When Can Positive Return Autocorrelation Arise in Rational Expectations Models?

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Abstract

In a dynamic trading model with rational expectations, we show that the interaction between public information and liquidity shocks can generate positive return autocorrelation. In the absence of anticipated future liquidity shocks, rational uninformed traders tend to follow public signals and act as momentum traders—chasing price trends and generating return autocorrelation. When future liquidity shocks are anticipated, informed traders switch to momentum-based strategies in response to sequential public information, generating positive return autocorrelation prior to the shocks' arrival. Our model generates a rich set of return patterns, including short-term momentum and long-term reversal. These insights help explain observed return dynamics without relying on behavioral biases.

JEL Classification Codes: B41, D8, D53, D61, D82, G12, G14, G18

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1 Introduction

Short-run time-series momentum in equity returns is a well-documented anomaly in financial markets. Although extensive empirical evidence supports its existence, explaining this phenomenon within rational expectations equilibrium (REE) models has remained a challenge. In the paper, we present a dynamic trading model with rational traders and show that the interaction between public information and liquidity shocks can generate time-series momentum in returns.

Return autocorrelation is typically negative in rational expectations equilibrium (REE) models, as liquidity trading pushes prices away from fundamental values before they eventually revert. In contrast, we show that with the arrival of public information, both informed and uninformed traders adjust their positions, and their subsequent trading can generate positive return autocorrelation. In the absence of future liquidity shocks, rational uninformed traders tend to follow the direction of the public signal—buying in response to positive news and selling in response to negative news. Informed traders—who previously traded based on private information and liquidity needs—often switch roles to become liquidity providers. Furthermore, traders who optimally bought (or sold) prior to the public information release continue trading in the same direction afterward, leading to autocorrelated trades and positive return autocorrelation.

In our model, informed traders optimally establish positions based on both private information and liquidity needs. Their liquidity-driven trading pushes prices away from fundamental values, as hedging distorts their optimal response to private information. In the absence of future liquidity shocks, rational uninformed traders—who initially provide liquidity to informed traders—anticipate that prices will eventually revert to fundamentals. Upon the arrival of public information, they interpret the signal as an indication of whether prices are too high or too low relative to fundamental values, and trade in its direction, expecting prices to converge. They buy when prices rise and sell when prices fall, thereby acting as momentum traders. Informed traders, in turn, act as contrarian traders—optimally adjusting their positions at more favorable prices by buying when prices fall and selling when prices rise.

We then examine how the anticipation of future liquidity shocks influences traders' behavior and return autocorrelation prior to the shocks' arrival. We find that return dynamics critically depend on the volatility of these future shocks. When future shocks are much less volatile than the initial shock, their expected impact is small, and their anticipated arrival has little influence on traders' behavior beforehand. As a result—similar to the case without anticipated future shocks—rational uninformed traders continue to trade in the direction of the public signal, buying in response to positive news and selling in response to negative news. This behavior leads to autocorrelated trades and positive return autocorrelation.

In contrast, when future shocks are highly volatile, informed traders alternate their trading direction from one period to the next in response to public information releases, due to uncertainty about whether the upcoming shock will be positive or negative. To avoid accumulating large long or short positions, they behave as contrarian traders—buying when past prices are low and selling when past prices are high. In doing so, they consume liquidity by pushing prices up when buying and down when selling. Rational uninformed traders, while providing liquidity to informed traders, also reverse their trading direction each period in response to public signals. This period-by-period reversal in trading direction generates negative return autocorrelation.

When the volatility of future liquidity shocks is comparable to that of the initial shock, traders behave differently from the case without anticipated future shocks. In response to public information releases, rational uninformed traders no longer act as momentum traders. They are concerned that future liquidity shocks may drive prices away from fundamental values, so a rising price does not necessarily signal convergence toward fundamentals. As a result, they refrain from buying simply because prices are increasing. Instead, they act as contrarian traders and provide liquidity by buying when prices are low and selling when prices are high. Informed traders, who initially buy (or sell) based on private information and liquidity needs, reverse their position once following the initial round of public information. After this adjustment, they continue trading in the same direction in response to subsequent public information releases. As more public information becomes available, their uncertainty about the asset's payoff—and thus about the payoff of the associated non-traded risky asset—diminishes. When the anticipated future shocks have moderate volatility, informed traders optimally adjust their hedging positions by reducing the size of their initial long or short exposures. Thus, in contrast to the case without future liquidity shocks, informed traders exhibit momentum trading behavior and demand liquidity, which gives rise to positive return autocorrelation.

Therefore, we show that positive return autocorrelation can arise in rational expectations equilibrium (REE) models under simple assumptions. Even in the absence of information asymmetry, a sequence of public information releases following a liquidity shock can generate positive return autocorrelation, as the coefficient on the liquidity shock in the risk premium term decreases over time. In the presence of information asymmetry but without future liquidity shocks, uninformed traders behave as momentum traders—chasing price trends and reinforcing return autocorrelation. When future liquidity shocks are anticipated and their volatility is comparable to that of the initial shock, informed traders adopt momentum-based strategies, and their trading behavior also gives rise to positive return autocorrelation.

By analyzing the interaction between public information and liquidity shocks, our model generates rich patterns of return dynamics—such as short-term momentum and long-term reversal—that align with empirical observations in financial markets. This framework provides a novel explanation for return autocorrelation within a fully rational expectations setting with sequential information arrivals. Although the model assumes, for simplicity, that public information arrives sequentially, the results remain robust even when informed traders interpret public signals to form private beliefs and trade on them prior to the arrival of new liquidity shocks. Therefore, the model remains applicable to real-world settings, where both the flow of public information and trading motivated by liquidity shocks (e.g., rebalancing needs) are prevalent.

Our model implies that higher levels of pre-announcement liquidity sales (purchases) predict positive (negative) returns and positive return autocorrelation following the announcement. Consistent with this implication, Levi and Zhang (2015) show that liquidity trades, specifically pre-announcement liquidity sales, are a significant driver of positive announcement returns.

The case without future liquidity shocks is related to Brennan and Cao (1996) and Brennan and Cao (1997). Brennan and Cao (1996) analyze the value of improved trading opportunities—either through more frequent trading in the underlying asset or through access to derivative securities. They show that trading in an appropriately designed derivative can achieve Pareto efficiency in a single trading round. In their model, well-informed investors also sell when prices are high and buy when prices are low. Similarly, Brennan and Cao (1997) demonstrate that domestic investors with a cumulative informational advantage over foreign investors tend to buy foreign assets when foreign returns are high and sell when returns are low. While their focus is on cross-border portfolio flows, our paper studies return dynamics by examining the interaction between public information and liquidity shocks. In contrast to both papers, we examine how the anticipation of future liquidity shocks affects traders' behavior and return dynamics prior to the shocks' arrival.

Our paper contributes to the theoretical literature on time-series return momentum (i.e., positive return autocorrelation). Since explaining momentum within rational expectations equilibrium (REE) models is challenging, many existing theories attribute it to irrational behavior or bounded rationality. For example, Hong and Stein (1999) suggest that slow information diffusion prevents agents from fully learning from prices. Kyle, Obizhaeva, and Wang (2023) study return predictability assuming traders agree to disagree about the precision of private information. In addition to trading on long-term fundamental value, traders also trade on perceived short-term opportunities arising from foreseen future disagreement. This short-term speculation dampens price fluctuations and generates time-series momentum. Barberis and Shleifer (2003) and Bogousslavsky (2016) model agents as following predefined trading strategies, such as trend-following, which induce autocorrelated returns. These frameworks assume deviations from rational expectations as a prerequisite for momentum. In contrast, we demonstrate that positive return autocorrelations can arise even when all agents are fully rational, as long as public information updates occur following liquidity shocks.

Allen, Morris, and Shin (2006) attribute price drift to iterated expectations in a market where traders share a common prior but observe different information. Banerjee, Kaniel, and Kremer (2009), however, point out that in such a noisy rational expectations model, when traders learn from prices, returns should exhibit mean reversion—not momentum—due to the noise in prices. Some existing rational expectations models explore how the risk premium may generate momentum, often emphasizing private information as a key driver. For example, Cespa and Vives (2012), Andrei and Cujean (2017), and Ai, Bansal, and Han (2021) argue that trading heterogeneity induced by private signals is necessary for momentum to arise, suggesting that public information alone cannot produce the price predictability required for return autocorrelation. Albuquerque and Miao (2014) show that when informed investors privately receive advance information about future earnings, the expected risk premium rises, generating short-run momentum. In contrast, our model demonstrates that neither private information nor trading heterogeneity is essential. For instance, in the absence of future shocks, when public information follows a liquidity shock, uninformed traders engage in momentum trading, consume liquidity, and generate positive return autocorrelation.

Berk, Green, and Naik (1999) and Johnson (2002) examine how a positive correlation between current dividend payments and future growth can lead to persistent fundamentals and, consequently, positive return autocorrelation. These models focus on the intrinsic dynamics of firm fundamentals rather than on investors' learning or trading behavior. In contrast, we examine how public information released after liquidity shocks influences traders' expectations, generating positive return autocorrelation even in the absence of fundamental changes.

The paper is structured as follows. Section 2 presents a dynamic trading model. Section 3 solves the equilibrium. Section 4 examines return dynamics under various market conditions. Section 5 concludes. All proofs are in the Appendix.

2 The Model

We consider a multi-period trading model with a continuum of identical informed investors (I-type) of mass α and a continuum of identical uninformed investors (U-type) of mass $1-\alpha$. Both informed and uninformed investors are perfectly competitive. Traders can trade two assets: a risk-free asset and a risky asset (the stock) at trading periods $t = 0, 1, ..., T$. The risk-free asset is in perfectly elastic supply, serves as the numeraire, and has its price normalized to one. The total supply of the stock is Θ. Each share of the stock pays a liquidation value of $V \sim \mathcal{N}(\bar{V}, \tau_V^{-1})$ at period $T+1$, where $\bar{V}$ is a constant, and $\mathcal{N}$ denotes the normal distribution.

At trading period $t = 0$, informed traders may observe a signal

$$v = V + \eta, \tag{1}$$

about the liquidation value V, where $\eta \sim \mathcal{N}(0, \tau_\eta^{-1})$ is an independent noise. At period t, we assume that each informed investor is endowed with X_t total number of shares of a nontradable risky asset,[1]

$$X_t = X_{t-1} + x_t, \tag{2}$$

where $X_{-1} = 0$, $x_t \sim \mathcal{N}(0, \tau_{x,t}^{-1})$, x_t is the number of shares received by each informed trader at period t and is independent of V and η. The nontradable risky asset can be liquidated only at period $T+1$. Without loss of generality, we assume that each unit of the nontradable risky asset pays a liquidation value of $V - \bar{V}$ at the final period $T+1$.[2] If $\tau_{x,t}^{-1} = 0$, then no new liquidity shock occurs at period t.

In addition to the private signal at $t = 0$, at each $t = 1, 2, ..., T$, all traders observe a public

[1]Similar to Vayanos and Wang (2012), Goldstein, Li, and Yang (2014), and Chen and Wang (2025), we assume that informed investors have two trading motives to prevent information from being fully revealed in equilibrium. For tractability, we assume that all informed traders share the same private information and experience the same liquidity shock.

[2]We can also assume that the nontradable asset has a payoff correlated with V. This correlation generates a hedging demand for the security. Such a generalization would not lead to qualitatively different results.

signal

$$s_t = V + \varepsilon_t, \tag{3}$$

where $\varepsilon_t \sim \mathcal{N}(0, \tau_{\varepsilon,t}^{-1})$ is an independent noise term.[3]

Define $\underline{Z}_t := (Z_0, Z_1, ..., Z_t)$ for any stochastic process $\{Z_t\}$, representing the history of Z_t up to t. On each date t, each informed investor chooses a demand schedule $\Theta_t^I(v, X_t, \underline{s}_t; \cdot)$. Each uninformed trader chooses a demand schedule $\Theta_t^U(\underline{s}_t; \cdot)$. The schedules Θ_t^I and Θ_t^U are traders' strategies. Given prices $\underline{P}_t$, the quantities demanded by informed and uninformed investors can be written as $\theta_t^I = \Theta_t^I(v, \underline{X}_t, \underline{s}_t, \underline{P}_t)$ and $\theta_t^U = \Theta_t^U(\underline{s}_t, \underline{P}_t)$. Investor i's information set, $\mathscr{F}_t^i$ can be written as

$$\mathscr{F}_t^I = \{v, \underline{s}_t, \underline{X}_t, \underline{P}_t\}, \quad \mathscr{F}_t^U = \{\underline{s}_t, \underline{P}_t\}. \tag{4}$$

Given prices $\underline{P}_t$, the problem for a type $i \in \{I, U\}$ investor is to solve:

$$\max_{\theta_t^i} \mathrm{E}[-e^{-A^i W_{T+1}^i} \mid \mathscr{F}_t^i], \tag{5}$$

where W_{T+1}^i and A^i represent the final wealth and risk aversion coefficient of type i traders, respectively. The final wealth is given by:

$$W_{T+1}^i = W_{-1}^i - \sum_{t=0}^{T} (\theta_t^i - \theta_{t-1}^i) P_t + \theta_T^i V + X_t^i (V - \bar{V}), \tag{6}$$

where $W_{-1}^i = \theta_{-1}^i \bar{V}$ is the initial wealth of type i-investors, $X_t^I = X_t$, and $X_t^U = 0$.

The definition of a perfect Bayesian equilibrium is given as follows.

Definition 1. *An equilibrium* $\left(\Theta_t^I(P_t), \Theta_t^U(P_t), P_t\right)$ *is such that*

1. *given all prices up to time t, $\Theta_t^i(P_t)$ solves the problem (5) for a type-i investor, $i \in \{I, U\}$, where the information sets of informed and uninformed traders are as given in equation (4);*
2. *the price P_t clears the risky asset market: $\Theta = \alpha\Theta_t^I(P_t) + (1-\alpha)\Theta_t^U(P_t)$, and the risk-free*

[3] If the signal s_t ($t \geq 1$), is privately observed by informed traders and there are no future liquidity shocks, uninformed traders can infer s_t from observed prices and thus s_t becomes public. We also verify that our results remain qualitatively similar even when s_t is privately observed and additional liquidity shocks occur in future periods.

asset market is also cleared;

3. *for every realization of the signals v, $\{\underline{s}_t\}$ and $\{\underline{X}_t\}$, the beliefs of all investors are consistent with the joint conditional probability distribution in equilibrium.*

3 The Equilibrium

In this section, we solve for the dynamic equilibrium prices and trading in closed form. We focus on a linear equilibrium in which trading prices are expressed as linear functions of the model's state variables. At $t = 0$, both informed investors' private signal v and liquidity shock X_0 affect informed investors' trading and thus the equilibrium price. From market price P_0, other investors can only infer the value of the composite signal

$$s_0 := v - hx_0 = V + \varepsilon_0, \tag{7}$$

where

$$\varepsilon_0 = \eta - hx_0 \sim \mathcal{N}(0, \tau_0^{-1}), \quad h = A^I \tau_\eta^{-1}, \quad \tau_0^{-1} = \tau_\eta^{-1} + h^2 \tau_{x,0}^{-1}. \tag{8}$$

Therefore, P_0 is informationally equivalent to the composite signal s_0, which is a linear combination of informed investors' private signal and the amount of the nontradable risky asset. Therefore, the information set at period t for uninformed investors $\mathscr{F}_t^U = \{s_0, \underline{s}_t\}$.

We obtain the following lemma on the dynamic Kalman filtering formulas.

Lemma 1. *(Kalman filtering). At t, for $i \in \{I, U\}$, trader i's estimate of the stock's liquidation value $\hat{V}_t^i := \mathrm{E}[V \mid \mathscr{F}_t^i]$ and trader U's estimate of the liquidity shocks $\hat{X}_t^U := \mathrm{E}[X_t \mid \mathscr{F}_t^U]$ are*

$$\hat{V}_t^I = \hat{V}_{t-1}^I + o_{V,t}^I \tau_{\varepsilon,t} e_t^I, \quad \hat{V}_t^U = \hat{V}_{t-1}^U + o_{V,t}^U \tau_t e_t^U, \quad \hat{X}_t^U = \hat{X}_{t-1}^U + K_{X,t}^U e_t^U, \tag{9}$$

where

$$e_t^i = s_t - \hat{V}_{t-1}^i, \ \text{for } i \in \{I, U\} \text{ and } t \geq 1, \ \text{and } e_0^I = v - \bar{V}, \ e_0^U = s_0 - \bar{V}, \tag{10}$$

$\hat{X}_{-1}^U = 0$, $\hat{V}_{-1}^I = \hat{V}_{-1}^U = \bar{V}$, the coefficient $K_{X,t}^U$ is defined in (A-6) in the Appendix, and the condi-

tional variances of V for informed and uninformed traders at t are,

$$o_{V,t}^{I} := \mathrm{Var}[V \mid \mathscr{F}_t^I] = \big(\tau_V + \sum_{s=0}^{t} \tau_{\varepsilon,s}\big)^{-1}, \quad o_{V,t}^{U} := \mathrm{Var}[V \mid \mathscr{F}_t^U] = \big(\tau_V + \sum_{s=0}^{t} \tau_{s}\big)^{-1},$$

where $\tau_{\varepsilon,0} = \tau_\eta$, $\tau_0 := (\tau_\eta^{-1} + h^2\tau_{x,0}^{-1})^{-1}$, and $\tau_s = \tau_{\varepsilon,s}$ for $s \geq 1$.

Proof. See Appendix A.1. □

Lemma 1 shows that informed traders' estimate $\hat{V}_t^I$ can be written as a recursive equation of $\hat{V}_{t-1}^I$ with an innovation term $s_t - \hat{V}_{t-1}^I$ and uninformed traders' estimate $(\hat{V}_t^U, \hat{X}_t^U)$ can be expressed as a recursive equation of $(\hat{V}_{t-1}^U, \hat{X}_{t-1}^U)$ with an innovation term $s_t - \hat{V}_{t-1}^U$. The conditional variance and mean of the security's payoff for traders have intuitive expressions. Specifically, the conditional variance of the payoff corresponds to the inverse of the total precision of traders' signals, while the conditional mean of the payoff (or the estimate of V) is determined by the precision-weighted average of signal innovations.

