma))^2} \right) + \left( \frac{(1 - \beta)c(1 - \beta)}{2M(\gamma + (1 - \beta)(1 - \gamma))^2} \right) & \text{if } \pi_0 = 1 \\
- \left( \frac{\beta \delta c(\alpha - \beta)}{2M(\gamma \alpha + \beta(1 - \gamma))^2} \right) + \left( \frac{(1 - \beta)c((1 - \alpha) - (1 - \beta))}{2M(\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma))^2} \right) & \text{if } \pi_1 = 1 \\
- \left( \frac{\alpha(1 - \beta)}{2M(1 - \gamma)^2} \right) & \text{if } \pi_0 = 0, \pi_1 < 1
\end{cases}
\]

The sign on the first case is determined by the sign on,

\[
- \delta + \left( \frac{\gamma + \beta(1 - \gamma)}{\gamma + (1 - \beta)(1 - \gamma)} \right)^2,
\]

which achieves its largest value at \( \delta = \bar{\delta} \) and equals,

\[
- \frac{\alpha(\gamma + (1 - \gamma)\beta)}{(1 - \alpha)(\gamma + (1 - \gamma)(1 - \beta))} + \left( \frac{\gamma + \beta(1 - \gamma)}{\gamma + (1 - \beta)(1 - \gamma)} \right)^2,
\]

which simplifies to,

\[
- \frac{\alpha}{(1 - \alpha)} + \left( \frac{\gamma + \beta(1 - \gamma)}{\gamma + (1 - \beta)(1 - \gamma)} \right),
\]

which can be negative or positive as in (A136), so the comparative static \( \frac{\partial \rho_\beta}{\partial \gamma} \) is equivocal when \( \pi_0 = 1 \).

The sign on the case in which \( \pi_0 = 0 \) and \( \pi_1 = 1 \) is determined by the sign on,

\[
- \delta + \left( \frac{1 - \beta}{\beta} \right) \left( \frac{((1 - \alpha) - (1 - \beta))}{(\alpha - \beta)} \right) \left( \frac{\gamma \alpha + \beta(1 - \gamma)}{\gamma(1 - \alpha) + (1 - \beta)(1 - \gamma)} \right)^2.
\]

Note that \( -\delta \) is negative and the first and third-portions of the second term are positive. The middle term,

\[
\frac{((1 - \alpha) - (1 - \beta))}{(\alpha - \beta)},
\]

is weakly negative. Therefore, both terms in (A155) are weakly negative and (A155) is negative. Therefore the comparative static of \( \rho_\beta \) with respect to \( \gamma \) is equivocal. ■