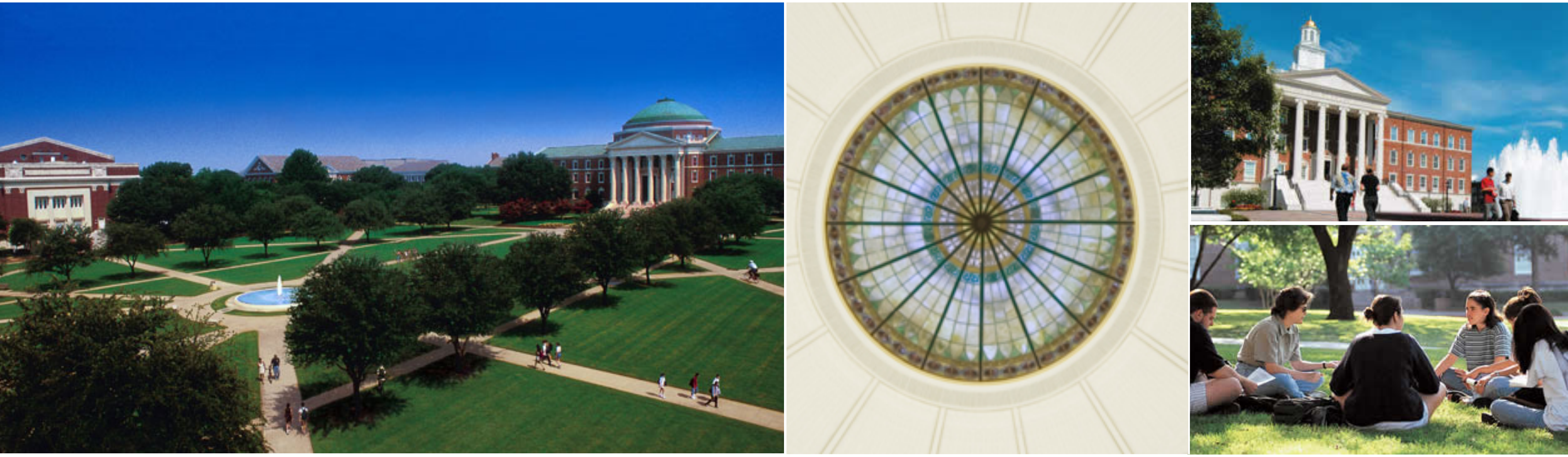
SMU



Using real objects and manipulatives to solve problems: A focus on factoring quadratics

Pooja Shivraj, M.S., Ph.D.

Educational Assessment Researcher

Southern Methodist University

Agenda

- The mathematical process standards: Why using manipulatives to solve mathematical problems is important
- An introduction to the CRA method: an instructional strategy incorporating the use of manipulatives
- Intertwining the process standards with mathematical content:
  - Using manipulatives and visual representations to factor quadratics
- Going beyond factoring quadratics with the CRA method
- A brainstorm session on how you can use this method in your classroom

The mathematical process standards

- Process standards describe *ways* in which students are expected to engage in the content; integrated at every grade level and in every course.
- There lies an expectation within the Texas Essential Knowledge and Skills for Mathematics (TEKS-M) that students will “select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate… to solve problems.” (Texas Education Agency [TEA], 2012)
- The process standards also indicate that students are expected to “…communicate mathematical ideas…using multiple representations…” (TEA, 2012).

The importance of using manipulatives to solve mathematical problems

- Stress on using manipulatives in process standards is based on research that demonstrates that using concrete objects to teach abstract concepts can help reinforce students' understanding of those mathematical concepts
- A research-based instructional strategy that can be used to help students grasp and strengthen their understanding of abstract concepts is called the CRA method
- Concrete-Representation-Abstract (CRA) – graduated instructional sequence

(Butler, et al., 2003; Flores, 2009; Gersten et al., 2009; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Miller & Hudson, 2006; Scheuermann et al., 2009; Witzel et al., 2003; Witzel, 2005)