The sufficient statistic $\hat{V}_t^I - \mu_t X_t$ is a composite signal of informed traders, capturing their dual trading motives: speculating based on private information and hedging against risks from liquidity shocks. The noise-to-signal ratios μ_t is

$$\mu_t = A^I o_{V,t}^I = A^I \big(\tau_V + \sum_{s=0}^{t} \tau_{\varepsilon,s}\big)^{-1}. \tag{11}$$

Since the sufficient statistic $\hat{V}_t^I - \mu_t X_t$ can be inferred by uninformed traders from prices, it follows that $\hat{V}_t^I - \mu_t X_t = \hat{V}_t^U - \mu_t \hat{X}_t^U$. Equation (11) implies that the noise-to-signal ratio for uninformed traders μ_t decreases over time, as prices progressively convey more information about informed traders' estimation of the stock value.

We provide closed-form expressions for equilibrium prices and trading quantities in the following proposition.

Proposition 1. *1. The equilibrium price at time t is*

$$P_t = \omega_t P_t^I + (1-\omega_t) P_t^U - \frac{\Theta}{\frac{\alpha}{\gamma_t^I} + \frac{1-\alpha}{\gamma_t^U}}, \tag{12}$$

where $\omega_t = \frac{\frac{\alpha}{\gamma_t^I}}{\frac{\alpha}{\gamma_t^I}+\frac{1-\alpha}{\gamma_t^U}}$. *Parameters* γ_t^i, *for* $i \in \{I, U\}$ *and* $t < T$, *are positive and can be computed recursively using Equations* (A-38) *and* (A-46) *and their values at* $t = T$ *are* $\gamma_T^i = A^i o_{V,T}^i$. *The non-participation prices of informed and uninformed traders are given as,*

$$\begin{aligned} P_t^I &= \bar{V} + \xi_t^I(\hat{V}_t^I - \mu_t X_t - \bar{V}) + \psi_t^I(\hat{V}_t^U - \bar{V}) - f_t^I \Theta, \\ P_t^U &= \bar{V} + \xi_t^U(\hat{V}_t^I - \mu_t X_t - \bar{V}) + \psi_t^U(\hat{V}_t^U - \bar{V}) - f_t^U \Theta. \end{aligned} \tag{13}$$

The coefficient μ_t *is as defined in Equation (11) and the coefficients* ξ_t^i, ψ_t^i, *and* f_t^i, *for* $i \in \{I, U\}$ *and* $t < T$ *are all positive and can be computed recursively using Equations* (A-38), (A-39), (A-46), *and* (A-47). *Their values at* $t = T$ *are* $\xi_T^I = 1$, $\xi_T^U = 0$, $\psi_T^I = 0$, $\psi_T^U = 1$, *and for* $i \in \{I, U\}$, $f_T^i = 0$, *implying that the non-participation prices at T are*

$$P_T^I = \hat{V}_T^I - \mu_T X_T \quad \text{and} \quad P_T^U = \hat{V}_T^U. \tag{14}$$

2. *The informed and uninformed traders' equilibrium holdings at t are*

$$\theta_t^I = \frac{P_t^I - P_t}{\gamma_t^I}, \quad \theta_t^U = \frac{P_t^U - P_t}{\gamma_t^U}. \tag{15}$$

Proof. See Appendix A.2. □

Equation (14) in Proposition 1 implies that, at period T, an informed trader's non-participation price is equal to the composite signal $\hat{V}_T^I - \mu_T X_T$. An uninformed trader's reservation price is equal to his estimate of the asset's liquidation value, $\hat{V}_T^U$, following the logic of a one-period model. Since the final payoff V is realized at period $T+1$, at period T informed traders adjust their positions based on their updated payoff estimates and the realized hedging demand. Uninformed traders adjust based on their updated payoff estimates. Neither group of traders has a dynamic speculative position related to the estimation of the next period's trading prices. Consequently, at period T, each trader's non-participation price is independent of the other type of traders' estimates.

In contrast, for $t < T$, both traders' non-participation prices increase with the informed traders' composite signal $\hat{V}_t^I - \mu_t X_t$ and the uninformed traders' estimate $\hat{V}_t^U$. This implies that each trader's non-participation price is influenced by other traders' estimates. This is because, at $t < T$, each trader's optimal demand consists of a long-term stock position and a short-term speculative position. The former is based on the trader's estimate of the asset's final payoff $\hat{V}_t^i$, while the latter is based on the trader's estimate of the next period's trading prices. Since the next period's trading prices are influenced by both traders' estimates, each trader's non-participation price at period t depends on the informed traders' composite signal $\hat{V}_t^I - \mu_t X_t$ and the uninformed traders' estimate $\hat{V}_t^U$. We show that each trader's reservation price is a function of both traders' estimates, the total supply Θ, as depicted in Equation (13).

Given that both the long-term stock position and short-term speculative position of each trader are based on his own estimate of the asset's payoff, and only the short-term speculative position is influenced by the other trader's estimates, the coefficient for $\hat{V}_t^I - \mu_t X_t$ in the informed traders' non-participation price is typically greater than that in the uninformed traders' reservation price. This implies that $\xi_t^I > \xi_t^U$ for $t < T$. Similarly, the coefficient for $\hat{V}_t^U$ in the uninformed traders' non-reservation price tends to be larger than that in the informed traders' reservation price, suggesting $\psi_t^U > \psi_t^I$ for $t < T$.[4]

4 Return Dynamics

To understand the key mechanism behind positive return autocorrelation, we examine three cases. We begin with the simplest setting: symmetric information and a one-time initial liquidity shock. Next, we include asymmetric information while still considering only an initial liquidity shock. Finally, we analyze the most general case, incorporating both information asymmetry and anticipated future liquidity shocks.

[4] We conduct an extensive numerical analysis across a wide range of parameters. We find that the parameter values consistently demonstrate $\xi_t^I > \xi_t^U$ and $\psi_t^U > \psi_t^I$ for $t < T$.

4.1 Return Dynamics with Symmetric Information and an Initial Liquidity Shock

We first consider the case without information asymmetry (i.e., $\tau_{\eta,0} = 0$), where informed traders receive only a liquidity shock at time $t = 0$. In this simplified setting, trading occurs only at $t = 0$. After establishing their optimal hedging positions, there is no heterogeneity between informed and uninformed traders. Consequently, no further trading takes place, although prices continue to update as new public information arrives over time.

The equilibrium prices and return autocorrelations are given in the following proposition.

Proposition 2. *1. The equilibrium price and return at time t are given by:*

$$\begin{aligned} P_t &= \hat{V}_t - \omega\mu_t\left(x_0 + \frac{\Theta}{\alpha}\right), \\ R_t &:= P_t - P_{t-1} = \omega\left(\mu_{t-1} - \mu_t\right)\left(x_0 + \frac{\Theta}{\alpha}\right) + \tau_{\varepsilon,t} o_{V,t}(s_t - \hat{V}_{t-1}). \end{aligned} \tag{16}$$

where $\omega := \frac{\alpha/A^I}{\alpha/A^I + (1-\alpha)/A^U}$, $\mu_t := A^I o_{V,t}$, and $\hat{V}_t$ and $o_{V,t}$ denote the conditional expectation and variance of V at time t, which are identical for both types of traders in the absence of information asymmetry.

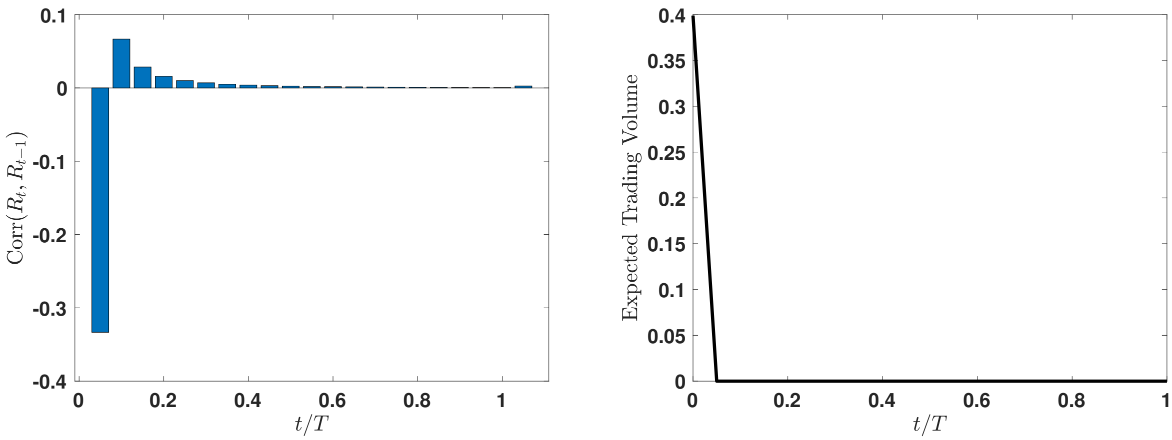
2. With the arrival of public information, μ_t decreases over time, and we have

$$\text{Cov}(R_t, R_{t-1}) = \omega^2(\mu_{t-1} - \mu_t)(\mu_{t-2} - \mu_{t-1})\text{Var}(x_0) > 0, \textit{ for } t \geq 2.$$

For $t = 1$, we have $\text{Cov}(R_1, R_0) < 0$, where $R_0 = P_0 - \bar{V}$.

The return autocorrelation at $t = 1$ is negative because an initial liquidity shock pushes prices away from their fundamental values, as in Rational Expectations models. As public information arrives, prices gradually revert toward fundamentals.

Proposition 2 shows that even in the absence of information asymmetry, a sequence of public information releases following a liquidity shock can generate positive return autocorrelation, as prices trend back toward fundamentals while the impact of the liquidity shock on the risk premium decreases over time.



(a) Return Autocorrelation

(b) Dynamics of Trading Volume

Figure 1. Symmetric Information with Initial Liquidity Shock Only. Parameters are $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 10^{-6}$, $\tau_{\varepsilon,t} = 1$, $\bar{V} = 1$, and $\tau_{x,t} = 10^6$ except for $\tau_{x,0} = 1$.

As illustrated in Figure 1, return autocorrelation at a one-lag interval is negative after the liquidity shock, but turns positive following the arrival of public information. In this simple case, trading activity only occurs in response to the initial liquidity shock, suggesting that trading itself is not the primary driver of time-series momentum in returns. Instead, it is the interplay between liquidity shocks and subsequent public information that generates positive return autocorrelation.

4.2 Return Dynamics with Asymmetric Information and Initial Liquidity Shocks

We next consider the case with asymmetric information and an initial liquidity shock. Since no additional liquidity shocks arrive after the initial period, any private information received after $t = 0$ is fully revealed to the market through equilibrium prices. As a result, the private signals v_t for $t = 1, 2, \ldots, T$ effectively become public information. In contrast, the initial private signal v_0 and the initial liquidity shock x_0 remain unknown to uninformed traders.

The equilibrium prices and return autocorrelations are given in the following proposition.

Proposition 3. *The equilibrium stock price is*

$$\begin{aligned} P_t &= \omega_t(\hat{V}_t^I - \mu_t X_t) + (1-\omega_t)\hat{V}_t^U - \omega_t \mu_t \frac{\Theta}{\alpha} \\ &= \hat{V}_t^U - \omega_t \mu_t \Big(\hat{X}_t^U + \frac{\Theta}{\alpha} \Big), \end{aligned} \tag{17}$$

where $\omega_t = \frac{\alpha}{A^I o_{V,t}^I} \left(\frac{\alpha}{A^I o_{V,t}^I} + \frac{1-\alpha}{A^U o_{V,t}^U} \right)^{-1}$*, and* $\hat{X}_t^U = \mathrm{E}_t^U[x_0]$*. The equilibrium stock holdings at* t *are:*

$$\theta_t^I = \frac{\Theta}{\alpha} - (1-\omega_t)\left(\hat{X}_t^U + \frac{\Theta}{\alpha} \right), \quad \theta_t^U = \frac{\alpha}{1-\alpha}(1-\omega_t)\left(\hat{X}_t^U + \frac{\Theta}{\alpha} \right). \tag{18}$$

Proof. See Appendix A.4. □

Proposition 3 implies that, in this simplified setting without future liquidity shocks, prices and trading quantities primarily depend on uninformed traders' expectations about the initial liquidity shock. Intuitively, if uninformed traders experienced the same liquidity shock as informed traders, their non-participation prices would be the same. Therefore, any difference in non-participation prices arises from differences in the liquidity shock, which are reflected by the difference in the estimation of the liquidity shock and the associated estimation risk premium.

We obtain the following proposition regarding the autocorrelation of returns.

Proposition 4. *1. The stock return for* $t = 1, 2, ..., T$ *is:*

$$R_t := P_t - P_{t-1} = a_t \left(\hat{X}_{t-1}^U + \frac{\Theta}{\alpha} \right) + b_t(s_t - \hat{V}_{t-1}^U), \tag{19}$$

where coefficients a_t *and* b_t *are given as*

$$a_t := \omega_{t-1}\mu_{t-1} - \omega_t \mu_t > 0, \quad b_t := \tau_{\varepsilon,t}\left(\omega_t o_{V,t}^I + (1-\omega_t) o_{V,t}^U \right) > 0. \tag{20}$$

2. For $t = 2, 3, ..., T$*, the covariance of returns is*

$$\mathrm{Cov}(R_t, R_{t-1}) = a_t a_{t-1} \mathrm{Var}(\hat{X}_{t-2}^U) + a_t b_{t-1} o_{VX,t-2}^U > 0. \tag{21}$$

For $t = 1$*, we have* $\mathrm{Cov}(R_1, R_0) < 0$*.*

Proof. See Appendix A.5. □

Proposition 4 implies that information arrivals following a liquidity shock generate time-series momentum in returns. To understand the underlying mechanism, we next examine how the arrival of public information influences traders' behavior. We obtain the following proposition.

Proposition 5. *1. Uninformed traders' time-t demand is:*

$$\Delta\theta_t^U := \theta_t^U - \theta_{t-1}^U = \alpha_t \left(\hat{X}_{t-1}^U + \frac{\Theta}{\alpha}\right) + \beta_t (s_t - \hat{V}_{t-1}^U),$$

where $\alpha_t = \frac{\alpha}{1-\alpha}(\omega_{t-1} - \omega_t) > 0$ *and* $\beta_t = \frac{\alpha}{1-\alpha}\frac{\tau_{\varepsilon,t}}{\mu_t}(1-\omega_t)(o_{V,t}^U - o_{V,t}^I) > 0$.

2. For $t = 2, 3, ..., T$*, the covariance of demands is*

$$\mathrm{Cov}\left(\Delta\theta_t^U, \Delta\theta_{t-1}^U\right) = \alpha_t \alpha_{t-1} \mathrm{Var}(\hat{X}_{t-2}^U) + \alpha_t \beta_{t-1} o_{VX,t-2}^U > 0.$$

For $t = 1$*, we also have* $\mathrm{Cov}\left(\Delta\theta_1^U, \Delta\theta_0^U\right) > 0$. *Since informed traders are counterparties to uninformed traders, trades exhibit positive autocorrelation for both types of traders.*

Proof. See Appendix A.6. □

Proposition 5 implies that following the arrival of public information, both informed and uninformed traders adjust their positions. Those who optimally bought (or sold) prior to the information release often continue trading in the same direction afterward, leading to autocorrelated trades and positive return autocorrelation.

We next examine which traders buy when prices rise and sell when prices fall, thereby behaving as momentum traders. Define

$$\kappa_t^i := \mathrm{Cov}\Big(\Delta\theta_t^i, R_{t-1}\Big), \quad \text{for} \ \ t = 1, 2, ..., T. \tag{22}$$

Since informed and uninformed traders are counterparties, it follows that $\kappa_t^U = -\kappa_t^I$. If $\kappa_t^i > 0$, type-i traders buy when price rise and sell when prices fall, thereby engaging in momentum trading. In contrast, if $\kappa_t^i < 0$, they engage in contrarian trading.

It is also helpful to examine which traders act as liquidity demanders—those whose sales drive down the concurrent price and whose purchases drive it up, thereby imposing a positive price impact. Following Vayanos and Wang (2012), we define the price impact of type-i traders' demand as

$$\lambda_t^i := \frac{\mathrm{Cov}\left(R_t, \Delta\theta_t^i\right)}{\mathrm{Var}\left(\Delta\theta_t^i\right)}, \quad \text{for } t = 0, 1, ..., T. \tag{23}$$

Similarly, since informed and uninformed traders are counterparties, it follows that $\lambda_t^U = -\lambda_t^I$. If $\lambda_t^i > 0$, type-i traders exert a positive price impact, indicating they are demanding liquidity. In contrast, if $\lambda_t^i < 0$, they are providing liquidity.

Proposition 6. *Following the arrival of public information:*

1. *Uninformed traders buy when prices rise and sell when prices fall, thereby behaving as momentum traders. In contrast, informed traders act as contrarian traders, i.e.,* $\kappa_t^I < 0$ *and* $\kappa_t^U > 0$*, for* $2 \leq t \leq T$*;*
2. *Uninformed traders demand liquidity: their sales drive down the concurrent price, and their purchases drive it up. Informed traders, by contrast, provide liquidity, i.e.,* $\lambda_t^I < 0$*, and* $\lambda_t^U > 0$*, for* $1 \leq t \leq T$*.*

Proof. See Appendix A.7. □

At $t = 0$, informed traders optimally establish positions based on both private information and liquidity needs. Their liquidity-driven trading pushes prices away from fundamental values, as hedging motives distort their optimal response to private signals. Proposition 6 implies that, in the absence of future liquidity shocks, rational uninformed traders—who initially provide liquidity to informed traders—anticipate that prices will eventually revert to fundamentals. When public information arrives, uninformed traders interpret it as an indication of whether current prices deviate from fundamental values and trade in the direction of the signal, expecting prices to converge. As a result, they chase the previous period's return—buying when prices rise and selling when prices fall—thereby behaving as momentum traders. Moreover, their positive price impact pushes prices higher when they buy and lowers prices when they sell, reinforcing time-series momentum in returns. Informed traders, by contrast, act as contrarian traders: they adjust their positions at more favorable prices by buying when prices fall and selling when prices rise.

