# Numeric Relational Reasoning: Anticipated and Unanticipated Strategies 

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## Executive Summary

The purpose of this report is to describe the anticipated and unanticipated strategies that we observed among students' responses from the Numeric Relational Reasoning (NRR) cognitive interviews conducted as part of the Measuring Early Mathematics and Reasoning Skills (MMaRS) project. See the Numeric Relational Reasoning Learning Progression Development and Protocol Development technical reports for details on the structure of the learning progression and the cognitive interviews (Technical Report No. 20-02; Technical Report No. 2004). This report details the process we used to collect various students' strategies from the cognitive interview data. We provide descriptions and examples of multiple strategies, both anticipated and unanticipated, that elementary students used to work through numeric relational reasoning tasks during the cognitive interviews. To find evidence of strategies in analyzing cognitive interview data, we use an adapted form of interaction analysis (Jordon \& Henderson, 1995). To conclude the report, we list various strategies by subcomponent across all core concepts.

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# Numeric Relational Reasoning: Anticipated and Unanticipated Strategies <br> <br> Introduction 

 <br> <br> Introduction}

The purpose of this report is to describe the anticipated and unanticipated strategies we observed from the qualitative analyses of the Numerical Relational Reasoning (NRR) Cognitive Interviews (CIs) for the Measuring Early Mathematics and Reasoning Skills (MMaRS) project. We found several strategies students used to work on the given tasks during cognitive interviews. We anticipated some strategies before analysis, based on a literature review and researchers' experience, and we referred to them as anticipated strategies. We also noticed some strategies that we did not anticipate and referred to them as unanticipated strategies among students' responses. We also listed all anticipated and unanticipated strategies observed for all subcomponents within a core concept.

## Research Questions

The cognitive interviews were designed to address four research questions related to gathering validity evidence for the NRR learning progression. These research questions rely on data collected directly from the cognitive interviews and from the fidelity of administration form. The research questions include:

- RQ 1: What level of evidence exists to confirm or disconfirm the ordering, content, and developmental appropriateness of the learning progressions?
- RQ 2: What are the characteristics of kindergarten, first- and second-grade students’ numeric relational reasoning within one-on-one cognitive interviews?
- RQ 3: What was the level of fidelity of implementation (fidelity by interviewer/observer)?
- RQ 4: What was the level of accessibility and comfort of students on all tasks within every learning progression?

In this report, we only focused on the second research question. To identify elementary students' numeric relational reasoning, we identified various strategies students used to work on the given problem during one-on-one cognitive interviews.

## Cognitive Interviews and Participants

This section briefly describes the cognitive interview protocols, participants, and data collection for the study. We have provided references to other technical reports for details of various components briefly discussed in this report.

## Cognitive Interviews

Cognitive interviews were conducted to gather one source of validity evidence to support the NRR LP's developmental appropriateness, ordering, conceptualization, and interconnections based on student responses to items aligned with the hypothesized LP. We analyzed students' correctness as aligned with the stipulated subcomponent skills and their demonstrated reasoning strategies. For more information on the development of the NRR learning progression, refer to the NRR Learning Progression Development technical report (Tech. Rep. No. 20-02). For more information on cognitive interview protocol development, including the protocol structure, development processes, and the refinement steps through student tryouts, refer to the NRR Cognitive Interview Protocol Development report (Tech. Rep. No. 20-04).

## Participants and Data Collection

In the cognitive interviews, 33 students in grades K-3 participated. The students were selected from three different private schools in a metropolitan area in a southern state. One kindergarten student was removed from the study due to the number of limited mathematical responses provided during the interview. The remaining 32 students remained as participants in the study. We asked teachers for the level of support each student needed when completing mathematics problems. We collected the level of support for each student to gauge the students' present level of understanding in early mathematics concepts.

The primary data sources collected from the cognitive interviews included audio and video recordings of the interviews, transcripts of interviews, and fidelity observation forms. The videos were analyzed alongside transcripts to determine students' correct responses, alignment to the learning progression, and student strategies to solve the given problems.

For complete details on participants, data collection, and the methods for purposeful sampling of participants, refer to the Numeric Relational Reasoning Cognitive Interviews: Methods and Quantitative Data Analyses (Technical Report No. 20-05).

## Student Sampling of Data

To optimize the number of interviews coded for student strategies and gather as much data as possible, we employed quota sampling (Emmel, 2013) as a technique to select interviews in a systematic way. Before identifying the smaller set of interviews, we assigned a number range to each student interview based on the number range that was used most often during the interview; see the Numeric Relational Reasoning Cognitive Interviews: Methods and Quantitative Data Analyses technical report for information on number ranges used in interviews (Technical Report No. 20-05). The distribution of selected interviews across different grade levels and number
ranges is displayed in Tables 1 and 2. Distribution of all selected interviews is available in Appendix A. We combined grades 2 and 3 together because grade 3 students were intended to represent end-of-year second-grade students.

## Table 1

Number of Properties of Operations Interviews

|  | Number Range |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Grade Level | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ | Grand Total |
| Kindergarten | 6 | 4 |  |  |  |  | 10 |
| Grade 1 |  | 6 | 4 |  |  |  | 10 |
| Grade 2 |  | 1 | 1 | 2 | 5 | 1 | 10 |
| Grade 3 |  |  |  |  | 1 | 1 | 2 |
| Grand Total | 6 | 11 | 5 | 2 | 6 | 2 | 32 |

## Table 2

Sample of Properties of Operations Interviews

|  | Number Range |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Grade Level | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ | Grand Total |
| Kindergarten | 2 | 2 |  |  |  |  | 4 |
| Grade 1 |  | 2 | 2 |  |  |  | 4 |
| Grade 2 |  | 0 | 0 | 1 | 2 | 0 | 3 |
| Grade 3 |  |  |  |  | 0 | 1 | 1 |
| Grand Total | 2 | 4 | 2 | 1 | 2 | 1 | 12 |
| $\quad$ Selected | 768 | 495 | 152 | 993 | 284 | 676 |  |
| $\quad$ SIDs | RK | 385 | 793 |  | 563 |  |  |
|  |  | 946 |  |  |  |  |  |
|  |  | 223 |  |  |  |  |  |

One student was selected from each number range per grade level from which we had a minimum of four Properties of Operations interviews. For the Relations and Composition and Decomposition targeted learning goals, three interviews were selected from each. The chain of decisions is as follows:

- Are there more than two interviews in a cell?
- $\mathrm{No}=$ Use those two interviews.
- Yes $=$ Use the two interviews with the highest number of ESSs represented.
- Is there a tie in the number of ESSs?
- $\mathrm{No}=$ Use those two interviews.
- $\quad$ Yes $=$ Use the interview with the higher number of skills within the targeted number range.
- If there is still a perfect match between interviews, use the interview of the student with a lower support level.

As a final decision point, the student with a lower support level was chosen for analysis; the overall project goal is that assessment use will provide teachers with meaningful data to make instructional decisions, even if students are not at a lower proficiency level. We only included interviews in the data set, where at least three-fourths of the interview protocol was completed. The final number of interviews selected for qualitative analysis is shown in Table 3.

## Table 3

Quota sample of interviews by grade level

| Subcomponent | K | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| Relations | 2 | 1 | 3 | 0 |
| Composition/Decomposition | 2 | 3 | 1 | 0 |
| Properties of Operations | 4 | 4 | 3 | 1 |

## Method

As informed by literature and experience, we anticipated multiple strategies for each subcomponent that elementary students may use to reason when working on given problems. We created a list of anticipated strategies for each subcomponent and included them with the subcomponent statements in interview protocols. During the analysis of cognitive interview data, our team identified some strategies students used that were not listed among anticipated strategies. Therefore, we created a list of such unanticipated strategies for each subcomponent. The purpose of collecting and reporting unanticipated strategies was to determine whether any prevalent student strategies may need to be considered for inclusion in conceptualizing the learning progression.

## Anticipated Strategies Codebook for Each Subcomponent

For each subcomponent, anticipated strategies were identified based on a literature review and the teaching and research experience of MMaRS team members. A table was developed for each subcomponent by adding anticipated strategies into the interview protocols. These tables were compiled in a document we referred to as the anticipated strategies codebook. Each table contained the following:

- Subcomponent number and description - This section carried identifying alphanumeric codes and descriptions of each subcomponent, as given in the interview protocols.
- Content question and reasoning question - All content and reasoning questions from the interview protocol were listed in this section of the table.
- Embedded and general tools relevant to the subcomponent - These were all mathematical tools used during the interviews, either embedded in the item or provided to students during the interview.
- Different way and other NRR subcomponents - "A different way" and other NRR subcomponents were created to list any unanticipated strategies students used and reference other subcomponents that were evident from student's responses.

An example of an anticipated strategies codebook is shown in Figure 1. The Anticipated Strategies Codebook is shown by targeted learning goal in Appendices B, C, and D as part of the Integrated Strategies document for Relations, Composition and Decomposition, and Properties.

## Figure 1

## Anticipated Strategies Codebook

NRR.A.1.a. Anticipated Strategies

| NRR.A.1.a. | Compare o quantities to find which is more/less using matching and counting strategies. |  |
| :---: | :---: | :---: |
| Content Question | (1) Find a day where Carla saw less clouds than she saw on Sunday. <br> (2) Find a day where Carla saw more clouds than she saw on Sunday. |  |
| Reasoning Question | Can you tell me or show me how you decided that this picture has more/less? |  |
| Anticipated Skills |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A1a_E_NOT-Cards | Pictures of groups of clouds are embedded into the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A1a_A_General tool | Child uses a general tool as listed above. |
|  | A1a_A_Count all | Child counts all clouds individually. [Counting] |
|  | A1a_A_Count by 2 s | Counting. Child counts clouds in groups of 2; ("2, 4, 6, 8") [Counting] |
|  | A1c_A_One to one (Individuals) | Child pairs each cloud on one card with a cloud on other card. [One-to-One correspondence of individual objects] |
|  | A1c_A_One to one (Groups) | Child pairs each group of clouds on one card with a group of clouds on other card. [One-to-One correspondence of Groups of objects] |
|  | A1c_A_Count pairs | Child counts pairs of corresponding clouds on each card ("1, 2, 3, 4") [Counting]. |
|  | Count Groups | Child counts number of groups to compare, not attending to individual clouds. |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements | Other NRR Skill Codes | NRR.C.8.a, NRR.C.8.b, NRR.C.8.c |

## Qualitative Analysis of Selected Subset of Data

The MMaRS coding team used an adapted form of interaction analysis (Jordon \& Henderson, 1995) to conduct the qualitative analysis of cognitive interview data. Traditionally, interaction analysis involves researchers from different projects meeting weekly to share a group analysis video. After introducing the research and video, the group watches and "stop[s] the tape whenever anything strikes them as significant" (p. 49). A collection of tapes is then compiled from edited clips to provide evidence of the phenomenon of interest.

For the MMaRS cognitive interviews, two teams, each consisting of a lead coder who provided an emic, or insider, perspective and a secondary coder providing an etic, or outsider, perspective analyzed the cognitive interviews. Lead coders were immersed in the learning progression development process, interview protocol development, and served as interviewers during the cognitive interviews. Each pair's secondary coder had little to no involvement with the learning progression development, interview protocol development, or interview process. Both the lead coder and secondary coder had sufficient content and pedagogical knowledge within the NRR construct.

The teams began by dividing each interview transcript into subcomponents and adding relevant non-verbal actions into the text to understand students' work better. Individual student talk turns, or lines of the text that signified students' speech, were identified as the unit of coding. Using the anticipated strategies codebook, coding teams marked strategies followed a coding guide, which included the following four steps:

- Step 1-Locate content and reasoning questions for each subcomponent: Both coders on a team read the interview transcripts simultaneously for each subcomponent to identify content and reasoning questions.
- Step 2-Identify and mark student strategies demonstrated by students in their responses: The two coders identified strategies evident in students' responses. By analyzing transcripts for the same subcomponent, the teams were able to employ a constant comparative method (Creswell, 2013) to revise and refine coded student strategies. While interacting with the transcripts, teams stopped to discuss any student talk turns of interest and decided whether or not the video was necessary for additional contextual information before moving on. The discussion was limited to five-to-seven minutes to mitigate lengthy discussions and ensure arguments were grounded in empirical evidence (Jordon \& Henderson, 1995). If a video was consulted, then all of that child's mathematical actions were documented to add context to the talk turn. Interviewer talk turns were considered as supporting evidence when a child used gestures or writing but did not provide any verbal explanations.
- Step 3-Create descriptions for any unanticipated strategies: Using the anticipated strategies codebook, students' strategies were coded as anticipated or unanticipated. After coding all selected transcripts for a subcomponent, team members discussed and reviewed coded strategies, anticipated and unanticipated, to ensure consistency of codes across time. Then coders crafted descriptions for all unanticipated strategies and compiled them by subcomponent. All anticipated and unanticipated strategies were collected in tables by grade level and number range within a subcomponent.
- Step 4-Extract examples of each unanticipated strategy from students' work: At the last step, coders added an example of students' strategies from the interview transcripts to describe the unanticipated strategies.


## Results

To understand the anticipated and unanticipated strategies students used during the cognitive interviews, we grouped and summarized all anticipated and unanticipated strategies for each subcomponent. We compiled the strategy summaries by subcomponent to provide teachers and leaders with information about what strategies students employ in early grades to solve numerical relational problems. The purpose of collecting various students' strategies was to understand elementary students' numeric relational reasoning characteristics within one-on-one cognitive interviews and to conceptualize all subcomponents of each targeted learning goal. Using the summary tables of anticipated and unanticipated strategies, we compiled synthesis tables that show which strategy was used more often for each subcomponent.

## Summary of Strategies

We summarized the anticipated and unanticipated strategies that students used when solving problems presented in the cognitive interview tasks. We compiled summaries of these strategies in tables for each subcomponent within each core concept.

The summary section for each subcomponent consists of four tables. The first table includes the subcomponent code, description, and grade range from the learning progression-it shows the subcomponent statement and the grade level boundaries. The second table, called "unanticipated strategies," includes the subcomponent, its description, and example(s) of all unanticipated strategies that coders observed for that subcomponent during interviews. Coders wrote this information after achieving an exact agreement. The examples of unanticipated strategies were the selected text from interview transcripts with student id and timestamp. The third and fourth tables carry all unanticipated and anticipated strategies listed by grade level and number ranges. Each table has names of all strategies along with student IDs within each grade level and number range. A snapshot of a summary document is shown in Figure 2.

We added subcomponent code, description, and appropriateness for a specific grade level in the first section of the summary table so that readers can link strategies to subcomponents and grade levels. In the second section, we included the name, description, and an example of all unanticipated strategies for the subcomponent. In the last section of the table, we added anticipated and unanticipated strategies for each grade level by their number ranges. Summaries for all subcomponents are available as part of integrated strategies documents in Appendices B, C, D for Relations, Composition \& Decomposition, and Properties, respectively.

## Synthesis of Strategies

To illuminate strategies that appeared more or less frequently within a core concept, we developed synthesis tables. The synthesis tables are simplified versions of the information presented in the summary tables. We organized synthesis tables by core concept; they show the individual and the total number of instances in which the given strategy was used within a subcomponent. Additionally, the synthesis tables reflect which strategies appeared at which grade level and number range more often. A snapshot of a synthesis of strategies table is shown in Figure 3.

## Figure 2

Organization of Summary Document

NRR_A.1.a. Summary
Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B | T |

Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Context. Students compared the open spaces or <br> on the cards rather than focusing on the <br> quantity of clouds. The example provided <br> demonstrates a child who used the house as a <br> reference point for determining more/less. | Well, I could tell that this one is more because it <br> has more sides...and then there's more around the <br> house than these (128, 9:19, 9:28). |
| A.1.c. Compare two quantities to find which is <br> more/less using mental images. | Because it looks bigger than that one... because it <br> has more clouds [child stated that Tuesday has <br> more and looks bigger than Monday, but did not <br> define Monday's quantity](337,07:08-07:20). |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (645) |  |  |  |  |  |
| 1 |  | A.1.C. (337) |  |  |  |  |
| 2 |  |  |  |  | Context (128) | - |

Unanticipated strategies by grade level and number range for NRR.A.1.a.

| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Count all (805) |  |  |  |  |  |
| 1 |  | - |  |  |  |  |
| 2 |  |  |  |  | - | Count groups <br> $(284,993)$ |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.1.a. activity

Synthesis tables for all core concepts are available as part of integrated strategies documents in Appendices B, C, D for Relations, Composition and Decomposition, and Properties, respectively. We created the integrated strategies document by combining the anticipated strategies codebook, summary of strategies document, and synthesis of strategies document. We organized the integrated strategies document by core concept. For each core concept, synthesis tables of anticipated and unanticipated strategies are provided first, then summary tables for anticipated and unanticipated strategies by subcomponent. Lastly, the anticipated strategies codebook tables for each subcomponent are provided.

## Organization of Strategies by Subcomponent

We used the integrated strategies document to compile the anticipated and unanticipated strategies students used by subcomponent. The purpose was to find which strategies appeared across multiple subcomponents. This information revealed which strategies are shared across subcomponents.

## Figure 3

Organization of Synthesis of Strategies

NRR.A.2. Synthesis Data
Unanticipated strategies: NRR.A.2.a-b

| Skill Code |  |  | A.2.a. | A.2.b. |
| :---: | :---: | :---: | :---: | :---: |
| Unanticipated |  |  | Counting |  |
| SID | Grade | Number Range |  |  |
| 645 | K | 0-5 | - | - |
| 805 | K |  | - | - |
| 337 | 1 | 0-10 | - | On |
| 128 | 2 | 0-99 | All | Down |
| 284 | 2 | 0-199 | All | Down, On |
| 993 | 2 |  | - | - |
| Strategies Total by Skill Code |  |  | 2 | 4 |

Anticipated strategies: NRR.A.2.a-b

| Skill Code <br> Anticipated Strategies |  |  |  | 2.a. | A.2.b. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | $\begin{aligned} & \hline \text { 100s } \\ & \text { Chart } \end{aligned}$ | Counting | - |
| SID | Grade | Number Range |  |  |  |
| 645 | K | 0-5 | - |  | - |
| 805 | K |  | - | All, On | - |
| 337 | 1 | 0-10 | 1 |  | - |
| 128 | 2 | 0-99 | 1 |  | - |
| 284 | 2 | 0-199 | 1 |  | - |
| 993 | 2 |  | 1 |  | - |
| Strategies Total by Skill Code |  |  | 4 | 2 | 0 |

All strategies were compiled in a spreadsheet with the following columns: Strategy Code, Strategy Description, Examples (only for unanticipated strategies), and subcomponent. For each subcomponent, a coder copied all anticipated strategy codes and their description into the relevant columns. The coder copied the name, description, and examples of all unanticipated strategies in appropriate columns in the next step. The number one was entered for anticipated strategies in the subcomponent columns, and the number two was entered for unanticipated strategies.

After completing a subcomponent, the coder moved to the next subcomponent in the Integrated Strategies document and repeated the above process. If a strategy code with the same definition already exists in the excel sheet, it was used for the next subcomponent as well; otherwise, a new entry was made in the excel sheet. A snapshot of the Strategies by Subcomponent spreadsheet is shown in Figure 4.

If any strategy appeared in the strategy tables without a definition, then the strategy's code was entered without a definition. In the description cell, it was noted that the description of this code is not available. If any subcomponent appeared as a strategy for any other subcomponent, then its name was entered as the subcomponent (e.g., A.4.c). After the primary coder completed an entire protocol, another team member verified the error-free transfer of information for $50 \%$ of the subcomponents. If an error was found, then the primary coder was informed, and the primary coder fixed the mistake and re-check all entries before sending it back for verification.

During the process of creating the strategies by subcomponent spreadsheet, it was identified that some strategies had the same strategy code name with similar definitions, like Number Relationships shown in Figure 4. Upon further review, where instances of the strategy code name and definition being closely aligned, the rows in the spreadsheet were combined, and the
definition was refined to focus less on the specific subcomponent and more on the generalization of the strategy.

Figure 4
Organization of Strategies by Subcomponent

| Strategy Code | Strategy Description | Example (only unanticipated strategies will hae an example) | $\begin{array}{\|c\|c\|} \hline \text { NRR. } \\ \text { A.3.a. } \end{array}$ | $\begin{array}{\|c\|c\|} \hline N R R . \\ \text { A.3.b. } \end{array}$ | $\begin{array}{\|l\|} \hline \text { NRR. } \\ \text { A.3.C } \end{array}$ | $\begin{array}{\|c\|} \hline \text { NRR. } \\ \text { A.3.d. } \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { NRRR. } \\ \text { A.3.e.e. } \\ \hline \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { NRRR. } \\ \text { A.3.f.f } \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \text { NRRR. } \\ \text { A.4.a } \\ \hline \end{array}$ | $\left\|\begin{array}{c} \text { NRR. } \\ \text { A.4.b } \end{array}\right\|$ | $\begin{array}{\|l\|} \mid N R R . \\ \text { A.4.C } \end{array}$ | $\begin{aligned} & \text { NRR. } \\ & \text { A.4.d } \end{aligned}$ | $\begin{aligned} & \text { NRR. } \\ & \text { A.4.e } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. Relationships | Child uses language of relationship like less/more, smaller/bigger, left/right |  |  |  |  |  |  |  |  | 1 |  |  |  |
| Number of Digits | Students focused on the number of digits on each card to identify the value of the number on the card. It is different from place value concept because students didn't refer to place value or compare digits at a specific place value on each card. Additionally, we didn't have a concept of place value in Relations protocol. | Because these [points to 76] have two digits, these [pointing to 100 and 103] have three $(128,49: 58$; NRR.A.3.e.). <br> Cause this [points to 76] has two numbers and this one [points to 100] has three, so I knew that this one [76] would go before this one [100] (993, 31:33; NRR.A.3.e.). <br> Student said: "It tells me that, cause this one (pointing to 107) has three, and this one (pointing to 78) only has two (128, 53:12; NRR.A.3.f.) |  |  |  |  | 2 | 2 |  |  |  |  |  |
| Number <br> relationships (1) | Child uses language of relationship like less/more, smaller/bigger, left/right |  |  |  |  | 1 |  |  |  |  |  | 1 |  |
| Number relationships (2) | Definition does not exist, only in data, not in description table. | (337; 128; 993; NRR.A.3.e.) |  |  |  |  | 1 |  |  |  |  |  |  |

## Conclusion

We analyzed a subset of cognitive interview data to find evidence of various strategies in elementary students' responses when solving numeric relational reasoning problems. Apart from anticipated strategies, elementary students used many strategies that we did not anticipate before analysis. The purpose of compiling anticipated and unanticipated strategies is for the conceptualization of the learning progression. Our work of anticipated and unanticipated strategies can also inform teachers and leaders about various strategies that elementary students tend to use to work on numerical relational problems. Additionally, we also reported on multiple strategies students frequently used across subcomponents.

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## Appendix A - Student Strategies Sampling Plan

## Relations Sampling Plan



| SID | $\begin{aligned} & \hline 645 \\ & 805 \end{aligned}$ | $\begin{aligned} & 337 \\ & 946 \end{aligned}$ |  |  | 128 | $\begin{aligned} & 993 \\ & 284 \end{aligned}$ |  | questions asked in targeted number range. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Table 7 Sample of Rel | ations | intervi | ws. |  |  |  |  | Exclude interviews that were less than $3 / 4$ completed. <br> One video was excluded based on this criterion. | To optimize number of videos coded and gather as much data as possible, video selection will consider how much of the interview was completed. Longer interviews will provide data points across more essentialized skill statements. |
|  | Colu | mn La | els: N | mber | ange |  |  |  |  |
| Grade <br> Level |  | 0-10 | 0-19 | 0-50 | 0-99 | 0-199 | Grand <br> Total |  |  |
| Kindergarten Grade 1 | 2 | 1 | $\begin{aligned} & 0 \\ & 0 \end{aligned}$ | $0$ |  |  | 2 1 |  |  |
| Grade 2 |  |  |  | 0 | 1 | 2 | 2 |  |  |
| Grade 3 |  |  |  |  |  | 0 | 0 |  |  |
| Grand <br> Total | 2 | 1 | 0 | 0 | 1 | 2 | 6 |  |  |
| SID | $\begin{aligned} & 645 \\ & 805 \end{aligned}$ | $337$ |  |  | 128 | $\begin{aligned} & 993 \\ & 284 \end{aligned}$ |  |  |  |

* Interview 946 excluded because it was less than $3 / 4$ completed.


## Composition \& Decomposition Sampling Plan



|  | Exclude interviews that were <br> less than $3 / 4$ completed. <br> No videos were excluded based <br> on this criterion. | video selection will consider how <br> much of the interview was <br> completed. Longer interviews will <br> provide data points across more <br> essentialized skill statements. |
| :--- | :--- | :--- |

## Properties of Operations Sampling Plan



|  | Exclude interviews that were <br> less than $3 / 4$ completed. <br> No videos were excluded based <br> on this criterion. |  |
| :--- | :--- | :--- |

## Appendix B - Integrated Strategies Document: Relations ${ }^{1}$

## NRR.A.1. Comparison Core Concept



[^0]Unanticipated strategies: NRR.A.1.a-g

| Skill Code |  |  | A.1.a. | A.1.b. | A.1.c. | A.1.d. | A.1.e. | A.1.f. | A.1.g. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unanticipated |  |  | - | - | - | Value of Digits |  |  |  |
| SID | Grade | Number Range |  |  |  |  |  |  |  |
| 645 | K | 0-5 | - | - | - | - | - | - | - |
| 805 | K |  | - | - | - | - | - | - | - |
| 337 | 1 | 0-10 | - | - | - | 1 | - | - | - |
| 128 | 2 | 0-99 | - | - | - | 1 | 1 | 1 | 1 |
| 284 | 2 | 0-199 | - | - | - | 1 | - | - | 1 |
| 993 | 2 |  | - | - | - | - | - | 1 | - |
| Strategies Total by Skill Code |  |  | 0 | 0 | 0 | 3 | 1 | 2 | 2 |

Anticipated strategies: NRR.A.1.a-d

| Skill Code |  |  | A.1.a. |  |  |  | c. | A.1. d. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategy |  |  | Counting | Height | Weight | NME | Counting |  |
| SID | Grade | Number Range |  |  |  |  |  |  |
| 645 | K | 0-5 | - | - | - | 1 | - | All |
| 805 | K |  | All | - | - | - | All | On |
| 337 | 1 | 0-10 | - | 1 | 1 |  | Groups | - |
| 128 | 2 | 0-99 | - | - | 1 | - | Skip (3) | - |
| 284 | 2 | 0-199 | Groups | - | - | - | Groups | - |
| 993 | 2 |  | Groups | 1 | - | - | Groups | - |
| Strategies Total by Skill Code |  |  | 3 | 2 | 2 | 1 | 5 | 2 |

Anticipated strategies: NRR.A.1.e-g

| Skill Code <br> Anticipated Strategy |  |  | A.1.e. |  |  | A.1.f. |  |  |  | A.1.g. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Written Number | NME | Left to Right | Create WNL | Left to Right | NME | Counting | NME | Unknown Meaning of | Left to Right |
| SID | Grade | Number Range | Line (WNL) |  |  |  |  |  |  |  | Inequality Symbols |  |
| 645 | K | 0-5 | - | 1 | - | - | - | 1 | - | - | 1 | - |
| 805 | K |  | - | - | - | 1 | - | - | All | 1 | - | - |
| 337 | 1 | 0-10 | Hashmarks | - | 1 | 1 | 1 | - | - | - | 1 | 1 |
| 128 | 2 | 0-99 | - | - | - | 1 | - | - | - | - | - | - |
| 284 | 2 | 0-199 | Intervals | - | - | 1 | 1 | - | 0 | - | - | - |
| 993 | 2 |  | Hashmarks Intervals | - | 1 | 1 | - | - | - | - | 1 | - |
| Strategies Total by Skill Code |  |  | 4 | 1 | 4 | 5 | 2 | 1 | 1 | 1 | 3 | 1 |
| Strategies Total by Core Concept: Counting: 10; |  |  |  | No Mathematical Evidence (NME): 4; |  |  |  |  | Left to Right: 7 |  |  |  |

NRR.A.1.a. Summary

## Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | B |
| Compare two quantities to find which is more/less using matching and |  |  |  |  |  |  |  |
| counting strategies. |  |  |  |  |  |  |  |

## Student Expectation

Students are expected to use both matching and counting strategies to compare the quantity of clouds in images on cards. Students are expected to justify reasoning with examples of how they know there are (or are not) the same number of clouds, which may include adding the totals or counting groups.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Context. Students compared the open spaces or <br> on the cards rather than focusing on the quantity <br> of clouds. The example provided demonstrates a <br> child who used the house as a <br> reference point for determining more/less. | Well, I could tell that this one is more because it <br> has more sides... and then there's more around the <br> house than these (128, 9:19, 9:28). |
| A.1.c. Compare two quantities to find which is <br> more/less using mental images. | Because it looks bigger than that one... because it <br> has more clouds [child stated that Tuesday has <br> more and looks bigger than Monday, but did not <br> define Monday's quantity](337, 07:08-07:20). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (645) |  |  |  |  |  |
| $\mathbf{1}$ |  | A.1.c. (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | Context (128) | - |

Unanticipated strategies by grade level and number range for NRR.A.1.a.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Count all (805) |  |  |  |  |  |
| $\mathbf{1}$ |  | - |  |  |  |  |
| 2 |  |  |  |  | - | Count groups <br> $(284,993)$ |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.1.a. activity

NRR.A.1.a. Anticipated Strategies

| NRR.A.1.a. | Compare o quantities to find which is more/less using matching and counting strategies. |  |
| :---: | :---: | :---: |
| Content Question | (1) Find a day where Carla saw less clouds than she saw on Sunday. <br> (2) Find a day where Carla saw more clouds than she saw on Sunday. |  |
| Reasoning Question | Can you tell me or show me how you decided that this picture has more/less? |  |
| Anticipated Skills |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A1a_E_NOT-Cards | Pictures of groups of clouds are embedded into the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A1a_A_General tool | Child uses a general tool as listed above. |
|  | A1a_A_Count all | Child counts all clouds individually. [Counting] |
|  | A1a_A_Count by 2s | Counting. Child counts clouds in groups of 2; ("2, 4, 6, 8") [Counting] |
|  | A1c_A_One to one (Individuals) | Child pairs each cloud on one card with a cloud on other card. [One-to-One correspondence of individual objects] |
|  | A1c_A_One to one (Groups) | Child pairs each group of clouds on one card with a group of clouds on other card. [One-to-One correspondence of Groups of objects] |
|  | A1c_A_Count pairs | Child counts pairs of corresponding clouds on each card ("1, 2, 3, 4") [Counting]. |
|  | Count Groups | Child counts number of groups to compare, not attending to individual clouds. |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill Statements | Other NRR Skill Codes | NRR.C.8.a, NRR.C.8.b, NRR.C.8.c |

## Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{F}$ | B | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ |
|  |  |  |  |  |  |  |  |

## Student Expectation

Students are expected to use a physical balance and explain how to determine one side weighs more than the other side. Students may use terms such as up/down, heavier/lighter, or tipping to describe the balance in regards to the height of a side and/or the weight of a side.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Size. Child did not refer to the balance. Instead <br> child referred to the size of the objects. | [The bear] is bigger ... [The cup] is little. (805, 3:53, <br> $4: 22)$. |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (645) <br> Size (805) |  |  |  |  |  |
| $\mathbf{1}$ |  | - |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | - | Skipped (284) |

Unanticipated strategies by grade level and number range for NRR.A.1.b.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - |  |  |  |  |  |
| $\mathbf{1}$ |  | Height (337) <br> Weight (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | Weight (128) | Height (993) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.1.b. activity.