The CRA method

Concrete

•Using manipulatives or models

•Learning by doing

Representation

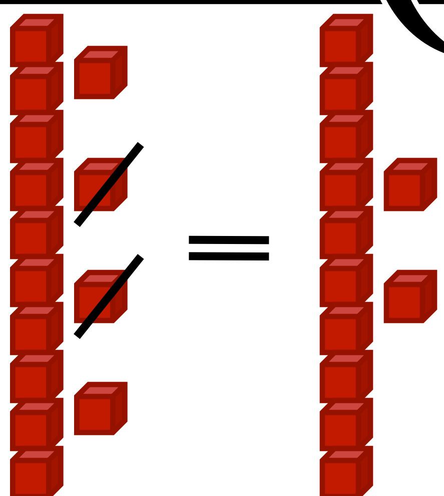
•Using visual representations like pictures/graphs

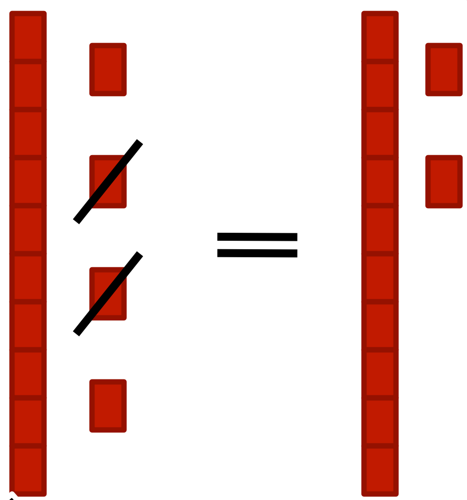
•Learning by visualizing

Abstract

•Using abstract mathematical notation

•Learning by translating





$$\begin{array}{r} 14 \\ -\ \ 2 \\ \hline 12 \end{array}$$

(Gersten et al., 2009)

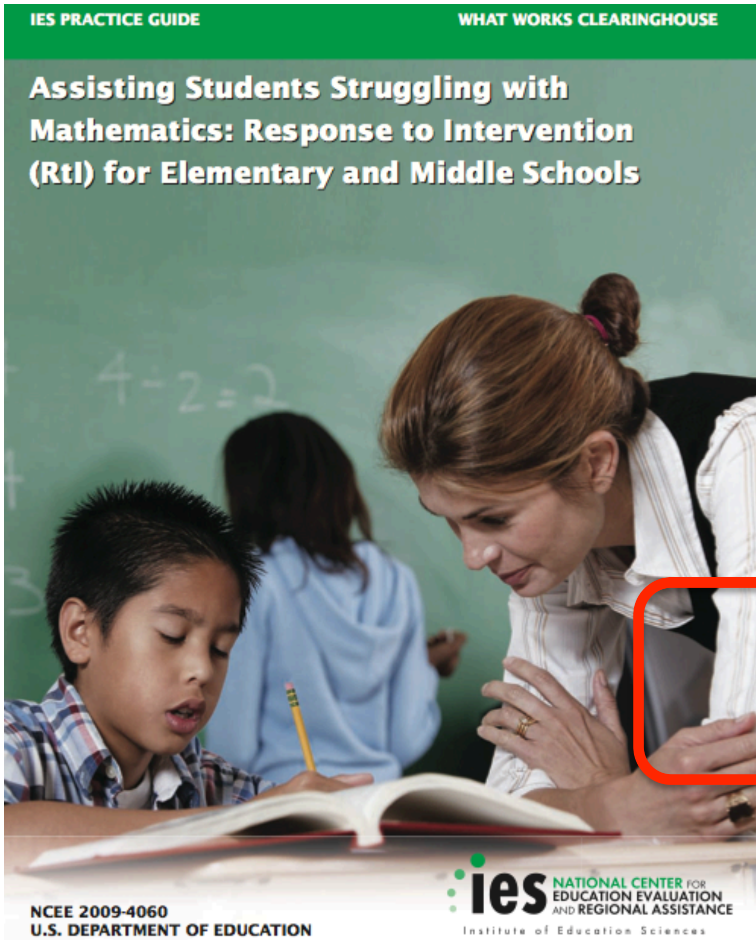
SMU

The CRA method for struggling students

- NCTM (2000) states that students are required to master skills and meet standards at every grade level, regardless of whether they have a learning disability
- Struggling students need enhanced strategies so they can perform at the same level as their peers
- Use of the CRA strategy can help bridge the gap between struggling and non-struggling students