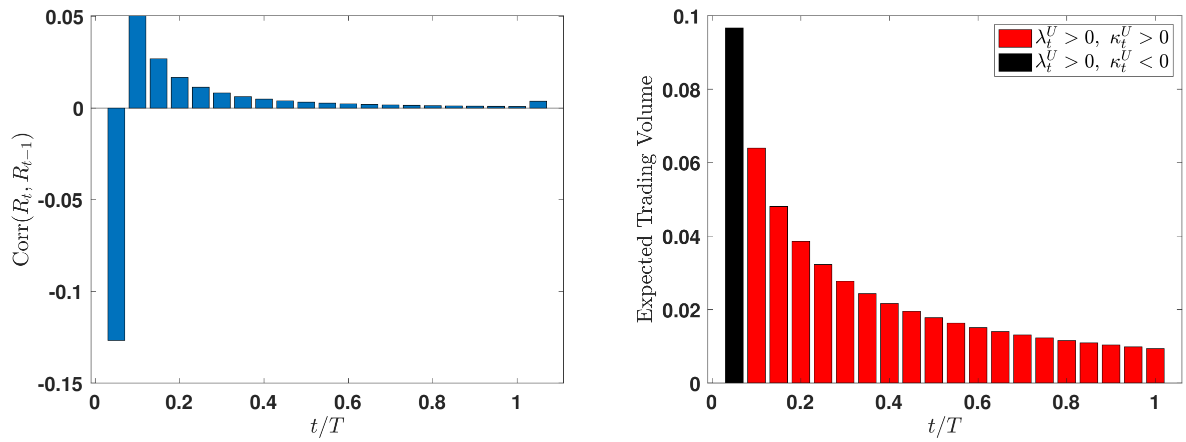


Figure 2. Parameters are $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 1$, $\tau_{\varepsilon,t} = 1$, $\bar{V} = 1$, and $\tau_{x,0} = 1$. In Panel (b), the red bars indicate that positive return autocorrelation occurs when $\lambda_t^U \kappa_t^U > 0$.

As Figure 2 illustrates, there is time-series momentum in returns for periods $t = 2, 3, ..., T$, during which uninformed traders behave as momentum traders and demand liquidity. In general, when $\lambda_t^i \kappa_t^i > 0$, momentum traders are also demanding liquidity: their tendency to buy following a positive return further drives prices upward, while their selling after a negative return pushes prices further downward. As a result, their trading behavior generates time-series momentum in returns.

Figure 3 shows that both return autocorrelation and trading volume tend to increase with the precision of public information. This positive relationship becomes more pronounced when informed traders' private signals are more precise (i.e., when $\tau_{\eta,0}$ is higher), as traders respond more aggressively to public information, leading to greater trading volume and stronger return autocorrelation.

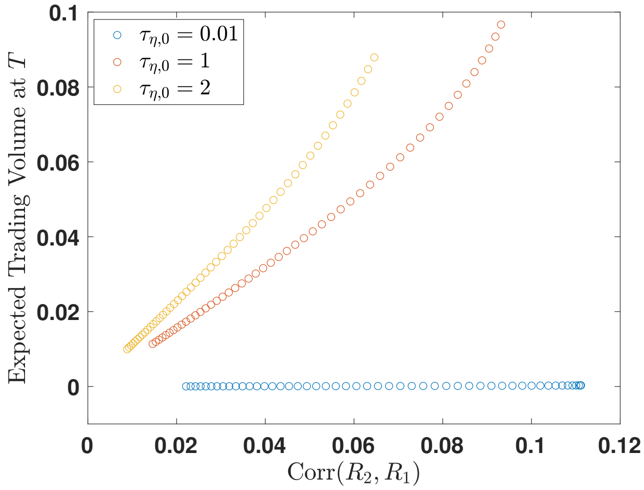


Figure 3. Return autocorrelation and trading volume as the precision of public information, $\tau_{\varepsilon,1}$ varies from 0.01 to 1. The parameter values are: $T = 1, \alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, $\tau_{x,0} = 1$, and $\tau_{\eta,0}$ as indicated in the legend.

4.3 Return Dynamics with Asymmetric Information and Future Liquidity Shocks

In this section, we demonstrate that this mechanism remains robust under more general settings involving multiple illiquid shocks. Based on Equation (**??**), the covariance between R_t and its one-period lag R_{t-1} can be expressed as follows:

$$\begin{aligned}\mathrm{Cov}(R_t, R_{t-1}) = {} & (g_t - g_{t-1})(g_{t-1} - g_{t-2})\mathrm{Var}(\hat{X}^U_{t-2}) + (\delta_t - \delta_{t-1})(\delta_{t-1} - \delta_{t-2})\mathrm{Var}(\hat{V}^U_{t-2}) \\ & + \left(\delta_{t-1} K^U_{V,t-1} - g_{t-1} K^U_{X,t-1}\right)\left[(\delta_t - \delta_{t-1}) K^U_{V,t-1} - (g_t - g_{t-1}) K^U_{X,t-1}\right]\Sigma^U_{t-1} \\ & - \left[(\delta_t - \delta_{t-1})(g_{t-1} - g_{t-2}) + (g_t - g_{t-1})(\delta_{t-1} - \delta_{t-2})\right]\mathrm{Cov}(\hat{V}^U_{t-2}, \hat{X}^U_{t-2}),\end{aligned} \tag{24}$$

where $\delta_{t-1} \geq \delta_t \geq 1$, $g_{t-1} \geq g_t \geq 0$, and $K^U_{V,t} > 0$. $K^U_{X,t} < 0$ if there is an illiquid shock at t, $K^U_{X,t} > 0$ otherwise. More specifically, $\delta_t = 1$ if there are no further illiquid shocks after time t, meaning time t is when the informed traders experience the last illiquid shock before the terminal time. Otherwise, $\delta_t > \delta_{t+1}$ if the informed traders anticipate additional illiquid shocks occurring after time t. g_t decreases only in response to an illiquid shock or the arrival of public information at time t; otherwise, it remains equal to g_{t-1}.

According to Equation (24), the covariance between R_t and its one-period lag R_{t-1} consists of four components: the variance of the innovation term Σ^U_{t-1}, the variance of uninformed traders' expectations about the fundamental $\text{Var}(\hat{V}^U_{t-2})$, the variance of their expectations about the cumulative illiquid shock $\text{Var}(\hat{X}^U_{t-2})$, and the covariance between these two expectations $\text{Cov}(\hat{V}^U_{t-2}, \hat{X}^U_{t-2})$. Compared to the benchmark case, where the covariance between R_t and its one-period lag R_{t-1} consists only two terms: the variance of the innovation term Σ^U_{t-1}, and the variance of uninformed traders' expectations about the cumulative illiquid shock $\text{Var}(\hat{X}^U_{t-2})$, the general case is more complex.

If informed traders receive the last illiquid shock at time t', meaning no additional illiquid shocks occur after t', then $\delta_t = 1$ for all $t \geq t'$. In this scenario, for $t > t'$, the coefficients of $\text{Var}(\hat{V}^U_{t-2})$ and $\text{Cov}(\hat{V}^U_{t-2}, \hat{X}^U_{t-2})$ become zero, simplifying the model to the benchmark case. As a result, uninformed traders become liquidity consumers, chasing the trend and rebalancing their positions upon the arrival of public information, which generates positive autocorrelation in returns over time.

If informed traders anticipate more illiquid shocks in the future, the extra terms in Equation (24), relative to the benchmark case, are $E_1 + E_2 + E_3$:

$$\begin{aligned}
&\underbrace{(\delta_t - \delta_{t-1})(\delta_{t-1} - \delta_{t-2})\text{Var}(\hat{V}^U_{t-2})}_{:=E_1>0} + \underbrace{\left(\delta_{t-1} K^U_{V,t-1} - g_{t-1} K^U_{X,t-1}\right)(\delta_t - \delta_{t-1}) K^U_{V,t-1} \Sigma^U_{t-1}}_{:=E_2<0} \\
&\quad \underbrace{- \left[(\delta_t - \delta_{t-1})(g_{t-1} - g_{t-2}) + (g_t - g_{t-1})(\delta_{t-1} - \delta_{t-2})\right] \text{Cov}(\hat{V}^U_{t-2}, \hat{X}^U_{t-2})}_{:=E_3>0}.
\end{aligned} \tag{25}$$

The terms E_1 and E_3 are positive, while the term E_2 is negative. These terms arise because traders' equilibrium positions depend on the expectations of uninformed traders regarding the fundamental value, due to the positive correlation between the stock and the illiquid asset. As shown in Equation (15), informed traders speculate on the expectations of uninformed traders and act as momentum traders: they take larger positions when uninformed traders' expectation is high and smaller positions when those expectation is low. The terms E_1 and E_3 capture such impact of informed traders' trading behavior on return autocorrelation, contributing to a more positive autocorrelation. At the same time, the future illiquid shock prompts informed traders to hedge in advance, causing them to act as contrarians by trading against news innovations. The term E_2 reflects this impact of informed traders' behavior on return autocorrelation, contributing to a more negative autocorrelation. Therefore, the dominant term among E_1, E_2 and E_3 depends on the volatility of the future illiquid shock, represented by $\tau_{x,t}^{-1}$. In the following paragraph, we discuss various scenarios based on different levels of future illiquid shock uncertainty.

If the future illiquid shock exhibits high volatility, indicated by a small $\tau_{x,10} = 0.5$ as shown in Figure 4, informed traders' speculation on uninformed traders' expectations dominates. In this scenario, they behave as momentum traders, consuming liquidity in the market before the arrival of future illiquid shock. This type of return momentum differs from the one in the benchmark case, as it does not originate from the risk premium component of the price but rather from the aggregate belief about the fundamental value. This unique intuition exists only in a non-stationary equilibrium, which is often overlooked by the existing literature, demonstrating that the aggregate belief about the fundamental value embedded in the price can generate time-series momentum within the REE framework.

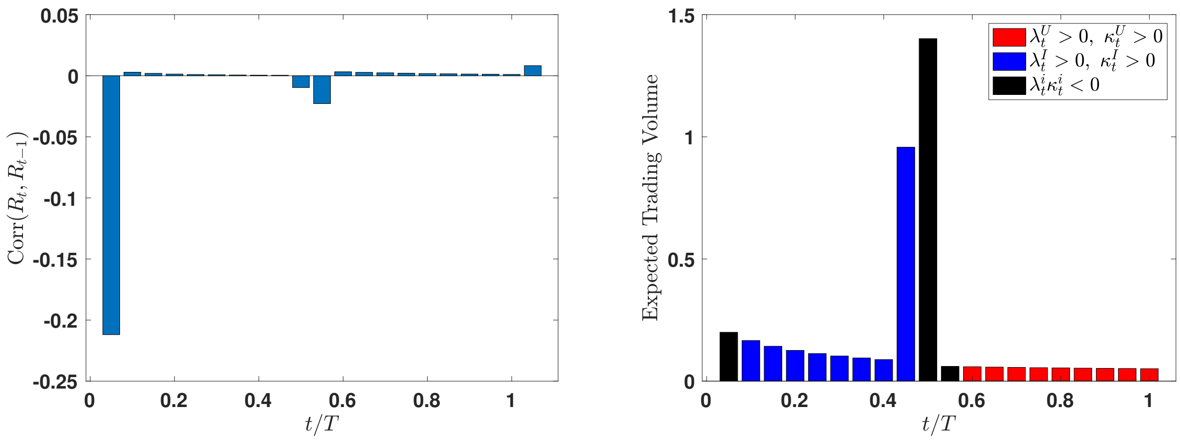
If the future illiquid shock becomes less volatile, indicated by a small $\tau_{x,10} = 1$ as shown in Figure 5, informed traders' advanced hedging against the future illiquid shock becomes dominant. In this scenario, they act as contrarians while still consuming liquidity in the market before the arrival of future illiquid shock. Therefore, in this scenario the momentum traders, served by unformed traders, are liquidity providers, resulting in no return momentum prior to the next illiquid shock.

If the future illiquid shock becomes even less volatile, indicated by a smaller $\tau_{x,10} = 5$ as shown in Figure 6, informed traders' advanced hedging against the future illiquid shock continues to dominate their speculation on uninformed traders' expectations. However, their hedging motive weakens, causing them to act as liquidity providers upon the arrivals of public information in the market. They consume liquidity only when they experience the illiquid shock. As a result, return momentum occurs during episodes of public information releases, driven primarily by uninformed traders.

In the extreme scenario where the future illiquid shock has very low volatility, indicated by $\tau_{x,10} = 10$ as shown in Figure 7, informed traders' advanced hedging becomes even weaker, converging toward the benchmark case. As a result, the time-series return autocorrelation remains positive after the first illiquid shock. This is driven by uninformed traders, who rebalance their positions upon receiving public information, acting as momentum traders and consuming liquidity.

In sum, the sequential arrival of illiquid shocks and public information is a necessary and sufficient condition for the emergence of serial momentum in returns. When an illiquid shock is followed by public information with no additional illiquid shocks expected, uninformed traders engage in momentum trading, consuming market liquidity. In contrast, when an illiquid shock is followed by public information with additional illiquid shocks anticipated, informed traders balance advanced hedging against future illiquid shocks with speculating on uninformed traders' expectations. When the future illiquid shock is highly volatile, informed traders prioritize speculation, acting as momentum traders and consuming liquidity, generating return momentum driven by aggregate beliefs about the fundamental value rather than risk premiums. As the future illiquid shock becomes less volatile, informed traders shift focus toward advanced hedging, behaving as contrarians and consuming liquidity, reducing return momentum. In scenarios of even lower volatility, informed traders' advanced hedging weakens further, causing them to become liquidity providers, with return momentum driven solely by public information releases. In the extreme case of minimal illiquid shock volatility, informed traders' advanced hedging motives nearly disappear, replicating the benchmark scenario where uninformed traders dominate, producing consistently positive return autocorrelation through their trend-chase trading.

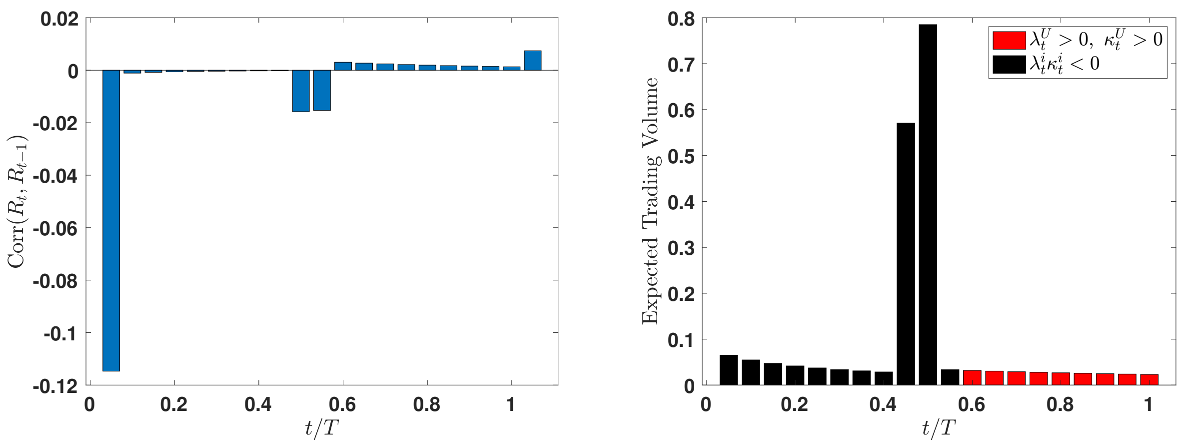
We then examine how the anticipation of future liquidity shocks influences traders' behavior and return autocorrelation prior to the shocks' arrival. We find that return dynamics critically depend on the volatility of these future shocks. When future shocks are much less volatile



(a) Return Autocorrelation

(b) Dynamics of Trading Volume

Figure 4. Informed traders experience illiquid shocks at both the beginning and midpoint of the trading duration. Parameters are $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 1$, $\tau_{\varepsilon,t} = 1$, $\bar{V} = 1$, and $\tau_{x,t} = 1e6$ except for $\tau_{x,0} = 2$, $\tau_{x,10} = 0.5$. In Panel (b), the red bars indicate that return momentum is driven by the trading of uninformed traders, while the blue bars indicate that it is driven by the trading of informed traders.