NRR.A.1.b. Anticipated Strategies

| NRR.A.1.b. | Compare two unspecified weights using balances to find which weighs more/less. |  |
| :---: | :---: | :---: |
| Content Question | (1) Which object weighs the most? <br> (2) Which object weighs the least? |  |
| Reasoning Question | NOT REQUIRED |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A1b_E_NOT-Balance | Different objects to check their weights using the given balance. (e.g., cube, toy, tape, paper clip, cup) |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
| Strategies | Code | Description |
|  | A1b_A_Weight | Child directly focuses on weight of each object and tells which object is heavier than other object |
|  | A1b_A_Height | Child first describes the position of balance (one side up other side down) then may links the position to the weight of the object. (It's not necessary to link height with weight) |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill Statements | Other NRR Skill Codes |  |

## NRR.A.1.c. Summary

Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B | T |
|  |  | Compare two quantities to find which is more/less using mental images. |  |  |  |  |  |  |

## Student Expectation

Students are expected to use mathematical reasoning and/or counting strategies to compare the quantity of clouds in images on cards. Students are expected to justify reasoning with examples of how they know there are (or are not) the same number of clouds, which may include adding the totals or counting groups.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| B.5.b. Compose a number with two parts. | ..this one's four and then you ot two more, its six <br> $(337,10: 08)$. |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :--- | :--- | :--- | :--- | :---: |
| $K$ | - |  |  |  |  |  |
| 1 |  | B.5.b. (337) |  |  |  |  |
| 2 |  |  |  |  | - | - |

Unanticipated strategies by grade level and number range for NRR.A.1.c.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Count all (805) <br> General tools <br> (805) <br> NME (645) |  |  |  |  |  |
| $\mathbf{1}$ |  | Count groups <br> (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | Count groups (128) <br> Compare groups <br> (128) | Count groups <br> (284, 993) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.1.c. activity.

| NRR.A.1.C. | Compare two quantities to find which is more/less using mental images. |  |
| :---: | :---: | :---: |
| Content Question | Is he/she correct? Why or why not? |  |
| Reasoning Question | Already embedded in content question. |  |
| Anticipated Skills |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A1c_E_NOT-Cards | Pictures of groups of clouds are embedded into the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A1c_A_General tool | Child uses a general tool as listed above. |
|  | A1c_A_Count all | Child counts all clouds individually. [Counting] |
|  | A1c_A_Count by 2 s | Counting. Child counts clouds in groups of 2; ("2, 4, 6, 8") [Counting] |
|  | A1c_A_One to one (Individuals) | Child pairs each cloud on one card with a cloud on other card. [One-to-One correspondence of individual objects] |
|  | A1c_A_One to one (Groups) | Child pairs each group of clouds on one card with a group of clouds on other card. [One-to-One correspondence of Groups of objects] |
|  | A1c_A_Count pairs | Child counts pairs of corresponding clouds on each card ("1, 2, 3, 4") [Counting]. |
|  | Count groups | Child counts the number of groups on the card. |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill Statements | Other NRR Skill Codes | NRR.C.8.a, NRR.C.8.b, NRR.C.8.c |


| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | $\mathbf{T}$ | F | B | $\mathbf{T}$ | F | B | $\mathbf{T}$ |
| Compare two numbers using mental number lines to determine which is more/less. |  |  |  |  |  |  |  |  |

## Student Expectation

Students are expected to recognize numbers and their value using number cards. Students are then expected to determine which numbers have greater value and compare multiple numbers in a row.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Value of digits. Children compared the <br> value of digits within numbers without <br> explicitly using place value. Children <br> correctly aligned numbers in the tens <br> and ones places when comparing <br> without explaining place value or <br> providing a unit value distinction <br> between the two numbers. | [Comparing 42 with 44 and 37 with 42]. Because 2 is less |
| [Comparing 37 with 42]. Because it's (37) the first number |  |
| you come to before 40 and 60 (284, 09:36). |  |$\quad$| [Comparing 5 and 7]. Because whenever I look at the |
| :--- |
| numbers, 5 is on over here and 7 is more up, that its more |
| further (337, 11:09). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - |  |  |  |  |  |
| $\mathbf{1}$ |  | Value of digits (337) |  |  |  |  |
| 2 |  |  |  |  | Value of <br> digits (128) | Value of digits (284) <br> A.3.e. (284) |

Unanticipated strategies by grade level and number range for NRR.A.1.d.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | General tool (645, 805) <br> Count all (645) <br> Count on (805) |  |  |  |  |  |
| $\mathbf{1}$ |  | $=$ |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | $=$ | - |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.1.d. activity.

NRR.A.1.d. Anticipated Strategies

| NRR.A.1.d. | Compare two numbers using mental number lines to determine which is more/less. |  |
| :---: | :---: | :---: |
| Content Question | (1) Is [numbe <br> (2) She saw than [num <br> (3) She saw [ more or le | for Monday] more or less than [number for Sunday]? <br> mber for Tuesday] butterflies on Tuesday [hold the Tuesday card]. Is that more or less er for Sunday]? <br> mber for Wednesday] butterflies on Wednesday [hold the Wednesday card]. Isthat than [number for Sunday]? |
| Reasoning Question | How do you know that [student's response number] is greater/less than [number]? |  |
| Anticipated Skills |  |  |
|  | Code | Description |
| Embedded <br> Mathematical Tools | A1d_E_NOT-Cards | No physical tools were embedded. Students were expected to use mental number line. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A1d_A_Place Value | Child uses Ones, Tens, , and hundreds places to compare number. |
|  | A1d_A_No. of digits | Child compare number of digits in each numbers. |
|  | A1d_A_Symbolic | Child recognizes symbolic representation of numbers without associating pictorial representation |
|  | Counting | Count on or count all when using a general tool as concrete representation of numbers |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements | Other NRR Skill Codes |  |

Essentialized Skill Statement

| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{F}$ | B | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ |
| Compare two numbers using written number lines to determine which is <br> more/less. |  |  |  |  |  |  |  |  |

## Student Expectation

Students are expected to determine where on a written number line two given numbers are placed. Students are then expected to determine based on their placement and relationship to the other number on the number line, which number is greater.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Value of digits. Children compared the value of digits <br> within numbers without explicitly using place value. <br> Children correctly aligned numbers in the tens and ones <br> places when comparing without explaining place value or <br> providing a unit value distinction between the two <br> numbers. | [Comparing 48 with 24]. Everyone knows <br> that 2 is less than 4 and 8 is more than 4 as <br> well (128, 19:20). |
| A.2.a. Without counting, use tools to find a unit <br> more/less than a given number. | Because its 10 below 130 (993, 11:20). |


| Grade | $0-5$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - |  |  |  |  |  |
| $\mathbf{1}$ |  | - |  |  |  |  |
| 2 |  |  |  |  | Value of <br> digits (128) | A.2.a. (993) |

Unanticipated strategies by grade level and number range for NRR.A.1.e.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (645) <br> Count all (805) |  |  |  |  |  |
| $\mathbf{1}$ |  | Hashmarks <br> (337) <br> LTR (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  | Hashmarks <br> (128) | Intervals (284, 993) <br> Hashmarks (993) <br> LTR (993) |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.1.e. activity.

| NRR.A.1.e. | Compare two numbers using written number lines to determine which is more/less. |  |
| :---: | :---: | :---: |
| Content Question | (1) Rene saw [number]stars at night. [Show card for Rene.] Rene wants to put her number on this number line. Can you show me where you would put this number on the number line? [Place the card on the number line.] Can you tell me why you put it here? <br> (2) Olivia saw [number]stars at night. [Show card for Olivia.] Can you show me where you would put this number on the number line? [Place the card on the number line.] Can you tell me why you put it here? <br> (3) Which person saw more stars? How do you know that [Olivia saw more than Rene OR Rene saw more than Olivia]? <br> (4) Can you tell me a number of stars that is less than [Rene's number]? How do you know that [student's response] is less than [Rene's number]? |  |
| Reasoning Question | Reasoning questions (italicized questions) are included in each content question row. |  |
| Anticipated Skills |  |  |
|  | Code | Description |
| Embedded <br> Mathematical Tools | A1e_E_NOT-Written Number Line | Written number line (with intervals marked) is embedded in the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A1e_A_Intervals | Child uses only marked intervals on the number line. |
|  | A1e_A_Hash Marks | Child uses hash marks on the number line. |
|  | A1e_A_Unit Places | Child uses unit, tenth, hundred places to compare numbers |
|  | A1e_A_Digits | Child compare number of digits in each numbers. |
|  | LTR | Left to right reading of numbers and ordering across number line |
|  | Counting | Count all or on, students use counting strategies when comparing numbers and ordering |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements | Other NRR Skill Codes |  |

## Essentialized Skill Statement

| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{F}$ | B | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | F | B | $\mathbf{T}$ |
| Compare two numbers using open number lines to determine which is <br> more/less. |  |  |  |  |  |  |  |  |

## Student Expectation

Students are expected to place numbers in order on an open number line (ONL) from while attending to an approximate distance between numbers to determine which number is more/less.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Value of digits. Child compared the value of <br> digits within numbers without explicitly using <br> place value. While it could be inferred that <br> place value is part of the child's rationale, the <br> child did not use terminology to indicate an <br> understanding. | [Comparing 54 and 40] Because four is less than five <br> and well, it's a zero so four is more than zero (128, <br> $22: 20)$. | | [Explaining why 50 is less than 54] 'Cause 50 has a 0 |
| :--- |
| right here and not a 4, and 0 is less than 4 (993, |
| $13: 34)$. |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - |  |  |  |  |  |
| $\mathbf{1}$ |  | - |  |  |  |  |
| 2 |  |  |  |  | Value of digits <br> $(128)$ | Value of digits (993) |

Unanticipated strategies by grade level and number range for NRR.A.1.f.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (645) <br> Create WNL (805) <br> General tool (805) <br> Count all (805) |  |  |  |  |  |
| $\mathbf{1}$ |  | Magnitude (337) <br> Create WNL (337) <br> Friendly numbers (337) <br> LTR (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | Create WNL <br> (128) | ONL (993) <br> Create WNL (284, <br> 993) <br> 9TR (284) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.1.f. activity.

| NRR.A.1.f. | Compare two numbers using open number lines to determine which is more/less. |  |
| :---: | :---: | :---: |
| Content Question | (1) The next night, Rene saw [number]stars at night. [Show card for Rene.] Now, she wants to put her number on this number line. Can you show me where you could put this number on the number line? [Place the card on the number line.] <br> (2) Olivia saw [number]stars at night. [Show card for Olivia.] Can you show me where you would put this number on the number line?[Place the card on the number line.] <br> (3) Who saw more stars? How do you know that [Olivia saw more than Rene OR Rene saw more than Olivia]? <br> (4) Can you tell me a number of stars that is less than [Rene's number]? How do you know that [student's response] is less than [Rene's number]? |  |
| Reasoning Question | Reasoning questions (italicized questions) are included in each content question row. |  |
| Anticipated Skills |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A1f_E_NOT-Cards | Open number lines embedded in the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A1f_A_ Use Unmarked Numbers | Child uses unmarked number line. |
|  | A1f_A_Create WNL | Child marks numbers on the open number line to use it. |
|  | A1f_A_Magnitude | Child focus on the distance between numbers taken into consideration |
|  | A1f_A_Order | Child places smaller numbers on Left side and larger numbers on Right side of the number line |
|  | A1f_A_Jumps | Child marks arcs to show "jump" between numbers (usually associated with skip cunting) |
|  | A1f_A_Friendly Numbers | Child marks the open number line with friendly number (e.g., $0,5,10 \mathrm{etc}$.$) .$ |
|  | LTR | Left to right reading of numbers and ordering across number line |
|  | Counting | Count all or on, students use counting strategies when comparing numbers and ordering |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill Statements | Other NRR Skill Codes | If a child does show another number, consider skill statements in core concept 8. |

Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B | T |
|  |  |  |  |  | Compare two numbers using symbols: >, <. |  |  |  |

## Student Expectation

Students are expected to use inequality symbols to express which number of two given numbers is the greatest. Students are expected to place the inequality symbol correctly between two numbers of different values.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Value of digits. Children compared the value of <br> digits within numbers without explicitly using <br> place value. Children correctly aligned numbers in <br> the tens and ones places when comparing <br> without explaining place value or providing a unit <br> value distinction between the two numbers. | [Comparing 130 and 196] Because there's a nine <br> and a six and three is less than nine and zero is less <br> than six (128, 23:40). |
| Because it's greater than 90 and it has a 6 and it's <br> greater than 0 . And 9 is greater than 3 (284, <br> $17: 29)$. |  |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - |  |  |  |  |  |
| 1 |  | - |  |  |  |  |
| 2 |  |  |  |  | Value of Digits <br> $(128)$ | Value of Digit <br> (284) |

Unanticipated strategies by grade level and number range for NRR.A.1.g.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Unknown <br> meaning (645) <br> NME (805) |  |  |  |  |  |
| $\mathbf{1}$ |  | LTR (337) <br> Unknown <br> meaning (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | - | Unknown meaning <br> (993) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.1.g. activity.

| NRR.A.1.g. | Compare two numbers using symbols: $>,<$. |  |
| :---: | :---: | :---: |
| Content Question | (1) Who saw more stars? <br> (2) Can you use one of these symbols [show symbol cards] to show that [student's response]? |  |
| Reasoning Question | How do you know that [student's response] saw more stars? |  |
| Anticipated Skills |  |  |
|  | Code | Description |
| Embedded <br> Mathematical Tools | A1g_E_NOT- (>,<) <br> Symbols | Greater than and Lesser than symbols ( >, < ) . |
| Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A1g_A_Unknown Meaning | Child doesn't know how to use >,< symbols to compare numbers, but has seen them and/or is willing to interact with them |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements | Other NRR Skill Codes | NRR.A.1.d(Mental Number Line), NRR.A.1.e(Written Number Line), NRR.A.1.f(Open Number Line), |


| 2. Ordinality |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.A.2.a. | Without counting, use tools to find a unit more/less than a given number. |  |  |  |  |  |  |  |  |
| NRR.A.2.b. | Without calculating, mentally find a unit more/less than a given number. |  |  |  |  |  |  |  |  |

## NRR.A.2. Synthesis Data

## Unanticipated strategies: NRR.A.2.a-b

| Skill Code |  |  | A.2.a. | A.2.b. |
| :---: | :---: | :---: | :---: | :---: |
| Unanticipated |  |  | Counting |  |
| SID | Grade | Number Range |  |  |
| 645 | K | 0-5 | - | - |
| 805 | K |  | - | - |
| 337 | 1 | 0-10 | - | On |
| 128 | 2 | 0-99 | All | Down |
| 284 | 2 | 0-199 | All | Down, On |
| 993 | 2 |  | - | - |
| Strategies Total by Skill Code |  |  | 2 | 4 |

Anticipated strategies: NRR.A.2.a-b

| Skill Code |  | A.2.a. |  | A.2.b. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated <br> Strategies | 100s <br> Chart | Counting | - |  |  |
| SID | Grade | Number <br> Range |  |  |  |
| 645 | K | $0-5$ | - |  | - |
| 805 | K | $0-$ | All, On | - |  |
| 337 | 1 | $0-10$ | 1 |  | - |
| 128 | 2 | $0-99$ | 1 |  | - |
| 284 | 2 | $0-199$ | 1 |  | - |
| 993 | 2 | 1 |  | - |  |
| Strategies Total by Skill Code | 4 | 2 | 0 |  |  |

Essentialized Skill Statement

| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{F}$ | B | $\mathbf{T}$ | $\mathbf{F}$ | B | $\mathbf{T}$ | $\mathbf{F}$ | B | T |
| Without counting, use tools to find a unit more/less than a given number. |  |  |  |  |  |  |  |  |

## Student Expectation

Students are expected to use a hundreds-chart to find a given number. Then students are expected to use the number chart to find 1 less, 1 more, 10 more, and 10 less than the given number. Students may or may not count out the numbers in between.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Count all. Children counted numbers on the chart <br> to find the number ten more or less than the <br> starting number. It was anticipated for children to <br> use units, number lines or embedded tools and <br> instead counted numbers to find the answer. | [Explaining how child knows a number is ten <br> more] ...we just start counting. One, two, three, <br> four, five, six, seven, eight, nine (128, 27:17). |
| Because I counted 10 and ended up at 97 (284, <br> $21: 01) .10 ~ m o r e, ~ y o u ~ c o u n t ~ 10 ~ o u t ~ a n d ~ s e e ~ w h a t ~$ <br> number you end up on (284, 22:04). |  |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - |  |  |  |  |  |
| 1 |  | - |  |  |  |  |
| 2 |  |  |  |  | Count all (128) | Count all (284) |

Unanticipated strategies by grade level and number range for NRR.A.2.a.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (645) <br> Count all (805) <br> Count on (805) |  |  |  |  |  |
| $\mathbf{1}$ |  | 100 's chart <br> (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | 100 's chart (128) | 100's chart (284, <br> 993) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.2.a. activity.

| NRR.A.2.a. | Without counting, use tools to find a unit more/less than a given number. |  |
| :---: | :---: | :---: |
| Content Question | (1) Can you show me where [number] is on the chart? <br> (2) What number is 1 less than [number]? How do you know it is one less? <br> (3) What number is 1 more than [number]? <br> (4) What number is 10 more than [number]? How do you know it is 10 more? <br> (5) What number is 10 less than [number]? |  |
| Reasoning Question | Reasoning questions (italicized questions) are included in each content question row. |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools |  | Since ESs explicitly states "Use Tools", code activities based on which tool was used and how |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A2a_A_WNL | Child uses Written Number Line (WNL) to answer the question. |
|  | A2a_A_MNL | Child uses Mental Number Line (MNL) to answer the question. |
|  | A2a_A_ONL | Child uses Open Number Line (ONL) to answer the question. |
|  | 100's chart | Student uses 100's chart to answer the question |
| A different way | 100s Chart | - Child uses 100s chart to answer the question. |
| NRR <br> Essentialized Skill <br> Statements | Other NRR Skill Codes | If a child does show another number, consider skill statements in core concept 8. |

Essentialized Skill Statement

| Kindergarten |  |  |  |  |  |  |  | Grade 1 |  | Grade 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ |  |  |  |
| Without calculating, mentally find a unit more/less than a given number. |  |  |  |  |  |  |  |  |  |  |  |

## Student Expectation

Students are expected to find 1 more, 2 less, and 10 more than a given number using mental strategies and counting. No hundreds-chart provided.

## Unanticipated Strategies

| Unanticipated Strategy. Justification/Description | Examples |
| :---: | :---: |
| Count on. Children counted on from a starting number, saying each number to arrive at the number ten more. While they may have employed a number line strategy, they relied heavily on counting single units. | [Explaining that ten is one more than eight] ...if you count up from one, its nine. Then you skip nine it's gonna be ten (337, 22:28). <br> So that would be... 92, 93.[counting], 99, 100 (284, 25:03). |
| Count down. Children counted backward from a starting number, saying each number to arrive at the number ten less. Some children counted aloud and others wrote the numbers to count down, writing first the ones then the tens place. | 90... and 89, 88... 87, 86... 85, 84, 83 (284, 24:4624:59). <br> Like, the numbers that are before 92 [writes out numbers RTL, ones then tens] 82 (128, 31:4132:57). |
| Value of digits. Children compared the value of digits within numbers without explicitly using place value. Children correctly aligned numbers in the tens and ones places when comparing without explaining place value or providing a unit value distinction between the two numbers. | [Interviewer describes] you were writing these numbers from right to left and ... filling in those numbers... fill in first then ones, then the tens place, right? [student nods affirmative] (128: 33:01). <br> When it's 10 less than a number, usually the first number goes 1 down and the next number stays the same (993, 22:10). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $0-99$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - | Count on (337) |  |  |  |  |
| $\mathbf{1}$ |  |  |  |  | Count down <br> (128) | Value of Digits (993) <br> Count down (284) <br> Count on (284) <br> Calculating (284) |
| $\mathbf{2}$ |  |  |  |  |  |  |

Unanticipated strategies by grade level and number range for NRR.A.2.b.

| Grade | $\mathbf{0 - 5}$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | General tool <br> (805) <br> NME (645) |  |  |  |  |  |
| $\mathbf{1}$ |  | - |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | - |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.2.b. activity.

| NRR.A.2.b. | Without calculating, mentally find a unit more/less than a given number. |  |
| :---: | :---: | :---: |
| Content Question | (1) Please find 1 more than the number $\qquad$ . <br> (2) Please find 2 less than the number $\qquad$ <br> (3) Please find 10 more than the number _. $\qquad$ <br> (4) Please find 10 less than the number $\qquad$ <br> (5) Please find 100 more than the number $\qquad$ |  |
| Reasoning Question | Please show me in pictures, words, or numbers how you found |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded <br> Mathematical Tools | A2b_E_NOT- ??? | No tools embedded, and use of any tool is not anticipated for this activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A2a_A_Calculating | Child use calculation strategy to find a number more/less. |
|  | A2a_A_Mental Number Line | Child uses mental number line to find a unit more/less. |
| A different way | Any Other Tool | - Use of any other strategy/tool (other than Counting and Mental Number Line) will be Unanticipated. |
| NRR <br> Essentialized Skill <br> Statements | Other NRR Skill Codes | If a child does show another number, consider skill statements in core concept 8. |

NRR.A.3. Transitivity Core Concept

| 3. Transitivity |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.A.3.a. | Compare two unspecified lengths (a) and (b) to a given reference length (c) to determine which is longer/shorter (a) or (b). |  |  |  |  |  |  |  |  |
| NRR.A.3.b. | Order unspecified quantities in a word problem. |  |  |  |  |  |  |  |  |
| NRR.A.3.c. | Order three unspecified weights using balances. |  |  |  |  |  |  |  |  |
| NRR.A.3.d. |  |  |  |  | Order three numbers using number relationships with tools. |  |  |  |  |
| NRR.A.3.e. |  |  |  |  | Order three numbers using number relationships without tools (i.e., mental strategies). |  |  |  |  |
| NRR.A.3.f. |  |  |  |  | Order three numbers in a word problem. |  |  |  |  |

## NRR.A.3. Synthesis Data

Unanticipated strategies: NRR.A.3.a-f

| Skill Code |  |  | A.3.a. | A.3.b. | A.3.c. | A.3.d. | A.3.e. | A.3.f. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unanticipated |  |  | Visual | - | - | A.3.e. | Number of Digits |  |
| SID | Grade | Number Range |  |  |  |  |  |  |
| 645 | K | 0-5 | - | - | - | - | - | - |
| 805 | K |  | 1 | - | - | - | - | - |
| 337 | 1 | 0-10 | - | - | - | - | - | - |
| 128 | 2 | 0-99 | 1 | - | - | 1 | 1 | 1 |
| 284 | 2 | 0-199 | - | - | - | - | - | - |
| 993 | 2 |  | - | - | - | 1 | 1 | - |
| Strategies Total by Skill Code |  |  | 2 | 0 | 0 | 2 | 2 | 1 |
| Strategies Total by Core Concept: Number of Digits $=3$ |  |  |  |  |  |  |  |  |

Anticipated strategies: NRR.A.3.a-c

|  |  |  | A.3.a. |  | A.3.b. |  |  | A.3.c. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | Cover | Align End and Mark with Finger | Draw Pictures | NME | Mut Com |  | Height | Weight | NME |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |
| 645 | K | 0-5 | 1 | - | - | 1 | - | - | - | - | 1 |
| 805 | K |  | 1 | - | 1 | - | - | - | - | - | 1 |
| 337 | 1 | 0-10 | - | 1 | 1 | - | 1 | 1 | 1 | 1 | - |
| 128 | 2 | 0-99 | - | - | 1 | - | - | - | - | 1 | - |
| 284 | 2 | 0-199 | - | 1 | 1 | - | 1 | - | 1 | 1 | - |
| 993 | 2 |  | - | 1 | - | - | 1 | 1 | 1 | 1 | - |
| Strategies Total by Skill Code |  |  | 2 | 3 | 4 | 1 | 3 | 2 | 3 | 4 | 4 |

Anticipated strategies: NRR.A.3.d-f

| Skill Code |  |  | A.3.d. |  |  | A.3.e. |  | A.3.f. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | NME | Tools | Number Relationship |  | NME |  | A.3.e. |
| SID | Grade | Number Range |  |  |  |  |  |
| 645 | K | 0-5 | 1 |  | - | - |  |  | 1 | 1 | - |
| 805 | K |  | 1 |  | - | - | 1 | 1 | - |
| 337 | 1 | 0-10 | - | Blank 100's Chart | 1 | 1 | - | - | - |
| 128 | 2 | 0-99 | - |  | - | 1 | - | - | 1 |
| 284 | 2 | 0-199 | - | 100's Chart | 1 | 1 | - | - | 1 |
| 993 | 2 |  | - |  | - | 1 | - | - | 1 |
| Strategies Total by Skill Code |  |  | 2 | 2 | 2 | 4 | 2 | 2 | 3 |
| Strategies Total by Core Concept: Number Relationship $=6$ |  |  |  | Multiple Comparison=5; |  | No Mathematical Evidence = 11; |  |  |  |

## Essentialized Skill Statement

## 3. Transitivity

| Code | Kindergarten |  |  | Grade 1 |  | Grade 2 |  |  |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ |
| NRR.A.3.a. | Compare two unspecified lengths (a) and (b) to a <br> given reference length (c) to determine which is <br> longer/shorter (a) or (b). |  |  |  |  |  |  |  |

## Student Expectation

children were expected to compare length of two objects (two pencils in this case) with a reference to the length of third object (felt strip in this case) and identify which object is longer and which one is shorter.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Visual. If a student didn't use the felt strip as <br> intended but used his/her visual observation to <br> compare the length of two objects, this is an <br> unanticipated strategy to solve this problem. | know it. I know this thing. I don't think about it <br> but I see it (805, 34:53). | | Because it looks more longer than this one and this |
| :--- |
| one looks shorter than this one (128, 36:49). |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Visual (805) |  |  |  |  |  |
| 1 |  | - |  |  |  |  |
| 2 |  |  |  |  | Visual (128) | - |

Unanticipated strategies by grade level and number range for NRR.A.3.a

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Cover <br> $(805,645)$ |  |  |  |  |  |
| $\mathbf{1}$ |  |  <br> Mark w/finger <br> $(337)$ |  |  | Use Middle <br> and Mark <br> w/Pen (128) |  <br> Mark w/finger <br> (993, 284) |
| $\mathbf{2}$ |  |  |  |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.3.a activity.

| NRR.A.3.a. | Compare two unspecified lengths (a) and (b) to a given reference length (c) to determine which is longer/shorter (a) or (b). |  |
| :---: | :---: | :---: |
| Content Question | Which pencil is longer? |  |
| Reasoning Question | How do you know that this pencil is longer? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A3a_E_NOT- Felt Strip | A felt strip to measure length of pencils |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  |  |  |
|  |  |  |
| Strategies | Code | Description |
|  | A3a_A_End | Child measures pencils by aligning end of the felt strip to an end of each pencil. |
|  | A2a_A_Middle | Child measures pencils using middle of the felt strip. |
|  | A2a_A_Mark | Child marks the felt strip either by holding with fingers or using a marker. |
|  | Cover | Child covers |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements | Other NRR Skill Codes |  |

## Essentialized Skill Statement

| 3. Transitivity |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten | $\mathbf{T}$ | Grade 1 | Grade 2 |  |  |  |  |  |
|  | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ |
| NRR.A.3.b. | Order unspecified quantities in a word problem |  |  |  |  |  |  |  |  |

## Student Expectation

Children were expected to compare three unspecified quantities given in a word problem and identify which one of the three quantities is largest and which one is smallest.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| In the selected data, no evidence of <br> unanticipated strategies were found. | N/A |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :--- | :--- | :--- | :--- |
| $K$ | - |  |  |  |  |  |
| 1 |  | - |  |  |  |  |
| 2 |  |  |  |  | - | - |

Unanticipated strategies by grade level and number range for NRR.A.3.b.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Draw Pictures <br> (805) <br> NME (645) |  |  |  |  |  |
| $\mathbf{1}$ |  | Draw Pictures (337) <br> Multiple <br> Comparison (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | Draw <br> Pictures <br> (128) | Draw Pictures (284) <br> Multiple Comparison <br> (284, 993) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.3.b. activity.

| NRR.A.3.b. | Order unspecified quantities in a word problem. |  |
| :---: | :---: | :---: |
| Content Question | Who has the most candy? |  |
| Reasoning Question | Please use words, pictures, or numbers to show how you figured out that ___ has the most candy. |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A3b_E_NOT- | No tools embeded |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | Draw Pictures | Student draw pictures of candies or bags |
|  | Creates Example | Child assigns number/quantities for each person's bag to determine order. (If child creates example then must code NRR.A.3.d and NRR.A.3.e) |
|  | Multiple Comparison | Student compares two objects in set of three, repeating the strategy until all objects are compared to one another two at a time. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill Statements | Other NRR Skill Codes | NRR.A.3.d and NRR.A.3.e |

