Grounding Justifications in Concrete Embodied Experience: The Link between Action and Cognition

Tangibility for the Teaching, Learning, and Communicating of Mathematics

MAGIC Research Group

University of Wisconsin - Madison

This work was made possible through the financial support of the National Science Foundation (DRL-0816406)

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Brief Framing

- Theories of **embodied cognition**
  - Mental processes rooted in perceptual and motor systems (Wilson, 2002)
  - Mathematical objects experiential, perception-based, and multimodal in nature (Barsalou, 1999; Lakoff & Nunez, 2000; Landy, Brooks, & Smout, 2012)
- Importance of **action** and **simulated action** for learning mathematical ideas (Abrahamson & Howsin, 2010; Martin & Schwartz, 2005; Nathan et al., 1992)
- **Gesture** as an instructional scaffold (Alibali et al., 2011; Alibali & Nathan 2007)

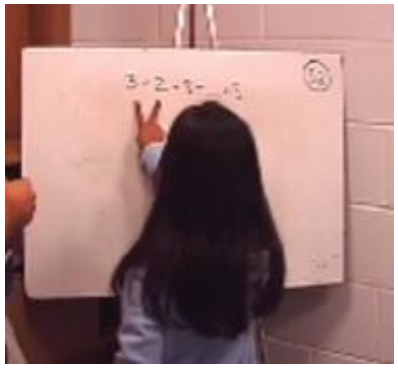
Directed Movement

- **Directed Action**

  (Thomas & Lleras, 2007, 2009)

- **Directing Gesture**

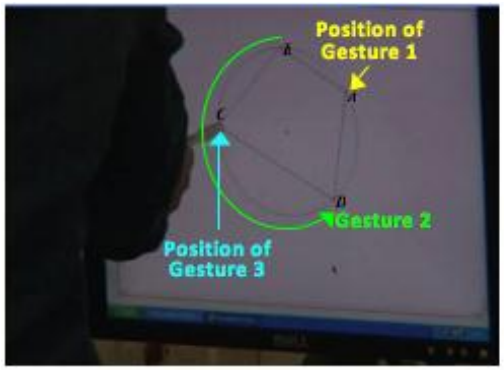
  (Goldin-Meadow, Cook, & Mitchell, 2009)

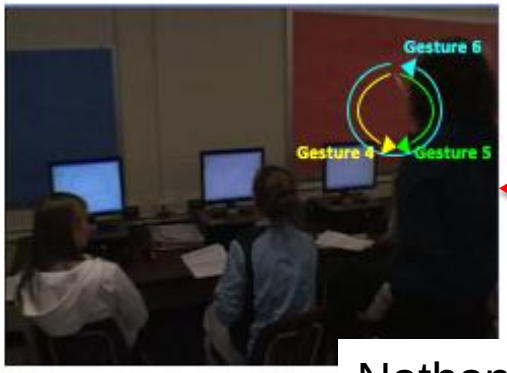


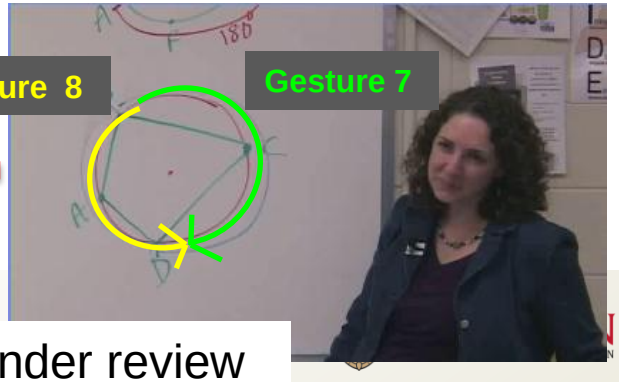
- **Directed action & gesture can implicitly influence cognition**