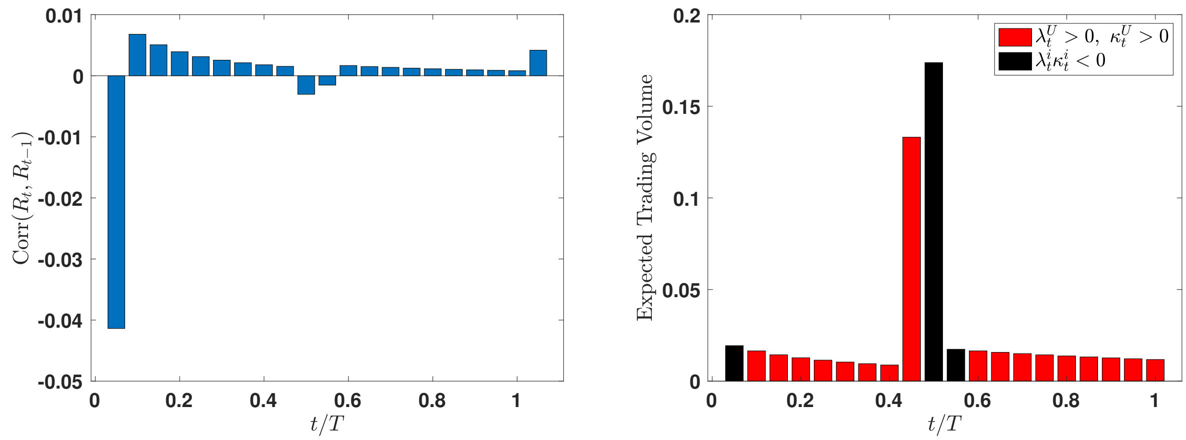


(a) Return Autocorrelation

(b) Dynamics of Trading Volume

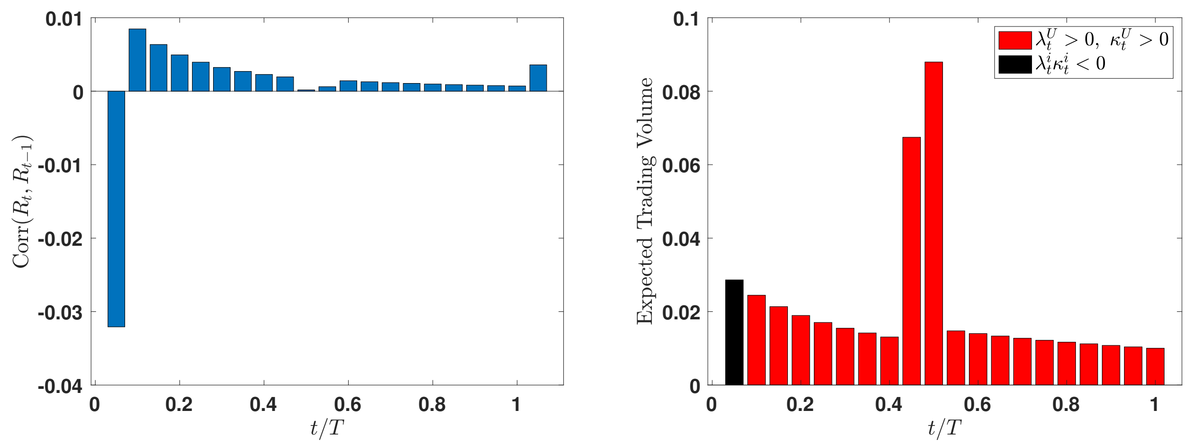
Figure 5. Informed traders experience illiquid shocks at both the beginning and midpoint of the trading duration. Parameters are $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 1$, $\tau_{\varepsilon,t} = 1$, $\bar{V} = 1$, and $\tau_{x,t} = 1e6$ except for $\tau_{x,0} = 2$, $\tau_{x,10} = 1$. In Panel (b), the red bars indicate that return momentum is driven by the trading of uninformed traders.

than the initial shock, their expected impact is small, and their anticipated arrival has little influence on traders' behavior beforehand. As a result—similar to the case without anticipated



(a) Return Autocorrelation (b) Dynamics of Trading Volume

Figure 6. Informed traders experience illiquid shocks at both the beginning and midpoint of the trading duration. Parameters are $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 1$, $\tau_{\varepsilon,t} = 1$, $\bar{V} = 1$, and $\tau_{x,t} = 1e6$ except for $\tau_{x,0} = 2$, $\tau_{x,10} = 5$. In Panel (b), the red bars indicate that return momentum is driven by the trading of uninformed traders.



(a) Return Autocorrelation (b) Dynamics of Trading Volume

Figure 7. Informed traders experience illiquid shocks at both the beginning and midpoint of the trading duration. Parameters are $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 1$, $\tau_{\varepsilon,t} = 1$, $\bar{V} = 1$, and $\tau_{x,t} = 1e6$ except for $\tau_{x,0} = 2$, $\tau_{x,10} = 10$. In Panel (b), the red bars indicate that return momentum is driven by the trading of uninformed traders.

future shocks—rational uninformed traders continue to trade in the direction of the public signal, buying in response to positive news and selling in response to negative news. This behavior leads to autocorrelated trades and positive return autocorrelation.

In contrast, when future shocks are highly volatile, informed traders alternate their trading direction from one period to the next in response to public information releases, due to uncertainty about whether the upcoming shock will be positive or negative. To avoid accumulating large long or short positions, they behave as contrarian traders—buying when past prices are low and selling when past prices are high. In doing so, they consume liquidity by pushing prices up when buying and down when selling. Rational uninformed traders, while providing liquidity to informed traders, also reverse their trading direction each period in response to public signals. This period-by-period reversal in trading direction generates negative return autocorrelation.

When the volatility of future liquidity shocks is comparable to that of the initial shock, traders behave differently from the case without anticipated future shocks. In response to public information releases, rational uninformed traders no longer act as momentum traders. They are concerned that future liquidity shocks may drive prices away from fundamental values, so a rising price does not necessarily signal convergence toward fundamentals. As a result, they refrain from buying simply because prices are increasing. Instead, they act as contrarian traders and provide liquidity by buying when prices are low and selling when prices are high. Informed traders, who initially buy (or sell) based on private information and liquidity needs, reverse their position once following the initial round of public information. After this adjustment, they continue trading in the same direction in response to subsequent public information releases. As more public information becomes available, their uncertainty about the asset's payoff—and thus about the payoff of the associated non-traded risky asset—diminishes. When the anticipated future shocks have moderate volatility, informed traders optimally adjust their hedging positions by reducing the size of their initial long or short exposures. Thus, in contrast to the case without future liquidity shocks, informed traders exhibit momentum trading behavior and demand liquidity, which gives rise to positive return autocorrelation.

The alternating episodes of illiquid shocks and public information releases create diverse patterns of time-series return autocorrelation, encompassing both short-term return momentum and long-term reversal dynamics. The covariance between R_t and its s-period lag R_{t-s} can be expressed as follows:

$$\begin{aligned}\mathrm{Cov}(R_t, R_{t-s}) &= (g_t - g_{t-1})(g_{t-s} - g_{t-s-1})\mathrm{Var}(\hat{X}^U_{t-s-1}) + (\delta_t - \delta_{t-1})(\delta_{t-s} - \delta_{t-s-1})\mathrm{Var}(\hat{V}^U_{t-s-1}) \\ &+ \left(\delta_{t-s} K^U_{V,t-s} - g_{t-s} K^U_{X,t-s}\right)\left[(\delta_t - \delta_{t-1}) K^U_{V,t-s} - (g_t - g_{t-1}) K^U_{X,t-s}\right]\Sigma^U_{t-s} \\ &- \left[(\delta_t - \delta_{t-1})(g_{t-s} - g_{t-s-1}) + (g_t - g_{t-1})(\delta_{t-s} - \delta_{t-s-1})\right]\mathrm{Cov}(\hat{V}^U_{t-s-1}, \hat{X}^U_{t-s-1}).\end{aligned} \tag{26}$$

As the lag s increases, the covariance may become more negative, depending on the specific patterns of illiquid shocks and public information releases. In the numerical example shown in Figure 8, there are 50 trading periods, with illiquid shocks occurring in the middle, spanning from period 30 to period 40. The figure shows that the return at period 35 is positively correlated with future returns for lags between 1 and 7, but becomes negatively correlated for longer lags. This pattern aligns with empirical evidence of short-term momentum and long-term reversal.

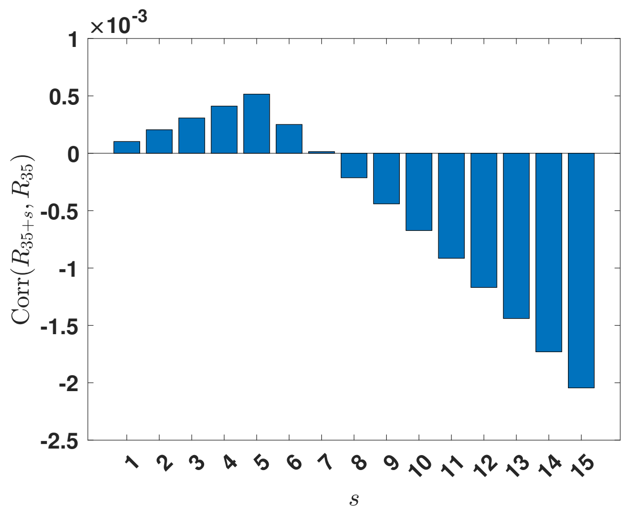


Figure 8. Illiquid shocks occur midway through the trading period, spanning from $t = 20$ to $t = 30$ within a total of $T = 50$ periods. The model parameters are set as follows: $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 2$, $\tau_{\varepsilon,t} = 3$, $\bar{V} = 1$, and $\tau_{x,t} = 1e6$, except during the illiquid shock period ($t = 20$ to $t = 30$), where $\tau_{x,t} = 5$.

5 Conclusion

This paper demonstrates that time-series return momentum can arise within a rational expectations equilibrium (REE) framework due to the interaction between illiquid shocks and public information releases. Our model reveals that even in the absence of private information or behavioral biases, positive return autocorrelation can emerge through a straightforward mechanism: traders' responses to sequential public information following an illiquid shock. Informed traders balance speculative and advanced hedging motives, shifting from momentum-driven trading to contrarian behavior depending on the volatility of future illiquid shocks. Uninformed traders, meanwhile, alternate between providing and consuming liquidity as they adjust their positions based on public signals. This dynamic generates distinct patterns of return autocorrelation, including short-term momentum and long-term reversal, consistent with empirical observations. By highlighting the critical role of liquidity events and public information updates, our findings offer a novel explanation for return momentum within a rational trading environment. This approach bridges theoretical models and observed market phenomena, expanding the understanding of how market structure and information flow shape asset price dynamics.

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A Appendix

A.1 Proof of Lemma 1

To prove Lemma 1, we first present the following lemma.

Lemma A-1. *Suppose z_t is an $n\times 1$ vector and it follows the process $z_t = a_{z,t}z_{t-1} + b_{z,t}\zeta_t$, where $\zeta_t \sim \mathcal{N}(0,\Sigma_{\zeta,t})$ is a $k\times 1$ vector, $a_{z,t}$ is an $n\times n$ matrix, and $b_{z,t}$ is an $n\times k$ matrix. The signal s_t is an $m\times 1$ vector, $s_t = a_{s,t}z_t + b_{s,t}\zeta_t$, where the noise of the signal is perfectly correlated with the noise of the process z_t, $a_{s,t}$ is an $m\times n$ matrix, and $b_{s,t}$ is an $m\times k$ matrix. Write the estimates of the first and second moments as $\hat{z}_t := \mathrm{E}_t[z_t]$, $o_t := \mathrm{E}[(z_t-\hat{z}_t)(z_t-\hat{z}_t)^\top]$, where $\mathrm{E}_t[.] = \mathrm{E}[.\mid \underline{s}_t]$ and $\underline{s}_t := \{s_0, s_1, s_2, ..., s_t\}$. Then the Kalman filtering formulas for $\hat{z}_t$ and o_t are*

$$\hat{z}_t = a_{z,t}\hat{z}_{t-1} + k_t\left[s_t - \mathrm{E}_{t-1}[s_t]\right], \quad o_t = (I_{n\times n} - k_t a_{s,t})(a_{z,t}o_{t-1}a_{z,t}^\top + b_{z,t}\Sigma_{\zeta,t}b_{z,t}^\top) - k_t b_{s,t}\Sigma_{\zeta,t}b_{z,t}^\top,$$

where k_t has been defined below in Equation (A-2).

Proof. Kalman filtering contains two parts: predict and update. The predicted (a priori) state estimate is $\hat{z}_{t|t-1} = \mathrm{E}_{t-1}[z_t] = a_{z,t}\hat{z}_{t-1}$, and the innovation (prefit residual) is

$$s_t - \mathrm{E}_{t-1}[s_t] = a_{s,t}z_t + b_{s,t}\zeta_t - a_{s,t}\hat{z}_{t|t-1} = a_{s,t}a_{z,t}(z_{t-1}-\hat{z}_{t-1}) + (a_{s,t}b_{z,t} + b_{s,t})\zeta_t.$$

The innovation (prefit residual) variance and covariance matrix are

$$\begin{aligned}\mathrm{Var}_{t-1}[s_t] &= a_{s,t}a_{z,t}o_{t-1}a_{z,t}^\top a_{s,t}^\top + (a_{s,t}b_{z,t} + b_{s,t})\Sigma_{\zeta,t}(a_{s,t}b_{z,t} + b_{s,t})^\top,\\ \mathrm{Cov}_{t-1}(z_t, s_t) &= a_{z,t}o_{t-1}a_{z,t}^\top a_{s,t}^\top + b_{z,t}\Sigma_{\zeta,t}(a_{s,t}b_{z,t} + b_{s,t})^\top,\end{aligned} \tag{A-1}$$

and the optimal Kalman gain k_t is

$$k_t = \mathrm{Cov}_{t-1}(z_t, s_t)(\mathrm{Var}_{t-1}[s_t])^{-1}, \tag{A-2}$$

where $\mathrm{Cov}_{t-1}(z_t, s_t)$ and $\mathrm{Var}_{t-1}[s_t]$ are as defined in Equation (A-1). Therefore, the updated (a posteriori) state estimate $\hat{z}_t$ and the updated (a posteriori) estimate covariance matrix o_t are

$$\hat{z}_t = a_{z,t}\hat{z}_{t-1} + k_t\left[s_t - \mathrm{E}_{t-1}[s_t]\right], \quad o_t = \mathrm{E}[(z_t-\hat{z}_t)(z_t-\hat{z}_t)^\top],$$

where $z_t - \hat{z}_t = (I_{n\times n} - k_t a_{s,t}) a_{z,t}(z_{t-1} - \hat{z}_{t-1}) + (b_{z,t} - k_t(a_{s,t} b_{z,t} + b_{s,t}))\zeta_t$. From Equation (A-2), we have $k_t \mathrm{Var}_{t-1}[s_t] k_t^\top = \mathrm{Cov}_{t-1}(z_t, s_t) k_t^\top$. Using Equations (A-1) and (A-2), the updated estimate covariance matrix can be simplified as

$$\begin{aligned} o_t =& (I_{n\times n} - k_t a_{s,t}) a_{z,t} o_{t-1} a_{z,t}^\top (I_{n\times n} - k_t a_{s,t})^\top + (b_{z,t} - k_t(a_{s,t} b_{z,t} + b_{s,t}))\Sigma_{\zeta,t}(b_{z,t} - k_t(a_{s,t} b_{z,t} + b_{s,t}))^\top \\ =& (I_{n\times n} - k_t a_{s,t})(a_{z,t} o_{t-1} a_{z,t}^\top + b_{z,t}\Sigma_{\zeta,t} b_{z,t}^\top) - k_t b_{s,t}\Sigma_{\zeta,t} b_{z,t}^\top. \end{aligned}$$

□

We now prove Lemma 1 using Lemma A-1. Informed traders estimate $\hat{V}_t^I$ using the signals v_t, with X_t being observable. Therefore, for informed traders, $z_t = V$, implying $a_{z,t} = 1, b_{z,t} = 0$. The signals are given by $v_t = V + \varepsilon_t$, which implies $a_{s,t} = b_{s,t} = 1$ and $\zeta_t = \varepsilon_t$. It follows that $\hat{V}_t^I$ are Gaussian Markov processes under $\mathscr{F}_t^I$,

$$\begin{aligned} &\hat{V}_t^I = \hat{V}_{t-1}^I + K_{V,t}^I e_t^I, \quad \text{where} \quad e_t^I = v_t - \hat{V}_{t-1}^I = V - \hat{V}_{t-1}^I + \varepsilon_t, \\ &e_t^I \mid \mathscr{F}_{t-1}^I \sim \mathcal{N}(0, \sigma_t^I), \quad \sigma_t^I = \mathrm{Var}_{t-1}^I(e_t^I) = o_{V,t-1}^I + \tau_{\varepsilon,t}^{-1}, \quad K_{V,t}^I = o_{V,t}^I \tau_{\varepsilon,t}, \end{aligned} \tag{A-3}$$

and the second moment estimation is $o_{V,t}^I = \left(\tau_V + \sum_{s=0}^t \tau_{\varepsilon,s}\right)^{-1}$.