(Butler, et al., 2003; Flores, 2009; Gersten et al., 2009; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Miller & Hudson, 2006; Scheuermann et al., 2009; Witzel et al., 2003; Witzel, 2005)

Research-Based recommendations for Tier II interventions



Recommendation	Level of evidence
Tier 1	
1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	Moderate
Tiers 2 and 3	
2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	Low
3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	Strong
4. Interventions should include instruction on solving word problems that is based on common underlying structures.	Strong
5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	Moderate
6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	Moderate
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Low
8. Include motivational strategies in tier 2 and tier 3 interventions.	Low

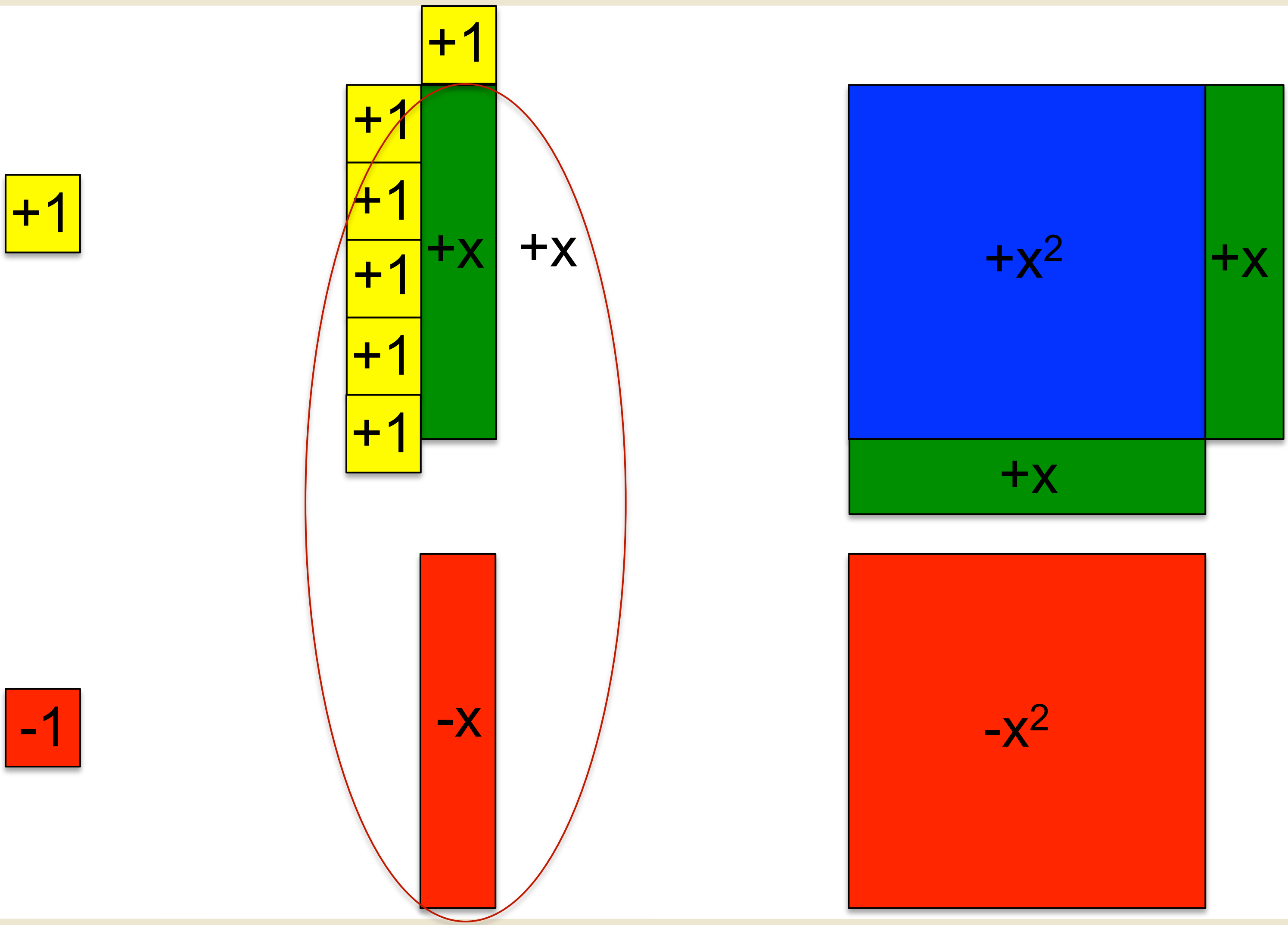
Source: Authors' compilation based on analysis described in text.