NRR.A.3.c. Summary

## Essentialized Skill Statement



## Student Expectation

children were expected to compare unspecified weights of three objects shown on pictures of a balance and identify which one of the three objects have most weight (heaviest) and least weight (lightest).

| Unanticipated Strategy. Justification/Description | Examples |
| :---: | :---: |
| Story. Student showed another example (which was not expected/asked) to explain the comparison of unspecified weights and to justify his/her response to the given problem. | When interviewer asked "Ben thinks bear is heaviest, how you would help Ben understand if he is right or wrong", student created another example and changed the position of sides of the balance and explained that if this new situation occurs then Ben will be right, and with current scenario Ben is wrong (993, 27:40) |
| Personal Experience. Student connect the given problem with personal experience of which object is heaviest and lightest. | Student compared the given situation of three stuffed animals on the balance with his/her personal experience with stuffed toys. Student expressed that sometimes I weight my stuffed animals and for some reason the toy cat is heaviest, it might be because of different materials (128, 44:19-45:16). |
| Labeling. Student label three unspecified weight objects as $1^{\text {st }}, 2^{\text {nd }}$, and $3^{\text {rd }}$, to mark which object is heaviest and which one is lightest. | Student labeled animals as $1^{\text {st }}$ and $2^{\text {nd }}$ on first balance and then labeled X and $2^{\text {nd }}$ on the second balance to represent which one is heavy on each balance, and concluded which two animals are heaviest in the two given scenarios ( $128,43: 27-44: 01$ ). |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - |  |  |  |  |  |
| $\mathbf{1}$ |  | - |  |  |  |  |
| 2 |  |  |  |  | Story (128) <br> Personal Experience <br> (128) <br> Labeling (128) | Another Way <br> (993) |

Unanticipated strategies by grade level and number range for NRR.A.3.c

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (805, <br> $645)$ |  | Weight (337) <br> Height (337) <br> Mult. Comparision <br> $(337)$ |  |  |  |
| $\mathbf{1}$ |  |  | Weight (128) <br> A.1.b. (128) | Weight (993, 284) <br> Height (993, 284) <br> Mult. |  |  |
| $\mathbf{2}$ |  |  |  | Comparision <br> (993) |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.3.c activity.

| NRR.A.3.C. | Order three unspecified weights using balances. |  |
| :---: | :---: | :---: |
| Content Question | (1) Which animal weighs the least? <br> (2) Which animal weighs the most? |  |
| Reasoning Question | Ben thinks the bear is the heaviest item. How would you help him understand if he is right or wrong? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A3c_E_NOT-Pictures of a balance | Pictures of the balance embedded into the activity |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A3c_A_Weight | Child directly focuses on weight of each object and tells which object is heavier/lighter than other object. |
|  | A3c_A_Height | Child first describes the position of balance (one side up other side down, tilt) then may links the position to the weight of the object. (It's not necessary to link height with weight) |
|  | Multiple comparison | Student compares two objects in set of three, repeating the strategy until all objects are compared to one another two at a time. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements | Other NRR Skill Codes |  |

NRR.A.3.d. Summary
Essentialized Skill Statement

| 3. Transitivity |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  | Grade 2 |  |  |  |  |
|  | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ |  |
| NRR.A.3.d. | Order three numbers using number relationships with <br> tools. |  |  |  |  |  |  |  |  |  |

## Student Expectation

Using number relationships and a mathematical tool (e.g., number line and hundreds chart), students were expected to compare three given numbers and arrange them in an increasing order.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| A.3.e. Order three numbers using number <br> relationships without tools (i.e., mental <br> strategies). | [Child selected 146 and placed to the right of 120] <br> Because it's more than 120 (128, 46:33-46:50). |
| [Interviewer asked if child wanted to use tools to <br> order numbers] No [child correctly ordered <br> numbers] (993, 29:12-29:33). |  |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - |  |  |  |  |  |
| $\mathbf{1}$ |  | - |  |  |  |  |
| 2 |  |  |  |  | A.3.e. (128) | A.3.e. (993) |

Unanticipated strategies by grade level and number range for NRR.A.3.d

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (805, 645) | Number Relationship <br> (337) <br> Blank 100s Chart (337) |  |  |  |  |
| $\mathbf{1}$ |  |  |  |  |  | 100s Chart (284) <br> Number Relationship <br> (284) |
| $\mathbf{2}$ |  |  |  |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.3.d activity.

| NRR.A.3.d. | Order three numbers using number relationships with tools. |  |
| :---: | :---: | :---: |
| Content Question | Can you put the numbers in order from least to greatest? Here are some tools you can use. [Show the hundreds chart and number line.] Which number is least? Which number is greatest? |  |
| Reasoning Question | Reasoning question listed in protocol does not aid in alignment for this skill code. |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A3d_E_NOT- Number line \& 100's Chart | Number line and 100's chart are embedded in the activity and students are expected to use only embedded tools to answer the question. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A3d_A_WNL | Child uses Written Number Line (WNL) to answer the question. |
|  | A3d_A_MNL | Child uses Mental Number Line (MNL) to answer the question. |
|  | A3d_A_ONL | Child uses Open Number Line (ONL) to answer the question. |
|  | Number relationships | Child uses language of relationship like less/more, smaller/bigger, left/right |
| A different way | Written 100s Chart Blank 100s Chart | - Child uses 100s chart to answer the question (written or blank) |
| NRR | Core Concept \# | If a child does show another number, consider skill statements in core concept 8. |
| Essentialized Skill | Other NRR Skill Codes |  |
| Statements | Non-NRR Skill Codes |  |

NRR.A.3.e. Summary
Essentialized Skill Statement

| 3. Transitivity |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  | Grade 2 |  |  |  |  |
|  | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ |  |
| NRR.A.3.e. | Order three numbers using number relationships <br> without tools (i.e., mental strategies). |  |  |  |  |  |  |  |  |  |

## Student Expectation

Using number relationships (Without Mathematical Tools), students were expected to compare three given numbers and arrange them in an increasing order. In this activity, it is explicit to Not use any mathematical tool.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Number of Digits. Students focused on the <br> number of digits on each card to identify the <br> value of the number on the card. It is different <br> from place value concept because students <br> didn't refer to place value or compare digits at a <br> specific place value on each card. Additionally, <br> we didn't have a concept of place value in <br> Relations protocol. | Because these [points to 76] have two digits, these <br> [pointing to 100 and 103] have three (128, 49:58). |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :--- | :--- | :--- | :---: |
| K | - |  |  |  |  |  |
| 1 |  | - |  |  |  |  |
| 2 |  |  |  |  | Number of Digits <br> (128) | Number of Digits <br> (993) |

Unanticipated strategies by grade level and number range for NRR.A.3.e

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (645, <br> $805)$ |  | Number <br> Relationship (337) |  |  |  |
| $\mathbf{1}$ |  |  |  |  | Number <br> Relationship <br> (128) | Number <br> Relationship (284, <br> 993) |
| $\mathbf{2}$ |  |  |  |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.3.e activity.

| NRR.A.3.e. | Order three numbers using number relationships without tools (i.e., mental strategies). |  |
| :---: | :---: | :---: |
| Content Question | Can you put the numbers in order from least to greatest? Which number is least? Which number is greatest? |  |
| Reasoning Question | How do you know that these numbers are in order from least to greatest? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A3e_E_NOT- | No tools are embedded in the activity and use of any tool is not anticipated for this activity. |
| Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | A3e_A_MNL | Child uses Mental Number Line (MNL) to answer the question. |
|  | A3e_A_Relationship | Child uses language of relationship like less/more, smaller/bigger, left/right |
|  | A3e_A_Place Values | Child compares using 10 s , and 100 s place values. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill Statements | Other NRR Skill Codes |  |

NRR.A.3.f. Summary
Essentialized Skill Statement

| 3. Transitivity |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T |  |  | T | F | B | T |
| NRR.A.3.f. |  |  |  | Order three numbers in a word problem. |  |  |  |  |  |

## Student Expectation

Students were given three numbers in a contextual situation, students were asked to place these numbers in order from least to greatest.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Number of Digits. Students focused on the |  |
| number of digits on each card to identify the |  |
| value of the number on the card. It is different | Student said: "It tells me that, cause this one |
| (pointing to 107) has three, and this one (pointing |  |
| fo 78) only has two (128, 53:12) |  |
| didn't refer value concept because students |  |
| specific place value on each campare digits at a |  |$\quad$.


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :--- | :--- | :--- | :---: |
| K | - |  |  |  |  |  |
| 1 |  | - |  |  |  |  |
| 2 |  |  |  |  | Number of Digits (128) | - |

Unanticipated strategies by grade level and number range for NRR.A.3.f.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (645, 805) |  |  |  |  |  |
| $\mathbf{1}$ |  | Mental <br> Number Line <br> (337) <br> A.1.d. (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | A.3.e. (128) | Place Value (993) <br> A.3.e. (284, 993) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.3.f. activity.


NRR.A.4. Representation of Order in Comparison Situations Core Concept


## NRR.A.4. Synthesis Data

Unanticipated strategies: NRR.A.4.a-e

| Skill Code |  |  | A.4.a |  | A.4.b. |  |  |  | A.4. c. |  | A.4.d. |  | A.4.e. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Unantic | pated | NME |  |  | Focus on A.4.a. Answer | Tool Not Used for ProblemSolving |  | Unfounded |  | ting | NME |  |
| SID | Grad e | Number Range |  |  |  |  |  |  |  |  |  | Used for ProblemSolving |
| 645 | K | 0-5 | 1 | 1 | 1 |  | - | - | - | - | - | - | - | - |
| 805 | K |  | 1 | 1 | 1 | - | - | - | - | - | - | - | - |
| 337 | 1 | 0-10 | - | - | - | 1 | 1 | 1 | - | - | - | - | - |
| 128 | 2 | 0-99 | - | - | - | 1 | 1 | - | - | All | - | - | 1 |
| 284 | 2 | 0-199 | - | - | - | - | 1 | - | 1 | - | On | 1 | - |
| 993 | 2 |  | - | - | - | - | 1 | 1 | - | On | On | - | - |
| Strategies Total by Skill Code |  |  | 2 | 2 | 2 | 2 | 4 | 2 | 1 | 2 | 2 | 1 | 1 |
| Strategies Total by Core Concept: No Mathematical Evidence $=5$; $\quad$ Counting $=4$; Tool Not Used for Problem-Solving $=7$ |  |  |  |  |  |  |  |  |  |  |  |  |  |

Anticipated strategies: NRR.A.4.a-e

| Skill Code |  |  | A.4.a. |  | A.4.b. | A.4.c |  | A.4.d. | A.4.e. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | Match groups | Count All | - | NME | A.1.c. |  | Open Number | NME |
| SID | Grade | Number Range |  |  |  |  |  |  | Line (ONL) |  |
| 645 | K | 0-5 | - | - | - | 1 | - | 1 | - | 1 |
| 805 | K |  | - | - | - | 1 | - | - | - | 1 |
| 337 | 1 | 0-10 | 1 | - | - | - | 1 | 1 | 1 | - |
| 128 | 2 | 0-99 | - | 1 | - | - | - | - | 1 | - |
| 284 | 2 | 0-199 | 1 | 1 | - | - | - | - | - | - |
| 993 | 2 |  | 1 | 1 | - | - | - | - | - | - |
| Strategies Total by Skill Code |  |  | 3 | 3 | 0 | 2 | 1 | 2 | 2 | 2 |

## Essentialized Skill Statement

| 4. Representations of Order in Comparison Situations |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  |  |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.A.4.a. | Find how much more/less between two <br> quantities using matching and counting <br> strategies. |  |  |  |  |  |  |  |  |

## Student Expectation

Understanding a given contextual situation, students were asked to compare two given quantities, identify which one is more/less, and find how much more/less between given quantities using matching and counting strategies.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Another Way. Student showed another way |  |
| (presented a possible solution) to the problem |  |
| which was not anticipated. Student suggested |  |
| how teacher could bring a group of 4 chairs to |  |
| accommodate every group of four students |  |
| until every child get a seat in the class. |  | | Student said: "If the teacher bring more and four |
| :--- |
| groups again, these four (students) got a chair, |
| these four got a chair (pointed to another group of |
| four students) and these four got a chair (pointed |
| to another group of four students). Then if she |
| bring two more, the whole class would have |
| enough chairs. |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | $\mathrm{NME}(645$, <br> $805)$ |  |  |  |  |  |
| $\mathbf{1}$ |  | - |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | - | Another way:balances (284) |

Unanticipated strategies by grade level and number range for NRR.A.4.a.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - |  |  |  |  |  |
| $\mathbf{1}$ |  | Match groups <br> (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | Count all <br> (128) | Count all (284, 993) <br> Match groups (284, 993) <br> B.5.c. (993) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.4.a. activity.

| NRR.A.4.a. | Find how much more/less between two quantities using matching and counting strategies. |  |
| :---: | :---: | :---: |
| Content Question | Can you show me how you could find out how many more children there are than chairs? How many more children are there than chairs? |  |
| Reasoning Question | NOT NEEDED |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A4a_E_NOT-Pictures of children and chairs | Pictures of children and chairs (in groups) are embedded in the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
|  |  |  |
| Strategies | Code | Description |
|  | Match Groups | Child matched groups to compare quantities as more/less |
|  | Counting | Child counted all or individual items in the situation to compare quantities or prove as more/less |
|  |  |  |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Core Concept \# | If a child does show another number, consider skill statements in core concept 8. |
| Essentialized Skill | Other NRR Skill Codes |  |
| Statements | Non-NRR Skill Codes |  |

## Essentialized Skill Statement

| 4. Representations of Order in Comparison Situations |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 | B | T | F | B | T |
|  | F | B | T | F | B |  |  |  |  |
| NRR.A.4.b. | Find how much more/less between two quantities using tools. |  |  |  |  |  |  |  |  |

## Student Expectation

Students were given pictures of children and chairs, it was expected that student will use mathematical tools to find how many children are more than the given number of chairs.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Tools Not Used for Problem Solving. This is unanticipated <br> category for the cases when students involve the given <br> mathematical tool in their work but students did not use <br> the tool for problem solving. For example, students used <br> Number line but instead of using it to identify the <br> magnitude between numbers they used it for writing <br> numbers on it. | Student write each number up to 18 (that <br> was total number of students in the <br> problem) on the open number line (284, <br> $44: 57)$. |
| Student used 100's chart and marked 10 <br> but again counted from 1 to 9 to identify <br> there are 9 children (128, 56:45) . |  |
| Focused on A.4.a. Answer. Since NRR.A.4.a and NRR.A.4.b <br> used same question but in A.4.b students were provided <br> mathematical tools to figure out their answers, students <br> didn't try to calculate or double-check their answer from | For NRR.A.4.b, student used the number <br> 10 (that was miscalculated in NRR.A.4.a) <br> and said "so there is 10 so there is at least <br> one, two, three, four, five, six, <br> s.4.a, instead, students focus on their previous answer <br> and try to show the same answer using tools. |
| can seat in there" (128, Nine children that $56: 38)$ |  |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (645, <br> $805)$ |  | Focus on A.4.a. answer <br> (337) <br> Tool not used for <br> problem solving (337) |  |  |  |
| $\mathbf{1}$ |  |  |  | Tool not used for <br> the problem <br> solving (128) <br> Focus on A.4.a. <br> answer (128) | Tool not used <br> for the problem <br> solving (284, <br> $993)$ |  |
| $\mathbf{2}$ |  |  |  |  |  |  |

Unanticipated strategies by grade level and number range for NRR.A.4.b.

| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| K |  |  |  |  |  |  |
| $\mathbf{1}$ |  | Blank 100's <br> chart (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.4.b. activity.

| NRR.A.4.b. | Find how much more/less between two quantities using tools. |  |
| :---: | :---: | :---: |
| Content Question | Can you show me how you could find out how many more children there are than chairs? How many more children are there than chairs? |  |
| Reasoning Question | NOT NEEDED |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A4b_E_NOT- Pictures of children and chairs | Pictures of children and chairs (in groups) are embedded in the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | Counting | Count all or on, students use counting strategies when comparing numbers and ordering |
|  | Blank 100's chart | Student uses 100's chart to answer the question |
|  | No. Relationships | Child uses language of relationship like less/more, smaller/bigger, left/right |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Core Concept \# | If a child does show another number, consider skill statements in core concept 8. |
| Essentialized Skill | Other NRR Skill Codes |  |
| Statements | Non-NRR Skill Codes |  |

## Essentialized Skill Statement

| 4. Representations of Order in Comparison Situations |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.A.4.c. |  | Find how much more/less between two numbers in a word problem using tools. |  |  |  |  |  |  |  |

## Student Expectation

Students were given a contextual situation in a word problem, it was expected that student will use mathematical tools to find how much one number is more/less than the other number (i.e., how many children are more than the given number of chairs)

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Tools Not Used for Problem-Solving. For this <br> activity, mathematical tools (100's chart) were <br> part of the skill code and it was anticipated that <br> students will use 100's chart to identify that <br> there are --- number of more seats than ---- <br> number of students. | Student put finger on number 23 and then count <br> all (backward) to 1 to confirm there are 22 <br> numbers. (128, 01:00:20-01:01:34) | | Students did not use the given mathematical tool |
| :--- |
| to solve the problem and said: "I pictured desks in |
| my head and then I pictured people in them" (337, |
| $44: 38-44: 42)$ |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ |  |  |  |  |  |  |
| $\mathbf{1}$ |  | Tools not used for <br> problem- solving <br> $(337)$ |  |  |  |  |
| $\mathbf{2}$ |  |  |  | Count all (128) <br> Tools not used <br> for problem- <br> solving (128) | Tools not used for <br> problem- solving (993) <br> A.4.d (284) Unfounded <br> (284) Count on (993) |  |

Unanticipated strategies by grade level and number range for NRR.A.4.c.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (645, 805) |  |  |  |  |  |
| $\mathbf{1}$ |  | A.1.c. Mental <br> Image (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | - | - |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.4.c. activity.

| NRR.A.4.c. | Find how much more/less between two numbers in a word problem using tools. |  |
| :---: | :---: | :---: |
| Content Question | Is Fran right or wrong? |  |
| Reasoning Question | How would you help Fran see if she is right or wrong? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A4c_E_NOT-Blank 100's Chart | Blank 100's number chart is embedded in the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | 100's chart | Student uses 100's chart to answer the question |
|  | Counting | Count all or on, students use counting strategies when comparing numbers and ordering |
|  | MNL | Child uses Mental Number Line (MNL) to answer the question. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Core Concept \# | If a child does show another number, consider skill statements in core concept 8. |
| Essentialized Skill | Other NRR Skill Codes |  |
| Statements | Non-NRR Skill Codes |  |

Essentialized Skill Statement

| 4. Representations of Order in Comparison Situations |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.A.4.d. | Find how much more/less between two numbers in a word problem. |  |  |  |  |  |  |  |  |

## Student Expectation

Understanding a given contextual situation in a word problem, students were asked to compare two given numbers (amount of dollars) and identify which one amount is more and why.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Count On: For this skill code, it was expected <br> that students will use numbers relationship and <br> computation to solve the given problem. It is <br> unanticipated that students will use any <br> Counting strategy here. | For the Mark and Jose money problem, student <br> use Count On strategy and drew mark for Count <br> On number to find who has more money. (993, <br> O3:48) |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - |  |  |  |  |  |
| $\mathbf{1}$ |  |  | - |  |  |  |
| $\mathbf{2}$ |  |  |  |  | NME (128) | Count On <br> (993,284) |

Unanticipated strategies by grade level and number range for NRR.A.4.d

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | A.1.c. (645) <br> A.1.a. (805) |  |  |  |  |  |
| $\mathbf{1}$ |  | A.1.c. (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.4.d activity.

| NRR.A.4.d. | Find how much more/less between two numbers in a word problem. |  |
| :---: | :---: | :---: |
| Content Question | Mark has \$_. Jose has \$_. How many more dollars does Mark have than Jose? |  |
| Reasoning Question | How do you know that Mark has___more dollars than Jose? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded <br> Mathematical Tools | A4d_E_NOT- ??? | No tool embedded in the activity. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
|  |  |  |
| Strategies | Code | Description |
|  | Number relationships | Child uses language of relationship like less/more, smaller/bigger, left/right |
|  | Counting | Count all or on, students use counting strategies when comparing numbers and ordering |
|  | MNL | Child uses Mental Number Line (MNL) to answer the question. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Core Concept \# | If a child does show another number, consider skill statements in core concept 8. |
| Essentialized Skill | Other NRR Skill Codes |  |
| Statements | Non-NRR Skill Codes |  |

Essentialized Skill Statement


## Student Expectation

For this skill code, students were given a benchmark number and two other numbers to find which one of the two numbers is closest to the benchmark using an open number line.

Understanding a given contextual situation in a word problem, students were asked to compare two given numbers (amount of dollars) and identify which one amount is more and why.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Tools Not Used for Problem Solving: This is unanticipated <br> category for the cases when students involve the given <br> mathematical tool in their work but students did not use <br> the tool for problem solving. For example, students used | Student write each number up to 18 (that <br> was total number of students in the <br> problem) on the open number line (284, <br> Number line but instead of using it to identify the <br> magnitude between numbers they used it for writing <br> numbers on it. |
| Student used 100's chart and marked 10 <br> but again counted from 1 to 9 to identify <br> there are 9 children (128, 56:45) |  |
| Unfounded: The student uses a reasoning that is incorrect <br> (a logical fallacy), or guesses at the solution. Response <br> may be be partially or fully developed (Crawford et al., <br> 2018). Student provided a logical explanation for the <br> solution of given problem (which <br> might be partially or fully incorrect) that make sense for <br> student. | Student said: "I will go around the <br> classroom (to count them) to see if they <br> looks more than $85 "(284, ~ 47: 03)$. |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :---: |
| K |  |  |  |  |  |  |
| $\mathbf{1}$ |  |  |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | Tools Not Used for <br> Problem Solving (128) | Unfounded <br> (993) |

Unanticipated strategies by grade level and number range for NRR.A.4.e

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (645, 805) |  |  |  |  |  |
| $\mathbf{1}$ |  | ONL (337) |  |  |  |  |
| $\mathbf{2}$ |  |  |  |  | ONL (128) | A.3.d. (284) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.A.4.e activity.

| NRR.A.4.e. | Compare two numbers to find which is [closest to/furthest from] a benchmark. |  |
| :---: | :---: | :---: |
| Content Question | Which number is closet to (\#)? (\#) or (\#)? [\# represent various numbers those were used based on the appropriate number ranges. For example, Which number is closest to 10 ? 7 or 12?] |  |
| Reasoning Question | NOT NEEDED |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | A4d_E_NOT- ??? | No tool embedded in the activity. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | ONL | Child uses the open number line to answer question without marking. |
|  | MNL | Child uses Mental Number Line (MNL) to answer the question. |
|  | Create WNL | Child marks numbers on the open number line to use it. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If adifferent strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements | Core Concept \# | If a child does show another number, consider skill statements in core concept 8. |
|  | Other NRR Skill Codes |  |
|  | Non-NRR Skill Codes |  |

## Appendix C - Integrated Strategies Document: Composition and Decomposition

NRR.B.5. Composition Core Concept

## 5. Composition

| Code | Kindergarten |  |  |  | Grade 1 |  |  | Grade 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F | F $\quad$ B | B T | - $\quad$ F | F ${ }^{\text {a }}$ | B $\quad$ T | T F | F ${ }^{\text {a }}$ | B $\quad$ T |
| NRR.B.5.a. | Compose a number with single objects. |  |  |  |  |  |  |  |  |
| NRR.B.5.b. | Compose a number with two parts. |  |  |  |  |  |  |  |  |
| NRR.B.5.c. | Compose a number with three or more parts. |  |  |  |  |  |  |  |  |
| NRR.B.5.d. | Compose a number with two or more parts using different number combinations. |  |  |  |  |  |  |  |  |
| NRR.B.5.e. | Compose a number with two or more parts using concepts of place value. |  |  |  |  |  |  |  |  |

NRR.B.5. Synthesis Data
Unanticipated strategies: NRR.B.5.a-c

| Skill | ode |  | B. 5 | .a. |  | 5.b. |  |  | B.5.c. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unan Strat | icipat gies |  | C | Counting | C | Counting | Writing | C | Construc ting | Counting | Writing |
| SID | Grade | Number Range |  |  |  |  |  |  | Numbers <br> Using <br> Digits |  |  |
| CK | K |  | 1 | - | 0 | - | - | 0 | 1 | - | - |
| 341 | K |  | 1 | - | 1 | - | Expression | 1 | - | - | Expression |
| 708 | 1 | 0-10 | 1 | Parts \& Combine Groups | 0 | Parts \& Combine Groups | - | 1 | - | Parts \& Combine Groups | - |
| 769 | 1 |  | 1 | Skip (10) | 1 | - | Equation | 1 | - | - | Equation |
| 223 | 1 | 0-19 | 1 | Skip (10) | 1 | - | - | 1 | - | Parts \& Combine Groups | - |
| 352 | 2 | 0-50 | 1 | Mental | $\begin{gathered} {[0-199]} \\ 1 \end{gathered}$ | - | - | $\begin{gathered} {[0-199]} \\ 1 \end{gathered}$ | - | - | - |
| Strategies Total by Skill Code |  |  | 6 | 4 | 4 | 1 | 2 | 5 | 1 | 2 | 2 |

Unanticipated strategies: NRR.B.5.d-e

| Skill Code |  |  | B.5.d. |  |  | B.5.e. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Una <br> Stra | icipat <br> gies |  | C | Compare Numbers | Counting | C | NME |
| SID | Grade | Number <br> Range |  | to Find Same |  |  |  |
| CK | K | -5 | 0 | - | - | $\begin{gathered} {[0-19]} \\ 0 \end{gathered}$ | - |
| 341 | K | 0-5 | 1 | 1 | - | $\begin{gathered} \hline[0-19] \\ 0 \end{gathered}$ | - |
| 708 | 1 | 0-10 | 1 | - | Parts \& Combine Groups | $\begin{gathered} {[0-19]} \\ 0 \end{gathered}$ | - |
| 769 | 1 |  | 1 | - | - | $\begin{gathered} {[0-19]} \\ 0 \end{gathered}$ | - |
| 223 | 1 | 0-19 | 1 | - | - | 0 | 1 |
| 352 | 2 | 0-50 | $\begin{gathered} {[0-199]} \\ 1 \\ \hline \end{gathered}$ | 1 | - | $\begin{gathered} {[0-199]} \\ 1 \end{gathered}$ | 1 |
| Strategies Total by Skill Code |  |  | 5 | 2 | 1 | 1 | 2 |

Anticipated strategies: NRR.B.5.a-b

| Skill Code <br> Anticipated <br> Strategies |  |  | B.5.a. |  |  |  |  | B.5.b. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | C | Add | Counting | Another Number | B.5.b. | C | Add | Equipartition | Counting | Tool |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |  |
| CK | K | 0-5 | 1 | - | All | 1 | - | 0 | 1 | - | - | (I) Square Tiles |
| 341 | K |  | 1 | - | All | 1 | - | 1 | 1 | Doubles | - | - |
| 708 | 1 | 0-10 | 1 | - | - | 1 | 1 | 0 | - | - | All | (I) Square Tiles |
| 769 | 1 |  | 1 | - | All | 1 | - | 1 | 1 | Doubles | All | (S) Fingers |
| 223 | 1 | 0-19 | 1 | - | All | 1 | - | 1 | - | Doubles | - | - |
| 352 | 2 | 0-50 | 1 | 1 | - | 1 | - | $\begin{gathered} {[0-199]} \\ 1 \end{gathered}$ | 1 | - | - | - |
| Strategies Total by Skill Code |  |  | 6 | 1 | 4 | 6 | 1 | 4 | 4 | 3 | 2 | 3 |

## Anticipated strategies: NRR.B.5.c-d

| Skill | ode |  |  |  | B.5.c. |  |  |  |  | B. 5 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Antic Strat | pated <br> gies |  | C | Add | Counting | Tool | Equipartition | B.5.b | C | Count ing | Tool | Compare | B.5.b | B.5.c. |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |  |  |  |
| CK | K |  | 0 | - | - | - | - | - | 0 | - | (I) Linking Cubes | - | 1 | - |
| 341 | K | 5 | 1 | - | - | - | - | 1 | 1 | - | - | Same Number | - | - |
| 708 | 1 | 0-10 | 1 | - | All | (S) Square Tiles | - | - | 1 | All | (I) Square Tiles | - | - | 1 |
| 769 | 1 |  | 1 | 1 | - | - | Triples | - | 1 | All, On | - | Sum | 1 | 1 |
| 223 | 1 | 0-19 | 1 | - | All | - | Triples | - | 1 | All, On | (I) Square Tiles | - | 1 | - |
| 352 | 2 | 0-50 | $\begin{array}{\|c} \hline[0-199] \\ 1 \\ \hline \end{array}$ | 1 | - | - | Doubles, Triples | - | $\begin{gathered} \hline[0-199] \\ 1 \\ \hline \end{gathered}$ | - | - | Same Number | - | - |
| Strategies Total by Skill Code |  |  | 5 | 2 | 2 | 1 | 4 | 1 | 5 | 5 | 3 | 3 | 3 | 2 |

## Anticipated strategies: NRR.B.5.e

| Skill Code <br> Anticipated Strategies |  |  | B.5.e. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | C | Place <br> Value | Counting | Tool | B.5.b. |
| SID | Grade | Number Range |  |  |  |  |  |
| CK | K |  | $\begin{array}{\|c} \hline[0-19] \\ 0 \end{array}$ |  | - | - | - |
| 341 | K | 0-5 | $\begin{gathered} {[0-19]} \\ 0 \end{gathered}$ |  | All | - | 1 |
| 708 | 1 | 0-10 | $\begin{gathered} {[0-19]} \\ 0 \end{gathered}$ |  | - | (I) Place Value Blocks | - |
| 769 | 1 |  | $\begin{array}{\|c\|} \hline[0-19] \\ 0 \\ \hline \end{array}$ | 1 | - | - | 1 |
| 223 | 1 | 0-19 | 0 | - | - | - | - |
| 352 | 2 | 0-50 | $\begin{gathered} {[0-199]} \\ 1 \end{gathered}$ | 1 | - | - | - |
| Strategies Total by Skill Code |  |  | 1 | 2 | 1 | 1 | 2 |

NRR.B.5.a. Summary
Essentialized Skill Statement

| Kindergarten |  | Grade 1 | Grade 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F |  |
| B |  | T |  |  |  |  |  |
| Compose a number with single objects. |  |  |  |  |  |  |  |

## Student Expectation

Students were expected to arrange tile counters into groups that could be combined and counted. Some students counted all counters as one group.