Projection

- Observed high school geometry classes (*N* = 17)
- Mathematical justification difficult practice to learn
- Mathematical ideas instantiated in different **contexts**
  - Computer lab (GSP) → Classroom (Discussion)
- Produce **cohesion** of mathematical ideas using **projection** (reference past/future activity)
- **Gesture** and **action** critical to cohesion production



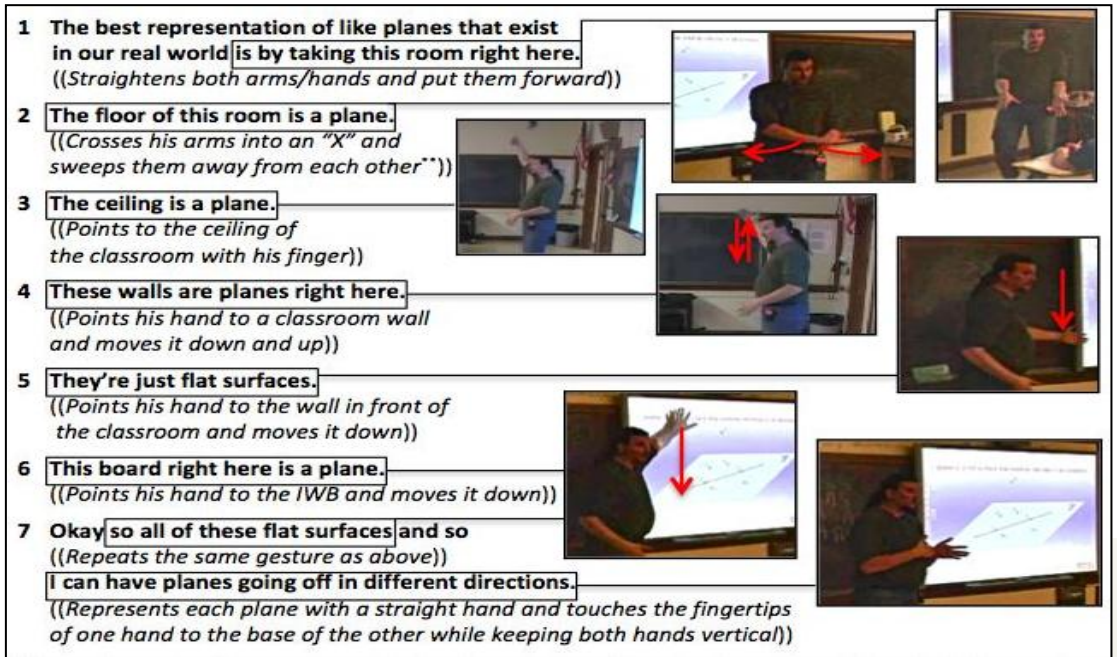




Nathan et al., under review

Viewpoint

- Gesturers express ideas with their bodies using different **viewpoints** (McNeill, 1992; Gerofsky, 2010)
  - ***Observer:*** Spectator of situation, third-person
  - ***Character:*** Agent in situation, first-person



Srisurichan et al., under review

Research Questions

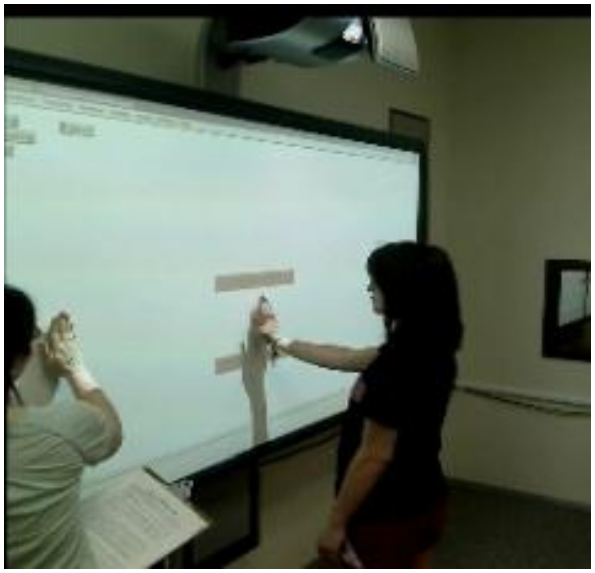
- How are **action** and **gesture** used spontaneously to support mathematical justification?
- Is there an ***implicit*** link between action and cognition that can support mathematical reasoning?
- Can ***explicitly*** linking actions to mathematical ideas using **projection** support mathematical reasoning?
- What is the effect of **viewpoint** condition? (character vs. observer)