Uninformed traders estimate both $\hat{V}_t^U$ and $\hat{X}_t^U$, the process z_t and the noise $\zeta_t \sim \mathcal{N}(0, \Sigma_{\zeta,t})$ are

$$z_t = \begin{pmatrix} V \\ X_t \end{pmatrix}, \quad \zeta_t = \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix}, \quad \Sigma_{\zeta,t} = \begin{pmatrix} \tau_{\varepsilon,t}^{-1} & 0 \\ 0 & \tau_{\eta,t}^{-1} \end{pmatrix}.$$

This implies that

$$z_t = a_{z,t} z_{t-1} + b_{z,t}\zeta_t, \quad a_{z,t} = I_{2\times 2}, \quad b_{z,t} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Uninformed traders' signal process is $s_t = a_{s,t} z_t + b_{s,t}\zeta_t$, $a_{s,t} = \begin{pmatrix} 1 & 0 \end{pmatrix}$, $b_{s,t} = \begin{pmatrix} 1 & -h_t \end{pmatrix}$. Uninformed traders' state variables $\hat{V}_t^U$ and $\hat{X}_t^U$ are Gaussian Markov processes under $\mathscr{F}_t^U$:

$$\hat{V}_t^U = \hat{V}_{t-1}^U + K_{V,t}^U e_t^U, \quad \hat{X}_t^U = \hat{X}_{t-1}^U + K_{X,t}^U e_t^U, \quad \text{where} \tag{A-4}$$

$$e_t^U = v_t - h_t\eta_t - \hat{V}_{t-1}^U = V - \hat{V}_{t-1}^U - h_t\eta_t + \varepsilon_t, \quad e_t^U \mid \mathscr{F}_{t-1}^U \sim \mathcal{N}(0, \Sigma_t^U),$$
$$\Sigma_t^U = \mathrm{Var}_{t-1}^U(e_t^U) = o_{V,t-1}^U + \tau_{\varepsilon,t}^{-1} + h_t^2\tau_{\eta,t}^{-1}, \quad K_{V,t}^U = o_{V,t-1}^U/\Sigma_t^U, \quad K_{X,t}^U = (o_{VX,t-1}^U - h_t\tau_{\eta,t}^{-1})/\Sigma_t^U, \tag{A-5}$$

and the second moment estimates are

$$o_{X,t}^U = o_{X,t-1}^U + \tau_{\eta,t}^{-1} - (o_{VX,t-1}^U - h_t\tau_{\eta,t}^{-1})^2/\Sigma_t^U, \quad o_{VX,t}^U = o_{VX,t-1}^U - (o_{VX,t-1}^U - h_t\tau_{\eta,t}^{-1})o_{V,t-1}^U/\Sigma_t^U,$$
$$o_{V,t}^U = (\tau_{\varepsilon,t}^{-1} + h_t^2\tau_{\eta,t}^{-1})o_{V,t-1}^U/\Sigma_t^U = \Big(\tau_V + \sum_{s=0}^{t}\big(\tau_{\varepsilon,s}^{-1} + h_s^2\tau_{\eta,s}^{-1}\big)^{-1}\Big)^{-1},$$

where $o_{VX,t}^U := \mathrm{Cov}(V, X_t \mid \mathscr{F}_t^U)$, the priors are $o_{V,-1}^U = \tau_V^{-1}, o_{X,-1}^U = 0$, and $o_{VX,-1}^U = 0$. The coefficients $K_{V,t}^U$ and $K_{X,t}^U$ can be written as

$$K_{V,t}^U = o_{V,t}^U\tau_t, \qquad K_{X,t}^U = o_{V,t}^U\tau_t\Big(\sum_{s=0}^{t-1}(h_s\tau_{\eta,s}^{-1} - h_t\tau_{\eta,t}^{-1})\tau_s - \tau_V h_t\tau_{\eta,t}^{-1}\Big). \tag{A-6}$$

Next, we figure out the coefficient h_t in the composite signal $s_t = v_t - h_t\eta_t$. We write the sufficient statistic of all the composite signals arriving from period 0 to t as $\hat{V}_t^I - \mu_t X_t$, which can be inferred by uninformed traders from prices. It follows that $\hat{V}_t^I - \mu_t X_t = \hat{V}_t^U - \mu_t\hat{X}_t^U$. From Equations (A-3) and (A-4), we have

$$\Big[K_{V,t}^I - (K_{V,t}^U - \mu_t K_{X,t}^U)\Big](v_t - \hat{V}_{t-1}^U) + \Big[(K_{V,t}^U - \mu_t K_{X,t}^U)h_t - \mu_t\Big]\eta_t + \big[(1 - K_{V,t}^I)\mu_{t-1} - \mu_t\big](X_{t-1} - \hat{X}_{t-1}^U) = 0. \tag{A-7}$$

Therefore, the coefficients of $v_t - \hat{V}_{t-1}^U$, η_t, and $X_{t-1} - \hat{X}_{t-1}^U$ need to be zero. Setting the coefficients of η_t and $X_{t-1} - \hat{X}_{t-1}^U$ to be zero yields

$$\mu_t/\mu_{t-1} = \Big(\tau_V + \sum_{s=0}^{t-1}\tau_{\varepsilon,s}\Big)\Big(\tau_V + \sum_{s=0}^{t}\tau_{\varepsilon,s}\Big)^{-1}, \quad \mu_t/h_t = \tau_{\varepsilon,t}\Big(\tau_V + \sum_{s=0}^{t}\tau_{\varepsilon,s}\Big)^{-1}. \tag{A-8}$$

Informed traders' expected utility at $t = T$ is

$$\mathrm{E}_T^I\big[-e^{-A^I W_{T+1}^I}\big] = -e^{-A^I\left[W_T^I + \theta_T^I(\hat{V}_T^I - P_T) + X_T\hat{V}_T^I - \frac{1}{2}A^I o_{V,T}^I(\theta_T^I + X_T)^2\right]},$$

where P_T denotes the trading price at $t = T$, taking the first-order condition with respect to θ_T^I yields

$$\theta_T^I = (\hat{V}_T^I - A^I o_{V,T}^I X_T - P_T)(A^I o_{V,T}^I)^{-1}.$$

It implies that informed traders' trading reveals the composite signal $\hat{V}_T^I - \mu_T X_T$, where

$$\mu_T = A^I o_{V,T}^I = A^I \Big(\tau_V + \sum_{s=0}^{T} \tau_{\varepsilon,s}\Big)^{-1}. \tag{A-9}$$

From Equations (A-8) and (A-9), we have

$$\mu_t = A^I o_{V,t}^I = A^I \Big(\tau_V + \sum_{s=0}^{t} \tau_{\varepsilon,s}\Big)^{-1}, \quad h_t = A^I / \tau_{\varepsilon,t}. \tag{A-10}$$

We then verify that the coefficient of $v_t - \hat{V}_{t-1}^U$ in Equation (A-7) is indeed zero.

A.2 Proof of Proposition 1

We first solve for prices and optimal trading quantities for $t = T$. The expected utility functions for informed and uninformed traders at $t = T$ are

$$\mathrm{E}_T^I \left[-e^{-A^I W_{T+1}^{I,i}} \right] = -e^{-A^I \left[W_T^I + \theta_T^I (\hat{V}_T^I - P_T) + X_T (\hat{V}_T^I - \bar{V}) - \frac{1}{2} A^I o_{V,T}^I (\theta_T^I + X_T)^2 \right]}, \tag{A-11}$$

$$\mathrm{E}_T^U \left[-e^{-A^U W_{T+1}^{U,j}} \right] = -e^{-A^U \left[W_T^U + \theta_T^U (\hat{V}_T^U - P_T) - \frac{1}{2} A^U o_{V,T}^U (\theta_T^U)^2 \right]}. \tag{A-12}$$

The first order condition with respect to θ_T^I and θ_T^U yields

$$\theta_T^I = \frac{P_T^I - P_T}{\gamma_T^I}, \qquad \theta_T^U = \frac{P_T^U - P_T}{\gamma_T^U}, \tag{A-13}$$

where the non-participating prices for informed and uninformed traders are

$$P_T^I = \hat{V}_T^I - \mu_T X_T, \quad P_T^U = \hat{V}_T^U, \tag{A-14}$$

and parameters μ_T, γ_T^I, and γ_T^U are given as

$$\mu_T = A^I o_{V,T}^I, \quad \gamma_T^I = A^I o_{V,T}^I, \quad \gamma_T^U = A^U o_{V,T}^U. \tag{A-15}$$

It follows that the equilibrium price at T is

$$P_T = \omega_T P_T^I + (1 - \omega_T) P_T^U - f_T \Theta, \tag{A-16}$$

where coefficients f_T and ω_T are given as

$$f_T = \left(\frac{\alpha}{\gamma_T^I} + \frac{1-\alpha}{\gamma_T^U}\right)^{-1}, \quad \omega_T = \frac{\alpha}{\gamma_T^I} f_T. \tag{A-17}$$

The equilibrium holdings are

$$\theta_T^I = -(1-\omega_T)\hat{X}_T^U + \frac{f_T}{\gamma_T^I}\Theta = (H_{\theta,T}^I)^\top \Phi_T^I, \quad \theta_T^U = \frac{\omega_T \mu_T}{\gamma_T^U}\hat{X}_T^U + \frac{f_T}{\gamma_T^U}\Theta = (H_{\theta,T}^U)^\top \Phi_T^U,$$

where $H_{\theta,T}^I$ and $H_{\theta,T}^U$ are given as

$$H_{\theta,T}^I = \left(0, 0, -(1-\omega_T), f_T/\gamma_T^I, 0\right)^T, \quad H_{\theta,T}^U = \left(0, \omega_T\mu_T/\gamma_T^U, f_T/\gamma_T^U, 0\right)^T. \tag{A-18}$$

Traders' value functions at T can be rewritten as

$$J_T^I = -\rho_T^I e^{-A^I[W_T^I + \frac{1}{2}(\Phi_T^I)^\top M_T^I \Phi_T^I]}, \quad J_T^U = -\rho_T^U e^{-A^U[W_T^U + \frac{1}{2}(\Phi_T^U)^\top M_T^U \Phi_T^U]}, \tag{A-19}$$

where $\rho_T^I = 1, \rho_T^U = 1$, Φ_T^I and Φ_T^U are defined as

$$\Phi_T^I := \left(\hat{V}_T^I, X_T, \hat{X}_T^U, \Theta, \bar{V}\right)^T, \quad \Phi_T^U := \left(\hat{V}_T^U, \hat{X}_T^U, \Theta, \bar{V}\right)^T, \tag{A-20}$$

and

$$M_T^I = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & -\gamma_T^I & 0 & 0 & -1 \\ 0 & 0 & \gamma_T^I(1-\omega_T)^2 & -(1-\omega_T)f_T & 0 \\ 0 & 0 & -(1-\omega_T)f_T & f_T^2/\gamma_T^I & 0 \\ 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \quad M_T^U = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{(\omega_T\mu_T)^2}{\gamma_T^U} & \frac{\omega_T\mu_T f_T}{\gamma_T^U} & 0 \\ 0 & \frac{\omega_T\mu_T f_T}{\gamma_T^U} & \frac{f_T^2}{\gamma_T^U} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The equilibrium price can be expressed as

$$P_T = (H_{P,T}^I)^\top \Phi_T^I = (H_{P,T}^U)^\top \Phi_T^U, \tag{A-21}$$

where $H_{P,T}^I$ and $H_{P,T}^U$ are given as

$$H_{P,T}^I = \left(1, -\mu_T, (1-\omega_T)\mu_T, -f_T, 0\right)^T, \quad H_{P,T}^U = \left(1, -\omega_T\mu_T, -f_T, 0\right)^T. \tag{A-22}$$

Next, we conjecture that traders' value functions at t are given as (A-19) where the subscript

T is replaced by t. Traders' state vectors Φ_t^I and Φ_t^U are defined as (A-20) where the subscript T is replaced by t.

Traders' holdings are

$$\theta_t^I = \frac{P_t^I - P_t}{\gamma_t^I}, \quad \theta_t^U = \frac{P_t^U - P_t}{\gamma_t^U}, \tag{A-23}$$

where P_t^I and P_t^U are informed and uninformed traders' non-participation prices. We also conjecture that traders' non-participation prices at t are

$$\begin{aligned} P_t^I &= \delta_t^I(\hat{V}_t^I - \mu_t X_t) + (1-\delta_t^I)\bar{V} + g_t^I \hat{X}_t^U - f_t^I \Theta, \\ P_t^U &= \delta_t^U \hat{V}_t^U + (1-\delta_t^U)\bar{V} - g_t^U \hat{X}_t^U - f_t^U \Theta, \end{aligned} \tag{A-24}$$

equation (A-60) implies that the coefficients at T, $\delta_T^I, \delta_T^U, g_T^I, g_T^U, f_T^I$, and f_T^U are

$$\delta_T^I = \delta_T^U = 1, \quad g_T^I = g_T^U = 0, \quad f_T^I = f_T^U = 0, \tag{A-25}$$

The equilibrium price can be written as

$$\begin{aligned} P_t &= \omega_t P_t^I + (1-\omega_t) P_t^U - f_t \Theta, \\ &= (H_{P,t}^I)^\top \Phi_t^I = (H_{P,t}^U)^\top \Phi_t^U, \end{aligned} \tag{A-26}$$

where

$$f_t = \left(\frac{\alpha}{\gamma_t^I} + \frac{1-\alpha}{\gamma_t^U} \right)^{-1}, \quad \omega_t = \frac{\alpha}{\gamma_t^I} f_t, \quad 1-\omega_t = \frac{1-\alpha}{\gamma_t^U} f_t, \tag{A-27}$$

and

$$H_{P,t}^I = \begin{pmatrix} \omega_t \delta_t^I + (1-\omega_t)\delta_t^U \\ -\mu_t \left[\omega_t \delta_t^I + (1-\omega_t)\delta_t^U\right] \\ \omega_t g_t^I + (1-\omega_t)(\delta_t^U \mu_t - g_t^U) \\ -(\omega_t f_t^I + (1-\omega_t) f_t^U + f_t) \\ 1 - \left[\omega_t \delta_t^I + (1-\omega_t)\delta_t^U\right] \end{pmatrix}, \quad H_{P,t}^U = \begin{pmatrix} \omega_t \delta_t^I + (1-\omega_t)\delta_t^U \\ \omega_t (g_t^I - \delta_t^I \mu_t) - (1-\omega_t) g_t^U \\ -(\omega_t f_t^I + (1-\omega_t) f_t^U + f_t) \\ 1 - \left[\omega_t \delta_t^I + (1-\omega_t)\delta_t^U\right] \end{pmatrix}. \tag{A-28}$$

The equilibrium holdings can be written as

$$\theta_t^I = (H_{\theta,t}^I)^\top \Phi_t^I, \quad \theta_t^U = (H_{\theta,t}^U)^\top \Phi_t^U, \tag{A-29}$$

where traders' state vectors Φ_t^I and Φ_t^U are defined as (A-20) where the subscript T is replaced by t. The coefficient vectors $H_{\theta,t}^I$ and $H_{\theta,t}^U$ are given as

$$H_{\theta,t}^I := \begin{pmatrix} \frac{1-\omega_t}{\gamma_t^I}(\delta_t^I - \delta_t^U) \\ -\frac{(1-\omega_t)\mu_t}{\gamma_t^I}(\delta_t^I - \delta_t^U) \\ \frac{1-\omega_t}{\gamma_t^I}(g_t^I + g_t^U - \delta_t^U \mu_t) \\ -\frac{1-\omega_t}{\gamma_t^I}(f_t^I - f_t^U) + \frac{f_t}{\gamma_t^I} \\ -\frac{1-\omega_t}{\gamma_t^I}(\delta_t^I - \delta_t^U) \end{pmatrix}, \quad H_{\theta,t}^U := \begin{pmatrix} -\frac{\omega_t}{\gamma_t^U}(\delta_t^I - \delta_t^U) \\ -\frac{\omega_t}{\gamma_t^U}(g_t^U + g_t^I - \delta_t^I \mu_t) \\ \frac{\omega_t}{\gamma_t^U}(f_t^I - f_t^U) + \frac{f_t}{\gamma_t^U} \\ \frac{\omega_t}{\gamma_t^U}(\delta_t^I - \delta_t^U) \end{pmatrix}. \tag{A-30}$$

Next, we solve the model recursively from time t to $t-1$ and verify our conjecture. We first solve the uninformed traders' optimization problem at $t-1$. Since

$$\Phi_t^U = \Phi_{t-1}^U + F_t^U e_t^U, \tag{A-31}$$

where $e_t^U = v_t - h_t\eta_t - \hat{V}_{t-1}^U$, $F_t^U = \left(K_{V,t}^U, K_{X,t}^U, 0, 0\right)^T$, $K_{V,t}^U$, and $K_{X,t}^U$ are as defined in equation (A-6).

Uninformed traders' value function (A-19) at t can be rewritten as

$$\begin{aligned} J_t^U &= -\rho_t^U e^{-A^U[W_{t-1}^U - \theta_{t-1}^U P_{t-1} + \theta_{t-1}^U P_t + \frac{1}{2}(\Phi_t^U)^\top M_t^U \Phi_t^U]}, \\ &= -\rho_t^U e^{-A^U\left[W_{t-1}^U - \theta_{t-1}^U P_{t-1} + \theta_{t-1}^U (H_{P,t}^U)^\top \Phi_{t-1}^U + \frac{1}{2}(\Phi_{t-1}^U)^\top M_t^U \Phi_{t-1}^U\right]} \\ &\quad \times e^{-A^U\left[\left(\theta_{t-1}^U (H_{P,t}^U)^\top F_t^U + (\Phi_{t-1}^U)^\top M_t^U F_t^U\right) e_t^U + \frac{1}{2} e_t^U (F_t^U)^\top M_t^U F_t^U e_t^U\right]}. \end{aligned} \tag{A-32}$$

The expectation at $t-1$ is

$$\begin{aligned} \mathrm{E}_{t-1}^U[J_t^U] &= -\rho_t^U e^{-A^U\left[W_{t-1}^U - \theta_{t-1}^U P_{t-1} + \theta_{t-1}^U (H_{P,t}^U)^\top \Phi_{t-1}^U + \frac{1}{2}(\Phi_{t-1}^U)^\top M_t^U \Phi_{t-1}^U\right]} \\ &\quad \times \sqrt{\frac{\Xi_t^U}{\Sigma_t^U}} e^{-A^U\left[-\frac{1}{2}A^U \Xi_t^U \left(\theta_{t-1}^U (H_{P,t}^U)^\top F_t^U + (\Phi_{t-1}^U)^\top M_t^U F_t^U\right)^2\right]}, \end{aligned} \tag{A-33}$$

where

$$\Xi_t^U := \left[(\Sigma_t^U)^{-1} + A^U (F_t^U)^\top M_t^U F_t^U\right]^{-1}. \tag{A-34}$$

The first order condition with respect to θ^U_{t-1} yields

$$-P_{t-1} + (H^U_{P,t})^\top \Phi^U_{t-1} = A^U \Xi^U_t (H^U_{P,t})^\top F^U_t (F^U_t)^\top \left[H^U_{P,t}\theta^U_{t-1} + M^U_t \Phi^U_{t-1} \right], \tag{A-35}$$

and the second order condition requires

$$A^U \Xi^U_t \left[(H^U_{P,t})^\top F^U_t \right]^2 > 0, \tag{A-36}$$

which is satisfied. Equation (A-35) implies that the non-participation price for uninformed traders is

$$P^U_{t-1} = \left[(H^U_{P,t})^\top - A^U \Xi^U_t (H^U_{P,t})^\top F^U_t (F^U_t)^\top M^U_t \right] \Phi^U_{t-1}. \tag{A-37}$$