## Unanticipated Strategies

| Unanticipated Strategy. Justification/Description | Examples |
| :---: | :---: |
| Counting: Skip by 10s. Children composed larger numbers than the given quantity of single objects by using each object to represent ten. Children counted by tens when explaining their stated composition. This was unanticipated as using an alternate unit for counting was not prompted. | l think I'll do 100 with these... $10,20,30,40,50$, 60, 70 80, 90,100 (223, 07:14-07:17). <br> [Interviewer asked what different numbers the child could count to with the counters] $10,20,30$, $40,50,60,70,80,90,100(769,00: 25 \& 00: 35)$ |
| Counting: Mental. Child provided the strategy of counting in head when asked how the total number was obtained. This reasoning strategy could be appropriate given the student was in Grade 2, but the child did not provide observable mathematic evidence. | [Interviewer asked how the child knew there were eleven] I counted in my head (352, 06:53). |
| Counting: Parts \& combine groups. Child created concrete representations of two parts with counters, counted each part, and combined the groups to compose the number. This strategy served as a variation of counting all and included necessary skills to compose numbers with single objects before composing with two parts. | [Interviewer describing 708's actions] There's eight in this row and then two in this row, and then you counted by pointing your finger at each of them (708, 01:14). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{K}$ |  |  |  |  |  |  |
| $\mathbf{1}$ |  |  <br> combine groups (708) | Counting: Skip <br> by 10s (223, <br> $769)$ |  |  |  |
| $\mathbf{2}$ |  |  |  | Counting: Mental <br> (352) |  |  |

Unanticipated strategies by grade level and number range for NRR.B.5.a.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{K}$ | Counting: All (CK, <br> 341) <br> Another Number <br> (CK, 341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Another number <br> (708, 769) <br> Counting: All (769) <br> B.5.b (708) | Another number <br> (223) <br> Counting: All <br> (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Another <br> Number (352) <br> Add (352) |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.5.a. activity.

| NRR.B.5.a. | Compose a number with single objects. |  |
| :---: | :---: | :---: |
| Content Question | What different numbers could you make using these counters? |  |
| Reasoning Questions | *Are there other numbers you can make using these counters? How do you know that there are $\qquad$ counters? |  |
| Anticipated Skills |  |  |
| Strategies | Code | Description |
| Embedded <br> Mathematical Tools | Square tiles | Square tiles of the same color are embedded into the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B5a_A_General tool | Child uses a general tool as listed above. |
|  | B5a_A_Add | Child adds groups of counters; differs from composing based on the child's vocabulary (e.g., add, addend, plus, sum, NOT make). Writing the + symbol may also be considered addition. |
|  | B5a_A_Combine groups | Child combines groups of counters |
|  | B5a_A_Count all | Child counts all square tiles individually |
|  | B5a_A_Count by 2s | Counting. Child counts square tiles in groups of 2 |
|  | B5a_A_Count on | Child counts one group, identifies the amount in the one group, then continues to count on. |
|  | B5a_A_Count parts | Child divides group of square tiles into parts then counts each part |
| A different way | B5a_D_Other number | The required reasoning question asks the child to make other numbers. |
| NRR <br> Essentialized Skills | Core Concept 6 | Consider evidence of decomposition skill statements alongside composition that was intended for this ESS. |
|  | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Skills evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

NRR.B.5.b. Summary
Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B | T |
| Compose a number with two parts. |  |  |  |  |  |  |  |  |

## Student Expectation

Students were expected to select two number cards that could then be combined using counting or addition strategies to represent a greater number.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Counting: Parts \& combine groups. Child created <br> concrete representations of two parts with <br> counters, counted each part, and combined the <br> groups to compose the number. This strategy <br> served as a variation of counting all and included <br> necessary skills to compose numbers with single <br> objects before composing with two parts. | [Described by interviewer] I see how you put some <br> green squares here for the six, and some blue <br> squares here for the two. And I like how you <br> pointed to each of them to count them (708, 05:06). |
| Writing: Expression. Child wrote mathematical <br> expression (not equations) to work on the given <br> task and to explain his/her mathematical <br> understanding. | In response to interviewer question, student <br> wrote and said Two plus two is four (and wrote <br> $2+2+4$ on the paper) (341, 03:46) |
| Writing: Equation. Child wrote mathematical <br> equation (not expression) to work on the given <br> task and to explain his/her mathematical <br> understanding. | Four plus four equals eight (student wrote 4+4=8) <br> (769, 08:38) |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $0-99$ | $0-199$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{K}$ | Writing: Expression <br> (341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Count Parts \& Combine <br> Groups (708) <br> Writing: Equation (769) |  |  |  |  |
| $\mathbf{2}$ |  |  |  | - |  |  |

Unanticipated strategies by grade level and number range for NRR.B.5.b.

| Grade | 0-5 | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :--- | :--- | :--- | :--- |
| $\mathbf{K}$ | Doubles (341) <br> Add (CK, 341) <br> Tools: (I) Sq. Tiles (CK) |  |  |  |  |  |
| $\mathbf{1}$ |  | Tool: (S) Fingers (769) <br> Tools: (I) Sq. Tiles (708) <br> Count all (708, 769) <br> Add (769) <br> Doubles (769) | Doubles <br> $(223)$ |  |  |  |
| $\mathbf{2}$ |  |  | Add <br> (352) |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.5.b. activity.

| NRR.B.5.b. | Compose a number with two parts. |  |
| :---: | :---: | :---: |
| Content Question | Here are some number cards. Please choose two cards. What number would you make if you add these numbers together? |  |
| Reasoning Question | *How did you figure out that $\qquad$ and $\qquad$ make the number $\qquad$ ? <br> What other numbers could you make using any two cards? <br> Show me what you saw in your head using pictures, words, or numbers? |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | - | No tools were embedded for this activity. Child was given cards with numbers. These cards were not intended as tools. However, examples of the cards as tools include and is not limited to, if the child uses the cards to create number sentences or places on an open number line. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B5b_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B5b_A_Add | Child adds numbers to find sum; differs from composing based on the child's vocabulary. Evidence of vocabulary use (e.g., add, plus, sum, NOT make) or writing symbols must be evident. Use of the word "addend" is unanticipated. |
|  | B5b_A_Count all | Child starts counting from one for the first card and continues to count each number individually. |
|  | B5b_A_Count on | Child recognizes number of first card, then continues to count on. |
|  | B5b_A_Doubles | Child uses doubles |
|  | B5b_A_Zero | Child recognizes the identity property of addition (i.e., the sum of zero and a number is that number). |
| A different way | * | The reasoning question that specifically asks for another number was not required. If the question was asked, then a different way was anticipated. If the question was not asked, then a different way was not anticipated. |
| NRR <br> Essentialized Skills | Core Concept 8 | If a child does show another number, consider skill statements from proeperties of operations, core concept 8. |
|  | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Skills evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

NRR.B.5.c. Summary

## Essentialized Skill Statement

| Gindergarten |  | Grade 2 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B |
| Compose a number with three or more parts. |  |  |  |  |  |  |  |

## Student Expectation

Students were expected to select three number cards that could then be combined using counting or addition strategies to represent a greater number.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Writing: Equation. Child wrote mathematical <br> equation (not expression) to work on the given <br> task and to explain his/her mathematical <br> understanding. | Student wrote 3 + 3 = 6; in response to the <br> question (769, 05:37). |
| Writing: Expression. Child wrote mathematical <br> expression (not equations) to work on the given <br> task and to explain his/her mathematical <br> understanding. | $[$ Student picked cards with zero, four, and six and <br> in response to interviewer's question\} student <br> wrote two mathematical expressions 0+4 and 4+6 <br> to show the different numbers (341, 0:38] |
| Count parts \& combine groups. Some children <br> created representations of parts and counted <br> single objects in parts before combining groups. <br> Counting single items to create groups and then | [Drew it out in groups and counted parts] One <br> two, one two, one two (223, 10:51). |
| putting the groups together demonstrated <br> variations or prerequisite skills of counting all or <br> counting on, anticipated strategies for this ESS. | you put three green squares, four yellow squares, <br> and two blue squares. And then you used your <br> finger to count, to point to each one and count. <br> (708, 08:05) |
| Adding on (Triples). Child created two 2-part <br> compositions to compose three parts, adding on <br> the third part to the first sum. While effective, it <br> was unanticipated to break the ESS into a multi- <br> step composition. | [Interviewer asked for proof that 10+10+10=30] So <br> 10 plus 10 is 20, and 20 plus 10 equals 30 (352, <br> $10: 07)$ |
| Constructing Numbers Using Digits. Student put <br> cards with different digits on them next to each <br> other to construct new numbers. | $[$ Interviewer asked so you put two, four, and five, <br> what numbers you can make] so this would be <br> fifty-four (putting cards of five and four together), |
| this would be fifty-two (putting cards of five and |  |
| two together). (CK, 04:56) |  |


| Grade | 0-5 | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Constructing <br> Numbers Using <br> Digits (CK) <br> Writing: <br> Expression (341) |  |  |  |  |  |
| $\mathbf{1}$ |  |  <br> combine groups <br> (708) <br> Writing: Equation <br> (769) |  <br> combine <br> groups (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Adding On <br> (Triples) (352) |  |  |

Unanticipated strategies by grade level and number range for NRR.B.5.c.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 -}$ <br> $\mathbf{9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Zero (341) <br> Writing: <br> Expression (341) <br> B.5.b (341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Tools: (S) Sq. Tiles <br> (708) <br> Counting: All (708) <br> Add (769) <br> Equipartition: Triples <br> (769) | Counting: All <br> (223) <br> Equipartition: <br> Triples (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Add (352) <br> Equipartition: <br> Doubles (352) <br> Equipartition: <br> Triples (352) |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.5.c. activity.

| NRR.B.5.c. | Compose a number with three or more parts. |  |
| :---: | :---: | :---: |
| Content Question | Now, choose three cards to make another number. What number did you make? |  |
| Reasoning Question | *How could you prove that $\qquad$ _, and $\qquad$ makes $\qquad$ ? <br> Can you show me in another way that $\qquad$ , and $\qquad$ makes $\qquad$ ? How did you figure out that $\qquad$ and m makes ? $\qquad$ |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded <br> Mathematical Tools |  | No tools were embedded for this activity. Child was given cards with numbers. These cards were not intended as tools. However, examples of the cards as tools include and is not limited to, if the child uses the cards to create number sentences or places on an open number line. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B5c_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B5c_A_Add | Child adds numbers to find sum; differs from composing based on the child's vocabulary. Evidence of vocabulary use (e.g., add, plus, sum, NOT make) or writing symbols must be evident. Use of the word "addend" is unanticipated. |
|  | B5c_A_Count all | Child starts counting from one for the first card and continues to count each number individually. |
|  | B5c_A_Count on | Child recognizes number of first card, then continues to count on for second and third cards. |
|  | B5c_A_Doubles | Child uses doubles |
|  | B5c_A_Zero | Child recognizes the identity property of addition (i.e., the sum of zero and a number is that number). |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, consider the strategy as unanticipated. |
| NRR <br> Essentialized Skills | Core Concept 8 | If a child does show another number, consider skill statements in core concept 8. |
|  | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |


| Kindergarten | Grade 1 | Grade 2 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ |
| Compose a number with two or more parts using different number |  |  |  |  |  |  |  |
| combinations. |  |  |  |  |  |  |  |

## Student Expectation

Students were expected to use counting or adding strategies to determine that the number combinations written from the prompt represent the same amount.

## Unanticipated Strategies

| Unanticipated Strategy. Justification/Description | Examples |
| :---: | :---: |
| Draw it out. Child drew circles as single objects to represent two parts. These served as a substitution for general tools that were available to the student, making the strategy similar but unanticipated. | [Draws a row of 6 circles and a row of 4 , then counts them] ...4, 5, 6. 1, 2, 3, 4, 1, 2, 3, 4, 5, 6, 7, $8,9,10,9,10-(769,07: 08)$. |
| Counting parts \& combine groups. Children showed a representation of parts and whole with single objects, counted parts independently, then combined the groups. It was unexpected for children to count groups independently before combining them to compose the number. | [Child showed a representation of two-part composition, interviewer described] ...you put six yellow and four blue out... (708, 12:07). <br> So it's 60 and 30 and 30 . So 60 and 30 and 30 . So 30,30 isn't that the same .... The adding. It's not the same adding (352, 13:23) |
| Compare Numbers to Find Same. Student compared one numbers with other (or part of the other number) to compare two numbers to decide which one is bigger and/or smaller among given three numbers. | [In comparing 11, 3 , and 23 ] student said: "They kind of did because they're both the same. Three and three (check mark 3 and 3 of 23) (341, 09:17). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{K}$ | Compare Numbers <br> to Find Same (341) |  |  |  |  |  |
| $\mathbf{1}$ |  |  <br> combine groups (708) <br> Draw a Picture (769) | - |  |  |  |
| $\mathbf{2}$ |  |  | Compare Numbers <br> to Find Same (352) |  |  |  |

Unanticipated strategies by grade level and number range for NRR.B.5.d.

| Grade | 0-5 | 0-10 | 0-19 | 0-50 | $\begin{aligned} & 0- \\ & 99 \end{aligned}$ | 0-199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | ```Tools: (I) Linking Cubes (CK) Same parts (341) B.5.b (CK)``` |  |  |  |  |  |
| 1 |  | Tools: (I) Sq. Tiles (708) <br> Counting: All $(708,769)$ <br> Counting On $(769,223)$ <br> Compare: Sum (769) <br> B.5.b. (769) B.5.c. <br> $(708,769)$ | Counting All (223) <br> Counting On (223) <br> Tools: (I) Sq. Tiles (223) <br> B.5.b. (223) |  |  |  |
| 2 |  |  |  | Compare: Same parts (352) |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.5.d. activity.

| NRR.B.5.d. | Compose a number with two or more parts using different number combinations. |  |
| :---: | :---: | :---: |
| Content Question | Ben and Carla were making numbers like you just did. Ben and Carla think that they made the same number. [follow-up questions included] What do you think? How do you know? |  |
| Reasoning Question | *Show me using pictures, words, or numbers how you found your answer. <br> Your friend doesn't think they have the same number. How could you use one of these tools to prove that they have the same number? |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools |  | No tools were embedded for this activity. Child was given cards with numbers. These cards were not intended as tools. However, examples of the cards as tools include and is not limited to, if the child uses the cards to create number sentences or places on an open number line. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
| Mathematical Tools | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B5d_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B5d_A_Add | Child adds numbers to find sum; differs from composing based on the child's vocabulary. Evidence of vocabulary use (e.g., add, plus, sum, NOT make) or writing symbols must be evident. Use of the word "addend" is unanticipated. |
|  | B5d_A_Compare Sums | Child compares the of Ben's cards to the sum of Carla's cards. |
|  | B5d_A_Count all | ild starts counting from one for the first card and continues to count each number individually. |
|  | B5d_A_Count on | Child recognizes number of first card, then continues to count on for second and third cards. |
|  | B5d_A_Digits | Name the number that the individual digits combine to make (e.g., 1 and 1 makes 11). |
|  | B5d_A_Same parts | Child identifies that part of Ben's cards are the same of part of Carla's cards. This strategy also includes skill code B.5.b and possibly B.5.c. |
| A different way | B5d_D_Combination | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, consider the strategy as unanticipated. <br> However, different number combinations were anticipated. If child was able to identify the two different number combinations as maintaining equality, then consider this as anticipated. |
| NRR | Core Concept 8 | If a child does show another number, consider skill statements in core concept 8. |
| Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |

NRR.B.5.e. Summary

## Essentialized Skill Statement

| Kindergarten | Grade 1 |  | Grade 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ |
| Compose a number with two or more parts using concepts of place value. |  |  |  |  |  |  |  |

## Student Expectation

Students were expected to listen to the prompt and use mathematical reasoning related to place value to explain what total number the combination of ones and tens would make. Students were also allowed to use place value blocks or other tools, if needed.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Write it out. Child wrote the number correctly <br> but could not provide reasoning for the <br> response. Breaking the whole number into units <br> and verbalizing how many of each unit is an <br> expected skill for Grade 2. | [Child stated same response twice without further |
| detail] One hundred and three (352, 15:49). |  |
| NME. Child requested markers to write and <br> drew an outdoor scene with no mathematical <br> relevance. | [Interviewer asked about drawing] A garden... I'm <br> going to make flowers.. it's the sky (223, 16:34- <br> 17:27) |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| K |  |  |  |  |  |  |
| $\mathbf{1}$ |  |  | NME (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Write it Out <br> (352) <br> NME (352) |  |  |

Unanticipated strategies by grade level and number range for NRR.B.5.e.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{K}$ | B.5.b. (341) <br> NME (CK) <br> Counting: All <br> (341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Tools: (I) Place-Value <br> Blocks (708) <br> B.5.b. (769) | - |  |  |  |
| $\mathbf{2}$ |  |  |  | - |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.5.e. activity.

| NRR.B.5.e. | Compose a number with two or more parts using concepts of place value. |  |
| :---: | :---: | :---: |
| Content Question | What number would 3 ones and [1,2,5 tens/ 1 hundred] make? |  |
| Reasoning Question | *You said that 3 ones and $\qquad$ tens makes $\qquad$ . How many ones do you need to make $\qquad$ ? How do you know that 3 ones and $\qquad$ tens makes $\qquad$ ? |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | - | No tools were embedded for this activity. Child was given cards with numbers. These cards were not intended as tools. However, examples of the cards as tools include and is not limited to, if the child uses the cards to create number sentences or places on an open number line. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B5e_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B5e_A_Add | Child adds numbers to find sum; differs from composing based on the child's vocabulary. Evidence of vocabulary use (e.g., add, plus, sum, NOT make) or writing symbols must be evident. Use of the word "addend" is unanticipated. |
|  | B5e_A_Compare Sums | Child compares the of Ben's cards to the sum of Carla's cards. |
|  | B5e_A_Count all | Child starts counting from one for the first card and continues to count each number individually. |
|  | B5e_A_Count on | Child recognizes number of first card, then continues to count on for second and third cards. |
|  | B5e_A_Representation | Child draws a representation such as tallies or dots and lines to aid in thinking. |
|  | B5e_A_Place value blocks | Count concrete place value blocks |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, consider the strategy as unanticipated. |
| NRR <br> Essentialized Skills | NRR.B.5.b. NRR.B.5.c. | Child might compose a number with two or more parts, but not necessarily use place value concepts. |
|  | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

## 6. Decomposition



NRR.B.6. Synthesis Data

Unanticipated strategies: NRR.B.6.a-d

|  |  |  | B.6.a. and B.6.b. |  |  |  |  |  | B.6.c. |  |  |  | B.6.d. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | nticip rateg |  | $\begin{array}{r} C \\ 6 a \end{array}$ | $\begin{array}{r} c \\ 6 b \end{array}$ | Construct <br> Numbers | Writing | Counting | Value of | C | $\begin{aligned} & N \\ & M \end{aligned}$ | Deconstruct Numbers Into | Writing | C | Writing |
| SID | Grade | Number Range |  |  | Using Digits |  |  | Digit |  | $E$ | Digits |  |  |  |
| CK | K | 0-5 | 0 | 0 | 1 | - | - | - | 0 | - | - | - | 1 | - |
| 341 | K |  | 0 | 0 | - | Expression | - | - | 0 | - | - | Numbers | 1 | - |
| 708 | 1 | 0-10 | 0 | 1 | - | - | - | - | 0 | 1 | - | - | 0 | - |
| 769 | 1 |  | 0 | 0 | 1 | - | - | - | 0 | - | 1 | - | 1 | - |
| 223 | 1 | 0-19 | 1 | 1 | - | - | Parts, All, | - | 8 | - | - | - | 1 | - |
| 352 | 2 | 0-50 | 0 | 1 | - | - | - | 1 | 1 | - | - | - | 0 | Equation |
| Strategies Total by Skill Code |  |  | 1 | 3 | 2 | 1 | 2 | 1 | 1 |  | 1 | 1 |  | 1 |

Unanticipated strategies: NRR.B.6.e-f

| Skill Code |  |  | B.6.e. |  |  |  | B.6.f. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | anticip trategi |  | C | Compare Numbers to Find | Construct <br> Numbers <br> Using Digits | Writing | C | Construct <br> Numbers Using Digits |
| SID | Grade | Number Range |  |  |  |  |  |  |
| CK | K | 0-5 | 0 | - | 1 | - | 8 | - |
| 341 | K |  | 1 | - | - | Expression | 8 | - |
| 708 | 1 | 0-10 | 0 | Different | - | - | 0 | - |
| 769 | 1 |  | 0 | - | - | - | 0 | 1 |
| 223 | 1 | 0-19 | 1 | - | - | - | 0 | - |
| 352 | 2 | 0-50 | 1 | Same | - | - | 1 | - |
| Strategies Total by Skill Code |  |  | 1 | 2 | 1 | 1 |  | 1 |

Anticipated strategies: NRR.B.6.a-c

| Skill Code |  |  | B.6.a. and B.6.b. |  |  |  |  | B.6.c. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Antic <br> Strat | pated gies |  | $\begin{array}{r} C \\ 6 a \end{array}$ | NRR. <br> B. 6 . | Tools | Counting | Basic Operat | $\begin{array}{r} C \\ 6 b \end{array}$ | Basic Opera | Tools | Zero | NRR.B. 5 |
| SID | Grade | Number Range |  |  |  |  | ions |  | tions |  |  |  |
| CK | K | 0-5 | 0 | - | (I) Linking Cubes | - | - | 0 | - |  | - | B.5.b |
| 341 | K |  | 0 | - | (I) Linking Cubes | - | - | 0 | - | - | - | - |
| 708 | 1 | 0-10 | 0 | B.6.b | (I) Linking Cubes | - | - | 0 | - | - | - | - |
| 769 | 1 |  | 0 | - | - | - | - | 0 | Add | - | 1 | B.5.b |
| 223 | 1 | 0-19 | 1 | B.6.b | (S) Fingers | - | - | 8 | - | - | - | - |
| 352 | 2 | 0-50 | 0 | - | (I) Written Number Line | On Skip(5) | Add | 1 | Add | - | - | $\begin{array}{\|l\|} \hline \text { B.5.c } \\ \text { B.5.e } \\ \hline \end{array}$ |
| Strategies Total by Skill Code |  |  | 1 | 2 | 5 | 2 | 1 | 1 | 2 | 1 | 1 | 4 |

Anticipated strategies: NRR.B.6.d-e

|  | Skill C | de | B.6.d |  |  |  |  |  | B.6.e. |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | $C$ | $\begin{array}{\|c\|} \hline \text { NRR. } \\ B .6 \end{array}$ | Counting | Distribute Equal | Zero | Different <br> Number | C | Basic Operat | Tools | Counting | Different <br> Number | $\begin{gathered} \hline N R R . \\ B .5 \end{gathered}$ | $\begin{gathered} \hline \text { NRR. } \\ B .6 \end{gathered}$ | Zero |
| SID | Grade | Number Range |  |  |  | Groups |  | Combina tions |  | ions |  |  | Combina tions |  |  |  |
| CK | K | 0-5 | 1 | B.6.b | On, Doubles | 1 | - | - | 0 | Add | - | All | - | B.5.b | - | - |
| 341 | K |  | 1 | B.6.b | - | 1 | - | - | 1 | - | - | On | - | - | - | - |
| 708 | 1 | 0-10 | 0 | B.6.c | - | 1 | 1 | - | 0 | - | (I) Linking Cubes | - | - | - | - | - |
| 769 | 1 |  | 1 | B.6.b | - | 1 | - | - | 0 | Add | - | - | 1 | - | $\begin{aligned} & \hline \text { B.6.b } \\ & \text { B.6.c } \end{aligned}$ | 1 |
| 223 | 1 | 0-19 | 1 | - | - | 1 | - | 1 | 1 | - | (S) Square Tiles | Parts, All | 1 | - | $\begin{aligned} & \hline \text { B.6.a } \\ & \text { B.6.b } \end{aligned}$ | - |
| 352 | 2 | 0-50 | 0 | $\begin{aligned} & \hline \text { B.6.a } \\ & \text { B.6.b } \end{aligned}$ | - | - | - | - | 1 | Add | - | - | 1 | $\begin{aligned} & \hline \text { B.5.b } \\ & \text { B.5.d } \end{aligned}$ | - | 1 |
| Strategies Total by Skill Code |  |  | 4 | 6 | 2 | 5 | 1 | 1 | 3 | 3 | 2 | 4 | 3 | 3 | 4 | 2 |

Anticipated strategies: NRR.B.6.f

| Skill Code |  |  | B.6.f |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Antici Strate |  |  | C | $\begin{aligned} & \text { NRR. } \\ & B .6 \end{aligned}$ | Counting | Tools | Basic Operations |
| SID | Grade | Number Range |  |  |  |  |  |
| CK | K |  | 8 | - |  |  | - |
| 341 | K | 0-5 | 8 | - |  |  | - |
| 708 | 1 | 0-10 | 0 | - | All | (I) Place Value Blocks | - |
| 769 | 1 |  | 0 | - |  |  | - |
| 223 | 1 | 0-19 | 0 | - | All |  | - |
| 352 | 2 | 0-50 | 1 | B.6.a | - | - | Add |
| Strategies Total by Skill Code |  |  | 1 | 1 | 2 | 1 | 1 |

NRR.B.6.a\&b. Summary

## Essentialized Skill Statement

| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | $\mathbf{T}$ | F | B | T | F | B |
| Decompose a number into two parts. |  |  |  |  |  |  |  |
|  | Decompose a number into two parts using equipartitioning. |  |  |  |  |  |  |

## Student Expectation

B.6.a. : Students were expected to listen to the prompt and select two number cards to combine in order to make the given even number. Doubles may or may not be used.
B.6.b. : Students were expected to listen to the prompt and select a different combination of two number cards in order to make the given even number. Doubles may or may not be used, depending on what was selected during B.6.a.