Participants and Procedure

- **Undergraduate students** (*N* = 107) enrolled in a psychology course at large Midwestern university
- **Think aloud** (Ericsson & Simon, 1993) with only scripted prompts by interviewer
- Provide **justifications** for 2 mathematical tasks
- Prior to being given task, directed to perform body-based actions **relevant** or **irrelevant** to solution

Environment

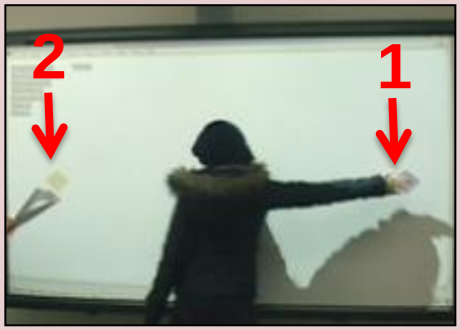
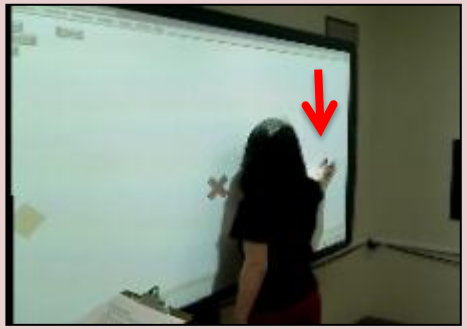
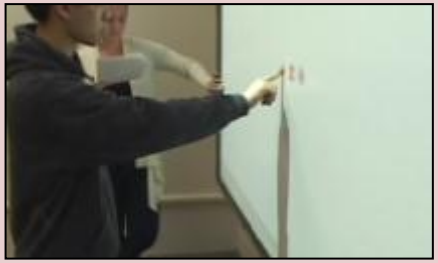
- Large interactive whiteboard
- Directed actions performed on images in GSP - scaled to body through initial measurements



Tasks

Triangle Task	Actions
Mary came up with the following conjecture: **“For any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side.”** Provide a justification as to why Mary’s conjecture is true or false.	**Character Viewpoint** **Relevant Actions**  **Irrelevant Actions**  **Observer Viewpoint** **Relevant Actions**  **Irrelevant Actions** 

Tasks

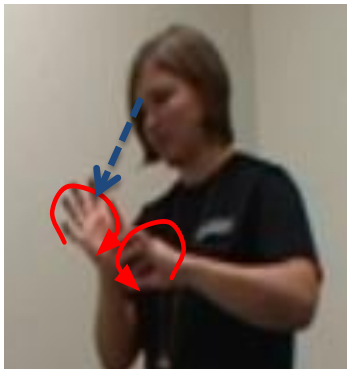
Gear Task	Actions
An unknown number of gears are connected in a chain. **If you know what direction the first gear turns, how can you figure out what direction the last gear will turn?** Provide a justification for your answer	**Character Viewpoint** **Relevant Actions**  **Irrelevant Actions**  **Observer Viewpoint** **Relevant Actions**  **Irrelevant Actions** 

Design

- **Relevant** action for one conjecture, **irrelevant** action for other
- One set of actions from **character viewpoint**, other from **observer viewpoint**
- No participants reported being aware of connection
- Backwards **projection** at end of session
  - Participants told that there is a connection between actions and task, opportunity to solve again