Equation (A-37) implies that parameter γ^U_{t-1}, and parameters $\delta^U_{t-1}, g^U_{t-1}, f^U_{t-1}$ in

$$P^U_{t-1} = \delta^U_{t-1} \hat{V}^U_{t-1} + (1-\delta^U_{t-1})\bar{V} - g^U_{t-1}\hat{X}^U_{t-1} - f^U_{t-1}\Theta,$$

can be written as

$$\begin{aligned}
\gamma^U_{t-1} &= A^U \Xi^U_t \left[(H^U_{P,t})^\top F^U_t \right]^2, \\
\delta^U_{t-1} &= \left[(H^U_{P,t})^\top - A^U \Xi^U_t (H^U_{P,t})^\top F^U_t (F^U_t)^\top M^U_t \right]_{1,1}, \\
g^U_{t-1} &= -\left[(H^U_{P,t})^\top - A^U \Xi^U_t (H^U_{P,t})^\top F^U_t (F^U_t)^\top M^U_t \right]_{1,2}, \\
f^U_{t-1} &= -\left[(H^U_{P,t})^\top - A^U \Xi^U_t (H^U_{P,t})^\top F^U_t (F^U_t)^\top M^U_t \right]_{1,3},
\end{aligned} \tag{A-38}$$

and it can be verified that

$$1-\delta^U_{t-1} = \left[(H^U_{P,t})^\top - A^U \Xi^U_t (H^U_{P,t})^\top F^U_t (F^U_t)^\top M^U_t \right]_{1,4}.$$

Since $\hat{X}^U_{t-1} = \frac{1}{\mu_{t-1}}\left(\hat{V}^U_{t-1} - (\hat{V}^I_{t-1} - \mu_{t-1}X_{t-1}) \right)$, the non-participation price for uninformed traders can thus be rewritten as in Equation (13) in Proposition 1. The coefficients ξ^U_{t-1} and ψ^U_{t-1} in P^U_{t-1} are

$$\xi^U_{t-1} := g^U_{t-1}/\mu_{t-1}, \quad \psi^U_{t-1} := \delta^U_{t-1} - g^U_{t-1}/\mu_{t-1}. \tag{A-39}$$

Next, we solve for the optimization problem for informed traders at $t-1$. Since

$$\Phi_t^I = H_t^I \Phi_{t-1}^I + F_t^I u_t^I, \tag{A-40}$$

where

$$H_t^I = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & K_{X,t}^U \mu_{t-1} & 1-K_{X,t}^U \mu_{t-1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad F_t^I = \begin{pmatrix} K_{V,t}^I & 0 \\ 0 & 1 \\ K_{X,t}^U & -K_{X,t}^U h_t \\ 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{A-41}$$

Informed traders' value function at t can be rewritten as

$$\begin{aligned} J_t^I &= -\rho_t^I e^{-A^I [W_{t-1}^I - \theta_{t-1}^I P_{t-1} + \theta_{t-1}^I P_t + \frac{1}{2}(\Phi_t^I)^\top M_t^I \Phi_t^I]} \\ &= -\rho_t^I e^{-A^I \left[W_{t-1}^I - \theta_{t-1}^I P_{t-1} + \theta_{t-1}^I (H_{P,t}^I)^\top H_t^I \Phi_{t-1}^I + \frac{1}{2}(H_t^I \Phi_{t-1}^I)^\top M_t^I H_t^I \Phi_{t-1}^I\right]} \\ &\quad \times e^{-A^I \left[\left(\theta_{t-1}^I (H_{P,t}^I)^\top F_t^I + (H_t^I \Phi_{t-1}^I)^\top M_t^I F_t^I\right) u_t^I + \frac{1}{2} u_t^I (F_t^I)^\top M_t^I F_t^I u_t^I\right]}. \end{aligned} \tag{A-42}$$

The expectation at $t-1$ is

$$\begin{aligned} \mathrm{E}_{t-1}^I[J_t^I] &= -\rho_t^I e^{-A^I \left[W_{t-1}^I - \theta_{t-1}^I P_{t-1} + \theta_{t-1}^I (H_{P,t}^I)^\top H_t^I \Phi_{t-1}^I + \frac{1}{2}(H_t^I \Phi_{t-1}^I)^\top M_t^I H_t^I \Phi_{t-1}^I\right]} \\ &\quad \times \sqrt{\frac{|\Xi_t^I|}{|\Sigma_t^I|}} e^{-A^I \left[-\frac{1}{2} A^I \left(\theta_{t-1}^I (H_{P,t}^I)^\top F_t^I + (H_t^I \Phi_{t-1}^I)^\top M_t^I F_t^I\right) \Xi_t^I \left(\theta_{t-1}^I (H_{P,t}^I)^\top F_t^I + (H_t^I \Phi_{t-1}^I)^\top M_t^I F_t^I\right)^\top\right]}, \end{aligned}$$

where $\Xi_t^I = \left[(\Sigma_t^I)^{-1} + A^I (F_t^I)^\top M_t^I F_t^I\right]^{-1}$.

The first order condition with respect to θ_{t-1}^I yields

$$-P_{t-1} + (H_{P,t}^I)^\top H_t^I \Phi_{t-1}^I = A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top \left[H_{P,t}^I \theta_{t-1}^I + M_t^I H_t^I \Phi_{t-1}^I\right], \tag{A-43}$$

and the second order condition requires

$$A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top H_{P,t}^I > 0, \tag{A-44}$$

which is satisfied under certain conditions. Equation (A-43) implies that the non-participation price for informed traders is

$$P_{t-1}^I = \left((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I\right) H_t^I \Phi_{t-1}^I. \tag{A-45}$$

Equation (A-45) implies that parameter γ_{t-1}^I, and parameters $\delta_{t-1}^I, g_{t-1}^I, f_{t-1}^I$ in

$$P_{t-1}^I = \delta_{t-1}^I(\hat{V}_{t-1}^I - \mu_{t-1}X_{t-1}) + (1-\delta_{t-1}^I)\bar{V} + g_{t-1}^I \hat{X}_{t-1}^U - f_{t-1}^I \Theta,$$

are given as

$$\begin{aligned}
\gamma_{t-1}^I &= A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top H_{P,t}^I, \\
\delta_{t-1}^I &= \left[\left((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I\right) H_t^I\right]_{1,1}, \\
g_{t-1}^I &= \left[\left((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I\right) H_t^I\right]_{1,3}, \\
f_{t-1}^I &= -\left[\left((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I\right) H_t^I\right]_{1,4},
\end{aligned} \tag{A-46}$$

and it can be verified that

$$\begin{aligned}
1-\delta_{t-1}^I &= \left[\left((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I\right) H_t^I\right]_{1,5}, \\
-\mu_{t-1}\delta_{t-1}^I &= \left[\left((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I\right) H_t^I\right]_{1,2}.
\end{aligned}$$

Since $\hat{X}_{t-1}^U = \frac{1}{\mu_{t-1}}\left(\hat{V}_{t-1}^U - (\hat{V}_{t-1}^I - \mu_{t-1}X_{t-1})\right)$, the non-participation price for informed traders can thus be rewritten as in Equation (13) in Proposition 1. The coefficients ξ_{t-1}^I and ψ_{t-1}^I in P_{t-1}^I are

$$\xi_{t-1}^I := \delta_{t-1}^I - g_{t-1}^I/\mu_{t-1}, \quad \psi_{t-1}^I := g_{t-1}^I/\mu_{t-1}. \tag{A-47}$$

Define $H_{P,t-1}^I, H_{P,t-1}^U, H_{\theta,t-1}^I$, and $H_{\theta,t-1}^U$ as in equations (A-28) and (A-30), replacing t by $t-1$, we can rewrite the equilibrium price and holdings as

$$P_{t-1} = (H_{P,t-1}^I)^\top \Phi_{t-1}^I = (H_{P,t-1}^U)^\top \Phi_{t-1}^U, \tag{A-48}$$

$$\theta_{t-1}^I = (H_{\theta,t-1}^I)^\top \Phi_{t-1}^I, \quad \theta_{t-1}^U = (H_{\theta,t-1}^U)^\top \Phi_{t-1}^U. \tag{A-49}$$

Therefore, the value functions of traders at $t-1$ can be expressed as

$$J_{t-1}^I = -\rho_{t-1}^I e^{-A^I[W_{t-1}^I + \frac{1}{2}(\Phi_{t-1}^I)^\top M_{t-1}^I \Phi_{t-1}^I]}, \quad J_{t-1}^U = -\rho_{t-1}^U e^{-A^U[W_{t-1}^U + \frac{1}{2}(\Phi_{t-1}^U)^\top M_{t-1}^U \Phi_{t-1}^U]},$$

where

$$
\begin{aligned}
M^I_{t-1} =&(H^I_t)^\top M^U_t H^I_t + H^I_{\theta,t-1}\left((H^I_t)^\top H^I_{P,t} - H^I_{P,t-1}\right)^\top + \left((H^I_t)^\top H^U_{P,t} - H^I_{P,t-1}\right)(H^I_{\theta,t-1})^\top \\
&- A^I\left[H^I_{\theta,t-1}(H^I_{P,t})^\top + (H^I_t)^\top M^I_t\right] F^I_t \Xi^I_t (F^I_t)^\top \left[H^I_{\theta,t-1}(H^I_{P,t})^\top + (H^I_t)^\top M^I_t\right]^\top,
\end{aligned}
\tag{A-50}
$$

$$
\begin{aligned}
M^U_{t-1} =&M^U_t + H^U_{\theta,t-1}\left(H^U_{P,t} - H^U_{P,t-1}\right)^\top + \left(H^U_{P,t} - H^U_{P,t-1}\right)(H^U_{\theta,t-1})^\top \\
&- A^U\left[H^U_{\theta,t-1}(H^U_{P,t})^\top + M^U_t\right] F^U_t \Xi^U_t (F^U_t)^\top \left[H^U_{\theta,t-1}(H^U_{P,t})^\top + M^U_t\right]^\top,
\end{aligned}
\tag{A-51}
$$

and

$$
\rho^I_{t-1} = \rho^I_t \sqrt{\frac{|\Xi^I_t|}{|\Sigma^I_t|}}, \quad \rho^U_{t-1} = \rho^U_t \sqrt{\frac{|\Xi^U_t|}{|\Sigma^U_t|}}. \tag{A-52}
$$

The equilibrium price can be written as

$$
P_t = \delta_t \hat{V}^U_t + (1-\delta_t)\bar{V} - g_t \hat{X}^U_t - \zeta_t \Theta, \tag{A-53}
$$

where the coefficients δ_t, g_t, and ξ_t are given as

$$
\delta_t = \omega_t \delta^I_t + (1-\omega_t)\delta^U_t, \quad g_t = -\omega_t(g^I_t - \delta^I_t \mu_t) + (1-\omega_t) g^U_t, \quad \zeta_t = \omega_t f^I_t + (1-\omega_t) f^U_t + f_t. \tag{A-54}
$$

Thus, the informed traders' holding can be expressed as

$$
N_I \theta^I_t = d_t(\hat{V}^U_t - \bar{V}) + k_t \hat{X}^U_t - l_t \Theta, \tag{A-55}
$$

where the coefficients d_t, k_t, and l_t are given as

$$
d_t := \frac{\omega_t}{\gamma^U_t / N_U}(\delta^I_t - \delta^U_t), \quad k_t := \frac{\omega_t}{\gamma^U_t / N_U}(g^I_t + g^U_t - \delta^I_t \mu_t), \quad l_t := \frac{\omega_t}{\gamma^U_t / N_U}(f^I_t - f^U_t) - \omega_t. \tag{A-56}
$$

A.3 Proof of Proposition 2

For $t \geq 2$, the return covariance has two components,

$$
\mathrm{Cov}(R_t, R_{t-1}) = \mathrm{Cov}(\hat{V}_t - \hat{V}_{t-1}, \hat{V}_{t-1} - \hat{V}_{t-2}) + \omega^2 \mathrm{Cov}\left((\mu_{-1} - \mu_t)x_0, (\mu_{t-2} - \mu_{t-1})x_0\right).
$$

The first component is driven by changes in the agent's belief about the fundamental value,

$$
\mathrm{Cov}(\hat{V}_t - \hat{V}_{t-1}, \hat{V}_{t-1} - \hat{V}_{t-2}) = 0, \tag{A-57}
$$

which is always zero because the agent's expectation about the fundamental can be iterated $\mathrm{E}_{t-1}[\hat{V}_t] = \hat{V}_{t-1}$. The second component is driven by the reduction in the market's required risk premium due to the arrival of public information, for $t \geq 2$,

$$\mathrm{Cov}\big((\mu_{t-1}-\mu_t)x_0, (\mu_{t-2}-\mu_{t-1})x_0\big) = (\mu_{t-1}-\mu_t)(\mu_{t-2}-\mu_{t-1})\tau_{x,0}^{-1}. \tag{A-58}$$

With the arrival of public information at both time $t-1$ and t, we have $\mu_{t-2} > \mu_{t-1} > \mu_t$. For $t = 1$, $\mathrm{Cov}(R_1, R_0) = \omega^2(\mu_1 - \mu_0)\mu_0 \mathrm{Var}(x_0) < 0$.

A.4 Proof of Proposition 3

As in the proof of Proposition 1 in Section A.2, we first solve for prices and optimal trading quantities for $t = T$. The expressions of prices and optimal holdings are the same as in the general case in Proposition 1.

$$\theta_T^I = \frac{P_T^I - P_T}{\gamma_T^I}, \qquad \theta_T^U = \frac{P_T^U - P_T}{\gamma_T^U}, \tag{A-59}$$

where the non-participating prices for informed and uninformed traders are

$$P_T^I = \hat{V}_T^I - \mu_T X_T, \quad P_T^U = \hat{V}_T^U, \tag{A-60}$$

and parameters μ_T, γ_T^I, and γ_T^U are given as

$$\mu_T = A^I o_{V,T}^I, \quad \gamma_T^I = A^I o_{V,T}^I, \quad \gamma_T^U = A^U o_{V,T}^U. \tag{A-61}$$

It follows that the equilibrium price at T is

$$P_T = \omega_T P_T^I + (1-\omega_T) P_T^U - \left(\frac{\alpha}{\gamma_T^I} + \frac{1-\alpha}{\gamma_T^U}\right)^{-1} \Theta, \tag{A-62}$$

where ω_T is given as

$$\omega_T = \frac{\frac{\alpha}{\gamma_T^I}}{\frac{\alpha}{\gamma_T^I} + \frac{1-\alpha}{\gamma_T^U}}. \tag{A-63}$$

The equilibrium holdings are

$$\theta_T^I = -(1-\omega_T)\hat{X}_T^U + \frac{\omega_T}{\alpha}\Theta, \quad \theta_T^U = \frac{\alpha}{1-\alpha}(1-\omega_T)\Big(\hat{X}_T^U + \frac{\Theta}{\alpha}\Big).$$

Since $\hat{V}_T^I - \mu_T X = \hat{V}_T^U - \mu_T \hat{X}_T^U$, the price P_T can be rewritten as

$$P_T = \hat{V}_T^U - \omega_T \mu_T (\hat{X}_T^U + \frac{\Theta}{\alpha}). \tag{A-64}$$

Plug the equilibrium price into the expected utility functions, we have informed traders' value function at $t = T$ as a function of state variable Φ_T

$$J_T^{I,i} = -e^{-A^I\left[W_T^{I,i} + \hat{V}_T^I X + \frac{1}{2}\mu_T(\theta_T^{I,i})^2 - \bar{V}X - \frac{1}{2}\mu_T X^2\right]}, \tag{A-65}$$

which can be rewritten as

$$J_T^{I,i} = -\rho_T^I e^{-A^I\left[W_T^{I,i} + \hat{V}_T^I X + \frac{1}{2}m_T^I\phi_T^2 + \frac{1}{2}\mu_T(1-\omega_T)(\hat{X}_T^U)^2\right]}, \tag{A-66}$$

where

$$m_T^I = -\mu_T\omega_T(1-\omega_T) = -\mu_T\omega_T\frac{f_T}{N_I}, \tag{A-67}$$

$$\rho_T^I = e^{A^I\left[\bar{V}X + \frac{1}{2}\mu_T X^2 - \frac{1}{2}\mu_T\omega_T(\frac{\Theta}{N_I})^2\right]}. \tag{A-68}$$

Similarly, uninformed traders' value function at $t = T$ can be written as:

$$J_T^{U,j} = -e^{-A^U\left[W_T^{U,j} + \frac{1}{2}\gamma_T^U(\theta_T^{U,j})^2\right]}, \tag{A-69}$$

which can be rewritten as

$$J_T^{U,j} = -\rho_T^U e^{-A^U\left[W_T^{U,j} + \frac{1}{2}m_T^U\phi_T^2\right]}, \tag{A-70}$$

where

$$m_T^U = \frac{N_I}{N_U}\mu_T\omega_T(1-\omega_T) = \mu_T\omega_T\frac{f_T}{N_U}, \tag{A-71}$$

$$\rho_T^U = 1. \tag{A-72}$$

Note that

$$N_I m_T^I + N_U m_T^U = 0. \tag{A-73}$$

We need to prove whether these forms of value functions are valid for time $t-1$ if they are held for time t. For uninformed traders,

$$\hat{V}_t^U = \hat{V}_{t-1}^U + K_{V,t}^U e_t^U, \tag{A-74}$$

$$\phi_t = \phi_{t-1} + K_{X,t}^U e_t^U, \tag{A-75}$$

$$P_t - P_{t-1} = a_t \phi_{t-1} + b_t e_t^U, \tag{A-76}$$

where

$$a_t = \omega_{t-1}\mu_{t-1} - \omega_t \mu_t, \tag{A-77}$$

$$b_t = K_{V,t}^U - \omega_t \mu_t K_{X,t}^U, \tag{A-78}$$

and

$$J_t^{U,j} = -\rho_t^U e^{-A^U\left[W_{t-1}^{U,j} + \theta_{t-1}^{U,j}(P_t - P_{t-1}) + \frac{1}{2} m_t^U \phi_t^2\right]} \tag{A-79}$$

the expected value function can be rewritten as

$$\mathrm{E}_{t-1}[J_t^{U,j}] = -\rho_t^U \sqrt{\Xi_t^U / \Sigma_t^U}\, e^{-A^U\left[W_{t-1}^{U,j} + a_t\phi_{t-1}\theta_{t-1}^{U,j} + \frac{1}{2} m_t^U \phi_{t-1}^2 - \frac{1}{2} A^U \Xi_t^U \left(m_t^U K_{X,t}^U \phi_{t-1} + b_t \theta_{t-1}^{U,j}\right)^2\right]}, \tag{A-80}$$

where

$$\Sigma_t^U = o_{V,t-1}^U + \tau_{\varepsilon,t}^{-1}, \tag{A-81}$$

$$\Xi_t^U = \left(\left(\Sigma_t^U\right)^{-1} + A^U m_t^U \left(K_{X,t}^U\right)^2\right)^{-1}. \tag{A-82}$$