## Unanticipated Strategies

| Unexpected Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Constructing Numbers Using digits: Child used <br> two digits and created a new number by placing <br> two digits together. | Three and three, Thirty-three. Four, three, Forty- <br> three (CK, 12:39) <br> In response to "so you selected 10 and 20, how <br> would you make 20 from 10 and 20" student said: <br> "so, because it's like when you smash it together, <br> when you put it on top, it makes it 20 (769, 12:28) |
| Writing: Expression. Child wrote mathematical <br> expression (not equations) to work on the given <br> task and to explain his/her mathematical <br> understanding. | In response to adding any two numbers to make <br> number 4, student said: "I should add- (and wrote <br> 4+5)" a mathematical expression instead of a <br> mathematical equation (341, 12:29) |
| Counting: All. Child counted all of the counters <br> (general tool) as a proof. Children in Grade 1 and <br> beyond should have skills to compose two | One, two, three, four, five, six, seven, eight, nine, <br> ten [counted all counters when asked if they made <br> numbers or by adding or counting on to replace <br> counting all. |
| ten] (223, 22:09). |  |
| Counting: Parts. In response to two parts of a <br> number, instead of Counting All numbers, the <br> child counted each parts. | In response to breaking apart 10 into 6 and 4, <br> student counted "One, two, three, four (for the art <br> 4) one, two, three, four, five, six (for the part 6) |
| Value of Digit. Children treated digits in multi- <br> digit numbers as independent when adding or <br> putting the digits in mathematically unsound <br> orders to create other numbers. Children should <br> recognize two-digit numbers as one number and <br> use composition skills to combine numbers. | Because three plus three equals six [explanation <br> for 30+30=60, without referencing units] (352, |
| $25: 30$ ) |  |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $0-99$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Construct <br> Numbers Using <br> Digits (CK) <br> Writing: <br> Expression (341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Construct <br> Numbers Using <br> Digits (769) | Counting: Parts <br> (223) <br> Counting: All <br> (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Value of Digit <br> (352) |  |  |

Unanticipated strategies by grade level and number range for NRR.B.6.a\&b
$\left.\begin{array}{|c|l|l|l|l|l|l|}\hline \text { Grade } & \mathbf{0 - 5} & \mathbf{0 - 1 0} & \mathbf{0 - 1 9} & \mathbf{0 - 5 0} & \begin{array}{c}\mathbf{0 - 1 9} \\ \mathbf{9 9}\end{array} & \begin{array}{c}\mathbf{0 -} \\ \mathbf{1 9 9}\end{array} \\ \hline \mathbf{K} & \begin{array}{c}\text { Tools: (I) } \\ \text { Linking } \\ \text { Cubes (CK, } \\ 341)\end{array} & & & & \\ \hline \mathbf{1} & & \begin{array}{c}\text { Tools: (I) } \\ \text { Linking Cubes } \\ \text { (708) } \\ \text { B.6.b. (708) }\end{array} & \begin{array}{c}\text { Tools: (S) } \\ \text { Fingers } \\ \text { (223) } \\ \text { B.6.b. (223) }\end{array} & & & \\ \hline \mathbf{2} & & & & \begin{array}{c}\text { Tools: (I) Written Number Line } \\ \text { (352) } \\ \text { Counting: On (352) }\end{array} \\ \text { Counting: Skip(5) (352) Basic } \\ \text { Operations: Add (352) }\end{array}\right]$

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.6.a\&b activity.

| NRR.B.6.a. | Decompose a number into two parts. |  |
| :---: | :---: | :---: |
| Content Question | I want to break apart the number___ What two numbers can you use to make the number___ |  |
| Reasoning Question | *How could you use one of these tools to show me that $\qquad$ breaks apart into_and_? Tell me more about how you figured out that $\qquad$ is the same as $\qquad$ -. |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | B6a_E_NOT-Part-part-whole | Part-part-whole figure was given to child. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | $\begin{aligned} & \text { B6a_A_General } \\ & \text { tool } \end{aligned}$ | Child uses a concrete object to demonstrate the skill. |
|  | B6a_A_Add | Child adds numbers to find sum; differs from composing based on the child's vocabulary. Evidence of vocabulary use (e.g., add, plus, sum, NOT make) or writing symbols must be evident. Use of the word "addend" is unanticipated. |
|  | B6a_A_Count on | Child acknowledges whole number, then counts on from a part then stops at whole number. |
|  | B6a_A_Doubles | Child uses doubles |
|  | B6a_A_Subtract | Child subtracts number from whole to find part. Evidence of vocabulary use (e.g., subtract, difference, minus, NOT take away) or writing symbols must be evident. Excluding "take away" to differentiate computational subtraction from the act of decomposing a number. |
|  | B6a_A_Unit | Child breaks given whole number into 1s, 10s, and/or 100s. |
|  | B6a_A_Zero | Child recognizes the identity property of addition (i.e., the sum of zero and a number is that number). |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, consider the strategy as unanticipated. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |


| NRR.B.6.b. | Decompose a number into two parts using equipartitioning. |  |
| :---: | :---: | :---: |
| Content Question | Content question asked depends on if child responded with doubles in the previous question. Equipartitioning: How would you break apart $\qquad$ into two parts that are the same number? What numbers would you use? Non-Equal groups: How would you break apart $\qquad$ into two parts using numbers that are not the same? What numbers would you use? |  |
| Reasoning Question | *How did you figure out that $\qquad$ breaks apart into $\qquad$ and ?$\qquad$ Show me what you saw in your head using pictures, words, or numbers. What made you decide to [child's decision]? |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | B6b_E_NOT-Part-partwhole | Part-part-whole figure was given to child. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B6b_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B6b_A_Add | Child adds numbers to find sum. Evidence of vocabulary use (e.g., add, addend, plus, sum, NOT make) or writing symbols must be evident. |
|  | B6b_A_Count on | Child acknowledges whole number, then counts on from a part then stops at whole number. |
|  | B6b_A_Doubles | Child uses doubles |
|  | B6b_A_Subtract | Child subtracts number from whole to find part. Evidence of vocabulary use (e.g., subtract, difference, minus, NOT take away) or writing symbols must be evident. Excluding "take away" to differentiate computational subtraction from the act of decomposing a number. |
|  | B6b_A_Unit | Child breaks given whole number into 1s, 10s, and/or 100s. |
|  | B6b_A_Zero | Child recognizes the identity property of addition (i.e., the sum of zero and a number is that number). |
| A different way |  | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, consider the strategy as unanticipated. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

NRR.B.6.c. Summary
Essentialized Skill Statement

| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B $\quad$ T

## Student Expectation

Students were expected to select three number cards that equal the same value of the given number in the prompt. The number is written as the whole in a part-part-whole chart and the student selects cards or writes numbers in as parts.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Deconstructing Numbers Into Digits: Child <br> separated both digits of a two-digit number to <br> break apart a given number into two or three <br> pieces. | In response to breaking apart 15 into three <br> numbers, student said "one and five" (769, 14:48) |
| Writing: Numbers. Child wrote mathematical <br> expression (not equations) to work on the given <br> task and to explain his/her mathematical <br> understanding. | In response to break number 3, student wrote 2 <br> and then wrote another 2 and said "I made a two. <br> Then I turned, then I made a two" (341, 16:59) |
| NME. Either the interviewer or child determined <br> that the content question was too difficult. Both <br> children were in Grade 1 and should have <br> prerequisite skills of decomposing a number into <br> two parts, making this question accessible <br> through scaffolding. | [Student gave interviewer 'thumbs down' to |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Writing: Numbers <br> (341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Deconstruct <br> Numbers Into <br> Digits (769) <br> NME (708) | - |  |  |  |
| $\mathbf{2}$ |  |  |  | - |  |  |

Unanticipated strategies by grade level and number range for NRR.B.6.c.

| Grade | 0-5 | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $0-99$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Tools: (I) Linking Cubes <br> (CK) <br> B.5.b. (CK) |  |  |  |  |  |
| $\mathbf{1}$ |  | Zero (769) <br> Basic Operations: Add <br> (769) | - |  |  |  |
|  |  | B.5.b. (769) |  |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.6.c. activity.

| NRR.B.6.C. | Decompose a number into three or more parts. |  |
| :---: | :---: | :---: |
| Content Question | What three numbers can you use to break apart the number____? |  |
| Reasoning Question | *How do you know that $\qquad$ breaks into $\qquad$ , $\qquad$ , and ?$\qquad$ What strategy do you like to use to break apart numbers? Tell me more about what you did... |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | B6c_E_NOT-Part-part-part-whole | Part-part-part-whole figure was given to child. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
| Mathematical Tools | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B6c_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B6c_A_Add | Child adds numbers to find sum. Evidence of vocabulary use (e.g., add, addend, plus, sum, NOT make) or writing symbols must be evident. |
|  | B6c_A_Count on | Child acknowledges whole number, then counts on from a part then stops at whole number. |
|  | B6c_A_Subtract | Child subtracts number from whole to find part. Evidence of vocabulary use (e.g., subtract, difference, minus, NOT take away) or writing symbols must be evident. Excluding "take away" to differentiate computational subtraction from the act of decomposing a number. |
|  | B6c_A_Unit | Child breaks given whole number into 1s, 10s, and/or 100s. |
|  | B6c_A_Zero | Child recognizes the identity property of addition (i.e., the sum of zero and a number is that number). |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, consider the strategy as unanticipated. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

## Essentialized Skill Statement

| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B | T |

## Student Expectation

Students were expected to think about the possible ways that the given number of cow pictures could be arranged into equal groups. Then students were expected to arrange the cow pictures into equal groups using counting and grouping strategies.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Writing: Equation. Child wrote an equation to <br> show a two-part composition. The child was <br> expected to interact with the tools to show <br> equipartitioning but student chose to write an <br> equation to explain thinking instead. | Student wrote the equation $12+12=24$ to <br> represent cows in two barns (352, 31:24) |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $K$ | - |  |  |  |  |  |
| $\mathbf{1}$ |  | - | - |  |  |  |
| $\mathbf{2}$ |  |  |  | Writing: <br> Equation (352) |  |  |

Unanticipated strategies by grade level and number range for NRR.B.6.d.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 -}$ <br> 99 | $\mathbf{0 - 1 9 9}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{K}$ | Distributed Equal Groups <br> (CK, 341) <br> B.6.b. (CK, 341) <br> Counting: On (CK) <br> Counting: Doubles (CK) |  |  |  |  |  |
| $\mathbf{1}$ |  | Distributed <br> Equal Groups <br> (708, 769) <br> B.6.c. (708) <br> B.6.b. (769) <br> Zero (708) | Distributed Equal <br> Groups (223) <br> Different Number <br> Combinations (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | B.5.a. (352) |  |  |

[^1]| NRR.B.6.d. | Decompose a number up to 25 into three or more parts using equipartitioning. |  |
| :---: | :---: | :---: |
| Content Question | Before you move the animals, how many animals do you think will be in each barn? |  |
| Reasoning Question | *What if there were $\qquad$ barns, how would that change how many animals are in each barn? How did you figure out that there are $\qquad$ animals in each barn? Can you prove that there would be $\qquad$ animals in each barn by using these things [point to the barns and animal cards]? |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | B6d_E_NOT-Pictures | Pictures of cows and barns were provided to child. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B6d_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B6d_A_Array | Child arranges cow in an array rather than a grouping. |
|  | B6d_A_Fair share | Child distributes one cow to each barn individually. |
|  | B6d_A_Groups | Child assigns groups of cows to each barn, then balances the number of cows in each barn. |
| A different way | B6d_D_Different | The required reasoning question adds a barn and asks that child how adding the barn changes the number of animals in each barn. A different way is coded if child is able to resdistribute the cows evenly with the additional barn. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | $T$ | F | B | T |
|  |  | Decompose a number into two or more parts using different number combinations. |  |  |  |  |  |  |

## Student Expectation

Students were expected to select number cards that represented at least two different ways of decomposing the given number.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Compare Numbers to Find Same/Different. <br> Student compared one numbers with other (or <br> part of the other number) to compare two <br> numbers to decide which one is bigger and/or <br> smaller among given three numbers. | In response to pick two numbers that will add up <br> to the number five, student picked two and one, <br> put them together to make twelve (CK, 21:42 - <br> $22: 26)$ |
| Construct Numbers Using Digits. Student put <br> cards with different digits on them next to each <br> other to construct new numbers. | In response to breaking a number, <br> Students said this one is two and this one is two, <br> and this one is three and four (Different) (708, <br> $32: 43)$ <br> Students compared two numbers to said they <br> both have five (Same) (352, 39:12) |
| Writing: Expression. Child wrote mathematical <br> expression (not equations) to work on the given <br> task and to explain his/her mathematical <br> understanding. | Student wrote "two plus two" and verbally said it <br> is four (341, 24;49) |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K |  |  |  |  |  |  |
| $\mathbf{1}$ |  | Doubles (769) | Doubles (223) <br> Count parts <br> (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Digits (352) |  |  |

[^2]| Grade | 0-5 | 0-10 | 0-19 | 0-50 | 0-99 | 0-199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Count on (341) <br> Add (CK) <br> Count all (CK) <br> B.5.b. (CK) Write it out (341) Digits (CK) |  |  |  |  |  |
| 1 |  | General tools (708) <br> Zero (769) <br> Add (769) <br> Different Way (769) <br> B.6.b. (769) <br> B.6.c. (769) | General tools (223) <br> Count all (223) <br> Different way (223) <br> B.6.a. (223) <br> B.6.b. (223) |  |  |  |
| 2 |  |  |  | $\begin{gathered} \hline \text { Add (352) } \\ \text { Zero. (352) } \\ \text { Different } \\ \text { way (352) } \\ \text { B.5.b. (352) } \\ \text { B.5.d. (352) } \\ \hline \end{gathered}$ |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.6.e. activity.

| NRR.B.6.e. | Decompose a number into two or more parts using different number combinations. |  |
| :---: | :---: | :---: |
| Content Question | Here are some number cards. Can you show me two different ways to break apart the number_? Are there other ways to break apart the number $\qquad$ ? |  |
| Reasoning Question | *What is similar about the number combinations you chose? How do you know that $\qquad$ and $\qquad$ is the same as $\qquad$ and $\qquad$ ? Show me what you saw in your head using pictures, words, or numbers. |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools |  | No tools were embedded for this activity. Child was given cards with numbers. These cards were not intended as tools. However, examples of the cards as tools include and is not limited to, if the child uses the cards to create number sentences or places on an open number line. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
| Mathematical Tools | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B6e_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B6e_A_Add | Child adds numbers to find sum. Evidence of vocabulary use (e.g., add, addend, plus, sum, NOT make) or writing symbols must be evident. |
|  | B6e_A_Count all | Child starts counting from one for the first card and continues to count each number individually until reaching the given whole number. |
|  | B6e_A_Count on | Child acknowledges whole number, then counts on from a part then stops at whole number. |
|  | B6e_A_Subtract | Child subtracts number from whole to find part. Evidence of vocabulary use (e.g., subtract, difference, minus, NOT take away) or writing symbols must be evident. Excluding "take away" to differentiate computational subtraction from the act of decomposing a number. |
|  | B6e_A_Zero | Child recognizes the identity property of addition (i.e., the sum of zero and a number is that number). |
| A different way | B6e_D_Different | The content question specifically asks for two different number combinations. A different way is coded if child is able to break apart the given number using at least two different number combinations. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

NRR.B.6.f. Summary
Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B | T |
|  |  | Decompose a number with two or more parts using concepts of place value. |  |  |  |  |  |  |

## Student Expectation

Students were expected to use mathematical reasoning and place value to determine the number of ones and tens needed to decompose a given number.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Construct Numbers Using Digits. Student | [Interviewer asked how many tens and ones to make 14, |
| put cards with different digits on them |  |
| next to each other to construct new |  |
| numbers. | child searched for number cards] 1. 1. 1... And two 4's... <br> because it makes 14 (769, 26:33-24:43). <br> Because 10 plus 10 equals 20, and 20 plus 4 equals 24 <br> $(352,40: 32)$. |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| K |  |  |  |  |  |  |
| $\mathbf{1}$ |  | Digits (769) |  |  |  |  |
| 2 |  |  |  | Units (352) |  |  |

Unanticipated strategies by grade level and number range for NRR.B.6.f.

| Grade | 0-5 | 0-10 | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (341, CK) |  |  |  |  |  |
| $\mathbf{1}$ |  | General Tool (708) <br> Count all (708) | Count all (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Add (352) <br> Doubles <br> (352) <br> B.6.a. (352) |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.6.f. activity.

| NRR.B.6.f. | Decompose a number into two or more parts using concepts of place value. |
| :--- | :--- | :--- |
| Content Question | How many ones and how many tens would you need to make__? ? |

## 7. Applying and Representing Composition and Decomposition

| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F | B | T | F | B | T | F | B | T |
| NRR.B.7.a. |  | Given one part of a number, identify the missing part. |  |  |  |  |  |  |  |
| NRR.B.7.b. |  | Given a unit, identify the missing part. |  |  |  |  |  |  |  |
| NRR.B.7.c. |  |  |  |  | Given one part of a number, identify two or more missing parts. |  |  |  |  |
| NRR.B.7.d. |  |  |  |  | Given on |  |  |  | parts |
| NRR.B.7.e. |  | Write an expression to represent the decomposition of a number. |  |  |  |  |  |  |  |

NRR.B.7.a. Synthesis

Unanticipated strategies: NRR.B.7.a-c

| Skill Code |  |  | B.7.a. |  |  | B.7.b. |  |  |  | B.7.c. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unanticipated Strategies |  |  | C | $\begin{gathered} N \\ M \\ E \end{gathered}$ | Unfounded: Take Away | C | $N$ $M$ | Constructing Numbers | Unfounded: Take Away | C | Constructing Numbers | Counting | Writing |
| SID | Grade | Number Range |  |  |  |  | E | Using Digits |  |  | Using Digits |  |  |
| CK | K | 0-5 | 1 | - | - | 1 | - | - | - | 0 | - | - | - |
| 341 | K |  | 1 | - | - | 1 | - | - | - | 0 | - | - | - |
| 708 | 1 | 0-10 | 0 | - | - | 0 | - | - | 1 | 8 | - | - | - |
| 769 | 1 |  | 0 | - | 1 | 0 | - | 1 | - | 0 | 1 | - | Equation |
| 223 | 1 | 0-19 | 0 | 1 | - | 0 | 1 | - | - | [0-10] 0 | - | All | - |
| 352 | 2 | 0-50 | 1 | - | - | 1 | - | - | - | 1 | - | - | - |
| Strategies Total by Skill Code |  |  | 3 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Unanticipated strategies: NRR.B.7.d-e

|  | Skill C |  | B.7.d. |  |  |  |  | B.7.e. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | nticip rategi |  | C | Writing | Counting | Constructing Numbers | NME | C | NME | Writing |
| SID | Grade | Number Range |  |  |  | Using Digits |  |  |  |  |
| CK | K |  | 0 | Numbers | - | - | - | 0 | - | - |
| 341 | K |  | 0 | - | - | - | - | 0 | - | - |
| 708 | 1 |  | 8 | - | - | - | - | 8 | - | - |
| 769 | 1 |  | 0 | - | - | 1 | - | 8 | - | - |
| 223 | 1 | 0-19 | [0-10] 0 | Numbers | All | - | - | [0-10] 0 | 1 | - |
| 352 | 2 | 0-50 | 0 | - | - | - | 1 | 1 | - | Equation |
| Strategies Total by Skill Code |  |  | 0 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |

Anticipated strategies: NRR.B.7.a-b

| Skill Code |  |  | B.7.a. |  |  |  |  |  | B.7.b. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | C | NME | Counting | Tool | Basic Operation | B.5.b. | C | NME | Counting | Tool | Basic Operation |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |  |  |
| CK | K |  | 1 | 1 | - | - | - | - | 1 | 1 | - | - | - |
| 341 | K | 0-5 | 1 | - | All | - | - | 1 | 1 | - | All | (I) Square Tiles | - |
| 708 | 1 | 0-10 | 0 | - | - | (I) Linking Cubes | - | - | 8 | - | - | (I) Linking Cube | - |
| 769 | 1 |  | 0 | - | - | - | - | - | 8 | - | - | - | - |
| 223 | 1 | 0-19 | 0 | - | - | - | - | - | 0 | - | - | - | - |
| 352 | 2 | 0-50 | 1 | - | - | - | Subtraction | - | 1 | - | - | - | Addition, Subtraction |
| Strategies Total by Skill Code |  |  | 3 | 1 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 2 | 2 |

Anticipated strategies: NRR.B.7.c-e

|  | Skill Cod |  |  |  | B.7.c. |  |  |  |  |  | B.7.d. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | C | NME | Tool | Counting | Basic Operation | B.5.b. | B.5.c. | C | Counting | Tool | Basic Operation | B.5.c. | C | NME |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| CK | K | 0-5 | 0 | 1 | - | - | - | - | - | 0 | - | - | - | - | 0 | 1 |
| 341 | K |  | 0 | - | (I) Linking Cubes | All | - | - | - | 0 | All | (I) Linking Cubes | - | - | 0 | 1 |
| 708 | 1 | 0-10 | 0 | 1 | - | - | - | - | - | 8 | - | - | - | - | 8 | - |
| 769 | 1 |  | 0 | - | - | - | Addition | 1 | - | 0 | - | - | Addition | 1 | 8 | - |
| 223 | 1 | 0-19 | $\begin{gathered} {[0-10]} \\ 0 \end{gathered}$ | - | (I) Square Tiles | - | - | - | - | $\begin{gathered} {[0-10]} \\ 0 \end{gathered}$ | - | (S) Square Tiles | - | - | $\begin{gathered} {[0-10]} \\ 0 \end{gathered}$ | - |
| 352 | 2 | 0-50 | 1 | - | - | - | Addition | - | 1 | 0 | - | - | - | - | 1 | - |
| Strategies Total by Skill Code |  |  | 1 | 2 | 2 | 1 | 2 | 1 | 1 | 0 | 1 | 2 | 1 | 1 | 1 | 2 |

NRR.B.7.a. Summary
Essentialized Skill Statement

| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B |
|  | Given one part of a number, identify the missing part. |  |  |  |  |  |  |

## Student Expectation

Students were expected to use deductive reasoning skills and addition/subtraction strategies, to determine the number of cubes that remain in a bag when told the total number and then given one part of the cubes.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| NME. Children in Grade 1 were expected to use <br> counting or subtraction strategies to generate <br> an answer. Child did not provide any <br> mathematical information or reasoning. | I don't know (223, 32:38, 32:45) |
| Unfounded: Take Away. Children used the term <br> "take away" to compare the initial quantity with <br> the remaining quantity after removing a portion. | In response to interviewer question 'there were 9 <br> marbles in the bag, I gave you 4, how many are <br> still left in this bag' student said '4, because when <br> lou take away 4, there was 4 more left' (769, <br> 29:19-29:51). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $0-99$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - |  |  |  |  |  |
| $\mathbf{1}$ |  | Unfounded: Take <br> Away (769) | NME (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | - |  |  |

Unanticipated strategies by grade level and number range for NRR.B.7.a.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (CK) <br> Count all (341) <br> B.5.b. (341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Tools: (I) Linking <br> Cubes (708) | - |  |  |  |
| $\mathbf{2}$ |  |  |  | Basic Operation: <br> Subtraction (352) |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.7.a. activity.

| NRR.B.7.a. | Given one part of a number, identify the missing part. |  |
| :---: | :---: | :---: |
| Content Question | There are_tiles in this bag. I'm going to give you_of the tiles. How many tiles are still in the bag? |  |
| Reasoning Question | *How do you know that there are $\qquad$ tiles still in the bag? Can you use one of these tools to prove that there are_cubes eft in the bag? Show me what you saw in your head using pictures words or numbers. |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | $\begin{aligned} & \text { B7a_E_NOT- } \\ & \text { tiles } \end{aligned}$ | Colored square tiles as counters. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B7a_A_Count all | Child attempts to count all counters in the bag. |
|  | B7a_A_Count on | Child starts with total of tiles removed from bag, then counts on until the total number of tiles that were in the bag |
|  | B7a_A_Subtract | Child subtracts tiles removed from total in bag to find number of tiles left in bag. Evidence of vocabulary use (e.g., subtract, difference, minus, NOT take away) or writing symbols must be evident. Excluding "take away" to differentiate computational subtraction from the actof decomposing a number. |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, consider the strategy as unanticipated. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

## Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B | T |
|  |  | Given a unit, identify the missing part. |  |  |  |  |  |  |

## Student Expectation

Students were expected to use deductive reasoning skills and addition/subtraction strategies, to determine the number of cubes that remain in a bag when told the total number and then given one part of the cubes.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| NME. Children in Grade 1 were expected to use <br> counting or subtraction strategies to generate <br> an answer. Child did not provide any <br> mathematical information or reasoning. | I don't know (223, 32:54, 33:03). |
| Constructing Numbers Using Digits. Student put <br> different digits next to each other to construct $a$ <br> new numbers. | Because uh.. um.. I forgot (708, 42:22- 73:03). |
| Unfounded: Take Away. Children used the term 5 and 4 make 54 (769, 30:42) <br> "take away" to compare the initial quantity with <br> the remaining quantity after removing a portion. | Because those 8 are here and you took away two, <br> and I think there is one more (708, 42:22) |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - |  |  |  |  |  |
| $\mathbf{1}$ |  | Constructing Numbers <br> Using Digits (769) <br> Unfounded: Take Away <br> (708) | NME (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Add (352) |  |  |

Unanticipated strategies by grade level and number range for NRR.B.7.b.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (CK, 708) <br> Tool: (I) Square <br> Tiles (341) <br> Counting: All <br> (341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Tools: (I) Linking <br> Cubes (769) Basic <br> Operations: Addition <br> (769) | Tools: (I) <br> Square Tiles <br> (223) |  | Basic Operations: <br> Addition, <br> Subtraction (352) |  |
| $\mathbf{2}$ |  |  |  |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.7.b. activity.

| NRR.C.7.b. | Given a unit, identify the missing part. |  |
| :---: | :---: | :---: |
| Content Question | Now there are_tiles in the bag. I am going to give you_tile. How many tiles are still in the bag? |  |
| Reasoning Question | *Tell me more about how you figured out that $\qquad$ Show me what you saw in your head using pictures words, or numbers. |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded <br> Mathematical Tools | $\begin{aligned} & \text { B7b_E_NOT- } \\ & \text { tiles } \end{aligned}$ | Colored square tiles as counters. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B7b_A_Count all | Child attempts to count all counters in the bag. |
|  | B7b_A_Count on | Child starts with total of tiles removed from bag, then counts on until the total number of tiles that were in the bag |
|  | B7b_A_Subtract | Child subtracts tiles removed from total in bag to find number of tiles left in bag. Evidence of vocabulary use (e.g., subtract, difference, minus, NOT take away) or writing symbols must be evident. Excluding "take away" to differentiate computational subtraction from the act of decomposing a number. |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, consider the strategy as unanticipated. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

NRR.B.7.c. Summary
Essentialized Skill Statement

| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ |
|  |  |  |  | Given one part of a number, identify two or more missing |  |  |  |  |
|  |  |  |  | parts. |  |  |  |  |

## Student Expectation

Students were expected to use deductive reasoning skills and addition/subtraction strategies to determine the missing part of the whole using the information given in the prompt. Students were shown a part-part-whole table with the total and one of three parts filled in.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Counting: All. Child counted all counters instead of <br> counting on from the given part of the whole. Child <br> recognized that the response was incorrect and <br> did not successfully correct, but the child <br> demonstrated a prerequisite skill to counting on. | One, three, four, five, six, seven, eight nine, <br> ten 35:01). |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - |  |  |  |  |  |
| $\mathbf{1}$ |  | - | Count all (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | - |  |  |

Unanticipated strategies by grade level and number range for NRR.B.7.c.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Tools: (I) Linking <br> Cubes (341) NME <br> (CK) <br> Counting: All (341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Basic <br> Operations: <br> Addition (769) <br> B.5.b. (769) | Tools: (I) <br> Square Tiles <br> (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Basic Operations: <br> Addition (352) <br> B.5.c. (352) |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.7.c. activity.

| NRR.B.7.c. | Given one part of a number, identify two or more missing parts. |  |
| :---: | :---: | :---: |
| Content Question | I want to use three numbers to make___ . I already have___. What other two numbers could I use to make___? |  |
| Reasoning Question | *How do you know that $\qquad$ , $\qquad$ , and $\qquad$ make $\qquad$ ? <br> How did you figure out that the other two numbers could be_and? Show me what you saw in your head using pictures, words, or numbers. |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded <br> Mathematical Tools | B7c_E_NOT-Part-part- whole | Part-part-whole figure was given to child. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B7c_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B7c_A_Add | Child adds numbers to find sum. Evidence of vocabulary use (e.g., add, addend, plus, sum, NOT make) or writing symbols must be evident. |
|  | B7c_A_Count on | Child acknowledges whole number, then counts on from a part then stops at whole number. |
|  | B7c_A_Doubles | Child uses doubles |
|  | B7c_A_Representation | Child draws a representation such as tallies or dots and lines to aid in thinking. |
|  | B7c_A_Subtract | Child subtracts number from whole to find part. Evidence of vocabulary use (e.g., subtract, difference, minus, NOT take away) or writing symbols must be evident. Excluding "take away" to differentiate computational subtraction from the act of decomposing a number. |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, consider the strategy as unanticipated. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

NRR.B.7.d. Summary
Essentialized Skill Statement

| Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B | T |
|  |  |  |  | Given one part of a number, identify two or more missing parts using different number combinations |  |  |  |  |

## Student Expectation

Students were expected to determine a different combination of numbers that total the same whole given in the previous prompt.

## Unanticipated Strategies

| Unanticipated Strategy. Justification/Description | Examples |
| :---: | :---: |
| NME. Child was expected to engage with the question based on age in Grade 2, but did not provide verbal or action evidence of understanding or reasoning. | [Child shook head no when asked to explain or draw; transcript and video showed student not responding to prompts and scaffolding] (352). |
| Counting: All. Child (grade 1, number rage 0-19) counted all counters instead of counting on from the given part of the whole. Child recognized that the response was incorrect and did not successfully correct, but the child demonstrated a prerequisite skill to counting on. | In response to 'can you make number 7 using any three numbers' student wrote and said, One, two, three, four, five, six, and seven (223, 37:58) |
| Constructing Numbers Using Digits. Student put different digits next to each other to construct a new numbers. | For the question to add given numbers 3,3 , and 1 , student said it will 31 and then said 3 plus 3 plus 1 will be 103 |
| Writing: Numbers. Children (KG and Grade 1) were expected to be construct a number with three parts but students wrote individual numbers/digits | In response to 'use three numbers to make number 5 ' student wrote 2,64 , and 70 and said because when I thinked in my head and then what I did was, I thinked in my then I thinked I might grow slow (CK, 31:55) |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $0-99$ | $0-199$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{K}$ | Writing: Numbers <br> (CK) |  | Constructing <br> Numbers Using <br> Digits (769) | Writing: <br> Numbers (223) <br> Counting: All <br> (223) |  |  |
| $\mathbf{1}$ |  |  |  | NME (352) |  |  |
| $\mathbf{2}$ |  |  |  |  |  |  |

Unanticipated strategies by grade level and number range for NRR.B.7.d.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{K}$ | NME (CK) <br> Tools: (I) Linking <br> Cubes (341) <br> Count all (341) |  |  |  |  |  |
| $\mathbf{1}$ |  | Basic Operations: <br> Addition (769) <br> B.5.b. (769) <br> NME (708) | Tools: (I) Sq. <br> Tiles (223) |  |  |  |
| $\mathbf{2}$ |  |  | Basic <br> Operations: <br> Addition (352) <br> B.5.c. (352) |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.7.d. activity.

| NRR.B.7.d. | Given one part of a number, identify two or more missing parts using different number combinations. |  |
| :---: | :---: | :---: |
| Content Question | Let's look back at the numbers you used to make $\square$ [number from last item]. I already have $\qquad$ . Can you think of two other numbers that we can use to make $\qquad$ ? |  |
| Reasoning Question | *Can you show me how $\qquad$ <br> Tell me more about how y Show me what you saw in | $\qquad$ and $\qquad$ and $\qquad$ and $\qquad$ can both make $\qquad$ ? your head using pictures, words or numbers. |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | B7d_E_NOT-Part-part-whole | Part-part-whole figure was given to child. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B7d_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B7d_A_Add | Child adds numbers to find sum. Evidence of vocabulary use (e.g., add, addend, plus, sum, NOT make) or writing symbols must be evident. |
|  | B7d_A_Count on | Child acknowledges whole number, then counts on from a part then stops at whole number. |
|  | B7d_A_Doubles | Child uses doubles |
|  | B7d_A_Representation | Child draws a representation such as tallies or dots and lines to aid in thinking. |
|  | B7c_A_Subtract | Child subtracts number from whole to find part. Evidence of vocabulary use (e.g., subtract, difference, minus, NOT take away) or writing symbols must be evident. Excluding "take away" to differentiate computational subtraction from the act of decomposing a number. |
| A different way | B7d_D_Different | The content question specifically asks for two different number combinations. A different way is coded if child is able to break apart the given number using at least two different number combinations. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

## Essentialized Skill Statement

| Kindergarten |  | Grade 1 |  | Grade 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | B | T | F | B | T | F | B |$|$| T |  |  |  |
| :--- | :---: | :---: | :---: |
|  |  | Write an expression to represent the decomposition of a number. |  |

## Student Expectation

Students were expected to write an equation using the numbers combined to make the whole in the previous two prompts.