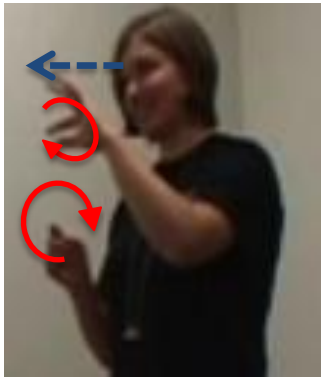
Findings

- **How are action and gesture used spontaneously to support mathematical justification?**
  - Action and gesture used in formulating (ascertaining) and communicating (persuading) mathematical justifications (Harel & Sowder, 1998)
  - Participants "**think with their bodies**"
  - Use action as an essential **modality** for mathematical communication



"If one gear was turning this way, then the spokes on it would push..."

(Later) "All the odd gears would be turning in the original direction."

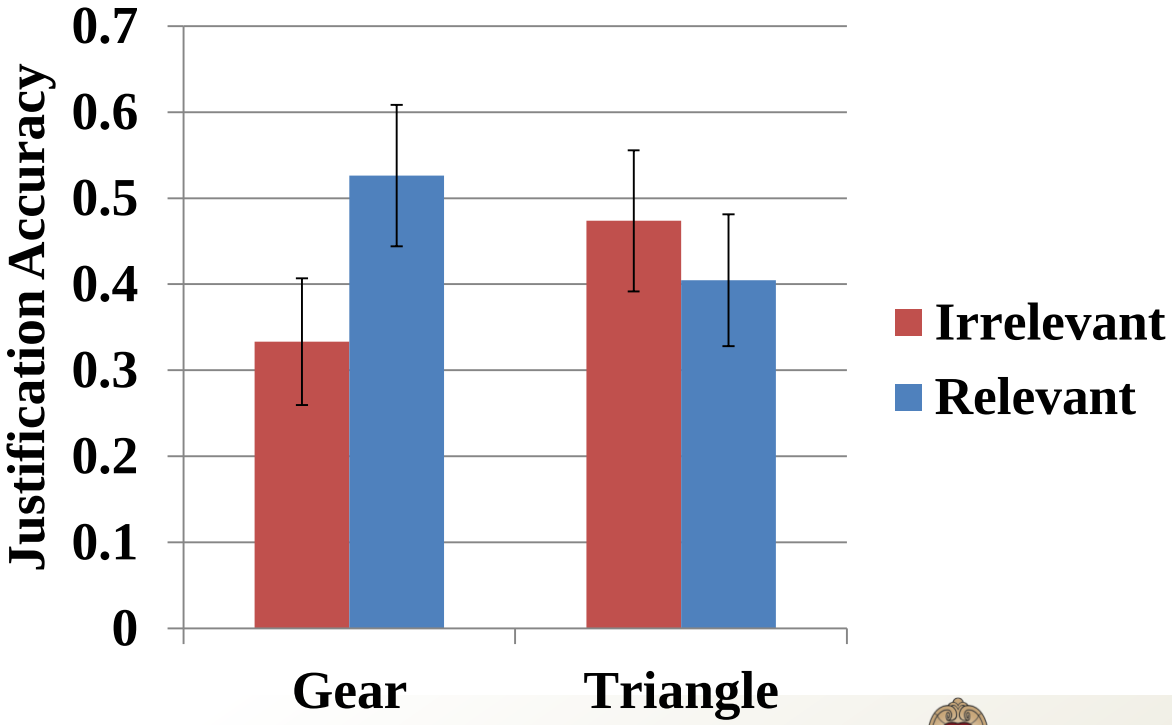


Findings

- Is there an ***implicit*** link between action and cognition that can support mathematical reasoning?