The first order condition with respect to $\theta_{t-1}^{U,j}$ gives

$$\theta_{t-1}^{U,j} = \frac{f_{t-1}}{N_U}\phi_{t-1}, \tag{A-83}$$

where

$$f_{t-1} = N_U \left(\frac{a_t}{A^U \Xi_t^U b_t^2} - \frac{m_t^U K_{X,t}^U}{b_t} \right). \tag{A-84}$$

The value function at time $t-1$ can be written as

$$J_{t-1}^{U,j} = -\rho_{t-1}^U e^{-A^U \left[W_{t-1}^{U,j} + \frac{1}{2} m_{t-1}^U \phi_{t-1}^2 \right]}, \tag{A-85}$$

where

$$\rho_{t-1}^U = \rho_t^U \sqrt{\Xi_t^U / \Sigma_t^U}, \tag{A-86}$$

$$m_{t-1}^U = m_t^U \left(1 - \frac{a_t K_{X,t}^U}{b_t} \right) + \frac{f_{t-1}}{N_U} a_t. \tag{A-87}$$

The second order condition requires that

$$A^U \Xi_t^U b_t > 0. \tag{A-88}$$

For informed traders, since

$$\hat{V}_t^I - \mu_t X = \hat{V}_t^U - \mu_t \hat{X}_t^U, \tag{A-89}$$

and

$$\hat{V}^I = \hat{V}_{t-1}^I + K_t^I e_t^I, \tag{A-90}$$

$$\phi_t = \phi_{t-1} + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U) + K_{X,t}^U e_t^I, \tag{A-91}$$

thus,

$$P_t - P_{t-1} = a_t \phi_{t-1} + b_t \mu_{t-1} (X - \hat{X}_{t-1}^U) + b_t e_t^I. \tag{A-92}$$

Since

$$J_t^{I,i} = -\rho_t^I e^{-A^I \left[W_{t-1}^{I,i} + \theta_{t-1}^{I,i} (P_t - P_{t-1}) + \hat{V}_t^I X + \frac{1}{2} m_t^I \phi_t^2 + \frac{1}{2} \mu_t (1-\omega_t) (\hat{X}_t^U)^2 \right]} \tag{A-93}$$

the expected value function can be rewritten as

$$\mathrm{E}_{t-1}^{I}[J_t^{I,i}] = -\rho_t^I \sqrt{\Xi_t^I/\Sigma_t^I} e^{-A^I\left[W_{t-1}^{I,i}+\hat{V}_{t-1}^I X+\theta_{t-1}^{I,i}\left(a_t\phi_{t-1}+b_t\mu_{t-1}(X-\hat{X}_{t-1}^U)\right)\right]}$$
$$\times e^{-A^I\left[\frac{1}{2}m_t^I\left(\phi_{t-1}+K_{X,t}^U\mu_{t-1}(X-\hat{X}_{t-1}^U)\right)^2+\frac{1}{2}\mu_t(1-\omega_t)\left(\hat{X}_{t-1}^U+K_{X,t}^U\mu_{t-1}(X-\hat{X}_{t-1}^U)\right)^2\right]}$$
$$\times e^{-A^I\left[-\frac{1}{2}A^I\Xi_t^I\left[\theta_{t-1}^{I,i}b_t+K_t^I X+m_t^I K_{X,t}^U\left(\phi_{t-1}+K_{X,t}^U\mu_{t-1}(X-\hat{X}_{t-1}^U)\right)+\mu_t(1-\omega_t)\left(\hat{X}_{t-1}^U+K_{X,t}^U\mu_{t-1}(X-\hat{X}_{t-1}^U)\right)\right]^2\right]}, \quad \text{(A-94)}$$

where

$$\Sigma_t^I = o_{V,t-1}^I + \tau_{\varepsilon,t}^{-1}, \quad \text{(A-95)}$$

$$\Xi_t^I = \left(\left(\Sigma_t^I\right)^{-1} + A^I\left(K_{X,t}^U\right)^2\left(m_t^I+\mu_t(1-\omega_t)\right)\right)^{-1}. \quad \text{(A-96)}$$

The first order condition with respect to $\theta_{t-1}^{I,i}$ gives

$$\theta_{t-1}^{I,i} = \left(\frac{a_t}{A^I\Xi_t^I b_t^2} - \frac{m_t^I K_{X,t}^U}{b_t}\right)\phi_{t-1} - \hat{X}_{t-1}^U = \frac{\Theta}{N_I} - \frac{f_{t-1}}{N_I}\phi_{t-1}, \quad \text{(A-97)}$$

which is due to the fact that

$$\mu_{t-1} = \frac{A^I\Xi_t^I K_t^I}{1-A^I\Xi^I(K_{X,t}^U)^2(m_t^I+\mu_t(1-\omega_t))} = A^I\Sigma_t^I K_t^I = A^I o_{V,t-1}^I, \quad \text{(A-98)}$$

and

$$K_t^I = K_{V,t}^U - \mu_t K_{X,t}^U, \quad \text{(A-99)}$$

$$K_t^I \mu_{t-1} = \mu_{t-1} - \mu_t. \quad \text{(A-100)}$$

The second order condition requires that

$$A^I\Xi_t^I b_t > 0. \quad \text{(A-101)}$$

In equilibrium, the market clearing condition gives

$$1-\frac{f_{t-1}}{N_I} = 1-\frac{N_U}{N_I}\left(\frac{a_t}{A^U\Xi_t^U b_t^2} - \frac{m_t^U K_{X,t}^U}{b_t}\right) = \frac{a_t}{A^I\Xi_t^I b_t^2} - \frac{m_t^I K_{X,t}^U}{b_t}, \quad \text{(A-102)}$$

which determines a_t, so ω_{t-1} can be determined.

After plugging in the equilibrium holdings, the informed traders' value function at time $t-1$

and the parameter m^I_{t-1} can be expressed as

$$J^{I,i}_{t-1} = -\rho^I_{t-1} e^{-A^I\left[W^{I,i}_{t-1}+\hat{V}^I_{t-1}X+\frac{1}{2}m^I_{t-1}\phi^2_{t-1}+\frac{1}{2}\mu_{t-1}(1-\omega_{t-1})(\hat{X}^U_{t-1})^2\right]}, \tag{A-103}$$

where

$$\rho^I_{t-1} = \rho^I_t \sqrt{\Xi^I_t/\Sigma^I_t} e^{A^I\left[\frac{1}{2}(\mu_{t-1}-\mu_t)X^2-\frac{1}{2}(\mu_{t-1}\omega_{t-1}-\mu_t\omega_t)\left(\frac{\Theta}{N_I}\right)^2\right]}, \tag{A-104}$$

$$m^I_{t-1} = m^I_t\left(1-\frac{a_t K^U_{X,t}}{b_t}\right) - \frac{f_{t-1}}{N_I}a_t. \tag{A-105}$$

We have

$$N_I m^I_{t-1} + N_U m^U_{t-1} = \left(N_I m^I_t + N_U m^U_t\right)\left(1-\frac{a_t K^U_{X,t}}{b_t}\right) = 0, \tag{A-106}$$

since

$$N_I m^I_T + N_U m^U_T = 0. \tag{A-107}$$

By using the fact that $N_I m^I_t + N_U m^U_t = 0$, we have

$$a_t = \frac{N_I b_t^2}{\frac{N_I}{A^I\Xi^I_t}+\frac{N_U}{A^U\Xi^U_t}} = \frac{N_I\left(\omega_t K^I_t + (1-\omega_t)K^U_{V,t}\right)^2}{\frac{N_I}{A^I\Sigma^I_t}+\frac{N_U}{A^U\Sigma^U_t}+\frac{N_I(1-\omega_t)}{\mu_t}(K^U_{V,t}-K^I_t)^2}. \tag{A-108}$$

Plug in the expression of ω_T, the recursively we have

$$\omega_t = \frac{\frac{N_I}{A^I o^I_{V,t}}}{\frac{N_I}{A^I o^I_{V,t}}+\frac{N_U}{A^U o^U_{V,t}}}. \tag{A-109}$$

Plug ω_t into the recursive expressions of m^I_t and m^U_t, the we can obtain

$$m^I_t = -\mu_t\omega_t(1-\omega_t), \tag{A-110}$$

$$m^U_t = \frac{N_I}{N_U}\mu_t\omega_t(1-\omega_t). \tag{A-111}$$

Plug m_t back to the formula of f_t, then we have

$$f_t = N_I(1-\omega_t). \tag{A-112}$$

A.5 Proof of Proposition 4

Proof. Some parameters can be rewritten as

$$a_t = \omega_{t-1}\mu_{t-1} - \omega_t\mu_t = \frac{N_I}{\frac{N_I}{A^I} + \frac{N_U}{A^U}} \frac{\tau_{\varepsilon,t}}{(\tau_V + \bar{\tau} + \tau_{t-1})(\tau_V + \bar{\tau} + \tau_t)} > 0, \tag{A-113}$$

for $t = 2, 3, ..., T$, and where

$$\bar{\tau} = \frac{\frac{N_I}{A^I}\tau_\eta + \frac{N_U}{A^U}\tau_0}{\frac{N_I}{A^I} + \frac{N_U}{A^U}}, \quad \tau_t = \sum_{i=1}^{t} \tau_{\varepsilon,i}. \tag{A-114}$$

And

$$\begin{aligned} b_t &= K^U_{V,t} - \omega_t\mu_t K^U_{X,t} = \omega_t K^I_t + (1-\omega_t)K^U_{V,t} \\ &= \tau_{\varepsilon,t}\left(\omega_t o^I_{V,t} + (1-\omega_t)o^U_{V,t}\right) = \frac{\tau_{\varepsilon,t}}{\tau_V + \tau_t + \bar{\tau}} > 0, \end{aligned} \tag{A-115}$$

$$K^U_{X,t} = \frac{1}{\mu_t}\left(K^U_{V,t} - K^I_t\right) = \frac{\tau_{\varepsilon,t}}{\mu_t}\left(o^U_{V,t} - o^I_{V,t}\right) > 0, \tag{A-116}$$

$$\Sigma^U_t = \frac{\tau_V + \tau_0 + \tau_t}{(\tau_V + \tau_0 + \tau_{t-1})\tau_{\varepsilon,t}} > 0, \tag{A-117}$$

for $t = 1, 2, ..., T$. Thus, price change can be rewritten as

$$\begin{aligned} P_{t+1} - P_t &= a_{t+1}\left(\hat{X}^U_t + \frac{\Theta}{N_I}\right) + b_{t+1}e^U_{t+1} \\ &= a_{t+1}\left(\hat{X}^U_{t-s} + \frac{\Theta}{N_I}\right) + a_{t+1}\sum_{i=0}^{s-1} K^U_{X,t-s}e^U_{t-i} + b_{t+1}e^U_{t+1}, \end{aligned} \tag{A-118}$$

and with the lag $s = 1, 2, ..., T-1$,

$$P_{t+1-s} - P_{t-s} = a_{t+1-s}\left(\hat{X}^U_{t-s} + \frac{\Theta}{N_I}\right) + b_{t+1-s}e^U_{t+1-s}. \tag{A-119}$$

Therefore, by the law of total variance for $t = 1, 2, ..., T$

$$\text{Var}(P_t - P_{t-1}) = a_t^2\text{Var}(\hat{X}^U_{t-1}) + b_t^2\Sigma^U_t. \tag{A-120}$$

Similarly, by the law of total covariance,

$$\text{Cov}(P_{t+1} - P_t, P_{t+1-s} - P_{t-s}) = a_{t+1}\left(a_{t+1-s}\text{Var}(\hat{X}^U_{t-s}) + b_{t+1-s}K^U_{X,t+1-s}\Sigma^U_{t+1-s}\right) > 0, \tag{A-121}$$

for $t = s, s+1, ..., T-1$, and where

$$\begin{aligned}\mathrm{Var}(\hat{X}_t^U) &= \sum_{s=1}^{t} \left(K_{X,s}^U\right)^2 \Sigma_s^U + \mathrm{Var}(\hat{X}_0^U) \\ &= \left(\frac{\tau_\eta - \tau_0}{A^I}\right)^2 \frac{\tau_t}{(\tau_V + \tau_0)(\tau_V + \tau_0 + \tau_t)} + \mathrm{Var}(\hat{X}_0^U).\end{aligned} \tag{A-122}$$

Since

$$\begin{aligned}\hat{V}_0^U &= \frac{\tau_0}{\tau_V + \tau_0}(V + \eta - hX) + \frac{\tau_V}{\tau_V + \tau_0}\bar{V}, \\ \hat{X}_0^U &= -\frac{\tau_h/h}{\tau_X + \tau_h}(V + \eta - hX) + \frac{\tau_h/h}{\tau_X + \tau_h}\bar{V},\end{aligned} \tag{A-123}$$

where

$$\tau_h = \frac{h^2}{\tau_V^{-1} + \tau_\eta^{-1}}, \tag{A-124}$$

for $t = s-1$,

$$\begin{aligned}\mathrm{Cov}(P_s - P_{s-1}, P_0) &= a_s \mathrm{Cov}\left(\hat{X}_0^U, \hat{V}_0^U - \omega_0 \mu_0 \hat{X}_0^U\right) \\ &= -\frac{a_s \tau_h/h}{\tau_X + \tau_h}\left(\frac{\tau_0}{\tau_V + \tau_0} + \frac{\omega_0 \mu_0 \tau_h/h}{\tau_X + \tau_h}\right)\left(\frac{1}{\tau_V} + \frac{1}{\tau_\eta} + \frac{h^2}{\tau_X}\right) < 0.\end{aligned} \tag{A-125}$$

Since

$$o_{V,T-s}^U - b_{T-s+1}\Sigma_{T-s+1}^U = \frac{\bar{\tau} - \tau_0}{(\tau_V + \tau_0 + \tau_{T-s})(\tau_V + \bar{\tau} + \tau_{T-s+1})} > 0, \tag{A-126}$$

for $t = T$,

$$\mathrm{Cov}(V - P_T, P_{T+1-s} - P_{T-s}) = a_{T+1-s}\omega_T \mu_T \mathrm{Var}(\hat{X}_{T-s}^U) + b_{T+1-s}\left(o_{V,T-s}^U - b_{T+1-s}\Sigma_{T+1-s}^U\right) > 0. \tag{A-127}$$

Therefore, we obtain the proposition

$$\kappa_{t,s} = \begin{cases} < 0, & t = s-1 \\ > 0, & t = s, s+1 ..., T \end{cases}$$

For $s = 1$, i.e., with one period lag

$$\kappa_{0,1} = -\frac{a_1\tau_h/h}{\tau_X+\tau_h}\sqrt{\frac{\tau_V^{-1}+\tau_\eta^{-1}+h^2\tau_X^{-1}}{a_1^2\text{Var}(\hat{X}_0^U)+b_1^2\Sigma_1^U}} < 0,$$

$$\kappa_{t,1} = \frac{a_{t+1}\left(a_t\text{Var}(\hat{X}_{t-1}^U)+b_tK_{X,t}^U\Sigma_t^U\right)}{\sqrt{a_{t+1}^2\text{Var}(\hat{X}_t^U)+b_{t+1}^2\Sigma_{t+1}^U}\sqrt{a_t^2\text{Var}(\hat{X}_{t-1}^U)+b_t^2\Sigma_t^U}} > 0, \quad \text{for } t = 1,2,\dots,T-1$$

$$\kappa_{T,1} = \frac{a_T\omega_T\mu_T\text{Var}(\hat{X}_{T-1}^U)+b_T(o_{V,T-1}^U-b_T\Sigma_T^U)}{\sqrt{\omega_T^2\mu_T^2\text{Var}(\hat{X}_T^U)+o_{V,T}^U}\sqrt{a_T^2\text{Var}(\hat{X}_{T-1}^U)+b_T^2\Sigma_T^U}} > 0. \tag{A-128}$$

□

A.6 Proof of Proposition 5

The trading volume is a normally distributed,

$$N_U(\theta_t^U-\theta_{t-1}^U) = (f_t-f_{t-1})\left(\hat{X}_{t-1}^U+\frac{\Theta}{N_I}\right)+f_tK_{X,t}^Ue_t^U, \tag{A-129}$$

with mean

$$\mu_{vol} = \text{E}\left[N_U(\theta_t^U-\theta_{t-1}^U)\right] = (f_t-f_{t-1})\frac{\Theta}{N_I}, \tag{A-130}$$

and variance for $t = 1,2,\dots,T$ can be computed by the law of total variance

$$\begin{aligned}\text{Var}\left(N_U(\theta_t^U-\theta_{t-1}^U)\right) &= \text{E}\left[\text{Var}_{t-1}^U\left((f_t-f_{t-1})\hat{X}_{t-1}^U+f_tK_{X,t}^Ue_t^U\right)\right]\\ &\quad+\text{Var}\left(\text{E}_{t-1}^U\left[(f_t-f_{t-1})\hat{X}_{t-1}^U+f_tK_{X,t}^Ue_t^U\right]\right)\\ &= \left(f_tK_{X,t}^U\right)^2\Sigma_t^U+(f_t-f_{t-1})^2\text{Var}(\hat{X}_{t-1}^U),\end{aligned} \tag{A-131}$$

where

$$\text{Var}(\hat{X}_{t-1}^U) = \sum_{s=1}^{t-1}\left(K_{X,s}^U\right)^2\Sigma_s^U+\text{Var}(\hat{X}_0^U), \tag{A-132}$$

can also be computed by the law of total variance. For $t = 0$,

$$\text{Var}\left(N_U(\theta_0^U-\theta_{-1}^U)\right) = f_0^2\text{Var}(\hat{X}_0^U). \tag{A-133}$$

Thus, the variance of the trading volume is

$$\sigma_{vol}^2 = \mathrm{Var}\left(N_U(\theta_t^U - \theta_{t-1}^U)\right) = \left(f_t K_{X,t}^U\right)^2 \Sigma_t^U + (f_t - f_{t-1})^2 \left[\sum_{s=1}^{t-1} \left(K_{X,s}^U\right)^2 \Sigma_s^U + \mathrm{Var}(\hat{X}_0^U)\right]. \quad \text{(A-134)}$$

Therefore,

$$\mathrm{Vol}_t = \sqrt{\frac{2}{\pi}}\sigma_{vol} e^{-\frac{\mu_{vol}^2}{2\sigma_{vol}^2}} + \mu_{vol}\mathrm{erf}\left(\frac{\mu_{vol}}{\sqrt{2}\sigma_{vol}}\right), \quad \text{(A-135)}$$

where erf() is the error function.