## Unanticipated Strategies

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| NME. Transcript not provided because <br> interviewer omitted question based on student <br> engagement or ability, or child did not respond | I don't know [response given 5 times to differently <br> scaffolding questions] (223) |
| to content questions with any mathematical |  |
| evidence of reasoning. |  |
| Writing: Equation. Children (Grade 2) were <br> expected to understand the difference between <br> mathematical expression and equation. Student <br> wrote a mathematical equation in response to <br> the question of writing expression involving <br> three numbers. | In response to writing an expression using 20, 10, <br> and 5 to represent 35, student wrote a <br> mathematical equation $20+10+5=35$. |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - |  |  |  |  |  |
| $\mathbf{1}$ |  | - | NME (223) |  |  |  |
| $\mathbf{2}$ |  |  |  | Writing: <br> Equation <br> $(352)$ |  |  |

Unanticipated strategies by grade level and number range for NRR.B.7.e.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (341, CK) |  |  |  |  |  |
| $\mathbf{1}$ |  | - | - |  |  |  |
| $\mathbf{2}$ |  |  |  | - |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.B.7.e. activity.

NRR.B.7.e. Anticipated Strategies

| NRR.B.7.e. | Write an expression to represent the decomposition of a number. |  |
| :---: | :---: | :---: |
| Content Question | Let's look back at the numbers you used to make $\qquad$ [number from last item]. .... <br> Can you write a number sentence to show that these three numbers make __? |  |
| Reasoning Question | *How do you know that this number sentence represents $\qquad$ ? Is there another way that you could write this number sentence? |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | B7e_E_NOT-Part-part- whole | Part-part-whole figure was given to child. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | B7e_A_General tool | Child uses a concrete object to demonstrate the skill. |
|  | B7e_A_Expression | Child adds numbers to find sum. Evidence of vocabulary use (e.g., add, addend, plus, sum, NOT make) or writing symbols must be evident. |
|  | B7e_A_Equation | Child acknowledges whole number, then counts on from a part then stops at whole number. |
| A different way | * | The reasoning question that specifically asks for another number sentence was not a required reasoning question. <br> If the question was asked, then a different way was anticipated. <br> If the question was not asked, then a different way was not anticipated. |
| NRR <br> Essentialized Skills | Other NRR Skill Codes | Consider ESSs not specifically listed here; refer to learning progressions. <br> - Skills prior to this skill code are considered anticipated. <br> - Skills beyond this skill code are considered unanticipated. |
|  | Non-NRR Skill Codes | Content evident in child's reasoning that are not captured in the current NRR learning progressions are considered unanticipated. |

## Appendix D - Integrated Strategies Document: Properties of Operations ${ }^{2}$

NRR.C.8. Equivalence of Quantity and Number Core Concept

| 8. Equivalence of Quantity and Number |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.C.8.a. | Given equivalent sets of quantities, recognize that the quantity of each set remains the same regardless of size, color, or arrangement. (conservation of number) |  |  |  |  |  |  |  |  |
| NRR.C.8.b. | Given a quantity broken into two parts, recognize that order does not change the quantity. (commutative property) |  |  |  |  |  |  |  |  |
| NRR.C.8.c. | Given a quantity, recognize that the quantity remains the same after joining/removing a part then removing/joining the same part. (undoing or additive inverse) |  |  |  |  |  |  |  |  |
| NRR.C.8.d. |  |  |  |  | Given two associated parts and another part, recognize that the quantity of the three parts remains the same if the parts are reassociated. (associative property) |  |  |  |  |
| NRR.C.8.e. |  | Given a quantity, recognize an equivalent expression that demonstrates one or more property of operations. |  |  |  |  |  |  |  |
| NRR.C.8.f. |  | Recognize two equivalent expressions that demonstrate one or more property of operations. |  |  |  |  |  |  |  |
| NRR.C.8.g. |  |  | Recognize two equivalent expressions that demonstrate decomposition and at least one property of operations. |  |  |  |  |  |  |

[^3]120

Unanticipated strategies: NRR.C.8.a-c

| Skill Code |  |  | C.8.a. |  |  | C.8.b |  | C.8.c |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unanticipated |  |  | C | Change in Total | $\begin{aligned} & N \\ & M \\ & E \end{aligned}$ | C | Unfoun ded | C | Compare Quantities Between Cards | Change in Total | $\begin{aligned} & N \\ & M \\ & E \end{aligned}$ |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |
| RK | K | 0-5 | 1 | - | 1 | 0 | 1 | 0 | 1 | - | - |
| 768 | K | 0-5 | 1 | - | - | 0 | - | 0 | 1 | - | - |
| 385 | K | 0-10 | 1 | Removing | - | 1 | - | 0 | 1 | - | - |
| 495 | K | 0-10 | 1 | - | - | 1 | - | 1 | - | - | - |
| 223 | 1 | 0-10 | 1 | - | - | 0 | - | 0 | - | - | 1 |
| 946 | 1 | 0-10 | 0 | - | - | 1 | - | 1 | 1 | - | - |
| 152 | 1 | 0-19 | 1 | - | - | 1 | - | 0 | 1 | - | - |
| 793 | 1 | 0-19 | 0 | - | 1 | 0 | - | 1 | - | - | - |
| 993 | 2 | 0-50 | 1 | Removing | - | 1 | - | 1 | - | - | - |
| 284 | 2 | 0-99 | 1 | - | - | 1 | - | 1 | - | - | - |
| 563 | 2 | 0-99 | 1 | - | - | 1 | - | 1 | - | - | - |
| 676 | 2 | 0-199 | 1 | Removing | - | 1 | - | 1 | - | Adding | - |
| Strategies Total by Skill Code |  |  |  | 3 | 2 |  | 1 |  | 5 | 1 | 1 |

Unanticipated strategies: NRR.C.8.d-g

| Skill Code |  |  | C.8.d |  | C.8.e |  | C.8.f |  | C.8.g |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unanticipated |  |  | C | NME | C | NME | C | Counting | C | Compare Quantities Between Cards | Unfoun ded |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |
| RK | K | 0-5 | 0 | - | 1 | - | 0 | - | 0 | - | - |
| 768 | K | 0-5 | 0 | - | 1 | 1 | 0 | - | 1 | - | - |
| 385 | K | 0-10 | 1 | - | 1 | - | 1 | - | 0 | - | - |
| 495 | K | 0-10 | 0 | - | 1 | 1 | 0 | All | 0 | 1 | - |
| 223 | 1 | 0-10 | 0 | 1 | 1 | 1 | 1 | All | 1 | - | 1 |
| 946 | 1 | 0-10 | 0 | - | 1 | - | 0 | - | 1 | - | - |
| 152 | 1 | 0-19 | 0 | - | 0 | - | 1 | All | 0 | - | 1 |
| 793 | 1 | 0-19 | 0 | - | 1 | 1 | 1 | - | 1 | - | - |
| 993 | 2 | 0-50 | 1 | - | 1 | - | 0 | - | 1 | - | - |
| 284 | 2 | 0-99 | 1 | - | 1 | - | 1 | - | 1 | - | - |
| 563 | 2 | 0-99 | 1 | - | 1 | - | 1 | - | 1 | - | - |
| 676 | 2 | 0-199 | 1 | 1 | 1 | - | 1 | 3 | 0 | - | - |
| Strategies Total by Skill Code |  |  |  | 2 |  | 4 |  |  |  | 1 | 3 |

Anticipated strategies: NRR.C.8.a-b

|  | kill Cod |  |  | C. 8 |  |  |  | C. 8 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Antic | pated |  | C | NRR.A. 1 | Counting | No Counting \& | $C$ | NRR.A. 1 | Match | Counting | Tell a |
| SID | Grade | Number Range |  |  |  | No Change in Numbers |  |  | Cards |  | Story |
| RK | K | 0-5 | 1 | - | - | - | 0 | - | - | - | - |
| 768 | K | 0-5 | 1 | A.1.a | - | - | 0 | A.1.a | - | - | - |
| 385 | K | 0-10 | 1 | - | - | - | 1 | A.1.a | 1 | - | 1 |
| 495 | K | 0-10 | 1 | - | - | 1 | 1 | - | 1 | - | - |
| 223 | 1 | 0-10 | 1 | - | - | 1 | 0 | - | - | All | - |
| 946 | 1 | 0-10 | 0 | - | All | - | 1 | - | - | - | 1 |
| 152 | 1 | 0-19 | 1 | - | - | 1 | 1 | A.1.a | - | - | 1 |
| 793 | 1 | 0-19 | 0 | - | - | - | 0 | - | - | - | 1 |
| 993 | 2 | 0-50 | 1 | - | - | - | 1 | - | - | - | - |
| 284 | 2 | 0-99 | 1 | - | - | - | 1 | - | - | - | 1 |
| 563 | 2 | 0-99 | 1 | - | - | 1 | 1 | - | 1 | Skip(3) | - |
| 676 | 2 | 0-199 | 1 | - | - | - | 1 | - | - | - | - |
| Strategies Total by Skill Code |  |  |  | 1 | 1 | 4 |  | 3 | 3 | 2 | 5 |

Anticipated strategies: NRR.C.8.c-d

|  | kill Code |  |  |  | 8.c. |  |  |  |  | .8.d |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Antici | pated |  | $C$ | NRR. | NRR. | Counting | Tell a | C | NRR. | NRR.B. 5 | Change in | Counting |
| SID | Grade | Number Range |  | A. 1 | B. 5 |  | Story |  | A. 1 |  | Total |  |
| RK | K | 0-5 | 0 | A.1.a | B.5.b | - | - | 0 | A.1.a | B.5.b; B.5.c | - | - |
| 768 | K | 0-5 | 0 | A.1.a | - | - | - | 0 | A.1.a | - | - | - |
| 385 | K | 0-10 | 0 | - | - | - | - | 1 | - | - | - | - |
| 495 | K | 0-10 | 1 | A.1.a | - | All | - | 0 | - | - | - | All |
| 223 | 1 | 0-10 | 0 | - | - | - | - | 0 | - | - | - | - |
| 946 | 1 | 0-10 | 1 | A.1.a | B.5.b | - | - | 0 | - | - | - | All |
| 152 | 1 | 0-19 | 0 | A.1.a | - | - | - | 0 | - | - | - | - |
| 793 | 1 | 0-19 | 1 | - | - | - | 1 | 0 | - | - | - | All |
| 993 | 2 | 0-50 | 1 | - | - | - | 1 | 1 | - | - | - | - |
| 284 | 2 | 0-99 | 1 | - | - | - | 1 | 1 | - | - | Removing | - |
| 563 | 2 | 0-99 | 1 | - | - | - | 1 | 1 | - | - | Removing | - |
| 676 | 2 | 0-199 | 1 | - | - | - | - | 1 | - | - | - | - |
| Strategies Total by Skill Code |  |  |  | 5 | 2 | 1 | 4 |  | 2 | 1 | 2 | 3 |

Anticipated strategies: NRR.C.8.e-f

| Skill Code |  |  | C.8.e. |  |  |  | C.8.f. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated |  |  | C | $\begin{array}{\|c\|} \hline \text { NRR. } \\ B .5 \end{array}$ | $\overline{N R R} \text {. }$c.8. | Connect <br> Pictures to <br> Expressions | C | $\begin{gathered} \hline \text { NRR. } \\ B .5 \end{gathered}$ | NRR C.8. | Connect <br> Pictures to <br> Numbers | Compare Sums |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |
| RK | K | 0-5 | 1 | - | C.8.b | - | 0 | - | - | - | - |
| 768 | K | 0-5 | 1 | - |  | - | 0 | - | - | - | - |
| 385 | K | 0-10 | 1 | - |  | 1 | 1 | B.5.b | C.8.b | - | 1 |
| 495 | K | 0-10 | 1 | - |  | - | 0 | $\begin{aligned} & \hline \text { B.5.b } \\ & \text { B.5.C } \end{aligned}$ | - | - | - |
| 223 | 1 | 0-10 | 1 | - |  | - | 1 | - | - | - | - |
| 946 | 1 | 0-10 | 1 | B.5.b |  | 1 | 0 | B.5.b | - | - | 1 |
| 152 | 1 | 0-19 | 0 | B.5.b |  | 1 | 1 | $\begin{aligned} & \hline \text { B.5.b } \\ & \text { B.5.c } \end{aligned}$ | - | 1 | - |
| 793 | 1 | 0-19 | 1 | - |  | - | 1 | - | - | - | 1 |
| 993 | 2 | 0-50 | 1 | B.5.b | C.8.b | - | 0 | - | C.8.b | - | 1 |
| 284 | 2 | 0-99 | 1 | - | C.8.c | 1 | 1 | - | C.8.b | - | 1 |
| 563 | 2 | 0-99 | 1 | - | C.8.c | 1 | 1 | - | - | - | - |
| 676 | 2 | 0-199 | 1 | B.5.b | C.8.c | 1 | 1 | B.5.b | C.8.b | 1 | - |
| Strategies Total by Skill Code |  |  |  | 4 | 5 | 6 |  | 7 | 4 | 2 | 5 |

## Anticipated strategies: NRR.C.8.g

| Skill Code |  |  | C.8.g. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated |  |  | C | NRR. <br> C.8 | Counting | Unfounded | Compare <br> Sums |
| SID | Grade | Number <br> Range |  |  |  |  |  |
| RK | K | $0-5$ | 0 | - | - | - | - |
| 768 | K | $0-5$ | 1 | - | - | - | - |
| 385 | K | $0-10$ | 0 | - | - | 1 | - |
| 495 | K | $0-10$ | 0 | - | - | - | - |
| 223 | 1 | $0-10$ | 1 | - | - | - | - |
| 946 | 1 | $0-10$ | 1 | - | - | - | - |
| 152 | 1 | $0-19$ | 0 | - | - |  | - |
| 793 | 1 | $0-19$ | 1 | - | On | - | 1 |
| 993 | 2 | $0-50$ | 1 | - | - | - | 1 |
| 284 | 2 | $0-99$ | 1 | - | - | - | 1 |
| 563 | 2 | $0-99$ | 1 | C.8.c | - | - | 1 |
| 676 | 2 | $0-199$ | 0 | C.8.c | - | - | 1 |
| Strategies Total by Skill Code |  | 2 | 1 | 1 | 5 |  |  |

NRR.C.8.a. Summary
Essentialized Skill Statement

## 8. Equivalence of Quantity and Number

| Code | Kindergarten | Grade 1 |  | Grade $\mathbf{2}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ |

NRR.C.8.a. Given equivalent sets of quantities, recognize that the quantity of each set remains the same regardless of size, color, or arrangement. (conservation of number)

## Student Expectation

Children were expected to recognize that the total number of ducks was the same as the total number of rabbits and that the quantity did not change when the rabbits were spread out.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Removing. Several students relied on the idea of <br> removal to explain why there were still the same <br> number of rabbits as ducks. | [There is still the same number] because you <br> pushed them back and not deleted one (385, <br> 02:16). |
|  | There is still the same number] because one's <br> not missing (993, 05:11). |
| [There is still the same number] because you |  |
| didn't take any away (676, 00:46). |  |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (RK) | Removing (385) |  |  |  |  |
| $\mathbf{1}$ |  | - | NME (793) |  |  |  |
| $\mathbf{2}$ |  |  |  | Removing (993) |  |  |

Unanticipated strategies by grade level and number range for NRR.C.8.a.

| Grade | 0-5 | 0-10 | 0-19 | 0-50 | 0-99 | 0-199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | 1a (768) | No Counting and No Change in Number (495) |  |  |  |  |
| 1 |  | No Counting and No Change in Number (223) <br> Count All (946) | No Counting and No Change in Number (152) |  |  |  |
| 2 |  |  |  | - | Matching Without Specific Number (284) <br> No Counting and No Change in Number (563) | Removing (676) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.8.a. activity

NRR.C.8.a. Anticipated Strategies

| NRR.C.8.a. | Given equivalent sets of quantities, recognize that the quantity of each set remains the same regardless of size, color, or arrangement. (conservation of number) |  |
| :---: | :---: | :---: |
| Content Question | (1) Are there the same number of rabbits as there are ducks? |  |
| Reasoning Question | How do you know th | ? |
| Anticipated Skills |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | C8a_E_NOT-Animals | Rabbits and ducks are embedded into the activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C8a_A_General tool | Child uses a general tool as listed above. |
|  | C8a_A_Count all | Child counts all rabbits and all ducks individually. [Counting] |
|  | C8a_A_Count by 2 s | Counting. Child counts animals in groups of 2; does not necessarily need to be a bunny paired with a duck. ("2, 4, 6, 8") [Counting] |
|  | C8a_A_One to one correspondence | Child pairs each rabbit to a duck. [Counting] |
|  | C8a_A_Count pairs | Child counts pairs ("1, 2, 3, 4") [Counting]. One to one correspondence should also be coded if child pairs a rabbit and a duck. |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Current Skill Code | Only specific student talk-turn within activity that is attributed to the current skill code. |
| Essentialized Skill Statements [Also consider ESSs not specifically listed here; refer to learning progressions.] | 01-Comparison | - Child compares total counted bunnies and total counted quantities. [Relations] <br> - Select skill code in core concept Comparison, that best fits child's approach. <br> - A counting strategy should also be selected within the current skill code. If theanticipated strategies are not reflective of what the child did, then code an unanticipated strategy to represent the child's s approach to comparing quantities. |

NRR.C.8.b. Summary
Essentialized Skill Statement

## 9. Equivalence of Quantity and Number

| Code | Kindergarten | Grade 1 |  | Grade $\mathbf{2}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F | B | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ |

NRR.C.8.b. Given a quantity broken into two parts, recognize that order does not change the quantity. (commutative property)

## Student Expectation

Children were expected to recognize that the total number of ducks did not change from one point in time to another point in time. First the big ducks when in then the little ducks, then later vice versa. All students were expected to be able to reason through this activity by acknowledging that the order in which the ducks entered the pond did not change the quantity.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Unfounded. When asked if there were the same <br> number of ducks in the pond, a Kindergarten <br> student focused on counting the bumps in the <br> ponds rather than the ducks. The child viewed <br> the ponds as different and therefore each side <br> could not be the same. | Because, but the pond is different in the shape, <br> though. Because, this one has ... let me see ... one, <br> two, three, four. It has four bumps, and this one <br> has one, two, three. That one has three (RK, 4:38). |


| Grade | $00-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Unfounded (RK) | - |  |  |  |  |
| $\mathbf{1}$ |  | - | - |  |  |  |
| $\mathbf{2}$ |  |  |  | - | - | - |

Unanticipated strategies by grade level and number range for NRR.C.8.b.

| Grade | 0-5 | 0-10 | 0-19 | 0-50 | 0-99 | 0-199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | 1a (768) | Tells a Story, Match Cards (385) <br> Match Cards (495) <br> 1a (385) |  |  |  |  |
| 1 |  | Count All (223) Tells a Story (946) | $\begin{gathered} \text { Tells a Story }(152,793) \\ 1 \mathrm{a}(152) \end{gathered}$ |  |  |  |
| 2 |  |  |  | Change of Order (993) | Tells a Story (284) <br> Count by 3s, Matching (563) | Change of Order (676) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.8.b. activity.

| NRR.C.8.b. | Given a quantity broken into two parts, recognize that order does not change the quantity. (commutative property) |  |
| :---: | :---: | :---: |
| Content Question | Are there more, less, or the same number of ducks in the pond now? |  |
| Reasoning Question | Can you make up a story that shows how the order doesn't change the total? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | C8b_E_NOT-Pictures | Pictures of ducks in groups. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C8b_A_General tool | Child uses a general tool as listed above. |
|  | C8b_A_Count all | Child counts all ducks on each card. [Counting] |
|  | C8b_A_Count groups | Child counts by groups of ducks. [Counting] |
|  | C8b_A_Count on | - Differs from count all. [Counting] <br> - If a child counts all ducks on one card, then proceeds to count the ducks on the next card, this is count all. However, if the child identifies the amount of ducks on the first card without individually counting, then starts from the total of the first card to continue counting on the next card, this is considered counting on. |
|  | C8b_A_Match cards | - Child matches the cards that have the same quantities. <br> - Child may either point to the two cards that are the same or line up the cards that are the same by switching the order |
|  | C8b_A_Immediate recognition | - Immediate recognition of same pattern or groupings. <br> - Child does not count images on cards to be able to provide a response. |
|  | C8b_A_Tell a story | - Child tells a story that demonstrates order does not change the total. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a differentstrategy was used, create an unanticipated node for the child's strategy. |
| NRR | Current Skill Code | Only specific student talk-turn within activity that is attributed to the current skill code. |


| Essentialized Skill | O5-Composition | - Add two or more quantities together. [Composition \& Decomposition] |
| :--- | :--- | :--- |
| Statements | - Select skill code in core concept Composition, that best fits child's approach. |  |
| [Also consider ESSs not |  | - A counting strategy should also be selected within the current skill code. If the anticipated strategies |
| specifically listed here; |  |  |
| are not reflective of what the child did, then code an unanticipated strategy to represent the child's s <br> refer to learning <br> progressions.] |  | approach to comparing quantities. |

NRR.C.8.c. Summary
Essentialized Skill Statement

| 8. Equivalence of Quantity and Number |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten | Grade 1 | Grade 2 |  |  |  |  |  |
|  | F | B | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | F | B |
| NRR.C.8.c. | Given a quantity, recognize that the quantity remains the same after joining/removing a <br> part then removing/joining the same part. (undoing or additive inverse) |  |  |  |  |  |  |  |

## Student Expectation

Children were expected to recognize that the total number of rabbits did not change from the beginning of the day to the end of the day after a quantity of rabbits were removed during the middle of the day and rejoined at the end of the day. All students were expected to be able to reason through this activity.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Adding to Total. The goal of the activity was for <br> students to recognize that the quantity <br> maintained the same given a situation involving <br> additive inverse. Considering an additional <br> amount to the total is beyond the additive <br> inverse property. | [There are the same number of rabbits] because <br> you didn't add any (676, 3:23). |
| Compare Quantities Between Cards. Instead of <br> comparing the quantities at different times of the <br> day, children compared quantities on the different <br> cards and misunderstood the intended question. | Because it has two and this one has one (768, <br> $8: 21)$. |
| Well this one has a little but [pointing to card with <br> one bunny] because they only have one. This one <br> has two because it has more [pointing to card with <br> two bunnies] (RK, $6: 00)$. |  |
|  | Because this square only has four...Oh, no, <br> because one has four and this one has five (152, <br> 11:39). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Compare Quantities <br> Between Cards (RK, <br> 768) | Compare Quantities <br> Between Cards (385) |  |  |  |
| $\mathbf{1}$ |  | NME (223) | Compare Quantities |  |  |
| Compare Quantities |  |  |  |  |  |
| Between Cards (946) |  |  |  |  |  |$\quad$| Between Cards (152) |
| :--- |

Unanticipated strategies by grade level and number range for NRR.C.8.c.

| Grade | 0-5 | 0-10 | 0-19 | 0-50 | 0-99 | 0-199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $\begin{gathered} 1 a, 5 a, 5 b(R K) \\ 1 a(768) \end{gathered}$ | Count All (495) <br> 1a, (495) |  |  |  |  |
| 1 |  | 1a, 5a, 5b (946) | 1a (152) <br> Additive Inverse Story (793) |  |  |  |
| 2 |  |  |  | Additive Inverse Story (993) | Additive Inverse <br> Story $(284,563)$ |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.8.c. activity.

| NRR.C.8.c. | Given a quantity, recognize that the quantity remains the same after joining/removing a part then removing/joining the same part. (undoing or additive inverse) |  |
| :---: | :---: | :---: |
| Content Question | Are there more, less, or the same number of rabbits at home now? |  |
| Reasoning Question | What makes you think that there are___rabbits now? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | C8c_E_NOT-Pictures | Pictures of rabbits in groups. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C8c_A_General tool | Child uses a general tool as listed above. |
|  | C8c_A_Act out | Child moves cards to act out their thinking. |
|  | C8c_A_Count all | Child counts all rabbits on each card. [Counting] |
|  | C8c_A_Count groups | Child counts by groups of rabbits. [Counting] |
|  | C8c_A_Count on | - Differs from count all. [Counting] <br> - If a child counts all rabbits on one card, then proceeds to count the rabbits on the next card, thisis count all. <br> - However, if the child identifies the amount of rabbits on the first card without individually counting, then starts from the total of the first card to continue counting on the next card, this is considered counting on. |
|  | C8c_A_Immediate recognition | - Immediate recognition of same pattern or groupings. <br> - Child does not count images on cards to be able to provide a response. |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Current Skill Code | Only specific student talk-turn within activity that is attributed to the current skill code. |


| Essentialized Skill | O5-Composition | - Add two or more quantities together. [Composition \& Decomposition] |
| :--- | :--- | :--- |
| Statements | - Select skill code in core concept Composition, that best fits child's approach. |  |
| [Also consider ESSs not |  | - A counting strategy should also be selected within the current skill code. If the anticipated strategies |
| specifically listed here; |  |  |
| are not reflective of what the child did, then code an unanticipated strategy to represent the child'ss <br> refer to learning <br> progressions.] |  | approach to comparing quantities. |

NRR.C.8.d. Summary
Essentialized Skill Statement

## 8. Equivalence of Quantity and Number

| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F | B | T | F | B | T | F | B |  |  |
| NRR.C.8.d. |  |  |  |  | Given two associated parts and another part, recognize that the quantity of the three parts remains the same if the parts are reassociated. (associative property) |  |  |  |  |  |

## Student Expectation

UPDATE

| $\begin{array}{l}\text { Unanticipated Strategy. } \\ \text { Justification/Description }\end{array}$ | Examples |
| :--- | :--- |
| Adding to or Subtracting from Total. The goal of |  |
| the activity was for students to recognize that the |  |
| quantity maintained the same given a situation |  |
| involving the associative property. Considering an |  |
| additional amount to the total or a removal from |  |
| the total is beyond the associative property. |  | \(\left.\begin{array}{ll}[The amount doesn't change] because these ducks <br>

are playing here but they didn't leave, they just <br>

went right here (563, 12:00).\end{array}\right]\)| [The amount doesn't change] because there is |
| :--- | :--- |
| ducks and rabbits and no more animals came, so |
| it's gonna be the same number (284, 12:19). |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K |  |  |  |  |  |  |
| $\mathbf{1}$ |  | NME (223) |  |  |  |  |
| 2 |  |  |  |  |  |  |

Unanticipated strategies by grade level and number range for NRR.C.8.d.

| Grade | 0-5 | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | 1a, 5b, 5c (RK) | Act Out (385) |  |  |  |
| $\mathbf{1}$ |  | Count All (495) |  |  |  |
| $\mathbf{2}$ |  | Count All (946) | Count All (793) |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.8.d. activity.

| NRR.C.8.d. | Given two associated parts and another part, recognize that the quantity of the three parts remains the same if the parts are reassociated. (associative property) |  |
| :---: | :---: | :---: |
| Content Question | If the ducks in the field join the ducks in the pond, will there be more, less, or the same number of animals? |  |
| Reasoning Question | Can you show me how you know that there will be [more, less, or the same]? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | C8d_E_NOT-Pictures | Pictures of ducks and rabbits. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C8d_A_General tool | Child uses a general tool as listed above. |
|  | C8d_Act out | Child moves cards to demonstrate thinking. |
|  | C8d_A_Count all | Child counts all animals on each card. [Counting] |
|  | C8d_A_Count groups | Child counts by groups of animals. [Counting] |
|  | C8d_A_ Count on | - Differs from count all. [Counting] <br> - If a child counts all animals on one card, then proceeds to count the animals on the next card, this is count all. |
|  |  | However, if the child identifies the amount of animals on the first card without individually counting, then starts from the total of the first card to continue counting on the next card, this is considered counting on. |
|  | C8d_A_Immediate recognition | Immediate recognition of same pattern or groupings. |
| A different way | - | A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Current Skill Code | Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | NRR.C.8.a. | Identify cards with the same values. Child might point to cards. [Properties of Operations] |


| Essentialized Skill | O5-Composition | - Add two or more quantities together. [Composition \& Decomposition] |
| :--- | :--- | :--- |
| Statements |  | - Select skill code in core concept Composition, that best fits child's approach. |
| [Also consider ESSs not |  | - A counting strategy should also be selected within the current skill code. If the anticipated strategies |
| specifically listed here; |  | are not reflective of what the child did, then code an unanticipated strategy to represent the child's s <br> refer to learning |
| progressions.] |  | approach to comparing quantities. |

NRR.C.8.e. Summary
Essentialized Skill Statement

## 8. Equivalence of Quantity and Number

| Code | Kindergarten | Grade 1 |  | Grade 2 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ |
| NRR.C.8.e. | Given a quantity, recognize an equivalent expression that demonstrates one or |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| more property of operations. |  |  |  |  |  |  |  |  |  |

## Student Expectation

Children were expected to use the total quantities of each card to connect to the expressions without having to compare the total of expressions to the total number of animals.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Count All. For 8.e., children were expected to use <br> the known quantities and connect the picture to <br> the expressions. To convince themselves that the <br> numbers were the same, children were still <br> counting the numbers. | There are one, two, three, four, five, six, seven, <br> eight, nine, ten, eleven, twelve, thirteen- (793, <br> 13:29). |
| Matching and Counting. Matching and counting <br> strategies were not expected for children who <br> were at least in first grade. | Because if count them altogether? I <br> counted 32 (152, 15:15, 15:35). <br> three which equals six and on here, as well, <br> because six plus six is 12, and same thing right <br> here because this one has 12 as well, so and then <br> 12 plus 12 equals eight, I mean, plus eight (152, <br> $16: 25)$. |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (768) | NME (495) |  |  |  |  |
| $\mathbf{1}$ |  | NME (223) | NME (793) |  |  |  |
| $\mathbf{2}$ |  |  |  |  |  |  |

Unanticipated strategies by grade level and number range for NRR.C.8.e.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | 8b (RK) | Connect Pictures to <br> Expressions (385) |  |  |  |  |
| $\mathbf{1}$ |  | Connect Pictures to <br> Expressions (946) | Connect Pictures to <br> Expressions (152) |  |  |  |
| $\mathbf{2}$ |  |  | $5 b$ (946) |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.8.e. activity.