N = 40

Note: All participants included report not being consciously aware that there was a connection at this stage of the session

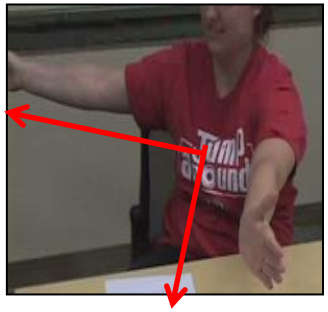


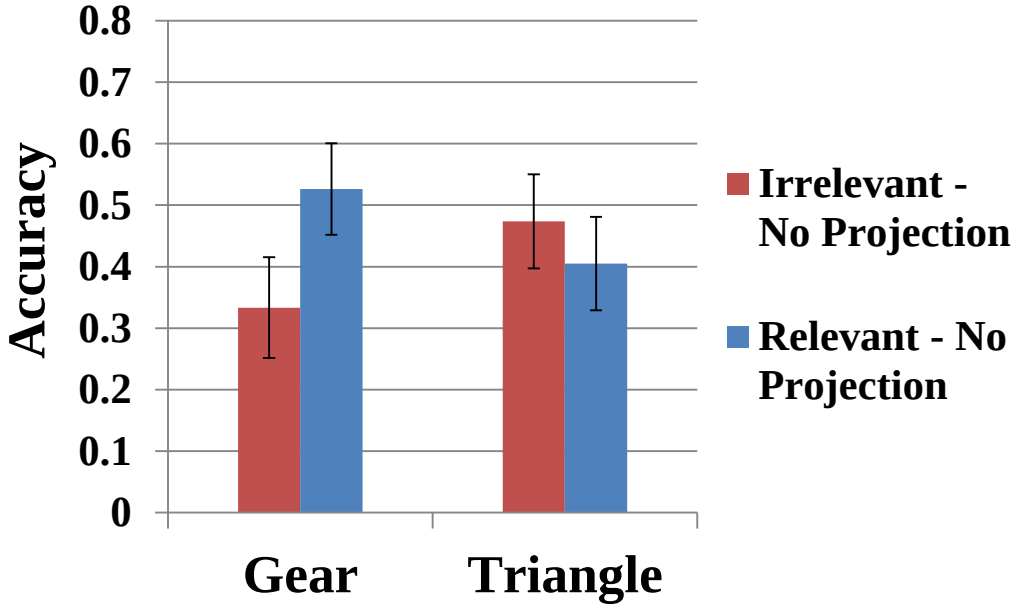
Findings

- Can ***explicitly*** linking actions to mathematical ideas using projection support mathematical reasoning?

"Oh! I see! If this was side A and this was side B…"

"They couldn't reach anything greater than A + B"