Is the method only for struggling students?

- Students differ in the way they process the information presented to them and differ in the way they eventually learn them
- Fleming's (1987) model, most widely used in education, has the following categories:
  - Kinesthetic
  - Visual
  - Reading/Writing
  - Auditory
- Teaching to each specific learning style has shown to have no effect on student achievement; the CRA method however encompasses different ways that students process information
- Most students have a hard time dealing making the transition from arithmetic concepts to abstract algebraic concepts, not just struggling students
- Using the CRA method to transition from manipulatives to pictorial representations of them to abstract notation can help all students with learning algebra

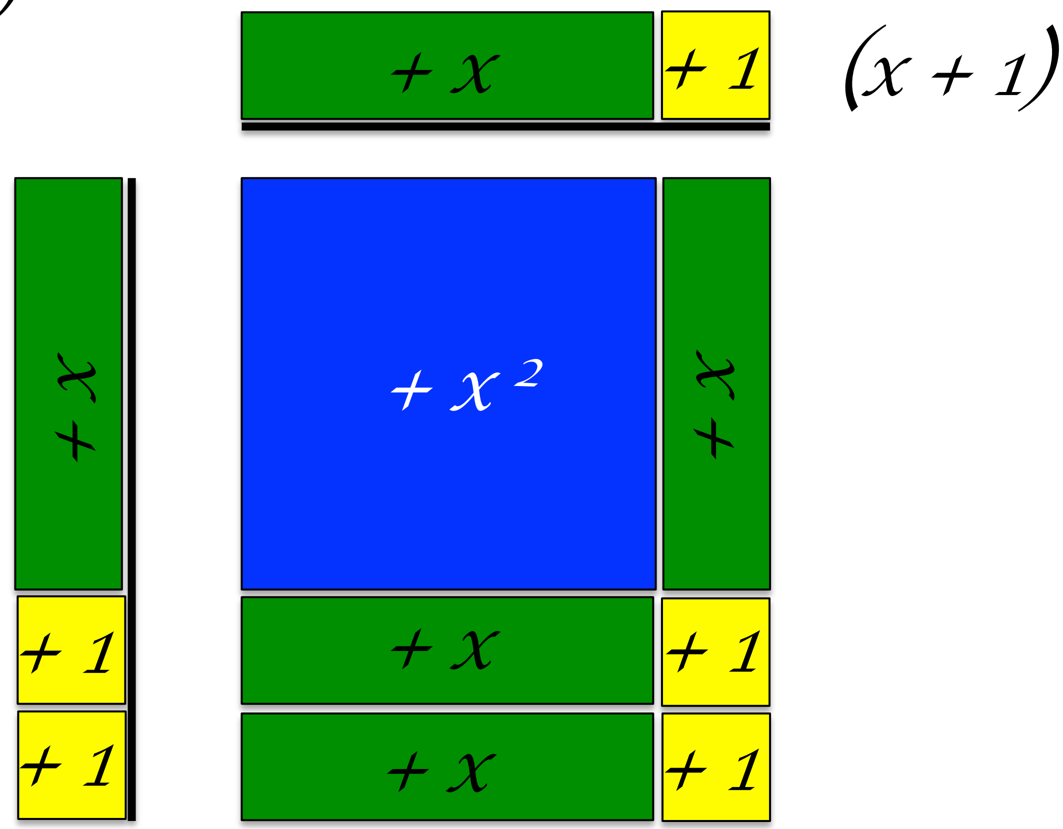
(Barbe, et al., 1979; Kolb, 1984; Fleming, 1987; James & Gardner, 1995)
(Rittle-Johnson & Star, 2009)