NRR.C.8.e. Anticipated Strategies

| NRR.C.8.e. | Given a quantity, recognize an equivalent expression that demonstrates one or more property of operations. |  |
| :---: | :---: | :---: |
| Content Question | Pick a number sentence that could describe all of the animals that Dasia saw. |  |
| Reasoning Question | How is that number sentence related to the animals? Is there another number sentence that describes the situation? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | C8e_E_NOT-Pictures | Pictures of ducks and rabbits. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Counting and computational strategies were excluded here since the focus is on the expressions. Evidence of counting/computational strategies can be considered unanticipated and should be coded accordingly. |  |
|  | Code | Description |
|  | C8e_A_General tool | Child uses a general tool as listed above. |
|  | C8e_A_Connect pictures to expressions | Explain how the numbers represented on the cards are related to the quantities of animals that Dasia saw. |
|  | C8e_A_ Move pictures | Rearrange picture cards to find an equivalent expression. |
| A different way | C8e_D_Another number sentence | Another number sentence was anticipated for this essentialized skill statement. All number sentences are representative of the images, but not all are aligned. As long as child selected another number sentence and was able to demonstrate an understanding of their selection then a different way should be assigned. |
| NRR | Current Skill Code | Only specific student talk-turn within activity that is attributed to the current skill code. |
| Essentialized Skill <br> Statements <br> [Also consider ESSs not | NRR.C.8.b. | Purposefully identify an expression demonstrating commutative property. [Properties of Operations] Child must demonstrate an understanding of the commutative property. Child may use vocabulary such as "switched", "swap" or child may move cards to connect to expression. |
| specifically listed here; refer to learning progressions.] | NRR.C.8.c. | Purposefully identify an expression demonstrating associative property. [Properties of Operations] Child must demonstrate an understanding of the associative property. Child may move cards and group ducks together to explain how the expression represents the images. |
|  | 05-Composition | - Add two or more quantities together. [Composition \& Decomposition] <br> - Select skill code in core concept Composition, that best fits child's approach. |

## Essentialized Skill Statement

| 8. Equivalence of Quantity and Number |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten | Grade 1 | Grade 2 |  |  |  |  |  |  |
|  | F | B | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | F | B | T |
| NRR.C.8.f. | Recognize two equivalent expressions that demonstrate <br> one or more property of operations. |  |  |  |  |  |  |  |  |

## Unanticipated Strategies

Children were expected to compare two equivalent expressions without comparing total sums.

| Strategy. Justification/Description | Examples |
| :---: | :---: |
| Compare Number of Addends. Child compared the number of addends in each expression to determine whether or not the expressions were the same/different. Child's reasoning was not mathematically sound, and provides confirming evidence that this ESS may not be suitable at the foundational level for Kindergarten. | Because these numbers [2+2+1] only have three, and these numbers $[1+4]$ only have two (RK, 14:24). |
| Count All. Children counted by 1s to determine the total. For this essentialized skill statement, counting all individually was not expected for grades 1 and 2 because children should be able to use strategies such as composition and properties of operations to determine equivalence of expressions. | Let me count out 4 and $4 . . .1,2,3,4 \ldots$ <br> $1,2,3,4,5,6$... So 4 plus 6 is $1,2,3,4,5,6,7,8,9,10$ (495, 10:32-12:08). <br> Six. One, two, three, four, five, six, seven, eight nine, 10. <br> Four, 4+4. One, two, three, four, five, six, seven, eight, nine, 10 (223, 13:06). <br> [Child pointed to each animal individually without verbalization] This one is the same as well....Because these two groups have together they make 24 and this one together they make eight, so if you were to put them together it would equal 32 (152, 17:17, 17:55). |
| Connect Pictures to Numbers. This activity was intended for use with two expressions without pictorial representations. Two children connected the expressions to the ducks and rabbits from the previous activity. Connect Pictures to Numbers. (cont.) | Because this one, well this one is right because over here there's 24 and eight right here (152, 17:10). <br> There's one group of ducks all together and there's one group of bunnies all together; so there's two numbers and there's two numbers and there's two groups of animals ( $676,7: 30$ ). |


| Write an Equation. Child wrote $8+2=10$ and 2 $+8=10$. This content is different than C.9.b. since quantities were not given on a balance or pictorial representation. The child was given two expressions. | $\begin{aligned} & {\left[\begin{array}{l} {[\text { Child writes } 8+2=10](385,13: 55)} \\ {[\text { Child also writes } 2+8=10)(385,14: 20)} \end{array}\right.} \end{aligned}$ |
| :---: | :---: |
| Blocks. Child described that a convincing explanation to a friend would include the use of blocks. While the blocks were always present on the table, the interviewer did not suggest that the child use the blocks for this activity; the child chose to select blocks without prompting. | I'll show them by using blocks, if I had some. And telling her which numbers are the same, because she has to know her numbers to know. So I would get the blocks and get six, and tell her the number sentences (284, 16:35). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | Compare Number <br> of Addends (RK) | Count All (495) <br> Write an Equation <br> (385) |  |  |  |  |
| $\mathbf{1}$ |  | Count All (223) | Count All (152) <br> Connect Pictures to <br> Numbers (152) |  |  |  |
| $\mathbf{2}$ |  |  |  | - | Blocks <br> (284) | Connect Pictures to <br> Numbers (676) |

Unanticipated strategies by grade level and number range for NRR.C.8.f.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - | Compare Sums (385) <br> $5 b / 8 b(385)$ <br> $5 b / 5 c(495)$ |  |  |  |  |
| $\mathbf{1}$ |  | Compare Sums (385) <br> $5 b(385)$ | Compare Sums <br> $(152)$ <br> $5 b / 5 c(152)$ |  |  |  |
| $\mathbf{2}$ |  |  |  | Compare Sums <br> $(993)$ <br> $8 b(993)$ | Compare Sums <br> $(284)$ <br> $8 b(284)$ | $5 b / 8 b$ <br> $(676)$ |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.8.f. activity.

| NRR.C.8.f. | Recognize two equivalent expressions that demonstrates one or more property of operations. |  |
| :---: | :---: | :---: |
| Content Question | Look at these two number sentences. Do they represent the same amounts or different amounts? |  |
| Reasoning Question | If one of your friends didn't believe that these two number sentences were the same, how would you convince them that you are right? |  |
| Anticipated |  |  |
| Strategies | Code | Description |
| Embedded Mathematical Tools | - | No tools embedded. The images were not intended to be associated with this activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C8e_A_General tool | Child uses a general tool as listed above. |
|  | C8f_A_ Compare sums | - Add numbers to find sums, compare <br> Write expressions out and calculate to find totals, then compare |
|  | C8f_A_Count on | - Count on from first addend to find total |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements <br> [Also consider ESSs not specifically listed here; refer to learning progressions.] | Current Skill Code | Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | NRR.C.8.b. | Identify numbers that are the same in both expressions and use concepts of commutative property to explain how the expressions are the same. (CP, NP) [Properties of Operations] |
|  | NRR.C.8.c. | Identify that the first number of each expression is the same and the combination of the last two addends in the literal expression is the same as the second addend of the associative property expression. (AP, L) |
|  | 06- Decomposition | - Decompose a number in an expression into two or more numbers present in the other expression. [Composition \& Decomposition] <br> - Select skill code in core concept Decomposition, that best fits child's approach. |

NRR.C.8.g. Summary
Essentialized Skill Statement

| 8. Equivalence of Quantity and Number |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  |  |  |  |  |  |  |
|  | F | B | $\mathbf{T}$ | Grade 1 | F | $\mathbf{B}$ | $\mathbf{T}$ | F | B |
| NRR.C.8.g. |  |  | Recognize two equivalent expressions that demonstrate <br> decomposition and at least one property of operations. |  |  |  |  |  |  |

## Student Expectation

Three expressions given to students were provided on two different cards; one card with one expression and the second card with two expressions to show the decomposition and commutative property. Children were expected to identify that all expressions were equivalent.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Unfounded. | l don't think [Olivia] wrote the right sentences <br> and [Rene] wrote the right...[Rene] wrote the <br> wrong and [Olivia] wrote the right...because I <br> think [Rene] added up too many numbers (152, <br> 20:41-21:02). |
| I want to count [all of the addends in each |  |
| expression] altogether (223, 17:23). |  |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $0-99$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K |  | Compared Quantities <br> (495) |  |  |  |  |
| $\mathbf{1}$ |  | Unfounded (223) | Unfounded (152) |  |  |  |
| $\mathbf{2}$ |  |  |  |  |  |  |

Unanticipated strategies by grade level and number range for NRR.C.8.g.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (768) NME <br> (RK) | Unfounded (385) |  |  |  |  |
| $\mathbf{1}$ |  | Compared Addends (946) | Compare Sums (793) <br> Count On (793) |  |  |  |
| $\mathbf{2}$ |  |  |  | Compare Sums (993) | Compare Sums <br> (284, 563) | Compare Sums <br> (676) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.8.g. activity.

| NRR.C.8.g. | Recognize two equivalent expressions that demonstrate decomposition and at least one property of operations. |  |
| :---: | :---: | :---: |
| Content Question | Dasia saw_animals playing in the field and_ducks in the pond. Rene wrote down these two number sentences to count the animals. They both agreed that there were_animals. Do you agree? |  |
| Reasoning Question | Why do you agree or disagree? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | - | No tools embedded. The images were not intended to be associated with this activity. |
| Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C8e_A_General tool | Child uses a general tool as listed above. |
|  | C8g_A_ Count on | - Count on from first addend to find total |
|  | C8g_A_Compare sums | - Add numbers to find sums, compare <br> - Write expressions out and calculate to find totals, then compare |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements <br> [Also consider ESSs not specifically listed here; refer to learning progressions.] | Current Skill Code | Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | NRR.C.8.b. | Identify numbers that are the same in both expressions and use concepts of commutative property to explain how the expressions are the same. [Properties of Operations] |
|  | NRR.C.8.c. | Identify that the second addend in Dasia's expression is the sum of the last two addends in Renee's expressions. |
|  | 06- Decomposition | - Decompose a number in an expression into two or more numbers present in the other expression. [Composition \& Decomposition] <br> - Select skill code in core concept Decomposition, that best fits child's approach. |

NRR.C.9. Equal Sign as a Relational Symbol Core Concept

| 9. Equal Sign as a Relational Symbol |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | B | T | F | B | T |
| NRR.C.9.a. | Recognize the equality between two quantities using a balance. |  |  |  |  |  |  |  |  |  |
| NRR.C.9.b. |  | Write a true equation using an equal sign to represent the relationship between given quantities on a balance or in a pictorial representation. |  |  |  |  |  |  |  |  |
| NRR.C.9.c. |  | Recognize true and not true equations with different equation structures: operations on the left side $(a+b=c)$; no operations $(a=a)$; operations on the right side $(c=a+b)$. |  |  |  |  |  |  |  |  |
| NRR.C.9.d. |  |  |  |  | Recognize true and not true equations with different equation structures: operations on the both sides ( $\mathrm{a}+\mathrm{b}=\mathrm{c}+\mathrm{d}$ ); multiple instances of a number. |  |  |  |  |  |

Unanticipated strategies: NRR.C.9.a-d

| Skill Code |  |  | C.9.a |  | $\begin{array}{\|c\|} \hline \text { C.9.b } \\ \hline c \\ \hline \end{array}$ | C.9.c |  |  |  | C.9.d |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Unantic | ated | C | NME |  | C |  | C | Unknown | C | C | Unknown |
| SID | Grade | Number Range |  |  |  | $a+b=c$ | $a=a$ | $c=a+b$ | Equation Structure | $a+b=c+d$ | $a+a=b+b$ | Equation <br> Structure |
| RK | K | 0-5 | 1 | - | - | - | 1 | - | - | - | - | - |
| 768 | K | 0-5 | 1 | 1 | - | - | 1 | 1 | - | 1 | - | - |
| 385 | K | 0-10 | 1 | - | - | 1 | - | - | - | - | - | - |
| 495 | K | 0-10 | 1 | - | - | - | - | - | - | - | 1 | - |
| 223 | 1 | 0-10 | 1 | - | - | 1 | 1 | - | $a=a$ | - | - | - |
| 946 | 1 | 0-10 | 1 | - | - | - | - | - | $a=a$ | - | - | - |
| 152 | 1 | 0-19 | 1 | - | - | 1 | - | 1 | $a=a$ | 1 | - | $a+a=b+b$ |
| 793 | 1 | 0-19 | 1 | - | - | 1 | - | - | $a=a$ | - | 1 | $a+b=c+d$ |
| 993 | 2 | 0-50 | 1 | - | - | 1 | - | 1 | $a=a$ | - | - | $\begin{aligned} & a+b=c+d ; \\ & a+a=b+b \end{aligned}$ |
| 284 | 2 | 0-99 | 1 | - | - | 1 | 1 | 1 | - | - | 1 | - |
| 563 | 2 | 0-99 | 1 | - | - | 1 | 1 | 1 | - | 1 | 1 | - |
| 676 | 2 | 0-199 | 1 | - | 1 | 1 | 1 | - | - | - | - | $a+b=c+d$ |
| Strategies Total by SkillCode |  |  |  |  |  |  |  |  | 5 |  |  | 5 |

Anticipated strategies: NRR.C.9.a-b

| Skill Code |  |  | C.9.a |  |  |  | c.9.b |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated |  |  | C | Quantity | Height | Weight | C | $\begin{aligned} & N \\ & M \\ & E \end{aligned}$ | Counting | Equation | Quantity | $\begin{array}{\|c\|} \hline \text { NRR. } \\ \text { B.5. } \\ \hline \end{array}$ | Height |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |  |  |
| RK | K | 0-5 | 1 | 1 | - | - | 0 | - | - | - | - | B.5.a | - |
| 768 | K | 0-5 | 1 | - | - | - | 0 | 1 | - | - | - | - | - |
| 385 | K | 0-10 | 1 | 1 | 1 | 1 | 0 | - | - | 1 | - | - | - |
| 495 | K | 0-10 | 1 | 1 | 1 | 1 | 0 | - | All | 1 | - | B.5.b | - |
| 223 | 1 | 0-10 | 1 | - | 1 | - | 0 | - | - |  | 1 | B.5.b | 1 |
| 946 | 1 | 0-10 | 1 | 1 | 1 | - | 0 | - | - | - | 1 | - | - |
| 152 | 1 | 0-19 | 1 | 1 | - | 1 | 0 | - | All | 1 | 1 | - | 1 |
| 793 | 1 | 0-19 | 1 | 1 | - | - | 0 | - | All | - | - | - | - |
| 993 | 2 | 0-50 | 1 | 1 | 1 | 1 | 0 | - | All | 1 | - | - | - |
| 284 | 2 | 0-99 | 1 | 1 | 1 | - | 0 | - | All | 1 | - | - | - |
| 563 | 2 | 0-99 | 1 | 1 | 1 | 1 | 0 | - | - | 1 | - | - | - |
| 676 | 2 | 0-199 | 1 | 1 | - | - | 1 | - | - | 1 | 1 | - | - |
| Strategies Total by Skill Code |  |  |  | 10 | 7 | 5 |  | 1 | 5 | 8 | 4 | 3 | 2 |

Anticipated strategies: NRR.C.9.c

| Skill Code <br> Anticipated |  |  | C.9.c |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | C | $C$ | C | N | Unknown | Count | Relational | Operational |
| SID | Grade | Number Range | $a+b=c$ | $a=a$ | $c=a+b$ | $M$ $E$ | Equation Structure |  |  |  |
| RK | K | 0-5 | 0 | 1 | 0 | 1 | - | - | - | - |
| 768 | K | 0-5 | 0 | 1 | 1 | 1 | - | - | - | - |
| 385 | K | 0-10 | 1 | 0 | 0 | - | $\mathrm{c}=\mathrm{a}+\mathrm{b} \mid \mathrm{a}=\mathrm{a}$ | On | - | $\mathrm{c}=\mathrm{a}+\mathrm{b} \mid \mathrm{a}=\mathrm{a}$ |
| 495 | K | 0-10 | 0 | 0 | 0 | - | $\mathrm{c}=\mathrm{a}+\mathrm{b} \mid \mathrm{a}=\mathrm{a}$ | - | - | - |
| 223 | 1 | 0-10 | 1 | 1 | 0 | - | - | - | - | $\mathrm{c}=\mathrm{a}+\mathrm{b} \mid \mathrm{a}+\mathrm{b}=\mathrm{c}$ |
| 946 | 1 | 0-10 | 0 | 0 | 0 | - | - | - | - | $c=a+b \mid a+b=c$ |
| 152 | 1 | 0-19 | 1 | 0 | 1 | - | - | - | - | $c=a+b \mid a+b=c$ |
| 793 | 1 | 0-19 | 1 | 0 | 0 | - | - | - | - | $c=a+b \mid a+b=c$ |
| 993 | 2 | 0-50 | 1 | 0 | 1 | - | - | - | - | $c=a+b \mid a+b=c$ |
| 284 | 2 | 0-99 | 1 | 1 | 1 | - | - | - | $\mathrm{a}=\mathrm{a}$ | $c=a+b \mid a+b=c$ |
| 563 | 2 | 0-99 | 1 | 1 | 1 | - | - | - | $\mathrm{a}=\mathrm{a}$ | $c=a+b \mid a+b=c$ |
| 676 | 2 | 0-199 | 1 | 1 | 0 | - | - | - | $\mathrm{a}=\mathrm{a}$ | $\mathrm{c}=\mathrm{a}+\mathrm{b} \mid \mathrm{a}+\mathrm{b}=\mathrm{c}$ |
| Strategies Total by Skill Code |  |  |  |  |  | 2 | 4 | 1 | 3 | 18 |

Anticipated strategies: NRR.C.9.d

|  | kill Cod |  | C.9.d |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated |  |  | $\begin{gathered} C \\ a+b=c+d \end{gathered}$ | $\begin{gathered} C \\ a+a=b+b \end{gathered}$ | $\begin{aligned} & N \\ & M \\ & M \end{aligned}$ | NRR. <br> B.5. | Unknown Equation Structure* | Relational | Operational |
| SID | Grade | Number Range |  |  |  |  |  |  |  |
| RK | K | 0-5 | 0 | 0 | - | - | $\mathrm{a}+\mathrm{b}=\mathrm{c}+\mathrm{d} \mid a+a=b+b$ | - | - |
| 768 | K | 0-5 | 1 | 0 | 1 | - | - | - | - |
| 385 | K | 0-10 | 0 | 0 | - | - | a+b=c+d \| $a+a=b+b$ | - | - |
| 495 | K | 0-10 | 0 | 1 | - | - | $a+b=c+d \mid a+a=b+b$ | - | - |
| 223 | 1 | 0-10 | 0 | 0 | - | - | - | - | - |
| 946 | 1 | 0-10 | 0 | 0 | - | - | $\mathrm{a}+\mathrm{b}=\mathrm{c}+\mathrm{d} \mid a+a=b+b$ | - | $\mathrm{a}+\mathrm{b}=\mathrm{c}+\mathrm{d} \mid a+a=b+b$ |
| 152 | 1 | 0-19 | 1 | 0 | - | - | - | - | $\mathrm{a}+\mathrm{b}=\mathrm{c}+\mathrm{d} \mid a+a=b+b$ |
| 793 | 1 | 0-19 | 0 | 1 | - | - | - | - | - |
| 993 | 2 | 0-50 | 0 | 0 | - | - | - | - | $\mathrm{a}+\mathrm{b}=\mathrm{c}+\mathrm{d} \mid a+a=b+b$ |
| 284 | 2 | 0-99 | 0 | 1 | - | - | - | - | $\mathrm{a}+\mathrm{b}=\mathrm{c}+\mathrm{d} \mid a+a=b+b$ |
| 563 | 2 | 0-99 | 1 | 1 | - | B.5.b | - | $a+b=c+d$ | $a+a=b+b$ |
| 676 | 2 | 0-199 | 0 | 0 | - | - | - | - | $\mathrm{a}+\mathrm{b}=\mathrm{c}+\mathrm{d} \mid a+a=b+b$ |
| Strategies Total by Skill Code |  |  |  |  | 1 | 1 | 8 | 1 | 11 |

NRR.C.9.a. Summary

## Essentialized Skill Statement

## 9. Equal Sign as a Relational Symbol

| Code | Kindergarten | Grade 1 |  | Grade 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{B}$ |
| NRR.C.9.a. | Recognize the equality between two quantities <br> using a balance. |  |  |  |  |  |  |  |

## Student Expectation

Children were expected to compare weights of three objects on each side of a balance, shown on a picture of the balance, and identify the equality between two quantities on both sides of the balance.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Different Terms for Balanced. Terms such as |  |
| balanced and same were expected for students to |  |
| use and all the grade levels. What was not | Itlls me that they're equal (RK_ICC, 19:17). |
| expected were students to connect to the term |  |
| equals, particularly at the Kindergarten level. One | (RK_ICC, 19:39). |
| Kindergarten student used the term equal to <br> describe what the balance says about the apples. |  |


| Grade | 0-5 | $0-10$ | $0-19$ | $0-50$ | $0-99$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (768) | - |  |  |  |  |
| $\mathbf{1}$ |  | - | - |  |  |  |
| $\mathbf{2}$ |  |  |  | - |  |  |

Unanticipated strategies by grade level and number range for NRR.C.9.a.


Anticipated strategies and skill codes used by grade level and number range for the NRR.C.9.a. activity.

NRR.C.9.a. Anticipated Strategies

| NRR.C.9.a. | Recognize the equality between two quantities using a balance. |  |
| :---: | :---: | :---: |
| Content Question | This is a picture of a balance. What does the balance tell you about the apples? |  |
| Reasoning Question | What tells you that this side is [not] the same as the other side? |  |
| Anticipated |  |  |
| Embedded <br> Mathematical Tools | Code | Description |
|  | C9a_E_NOT-Picture of balance | Picture of balance was embedded into the activity. |
|  | C9a_E_NOTPhysical balance | A physical balance was available for students as necessary. When a balance was used, counting bears were also included for this activity. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C9a_A_General tool | Child uses a general tool as listed above. |
|  | C9a_A_ Count all | Child counts apples on each side of the balance |
|  | C9a_A_ Height | - Explain that both sides are equal using words such as same height or level. <br> - Child's response to reasoning question might include words such as lower or higher. |
|  | C9a_A_ Weight | - Explain that both sides are equal using vocabulary such as same weight <br> - Child's response to reasoning question might include words such as heavier or lighter. |
|  | C9a_A_Immediate recognition | - Child might immediately recognize that the quantities on both sides of the balance areequivalent. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Current Skill Code | Only specific student talk-turn within activity that is attributed to the current skill code. |
| Essentialized Skill Statements [Also consider ESSs not specifically listed here; refer to learning progressions.] | NRR.C.8.a. | Child compares the quantity of each side of the balance. |

NRR.C.9.b. Summary
Essentialized Skill Statement

| 9. Equal Sign as a Relational Symbol |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  |  | Grade 1 |  |  |  |  |  |  |
|  | F | B | T |  | F | B | T | F | B |  | T |
| NRR.C.9.b. |  | Write a true equation using an equal sign to represent the relationship between given quantities on a balance or in a pictorial representation. |  |  |  |  |  |  |  |  |  |

## Student Expectation

Children were expected to write a true equation, such as $3=3$ or $3=1+2$ to represent the apples on the given picture of a balance. Physical balances were available as needed for Kindergarten students and first-grade students performing at lower proficiency levels.

| Unanticipated Strategy. Justification/Description | Examples |
| :--- | :--- |
| Additive Property of Zero. A reasoning question for this ESS | $[$ The equation 3=3] would have |
| asked if 3=3 represented the apples in the picture of the |  |
| balance or the physical balance with bears. The anticipated |  |
| responses were for students to either agree or disagree; it was |  |
| unanticipated for students to provide a deeper explanation. |  |
| One child explained using the additive property of zero which |  |
| pairs with the child's definition of number sentence. |  |$\quad$| right here and a zero right here, |
| :--- |
| [pointing to after the 3 on the right |
| side] (993, 22:19). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Associate a=a to <br> pictorial balance. <br> (RK) | - |  |  |  |  |
| $\mathbf{1}$ |  | - | - |  |  |  |
| $\mathbf{2}$ |  |  |  | Additive Property <br> (993) <br> Defining Number <br> Sentences (993) |  |  |

Unanticipated strategies by grade level and number range for NRR.C.9.b.

| Grade | 0-5 | 0-10 | 0-19 | 0-50 | 0-99 | 0-199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $\begin{aligned} & \text { 5a (RK) NME } \\ & (768) \end{aligned}$ | $\begin{gathered} \text { Count All }(495) \\ \text { Equation }(495,385) \\ 5 b(495) \end{gathered}$ |  |  |  |  |
| 1 |  | $\mathrm{c}=\mathrm{a}+\mathrm{b}$ Weight <br> Counting (793) <br> Expression <br> Equation (3+3=6) <br> Quantity <br> Height <br> Identify $\mathrm{a}=\mathrm{a}$ (223) <br> 5b (223) | Equation (152) <br> Quantity $(152,946)$ <br> Identify $\mathrm{a}=\mathrm{a}$ (152) <br> Count All <br> Height |  |  |  |
| 2 |  |  |  | Expressions <br> Equations <br> Counting <br> Connecting Pictures to Numbers (993) | Equation (284, 563) Count All (284) | Equation Quantity (676) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.9.b. activity

| NRR.C.9.b. | Write a true equation using an equal sign to represent the relationship between given quantities on a balance or in a <br> pictorial representation. |  |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| Content Question | Can you write a number sentence to describe the relationship between the apples on either side of the balance? |  |  |  |  |  |
| Reasoning Question | How do you know that this number sentence describes the apples on the balance? What <br> if we wrote 3=3. Could this number sentence represent the apples? |  |  |  |  |  |
| Anticipated |  |  |  |  |  |  |

refer to learning
progressions.]

NRR.C.9.c. Summary

## Essentialized Skill Statement

| 9. Equal Sign as a Relational Symbol |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.C.9.c. |  |  | Recognize true and not true equations with different equation structures: operations on the left side ( $a+b=c$ ); no operations ( $a=$ a); operations on the right side ( $c=a+b$ ). |  |  |  |  |  |  |

## Student Expectation

For Kindergarten students performing at a high proficiency level and beyond, children are expected to accurately recognize true and not true equations given different equation structures. While the goal of core concept nine is for students to recognize the equal sign as a relational symbol, it was still anticipated that students may interpret the equal sign operationally.

| Strategy. Justification/Description | Examples |
| :---: | :---: |
| Unknown Equation Structure. The grade band for NRR.C.9.c. begins at the target level of Kindergarten. It was not anticipated that by first grade students would still be struggling with the meaning of the equation structures, specifically $a=a$. | $\mathrm{a}=\mathrm{a}$ : <br> [ $8=8$ is true] because it is adding...because they're both the same, because that's why they put the equal (223, 25:16). <br> [Student read card, 8=8] Eight plus eight [is not true] because it still is the same...because you like make 8... [The equal sign means] like you be like something else...like make together (946, 20:52-21:46). <br> [Student read card, 16=16] Sixteen plus sixteen... (793, 26:36) <br> I don't know what it means (152, 26:24). <br> I'm not sure...because there's no like thing that they're equal to (993, 24:22, 24:27) |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - | - |  |  |  |  |
| $\mathbf{1}$ |  | Unknown Equation <br> Structure (a=a) (223, 946) | Unknown Equation <br> Structure (a=a) (152, <br> $793)$ |  |  |  |
| $\mathbf{2}$ |  |  |  | Unknown Equation <br> Structure (a=a) (993) |  |  |

Unanticipated strategies by grade level and number range for NRR.C.9.c.

| Grade | 0-5 | 0-10 | 0-19 | 0-50 | 0-99 | 0-199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (RK, 768) | Unknown Equation Structure $(c=a+b, a=a)(385,495)$ <br> Count on ( $c=a+b$ ) <br> (385) <br> Operational $(a=a, c=a+b)(385)$ |  |  |  |  |
| 1 |  | $\begin{gathered} \text { Operational }(a+b=c, c=a+b) \\ (223,946) \end{gathered}$ | ```Operational (c=a+b, a+b=c) (152, 793)``` |  |  |  |
| 2 |  |  |  | $\begin{gathered} \text { Operational }(a+b=c, \\ c=a+b)(993) \end{gathered}$ | Operational ( $a+b=c, c=a+b$ ) $(284,563)$ <br> Relational ( $\mathrm{a}=\mathrm{a}$ ) $(284,563)$ | Operational $(a+b=c, c=a+b)$ <br> (676) <br> Relational ( $\mathrm{a}=\mathrm{a}$ ) 676 |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.9.c. activity


| Essentialized Skill | 06 -Decomposition <br> Statements | $\bullet$ Add or subtract part or all of the expression |
| :--- | :--- | :--- |
| [Also consider ESSs not |  |  |
| specifically listed here; |  |  |
| refer to learning |  |  |
| progressions.] |  |  |

* NRR.C.9.c. and NRR.C.9.d. skill codes are combined and strategies are same.