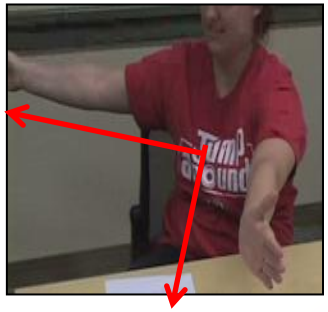
N = 40

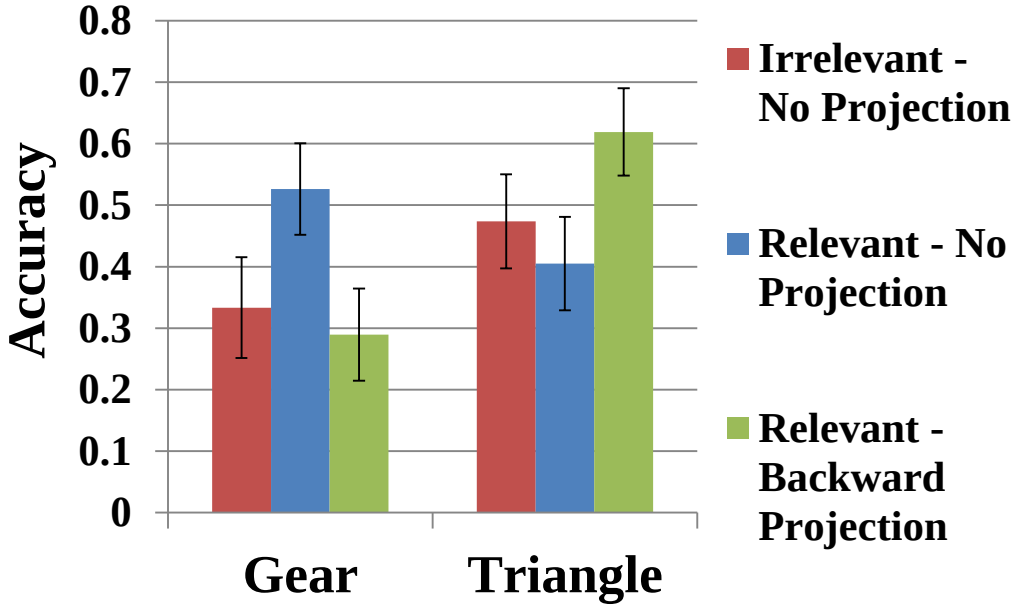
Findings

- **Can *explicitly* linking action-based interventions to mathematical ideas support mathematical reasoning?**

"Oh! I see! If this was side A and this was side B…"

"They couldn't reach anything greater than A + B"

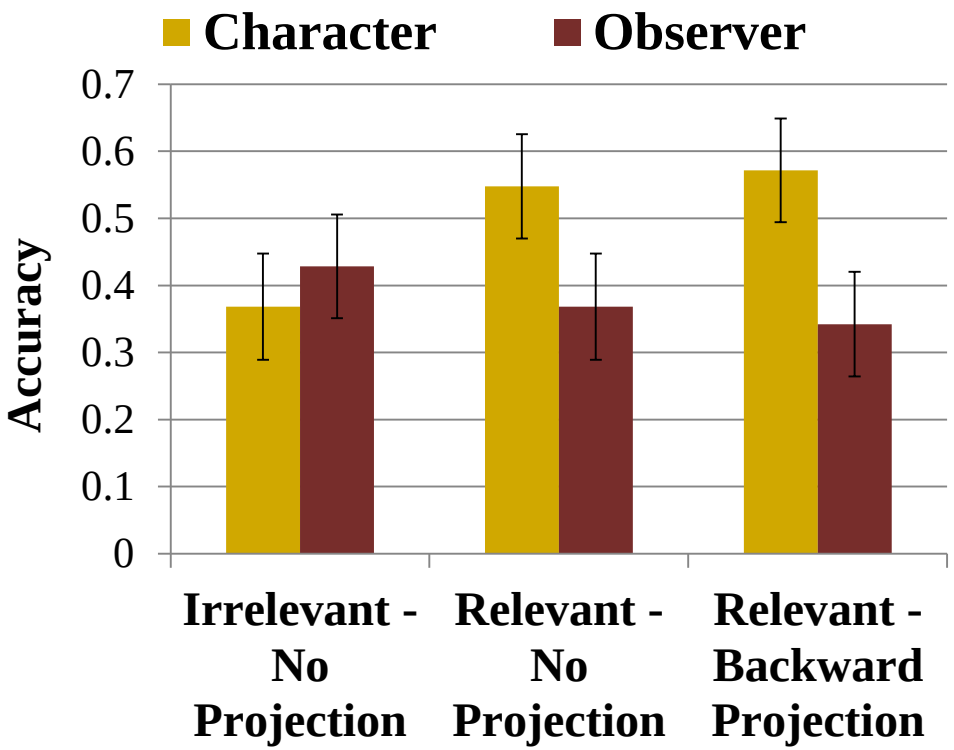




N = 40

Findings

- What is the effect of *viewpoint* condition? (character vs. observer)



N = 40

Implications

- **Gesture** and **action** play critical role in formulating and communicating mathematical justifications
- Directing students to perform **relevant actions** can support key mathematical insights
- Having students **generate connections** can be powerful, although some actions may work implicitly
- **Character viewpoint**, first-person embodied experience, especially effective support

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