Algebra Tiles



Multiplying out expressions

$(x + 2)(x + 1)$

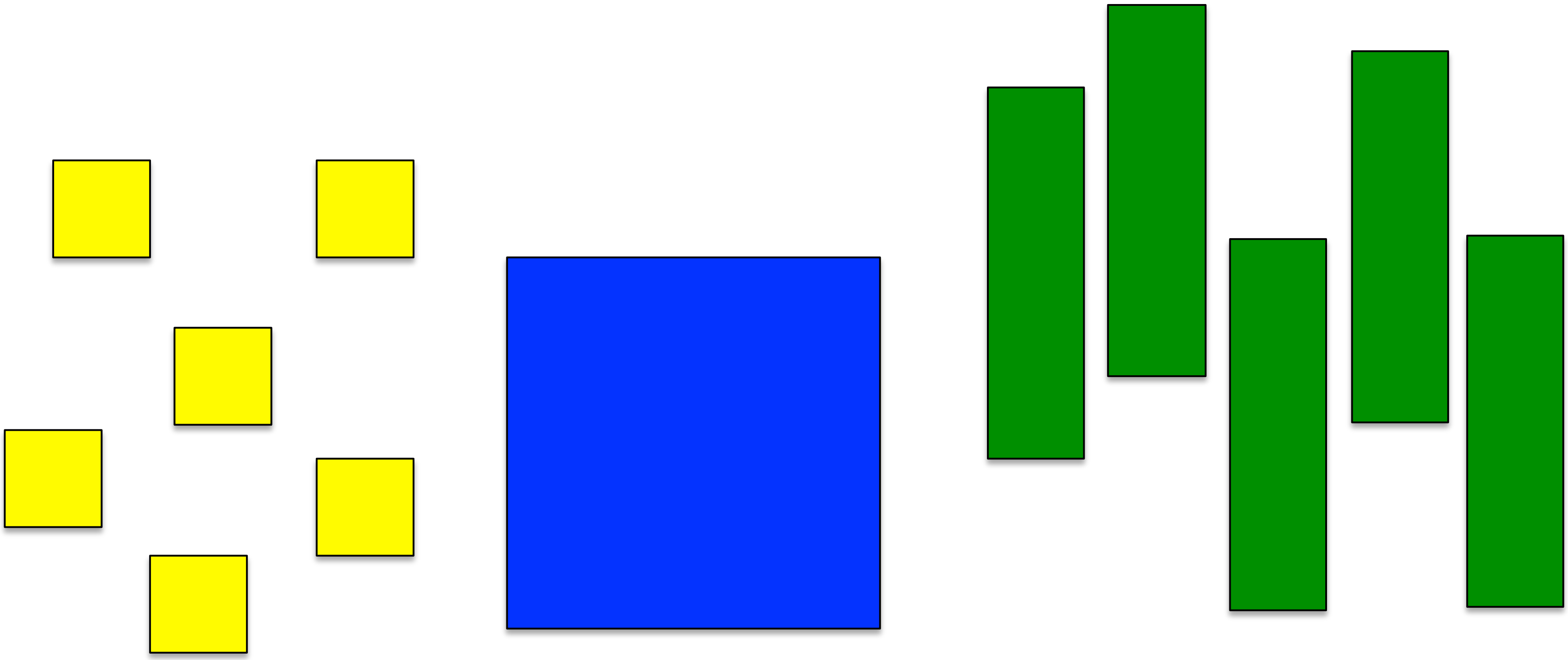


$x^2 + 3x + 2$

Multiplying out expressions

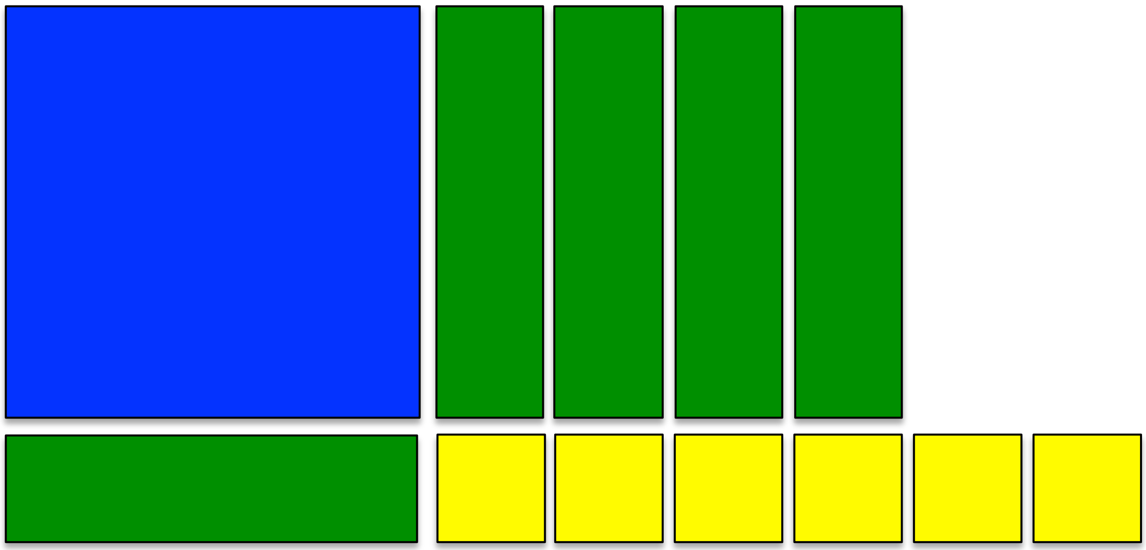
$$(x - 1)(x + 2)$$

Factoring Quadratics: Example



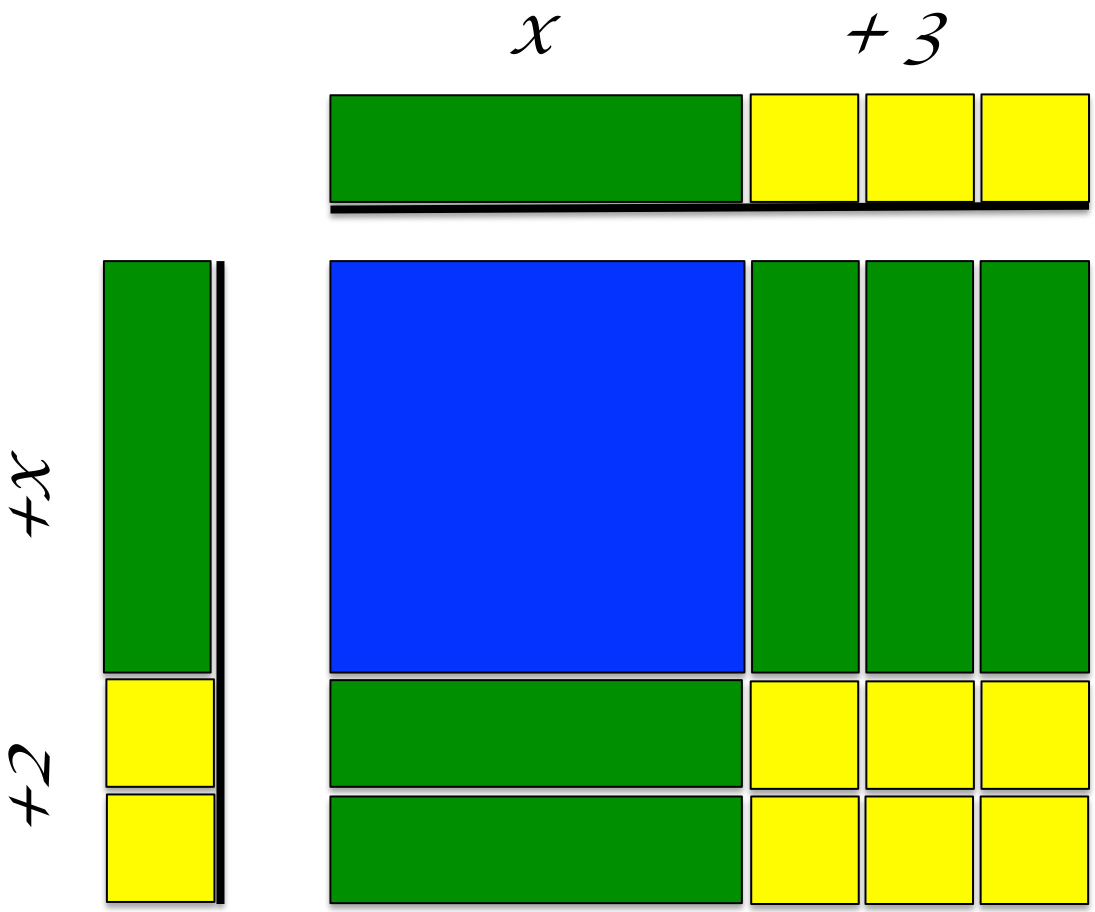
$x^2 + 5x + 6$

Factoring Quadratics: Example



$x^2 + 5x + 6$

Factoring Quadratics: Example



$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Demonstrations

SMU

Translating manipulatives to visual representations

- The process standards: Students are expected to: “select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate… to solve problems.” (Texas Education Agency [TEA], 2012)
- Lack of constant accessibility of concrete manipulatives
- Graduated release of concrete manipulatives
- Interpretations of different representations of the same concept – i.e., visualizing quadratics as area models
- LET’S DO EXAMPLES!

Other Lessons using Algebra Tiles

- Expanding or multiplying expressions (e.g., $(x + 1)(x + 2)$)
- Factoring expressions (e.g., $2x – 6 = 2 (x – 3)$)
- Solving single-step and multi-step equations (e.g., $2x + 2 = 4$)
- Substitutions (e.g., $2x + 2$ when $x = -1$)