Therefore, the covariance of demands is

$$\begin{aligned} &\mathrm{Cov}\left(N_U(\theta_t^U - \theta_{t-1}^U), N_U(\theta_{t-1}^U - \theta_{t-2}^U)\right) \\ &= f_{t-1}(f_t - f_{t-1})(K_{X,t-1}^U)^2 \Sigma_{t-1}^U + (f_t - f_{t-1})(f_{t-1} - f_{t-2})\mathrm{Var}(\hat{X}_{t-2}^U) > 0, \end{aligned} \quad \text{(A-136)}$$

for $t = 2, 3, ..., T$, and for $t = 1$,

$$\mathrm{Cov}\left(N_U(\theta_1^U - \theta_0^U), N_U(\theta_0^U - \theta_{-1}^U)\right) = f_0(f_1 - f_0)\mathrm{Var}(\hat{X}_0^U) > 0. \quad \text{(A-137)}$$

In this simple information structure, the demand autocorrelation is positive, since $f_t > f_{t-1} > 0$.

A.7 Proof of Proposition 6

Proof. Since

$$P_t - P_{t-1} = a_t\left(\hat{X}_{t-1}^U + \frac{\Theta}{N_I}\right) + b_t e_t^U, \quad \text{(A-138)}$$

and

$$\alpha(\theta_t^I - \theta_{t-1}^I) = -f_t K_{X,t}^U e_t^U - (f_t - f_{t-1})\left(\hat{X}_{t-1}^U + \frac{\Theta}{N_I}\right), \quad \text{(A-139)}$$

then by the law of total covariance,

$$\mathrm{Cov}(P_t - P_{t-1}, \theta_t^I - \theta_{t-1}^I) = -(f_t - f_{t-1})a_t \mathrm{Var}(\hat{X}_{t-1}^U)/\alpha - f_t K_{X,t}^U b_t \Sigma_t^U/\alpha, \quad \text{(A-140)}$$

and from Appendix A.6, we have,

$$\mathrm{Var}\left(\alpha(\theta_t^I - \theta_{t-1}^I)\right) = (f_t - f_{t-1})^2 \mathrm{Var}(\hat{X}_{t-1}^U) + \left(f_t K_{X,t}^U\right)^2 \Sigma_t^U. \quad \text{(A-141)}$$

Since all the information is public after time zero,

$$\frac{1}{o^I_{V,t}} = \frac{1}{o^I_{V,t-1}} + \tau_{\varepsilon,t}, \quad \frac{1}{o^U_{V,t}} = \frac{1}{o^U_{V,t-1}} + \tau_{\varepsilon,t}, \tag{A-142}$$

$$K^I_t = o^I_{V,t}\tau_{\varepsilon,t}, \quad K^U_{V,t} = o^U_{V,t}\tau_{\varepsilon,t}, \tag{A-143}$$

and

$$f_t = \alpha(1-\omega_t), \tag{A-144}$$

then we have

$$\begin{aligned} \frac{b_t}{a_t} &= \frac{f_t K^U_{X,t}}{f_t - f_{t-1}}, \\ \frac{a_t}{f_t - f_{t-1}} &= \left(\frac{A^I}{N_I} + \frac{A^U}{N_U}\right)\frac{1}{\tau_\eta - \tau_0} > 0. \end{aligned} \tag{A-145}$$

Therefore,

$$f_t - f_{t-1} > 0, \tag{A-146}$$

and

$$\lambda^I_t = \begin{cases} \frac{\frac{h\tau_0}{\tau_V+\tau_0} + \frac{\omega_0\mu_0\tau_h}{\tau_X+\tau_h}}{\frac{f_0\tau_h}{\tau_X+\tau_h}} > 0, & t = 0 \\ -\left(\frac{A^I}{N_I} + \frac{A^U}{N_U}\right)\frac{1}{\tau_\eta-\tau_0} < 0, & t = 1, 2, ..., T \end{cases}$$

Also by the law of total covariance,

$$\kappa^I_t = \text{Cov}(\alpha(\theta^I_t - \theta^I_{t-1}), R_{t-1}) = -(f_t - f_{t-1})\left[a_{t-1}\text{Var}(\hat{X}^U_{t-2}) + b_{t-1}K^U_{X,t-1}\Sigma^U_{t-1}\right]. \tag{A-147}$$

Since

$$f_t - f_{t-1} > 0, \quad a_t > 0, \quad b_t > 0, \quad K^U_{X,t} > 0, \tag{A-148}$$

we have

$$\kappa^I_t < 0, \quad \kappa^U_t > 0, \quad \text{for } \; t = 2, 3, ..., T. \tag{A-149}$$

For $t = 1$,

$$\kappa_1^I = \mathrm{Cov}(N_I(\theta_1^I - \theta_0^I), P_0)$$
$$= (f_1 - f_0)\frac{\omega_0\mu_0\tau_h/h}{\tau_X + \tau_h}\left(\frac{\tau_0}{\tau_V + \tau_0} + \frac{\tau_h/h}{\tau_X + \tau_h}\right)\left(\frac{1}{\tau_V} + \frac{1}{\tau_\eta} + \frac{h^2}{\tau_X}\right) > 0, \tag{A-150}$$

thus

$$\kappa_1^U < 0. \tag{A-151}$$

□

A.8 Return Autocorrelation

By the law of total variance and covariance,

$$\mathrm{Var}(\hat{V}_t^U) = \sum_{s=1}^{t}(K_{V,s}^U)^2\Sigma_s^U + \mathrm{Var}(\hat{V}_0^U), \tag{A-152}$$

$$\mathrm{Var}(\hat{X}_t^U) = \sum_{s=1}^{t}(K_{X,s}^U)^2\Sigma_s^U + \mathrm{Var}(\hat{X}_0^U), \tag{A-153}$$

$$\mathrm{Cov}(\hat{V}_t^U, \hat{X}_t^U) = \sum_{s=1}^{t}K_{V,s}^U K_{X,s}^U\Sigma_s^U + \mathrm{Cov}(\hat{V}_0^U, \hat{X}_0^U), \tag{A-154}$$

where

$$\begin{aligned}\mathrm{Var}(\hat{V}_0^U) &= \left(\frac{\tau_0}{\tau_V + \tau_0}\right)^2\left(\frac{1}{\tau_V} + \frac{1}{\tau_{\varepsilon,0}} + \frac{h_0^2}{\tau_{x,0}}\right),\\ \mathrm{Var}(\hat{X}_0^U) &= \left(\frac{\tau_{h,0}/h_0}{\tau_{x,0} + \tau_{h,0}}\right)^2\left(\frac{1}{\tau_V} + \frac{1}{\tau_{\varepsilon,0}} + \frac{h_0^2}{\tau_{x,0}}\right),\\ \mathrm{Cov}(\hat{V}_0^U, \hat{X}_0^U) &= -\left(\frac{\tau_0}{\tau_V + \tau_0}\right)\left(\frac{\tau_{h,0}/h_0}{\tau_{x,0} + \tau_{h,0}}\right)\left(\frac{1}{\tau_V} + \frac{1}{\tau_{\varepsilon,0}} + \frac{h_0^2}{\tau_{x,0}}\right),\end{aligned} \tag{A-155}$$

and

$$\tau_t = \frac{1}{\tau_{\varepsilon,t}^{-1} + h_t^2\tau_{x,t}^{-1}}, \quad \tau_{h,t} = \frac{h_t^2}{\tau_{\varepsilon,t}^{-1} + \tau_V^{-1}}. \tag{A-156}$$

Thus, the variance of return is

$$\begin{aligned}\mathrm{Var}(P_t - P_{t-1}) = &\left(a_t K_{V,t}^U + b_t K_{X,t}^U\right)^2\Sigma_t^U + \mathrm{Var}\left((a_1 - a_0)\hat{V}_0^U + (b_1 - b_0)\hat{X}_0^U\right)\\ &+ \sum_{s=1}^{t-1}\left((a_t - a_{t-1})K_{V,s}^U + (b_t - b_{t-1})K_{X,s}^U\right)^2\Sigma_s^U,\end{aligned} \tag{A-157}$$

for $t=0$,

$$\mathrm{Var}(P_0)=a_0^2\mathrm{Var}(\hat{V}_0^U)+b_0^2\mathrm{Var}(\hat{X}_0^U)+2a_0b_0\mathrm{Cov}(\hat{V}_0^U,\hat{X}_0^U), \tag{A-158}$$

and for $t=T$,

$$\mathrm{Var}(V-P_T)=o_{V,T}^U+(1-a_T)^2\mathrm{Var}(\hat{V}_T^U)+b_T^2\mathrm{Var}(\hat{X}_T^U)-2(1-a_T)b_T\mathrm{Cov}(\hat{V}_T^U,\hat{X}_T^U). \tag{A-159}$$

The s-order (where $s=1,2,\ldots,T-t$) autocorrelation of returns is

$$\begin{aligned}\mathrm{Cov}(P_{t+s}-P_{t+s-1},P_t-P_{t-1})&=\left(a_tK_{V,t}^U+b_tK_{X,t}^U\right)\left[(a_{t+s}-a_{t+s-1})K_{V,t}^U+(b_{t+s}-b_{t+s-1})K_{X,t}^U\right]\Sigma_t^U\\&+(a_{t+s}-a_{t+s-1})(a_t-a_{t-1})\mathrm{Var}(\hat{V}_{t-1}^U)+(b_{t+s}-b_{t+s-1})(b_t-b_{t-1})\mathrm{Var}(\hat{X}_{t-1}^U)\\&+\left[(a_{t+s}-a_{t+s-1})(b_t-b_{t-1})+(b_{t+s}-b_{t+s-1})(a_t-a_{t-1})\right]\mathrm{Cov}(\hat{V}_{t-1}^U,\hat{X}_{t-1}^U),\end{aligned} \tag{A-160}$$

for $t=0$

$$\begin{aligned}\mathrm{Cov}(P_s-P_{s-1},P_0)&=a_0(a_s-a_{s-1})\mathrm{Var}(\hat{V}_0^U)+b_0(b_s-b_{s-1})\mathrm{Var}(\hat{X}_0^U)\\&+\left[a_0(b_s-b_{s-1})+b_0(a_s-a_{s-1})\right]\mathrm{Cov}(\hat{V}_0^U,\hat{X}_0^U),\end{aligned} \tag{A-161}$$

and for $t=T+1-s$,

$$\begin{aligned}&\mathrm{Cov}(V-P_T,P_{T+1-s}-P_{T-s})\\&=\left(a_{T+1-s}K_{V,T+1-s}^U+b_{T+1-s}K_{X,T+1-s}^U\right)\left[o_{V,T-s}^U-\left(a_{T+1-s}K_{V,T+1-s}^U+b_{T+1-s}K_{X,T+1-s}^U\right)\Sigma_{T+1-s}^U\right]\\&\quad+(1-a_T)(a_{T+1-s}-a_{T-s})\mathrm{Var}(\hat{V}_{T-s}^U)-b_T(b_{T+1-s}-b_{T-s})\mathrm{Var}(\hat{X}_{T-s}^U)\\&\quad+\left[(1-a_T)(b_{T+1-s}-b_{T-s})-b_T(a_{T+1-s}-a_{T-s})\right]\mathrm{Cov}(\hat{V}_{T-s}^U,\hat{X}_{T-s}^U).\end{aligned} \tag{A-162}$$

The informed traders' holding can be expressed as

$$N_I\theta_t^I=d_t(\hat{V}_t^U-\bar{V})+k_t\hat{X}_t^U+l_t\Theta, \tag{A-163}$$

where

$$d_t = \frac{\omega_t}{\gamma_t^U / N_U}(\delta_t^I - \delta_t^U),$$
$$k_t = \frac{\omega_t}{\gamma_t^U / N_U}(g_t^I - g_t^U - \delta_t^I \mu_t), \tag{A-164}$$
$$l_t = \frac{\omega_t}{\gamma_t^U / N_U}(f_t^I - f_t^U) + \omega_t.$$

Thus, the variance of informed traders' demand is

$$\begin{aligned} \mathrm{Var}\left(N_I(\theta_t^I - \theta_{t-1}^I)\right) = \left(d_t K_{V,t}^U + k_t K_{X,t}^U\right)^2 \Sigma_t^U + \mathrm{Var}\left((d_1 - d_0)\hat{V}_0^U + (k_1 - k_0)\hat{X}_0^U\right) \\ + \sum_{s=1}^{t-1} \left((d_t - d_{t-1}) K_{V,s}^U + (k_t - k_{t-1}) K_{X,s}^U\right)^2 \Sigma_s^U, \end{aligned} \tag{A-165}$$

for $t = 0$,

$$\mathrm{Var}\left(N_I(\theta_0^I - \theta_{-1}^I)\right) = d_0^2 \mathrm{Var}(\hat{V}_0^U) + k_0^2 \mathrm{Var}(\hat{X}_0^U) + 2 d_0 k_0 \mathrm{Cov}(\hat{V}_0^U, \hat{X}_0^U). \tag{A-166}$$

The covariance of informed traders' current demand and previous demand is

$$\begin{aligned} &\mathrm{Cov}\left(N_I(\theta_t^I - \theta_{t-1}^I), N_I(\theta_{t-1}^I - \theta_{t-2}^I)\right) \\ &= \left(d_{t-1} K_{V,t-1}^U + k_{t-1} K_{X,t-1}^U\right) \left[(d_t - d_{t-1}) K_{V,t-1}^U + (k_t - k_{t-1} () K_{X,t-1}^U\right] \Sigma_{t-1}^U \\ &+ (d_t - d_{t-1})(d_{t-1} - d_{t-2}) \mathrm{Var}(\hat{V}_{t-2}^U) + (k_t - k_{t-1})(k_{t-1} - k_{t-2}) \mathrm{Var}(\hat{X}_{t-2}^U) \\ &+ [(d_t - d_{t-1})(k_{t-1} - k_{t-2}) + (k_t - k_{t-1})(d_{t-1} - d_{t-2})] \mathrm{Cov}(\hat{V}_{t-2}^U, \hat{X}_{t-2}^U), \end{aligned} \tag{A-167}$$

for $t = 0$

$$\begin{aligned} &\mathrm{Cov}\left(N_I(\theta_1^I - \theta_0^I), N_I(\theta_0^I - \theta_{-1}^I)\right) \\ &= d_0(d_1 - d_0) \mathrm{Var}(\hat{V}_0^U) + k_0(k_1 - k_0) \mathrm{Var}(\hat{X}_0^U) + [k_0(d_1 - d_0) + d_0(k_1 - k_0)] \mathrm{Cov}(\hat{V}_0^U, \hat{X}_0^U). \end{aligned} \tag{A-168}$$

The covariance of informed traders' demand and return is

$$\begin{aligned} \mathrm{Cov}\left(P_t - P_{t-1}, N_I \theta_t^I - N_I \theta_{t-1}^I\right) = \left(a_t K_{V,t}^U + b_t K_{X,t}^U\right)\left(d_t K_{V,t}^U + k_t K_{X,t}^U\right) \Sigma_t^U \\ + (d_t - d_{t-1})(a_t - a_{t-1}) \mathrm{Var}(\hat{V}_{t-1}^U) + (k_t - k_{t-1})(b_t - b_{t-1}) \mathrm{Var}(\hat{X}_{t-1}^U) \\ + [(d_t - d_{t-1})(b_t - b_{t-1}) + (k_t - k_{t-1})(a_t - a_{t-1})] \mathrm{Cov}(\hat{V}_{t-1}^U, \hat{X}_{t-1}^U), \end{aligned} \tag{A-169}$$

for $t = 0$

$$\mathrm{Cov}\left(P_0, N_I \theta_0^I\right) = a_0 d_0 \mathrm{Var}(\hat{V}_0^U) + b_0 k_0 \mathrm{Var}(\hat{X}_0^U) + (a_0 k_0 + b_0 d_0) \mathrm{Cov}(\hat{V}_0^U, \hat{X}_0^U). \tag{A-170}$$

Hence, the return autocorrelation $\kappa_{t,s}$ and informed traders' price impact λ_t can be computed.

To check whether uninformed traders are contrarian traders or momentum traders, we compute

$$\begin{aligned}\text{Cov}(N_I\theta^I_{t+1} - N_I\theta^I_t, P_t - P_{t-1}) = &\left(a_t K^U_{V,t} + b_t K^U_{X,t}\right)\left[(d_{t+1} - d_t)K^U_{V,t} + (k_{t+1} - k_t)K^U_{X,t}\right]\Sigma^U_t \\ &+(d_{t+1} - d_t)(a_t - a_{t-1})\text{Var}(\hat{V}^U_{t-1}) + (k_{t+1} - k_t)(b_t - b_{t-1})\text{Var}(\hat{X}^U_{t-1}) \\ &+ [(d_{t+1} - d_t)(b_t - b_{t-1}) + (k_{t+1} - k_t)(a_t - a_{t-1})]\,\text{Cov}(\hat{V}^U_{t-1}, \hat{X}^U_{t-1}),\end{aligned} \tag{A-171}$$

for $t = 0$

$$\begin{aligned}\text{Cov}(N_I\theta^I_1 - N_I\theta^I_0, P_0) = &\,a_0(d_1 - d_0)\text{Var}(\hat{V}^U_0) + b_0(k_1 - k_0)\text{Var}(\hat{X}^U_0) \\ &+ [a_0(k_1 - k_0) + b_0(d_1 - d_0)]\,\text{Cov}(\hat{V}^U_0, \hat{X}^U_0).\end{aligned} \tag{A-172}$$