Extending the CRA to concepts beyond algebra

- Learning slopes
- Rate of change (velocity/drip rate/fill rate)
- Can you brainstorm others?

Contact Information

Dr. Pooja Shivraj

Educational Assessment Researcher

Research in Mathematics Education

Southern Methodist University

Email : pshivraj@smu.edu

Phone: (214) 768-7642

Website: http://www.smu.edu/rme

THANK YOU!

References

- Barbe, W. B., Swassing, R. H., Milone, M. N. (1979). *Teaching Through Modality Strengths: Concepts and Practices. Columbus, Ohio: Zaner-Blosner.*
- Butler, F. M., Miller, S. P., Crehan, K., Babbitt, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice, 18*(2), 99-111.
- Fleming, N. D., & Mills, N. D. (1992). Not another inventory, rather a catalyst for reflection. *To Improve the Academy, 11*, 137.
- Flores, M. M. (2009). Teaching subtraction with regrouping to students experiencing difficulty in mathematics. *Preventing School Failure, 53*(3), 145-152.
- Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides/
- James, W.; Gardner, D. (1995). Learning styles: Implications for distance learning. *New Directions for Adult and Continuing Education, 67*, 19-32.
- Kolb, D. (1984). *Experiential learning: Experience as the source of learning and development.* Englewood Cliffs, NJ: Prentice-Hall.
- Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice, 15*(1), 10-21.
- Maccini, P., & Ruhl, K. L. (2000). Effects of a graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children,23*(4), 465-489.
- Miller, S. P., & Hudson, P. J. (2006). Helping students with disabilities understand what mathematics means. *TEACHING Exceptional Children, 39*(1),28-35.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.
- Rittle-Johnson, B., & Star, J. R. (2009). Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving. *Journal of Educational Psychology, 101*(3), 529.
- Scheuermann, A. M., Deshler, D. D., & Schumaker, J. B. (2009). The effects of the explicit inquiry routine on the performance of students with learning disabilities on one-variable equations. *Learning Disability Quarterly, 32*(2), 103-120.
- Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research and Practice*, *18*(2), 121-131.
- Witzel, B. S. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities: A Contemporary Journal, 3*(2), 49-60.