NRR.C.9.d. Summary
Essentialized Skill Statement

| 9. Equal Sign as a Relational Symbol |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.C.9.d. |  |  |  |  | Recognize true and not true equations with different equation structures: operations on the both sides ( $a+b=c+d)$; multiple instances of a number. |  |  |  |  |

## Student Expectation

For first-grade students performing at mid-level proficiency and beyond, children are expected to accurately recognize true and not true equations given different equation structures. While the goal of core concept nine is for students to recognize the equal sign as a relational symbol, it was still anticipated that students may interpret the equal sign operationally.

| Unanticipated Strategy. Justification/Description | Examples |
| :---: | :---: |
| Unknown Equation Structure. The grade band for NRR.C.9.d. begins at the bridging level of first-grade. It was not anticipated that by second grade students would still be struggling with the meaning of the equation structures with operations on both sides. | $a+b=c+d:$ <br> [11+5=14+2 is not true] because eleven plus five equals fourteen is not two. (793, 27:31) <br> I've never seen this before... with the equal sign in the middle...usually the equal sign is over here [pointing to plus sign after first addend on left side] and over there [pointing to plus sign before last addend on right side] (993, 26:03, 26:06, 26:15). <br> [l'm not sure about $151+5=150+6]$ because there's too much numbers everywhere and it doesn't make sense... well, you can have enough numbers, but it just seems confusing right now because [151] is a bigger than [150], so that can't equal (676, 18:09, 18:18). $a+a=b+b:$ <br> ...I don't know what [the +8 ] means [in the equation $4+4=8$ +8] (152, 25:56). <br> This also has the equal sign in the middle...so I'm not sure (993, 26:46, 26:52). |


| Grade | $\mathbf{0 - 5}$ | 0-10 | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ |  | - |  |  |  |  |
| $\mathbf{1}$ |  |  | Unknown Equation <br> Structure (152, 793) |  |  |  |
| $\mathbf{2}$ |  |  |  | Unknown Equation <br> Structure (993) |  |  |

Unanticipated strategies by grade level and number range for NRR.C.9.d.

| Grade | 0-5 | 0-10 | 0-19 | 0-50 | 0-99 | 0-199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Unknown Equation Structure (RK) NME (768) | Unknown Equation Structure $(385,495)$ |  |  |  |  |
| 1 |  | Unknown Equation Structure (946) Operational (946) | Operational (152) |  |  |  |
| 2 |  |  |  | Operational (993) | ```Operational (284) 5b (563) Relational (a+b=c+d) (563) Operational (a+a=b+b) (563)``` | Operational (676) <br> Comparison (676) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.9.d. activity.

| *NRR.C.9.c. NRR.C.9.d. <br> Content Question* | Recognize true and not true equations with different equation structures: operations on the left side $(a+b$ $=\mathrm{c})$; no operations $(\mathrm{a}=\mathrm{a})$; operations on the right side $(\mathrm{c}=\mathrm{a}+\mathrm{b})$. <br> Recognize true and not true equations with different equation structures: operations on the both sides ( $a+b=c+$ d); multiple instances of a number. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | I'm going to give you a number sentence [equation]. If you think the number sentence is true, then place the card in this true area. If you think this is not true, then place the card in the not true area. [Motion to those areas on the work mat.]. If you are not sure about a card then put it in the center[unsure]area. |  |  |  |
|  | $a+b=c$ Not True | $\mathrm{a}=\mathrm{a}$ : True <br> $\mathrm{c}=\mathrm{a}+\mathrm{b}$ : Not True | $a+b=c+d$ : True | $\begin{aligned} & a+a+. . \\ & \ldots+b: N \end{aligned}$ |
| Reasoning Question | Why is this card not true? What can you tell me about this card? |  |  |  |
| Anticipated |  |  |  |  |
|  | Code | Description |  |  |
| Embedded <br> Mathematical Tools | No tools were embedded for this activity. |  |  |  |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |  |  |
|  | Square tiles | Colored square tiles as counters |  |  |
|  | Linking cubes | Linking cubes as counters |  |  |
|  | Fingers | Fingers as counters |  |  |
| Strategies | Code | Description |  |  |
|  | C9cd_A_General tool | Child uses a general tool as listed above. |  |  |
|  | C9cd_A_ operational | - Child uses the equal sign as an operation (i.e., equal sign means something "to do"). |  |  |
|  | C9cd_A_relational | - Child uses the equal sign to show both sides are the same. |  |  |
|  | C9cd_A_Count on | - Count on from one number in an equation to find the total [counting] |  |  |
|  | C9cd_A_Identify symbols | - Identify numbers and symbols in the equation |  |  |
| A different way |  | - A different strategy was not anticipated for this essentialized skill statement. If a differentstrategy was used, create an unanticipated node for the child's strategy. |  |  |
| NRR | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |  |  |
|  | 05-Composition | - Compose a number with two parts. [Composition \& Decomposition] |  |  |


| Essentialized Skill | 06 -Decomposition <br> Statements | $\bullet$ Add or subtract part or all of the expression |
| :--- | :--- | :--- |
| [Also consider ESSs not |  |  |
| specifically listed here; |  |  |
| refer to learning |  |  |
| progressions.] |  |  |

* NRR.C.9.c. and NRR.C.9.d. skill codes are combined.

NRR.C.10. Maintaining Equality Core Concept

| 10. Maintaining Equality |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.C.10.a. | Given a contextual situation with known quantities, use one or more properties of operations to recognize when equality is maintained. |  |  |  |  |  |  |  |  |
| NRR.C.10.b. |  | Given a contextual situation with unknown quantities, use one or more properties of operations to recognize when equality is maintained. |  |  |  |  |  |  |  |
| NRR.C.10.c. |  |  |  |  | Given a contextual situation with known quantities that models one or more properties of operations, write a true equation to represent the situation. |  |  |  |  |
| NRR.C.10.d. |  |  |  |  | Recognize true and not true equations with known numbers using one or more properties of operations. |  |  |  |  |

Unanticipated strategies: NRR.C.10.a-d

|  | Skill C |  | C.10.a | C.10.b | C.10.c |  | C.10.d |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Unantici | ated | C | c | C | c | c | c | C | Unknown |
| SID | Grade | Number Range |  |  |  | $a+b=b+a$ | $a+b-b=a+0$ | $a-b+b=a$ | $a+b+c=a+d$ | Equation <br> Structure |
| RK | K | 0-5 | 1 | - | - | - | - | - | - | - |
| 768 | K | 0-5 | - | - | - | 1 | - | 1 | - | - |
| 385 | K | 0-10 | 1 | 1 | 1 | - | - | - | - | - |
| 495 | K | 0-10 | 1 | - | - | - | - | 1 | 1 | - |
| 223 | 1 | 0-10 | 1 | - | - | - | - | 1 | - | - |
| 946 | 1 | 0-10 | 1 | - | - | 1 | - | 1 | - | - |
| 152 | 1 | 0-19 | 1 | 1 | 1 | - | - | - | - | - |
| 793 | 1 | 0-19 | 1 | - | - | - | 1 | - | 1 | - |
| 993 | 2 | 0-50 | 1 | 1 | - | 1 | 1 | 1 | 1 | - |
| 284 | 2 | 0-99 | 1 | 1 | 1 | - | 1 | - | 1 | - |
| 563 | 2 | 0-99 | 1 | 1 | 1 | 1 | - | - | - | - |
| 676 | 2 | 0-199 | 1 | 1 | - |  | 1 | - | - | 1 |
| Strategies Total by Skill Code |  |  |  |  |  |  |  |  |  | 1 |


|  | Code |  |  | C. 1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticip |  |  | C | Counting | Match Groups | Change | NRR.C.8. |
| SID | Grade | Number Range |  |  |  | Order |  |
| RK | K | 0-5 | 1 | All | - | - | - |
| 768 | K | 0-5 | 0 | - | - | - | - |
| 385 | K | 0-10 | 1 | - | 1 | - | - |
| 495 | K | 0-10 | 1 | All | - | - | - |
| 223 | 1 | 0-10 | 1 | All | 1 | - | - |
| 946 | 1 | 0-10 | 1 | All | 1 | - | - |
| 152 | 1 | 0-19 | 1 | - | 1 | - | - |
| 793 | 1 | 0-19 | 1 | All | - | - | - |
| 993 | 2 | 0-50 | 1 | - | 1 | - | - |
| 284 | 2 | 0-99 | 1 | - | 1 | - | - |
| 563 | 2 | 0-99 | 1 | - | 1 | 1 | C.8.b |
| 676 | 2 | 0-199 | 1 | - | - | 1 | C.8.b |
| Strategies Total by Skill Code |  |  |  | 5 | 7 | 2 | 2 |

Anticipated strategies: NRR.C.10.b

|  | ode |  |  | C. 1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Antic |  |  | C | Counting | Match | Change Order | NRR.C. 8 |
| SID | Grade | Number Range |  |  | Groups |  |  |
| RK | K | 0-5 | 0 | All | - | - | - |
| 768 | K | 0-5 | 0 | - | - | - | - |
| 385 | K | 0-10 | 1 | - | 1 | - | - |
| 495 | K | 0-10 | 0 | All | - | - | - |
| 223 | 1 | 0-10 | 0 | All | 1 | - | - |
| 946 | 1 | 0-10 | 0 | All | 1 | - | - |
| 152 | 1 | 0-19 | 1 | - | 1 | - | - |
| 793 | 1 | 0-19 | 0 | All | - | - | - |
| 993 | 2 | 0-50 | 1 | - | 1 | - | - |
| 284 | 2 | 0-99 | 1 | - | 1 | - | - |
| 563 | 2 | 0-99 | 1 | - | 1 | 1 | - |
| 676 | 2 | 0-199 | 1 | - | - | 1 | C.8.b |
| Strategies Total by Skill Code |  |  |  | 5 | 7 | 2 | 1 |

Anticipated strategies: NRR.C.10.c

| Skill Code |  | C.10.c |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated | C | NME | Counting | NRR.C.8 |  |  |
| SID | Grade | Number <br> Range |  |  |  |  |
| RK | K | $0-5$ | 0 | 1 | - | - |
| 768 | K | $0-5$ | 0 | 1 | - | - |
| 385 | K | $0-10$ | 1 | - | - | C.8.b |
| 495 | K | $0-10$ | 0 | - | All | - |
| 223 | 1 | $0-10$ | 0 | 1 | - | - |
| 946 | 1 | $0-10$ | 0 | - | All | - |
| 152 | 1 | $0-19$ | 1 | - | - | C.8.b |
| 793 | 1 | $0-19$ | 0 | - | - | C.8.b |
| 993 | 2 | $0-50$ | 0 | - | - | C.8.b |
| 284 | 2 | $0-99$ | 1 | - | - | C.8.b |
| 563 | 2 | $0-99$ | 1 | - | - | C.8.b |
| 676 | 2 | $0-199$ | 0 | - | - | C.8.b |
| Strategies Total by Skill Code |  | 3 | 2 | 7 |  |  |

Anticipated strategies: NRR.C.10.d

| Skill | ode |  |  | C.10.d |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anti | pated |  | c | c | C | C | N | Unknown | Operational |
| SID | Grade | Number Range | $a+b=b+a$ | $a+b-b=a+0$ | $a-b+b=a$ | $a+b+c=a+d$ | ME | Equation Structure |  |
| RK | K | 0-5 | 0 | 0 | 0 | 0 | 1 | - | - |
| 768 | K | 0-5 | 1 | 0 | 1 | 0 | 1 | - | - |
| 385 | K | 0-10 | 0 | 0 | 0 | 0 | 1 | - | - |
| 495 | K | 0-10 | 0 | 0 | 1 | 1 | - | 1 | - |
| 223 | 1 | 0-10 | 0 | 0 | 1 | 0 | 1 | - | - |
| 946 | 1 | 0-10 | 1 | 0 | 1 | 0 | - | - | - |
| 152 | 1 | 0-19 | 0 | 0 | 0 | 0 | - | 1 | 1 |
| 793 | 1 | 0-19 | 0 | 1 | 0 | 1 | - | 1 | 1 |
| 993 | 2 | 0-50 | 1 | 1 | 1 | 1 | - | - | - |
| 284 | 2 | 0-99 | 0 | 1 | 0 | 1 | - | - | 1 |
| 563 | 2 | 0-99 | 1 | 0 | 0 | 0 | - | - | 1 |
| 676 | 2 | 0-199 | 0 | 1 | 0 | 0 | - | - | - |
| Strategies Total by Skill Code |  |  |  |  |  |  | 4 | 3 | 4 |

NRR.C.10.a. Summary
Essentialized Skill Statement

| 10. Maintaining Equality |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  | Grade 1 | Grade 2 | B |  |  |  |
|  | F | B | T | F | B | T | F |  |
| NRR.C.10.a. | Given a contextual situation with known <br> quantities, use one or more properties of <br> operations to recognize when equality is <br> maintained. |  |  |  |  |  |  |  |

## Student Expectation

UPDATE

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| In the selected data, no evidence of <br> unanticipated strategies found. | $\mathrm{N} / \mathrm{A}$ |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |

Unanticipated strategies by grade level and number range for NRR.C.10.a.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | Count All (RK) 5a <br> (768) | Count All (495) <br> Match Groups (385) <br> 1a (946) |  | $0-99$ |  |
| $\mathbf{1}$ |  | Count All, Match Groups <br> (223, 946) | Count All (793) |  |  |
| $\mathbf{2}$ |  |  |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.10.a. activity.

| NRR.C.10.a. | Given a contextual situation with known quantities, use one or more properties of operations to recognize when equality is maintained. |  |
| :---: | :---: | :---: |
| Content Question | Julie thinks that more students rode on this bus since more students got on first. What do you think? Are there the same, more, or less number of students in the buses? |  |
| Reasoning Question | Can you show me how you know that there are the same, more, or less? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | C10a_E_NOTPicture of students | Quantities displayed on cards with pictures of students. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C10a_A_General tool | Child uses a general tool as listed above. |
|  | C10a_A_Count all | - Child counts all students |
|  | C10a_A_ Count groups | - Child counts groups of students |
|  | C10a_A_ Match groups | - Match groupings visually with or without counting |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill <br> Statements <br> [Also consider ESSs not specifically listed here] | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | NRR.C.8.b. | - Identify that there are the same amount of students using words such as switch, swap, different order. |
|  | $\begin{aligned} & \text { 05- Composition } \\ & 06 \text { - Decomposition } \\ & 07 \text { - Applying... } \end{aligned}$ | - Compose a number with two parts. [Composition \& Decomposition] <br> - Add to find sums and compare <br> - Select skill code in Composition \& Decomposition, that best fits child's approach. |

## NRR.C.10.b. Summary

## Essentialized Skill Statement

| 10. Maintaining Equality |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.C.10.b. |  | Given a contextual situation with unknown quantities, use one or more properties of operations to recognize when equality is maintained. |  |  |  |  |  |  |  |

## Student Expectation

Students were expected to be able to recognize that equality maintained the same in a situation involving the additive inverse property.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| In the selected data, no evidence of <br> unanticipated strategies found. | $\mathrm{N} / \mathrm{A}$ |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $K$ |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

Unanticipated strategies by grade level and number range for NRR.C.10.b.
$\left.\begin{array}{|c|c|c|c|c|c|c|}\hline \text { Grade } & \mathbf{0 - 5} & \mathbf{0 - 1 0} & \mathbf{0 - 1 9} & \mathbf{0 - 5 0} & \mathbf{0 - 9 9} & \\ \hline \text { K } & \begin{array}{c}\text { Count All (RK) 5a } \\ \text { (768) }\end{array} & \text { Count All (495) } \\ \text { Match Groups (385) }\end{array}\right]$

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.10.b. activity.

| NRR.C.10.b. | Given a contextual situation with unknown quantities, use one or more properties of operations to recognize when equality is maintained. |  |
| :---: | :---: | :---: |
| Content Question | [KB-1B] Does he have the same, more, or less keys than he did before the school year started? [1T-2T] Does Mr. King have the same, more, or less keys than he did before he gave Tina his keys? |  |
| Reasoning Question | Can you show me how you know that there are the same, more, or less? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded <br> Mathematical Tools | - | No tools were embedded for this activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C10b_A_General tool | Child uses a general tool as listed above. |
|  | C10b_A_Act out | - Model with concrete objects. Also assign a mathematical tool |
|  | C10b_A_ Create example | - Since both problems include unknown numbers, child might create a similar problem with known quantities to explain thinking. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
| Essentialized Skill Statements <br> [Also consider ESSs not specifically listed here; refer to learning progressions.] | NRR.C.8.c. | - Additive inverse with known quantities. |

NRR.C.10.c. Summary
Essentialized Skill Statement

| 10. Maintaining Equality |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  |  | Grade 2 |  |  |
|  | F | B | T | F | B |  | T | F | B | T |
| NRR.C.10.c. | Given a contextual situation with known quantities that models one or more properties of operations, write a true equation to represent the situation. |  |  |  | Given a contextual situation with known quantities that models one or more properties of operations, write a true equation to represent the situation. |  |  |  |  |  |

## Student Expectation

Children were expected to write a true equation incorporating at least on property of operation to represent the given context.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| In the selected data, no evidence of <br> unanticipated strategies were found. | $\mathrm{N} / \mathrm{A}$ |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - | - |  |  |  |  |
| 1 | - | - |  |  |  |  |
| 2 |  |  |  | - | - | - |

Unanticipated strategies by grade level and number range for NRR.C.10.c

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (768,RK) | Count all (495) Same <br> Numbers, 8.b (385) |  |  |  |  |
| $\mathbf{1}$ |  | Count all (946) <br> Order, 8.b (793) <br> NME (223) | $8 . b$ (152) |  |  |  |
| $\mathbf{2}$ |  |  | $8 . b$ (993) | Story, 8.b (563) <br> $8 . b ~(284) ~$ | 8.6 (676) |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.10.c activity.

| NRR.C.10.c. | Given a contextual situation with known quantities that models one or more properties of operations, write a true equation to represent the situation. |  |
| :---: | :---: | :---: |
| Content Question | 1) What would this box look like as a number sentence? <br> 2) What would this box look like as a number sentence? |  |
| Reasoning Question | What can you tell me about the two number sentences? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded <br> Mathematical Tools | - | No tools were embedded for this activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C10c_A_General tool | Child uses a general tool as listed above. |
|  | C10c_A_Count all | Child counts all cupcakes to determine that the quantities are the same. |
|  | C10c_A_Count groups | Child counts by groupings, 4 s , $5 \mathrm{~s}, 10 \mathrm{~s}$ |
|  | C10c_A_Expression | Child writes an expression to represent the total number of cupcakes (i.e. no equal sign present) |
|  | C10c_A_Equation | Child writes an equation to represent the total number of cupcakes (i.e., 3+4=7) |
|  | C10c_A_One-to-one | - Child matches pictures of cupcakes |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | NRR.C.8.c. | - Additive inverse with known quantities. |
| [Also consider ESSs not specifically listed here; refer to learning progressions.] | 05 - Composition | - Add cupcakes together to find sum <br> - Add cupcakes by groups (e.g. $5+5=10,10+5=15$ ) <br> - Select skill code in Composition that best fits child's approach. |

NRR.C.10.d. Summary
Essentialized Skill Statement

| 10. Maintaining Equality |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.C.10.d. |  |  |  |  |  | Recognize true and not true equations with known numbers using one or more properties of operations. |  |  |  |

## Student Expectation

For this essentialized skill statement, students were given four equation structures to determine whether or not the equation was true. The four equation structures included: (1) $a+b=b+a ;(2) a+b-b=a+0$; (3) $a-b+b=a$; and (4) $a+b+c=a+d$.

| Unanticipated Strategy. Justification/Description | Examples |
| :---: | :---: |
| Unknown Equation Structure. First-grade students at a high-proficiency level and beyond were expected to be able to use relational thinking to determine if each equation of the different equation structures were true or not true. The examples provided indicate confusion based on the equation structures. | [76 + 87 = 87 + 76] So I don't really know what's going on because ... I wanna know why they put these in different ... like why did they swap them? (676, 24:43). <br> It's again they have the same numbers and it's confusing that they are just different sections. 'Cause I do not think that $134-56+56=134$ ( $676,25: 43$ ). |
| Compare Sums. Child computed each side to determine whether each side had the same amount. In the example, the child made an error in computation; however, the strategy used was to compute each side and compare sums. At the second-grade level, children were expected to be able to either use a combination of associative and commutative properties without computation to determine whether or not the equation was true or not true. | $[17+12+12=24+12] 36$ for these two and 41 here and ... I forgot what I said for this one ... 36 ... They're not true (993, 40:45). <br> Interviewer Response: <br> I like how you added [the left side], and then you added all [of the right side] to find the same (EK, 41:30). |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | - | - |  |  |  |  |
| $\mathbf{1}$ |  | - | - |  |  |  |
| $\mathbf{2}$ |  |  |  | Compare Sums $[\mathrm{a}+\mathrm{b}-\mathrm{b}=\mathrm{c}+\mathrm{d}](993)$ | - | Unknown Equation <br> Structure (676) |

Unanticipated strategies by grade level and number range for NRR.C.10.d.
$\left.\begin{array}{|c|c|c|c|c|c|c|}\hline \text { Grade } & \mathbf{0 - 5} & \mathbf{0 - 1 0} & \mathbf{0 - 1 9} & \mathbf{0 - 5 0} & \mathbf{0 - 9 9} \\ \hline \text { K } & \begin{array}{c}\text { NME (768, } \\ \text { RK) }\end{array} & \begin{array}{c}\text { Unknown Equation } \\ \text { Structure v(495) NME } \\ (385)\end{array} & & & \\ \hline \mathbf{1} & & \text { (946) } \\ \text { NME (223) }\end{array} \quad \begin{array}{c}\text { Operational, } \\ \text { Unknown Equation } \\ \text { Structure (152, 793) }\end{array}\right)$

Ananticipated strategies by grade level and number range for NRR.C.10.d.

NRR.C.10.d. Anticipated Strategies

| NRR.C.10.d. | Recognize true and not true equations with known numbers using one or more properties of operations. |  |
| :---: | :---: | :---: |
| Content Question* | Here are some cards with different number sentences. Please sort them into a pile of cards you think are true and a pile you think are not true. Try to figure it out without adding up the numbers. If you're not sure about a card then put it in the center [unsure] pile. |  |
|  | $a+b=b+a$ : True |  |
| Reasoning Question | What can you tell me about this card? Why did you put it here? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | - | No tools were embedded for this activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C10d_A_General tool | Child uses a general tool as listed above. |
|  | C10d_A_Count on | Count on from one number in an equation to find the total [counting] |
|  | C10d_A_ Identify symbols | - Identify numbers and symbols in the equation |
|  | C10d_A_ Operational | - Child uses the equal sign as an operation (i.e., equal sign means something "to do"). |
|  | C10d_A_ Relational | - Child uses the equal sign to show both sides are the same. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill Statements <br> [Also consider ESSs not specifically listed here; refer to learning progressions.] | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | NRR.C.8.b. | - Commutative property |
|  | NRR.C.8.c. | - Additive inverse with known quantities. |
|  | NRR.C.8.d. | - Associative property |
|  | 05 -Composition | - Add numbers to find partial sums <br> - Select skill code in Composition that best fits child's approach. |


| 11. Solving for Unknown Values |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  |  | Grade 2 |  |  |
|  |  |  | T | F | B | B | T | F | B | T |
| NRR.C.11.a. |  |  | Solve for an unknown value in a true equation using a relational definition of equal sign. |  |  |  |  |  |  |  |
| NRR.C.11.b. |  |  | Given a contextual situation modeling a true equation, apply one or two properties of operations or property of equality to solve for an unknown value using concrete objects. |  |  |  |  |  |  |  |
| NRR.C.11.c. |  |  | Given a contextual situation modeling a true equation, apply one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |  |  |  |  |  |  |
| NRR.C.11.d. |  |  | Apply one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |  |  |  |  |  |  |
| NRR.C.11.e. |  |  |  |  | Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value using concrete objects. |  |  |  |  |  |
| NRR.C.11.f. |  |  |  |  | Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |  |  |  |  |
| NRR.C.11.g. |  |  |  |  | Apply decomposition with one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |  |  |  |  |

Unanticipated strategies: NRR.C.11.a-g

| Skill Code |  |  | C.11.a |  |  |  |  |  | 1.d | C.11.e |  | .11.f |  | C.11.g |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unanticipated |  |  | C | C | $\begin{gathered} N \\ M \\ E \end{gathered}$ | C | $\begin{gathered} N \\ M \\ E \end{gathered}$ | C | $\begin{array}{\|l\|} \hline N \\ M \\ E \end{array}$ | c | C | Counting | $\begin{gathered} \hline N \\ M \\ E \end{gathered}$ | C |
| SID | Grade | Number Range |  |  |  |  |  |  |  |  |  |  |  |  |
| RK | K | 0-5 | 0 | 0 | - | 0 | - | 0 | - | 0 | 8 | - | - | 8 |
| 768 | K | 0-5 | 0 | 0 | - | 0 | - | 0 | - | 0 | 0 | - | - | 0 |
| 385 | K | 0-10 | 0 | 1 | - | 0 | - | 0 | - | 1 | 0 | - | - | 0 |
| 495 | K | 0-10 | 1 | 0 | - | 0 | - | 0 | - | 0 | 0 | - | - | 0 |
| 223 | 1 | 0-10 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 8 | - | - | 8 |
| 946 | 1 | 0-10 | 0 | 0 | 1 | 0 | - | 0 | - | 0 | 0 | - | - | 0 |
| 152 | 1 | 0-19 | 0 | 1 | - | 0 | - | 0 | - | 1 | 1 | - | - | 1 |
| 793 | 1 | 0-19 | 0 | 8 | - | 8 | - | 8 | - | 8 | 8 | - | 1 | 8 |
| 993 | 2 | 0-50 | 0 | 1 | - | 0 | - | 0 | - | 1 | 0 | - | - | 8 |
| 284 | 2 | 0-99 | 0 | 1 | - | 0 | - | 1 | - | 0 | 0 | All | - | 1 |
| 563 | 2 | 0-99 | 1 | 1 | - | 0 | - | 1 | - | 1 | 1 | - | - | 1 |
| 676 | 2 | 0-199 | 0 | 1 | - | 0 | - | 1 | - | 1 | 0 | - | 1 | 0 |
| Strategies Total by Skill Code |  |  |  |  | 2 |  | 1 |  | 1 |  |  | 1 | 1 |  |

Anticipated strategies: NRR.C.11.a-b

| Skill Code |  |  | c.11.a. |  |  |  |  |  | C.11.b. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | icipat rategie |  | C | NME | Counting | Relational | Operational | NRR.B.5. | C | NME | NRR.C.8. | Operational |
| SID | Grade | Number <br> Range |  |  |  |  |  |  |  |  |  |  |
| RK | K | 0-5 | 0 | 1 | - | - | - | - | 0 | 1 | - | - |
| 768 | K |  | 0 | 1 | - | - | - | - | 0 | 1 | - | - |
| 385 | K | 0-10 | $\begin{gathered} {[0-5]} \\ 0 \end{gathered}$ | 1 | - | - | - | - | $\begin{gathered} {[0-5]} \\ 1-1 \end{gathered}$ | - | C.8.c. | - |
| 495 | K |  | 1-0 | 1 | - | - | - | - | 0 | - | - | - |
| 223 | 1 | 0-10 | 0 | - | All | - | - | - | $\begin{gathered} {[0-5]} \\ 0 \end{gathered}$ | 1 | - | - |
| 946 | 1 |  | 0 | - | - | - | - | - | 0 | 1 | - | - |
| 152 | 1 | 0-19 | 0 | - | - | - | - | - | 1-1 | - | C.8.c. | - |
| 793 | 1 |  | 0 | - | On | - | - | - | S | S | - | - |
| 993 | 2 | 0-50 | 0 | - | - | - | - | B.5.b. | 1-1 | - | C.8.c. | - |
| 284 | 2 | 0-99 | 0 | - | All | - | - | - | 1-1 | - | C.8.c. | - |
| 563 | 2 |  | 1-1 | - | - | 1 | - | B.5.d. | 1-0 | - | - | Computation |
| 676 | 3 | 0-199 | 0 | - | - | - | Left Side | - | $\begin{gathered} {[0-99]} \\ 1-0 \end{gathered}$ | - | C.8.c. | - |
| Strategies Total by Skill Code |  |  | 2-1 | 4 | 3 | 1 | 1 | 2 | 6-4 | 4 | 5 | 1 |

Anticipated strategies: NRR.C.11.c

|  | Skill C |  | C.11.c. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | C | NME | Counting | Operational | NRR.B.5. | NRR.B. 6 |
| SID | Grade | Number Range |  |  |  |  |  |  |
| RK | K | 0-5 | 0 | 1 | - | - | - | - |
| 768 | K |  | 0 | 1 | - | - | - | - |
| 385 | K | 0-10 | $\begin{array}{r} {[0-5]} \\ 0 \end{array}$ | 1 | - | - | - | - |
| 495 | K |  | 0 | 1 | - | - | - | - |
| 223 | 1 | 0-10 | 0 | 1 | - | - | - | - |
| 946 | 1 |  | 0 | - | - | - | - | - |
| 152 | 1 | 0-19 | 0 | - | On | - | - | - |
| 793 | 1 |  | S | S | - | - | - | - |
| 993 | 2 | 0-50 | 0 | - | - | - | B.5.c. | B.6.f. |
| 284 | 2 | 0-99 | 0 | - | On, Skip (5) | - | - | - |
| 563 | 2 |  | 0 | - | - | Left Side | B.5.c. | - |
| 676 | 3 | 0-199 | $\begin{gathered} {[0-99]} \\ 0 \end{gathered}$ | - | - | Left Side | B.5.c. | - |
| Strategies Total by Skill Code |  |  | 0-11 | 5 | 3 | 2 | 3 | 1 |

Anticipated strategies: NRR.C.11.d

|  | Skill C |  |  |  | .d. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | $C$ | NME | Counting | Operational | NRR.C. 8. |
| SID | Grade | Number Range |  |  |  |  |  |
| RK | K | 0-5 | 0 | 1 | - | - | - |
| 768 | K |  | 0 | 1 | - | - | - |
| 385 | K | 0-10 | $\begin{gathered} {[0-5]} \\ 0 \\ \hline \end{gathered}$ | 1 | - | - | - |
| 495 | K |  |  | 1 | - | - | - |
| 223 | 1 | 0-10 | 0 | 1 | - | - | - |
| 946 | 1 |  | 0 | - | All | - | - |
| 152 | 1 | 0-19 | 0 | - | - | Left Side | - |
| 793 | 1 |  | S | S | - | - | - |
| 993 | 2 | 0-50 | 0 | - | - | Left Side | - |
| 284 | 2 | 0-99 | 1-1 | - | On | - | - |
| 563 | 2 |  | 1-1 | - | - | - | C.8.b. |
| 676 | 3 | 0-199 | $\begin{gathered} {[0-99]} \\ 1-0 \end{gathered}$ | - | - | - | - |
| Strategies Total by Skill Code |  |  |  | 5 | 2 | 2 | 1 |

Anticipated strategies: NRR.C.11.e

|  | Skill C |  |  |  | C.11.e. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | C | NME | Counting | Relational | Operational | NRR.B.6. |
| SID | Grade | Number Range |  |  |  |  |  |  |
| RK | K | 0-5 | 0 | 1 | - | - | - | - |
| 768 | K |  | 0 | 1 | - | - | - | - |
| 385 | K | 0-10 | $\begin{gathered} {[0-5]} \\ 1-0 \end{gathered}$ | - | - | - | - | - |
| 495 | K |  | 0 | - | - | - | - | - |
| 223 | 1 | 0-10 | 0 | 1 | - | - | - | - |
| 946 | 1 |  | 0 | 1 | - | - | - | - |
| 152 | 1 | 0-19 | 1-1 | - | On | - | - | - |
| 793 | 1 |  | S | S | - | - | - | - |
| 993 | 2 | 0-50 | 1-0 | - | - | 1 | Computation | - |
| 284 | 2 | 0-99 | 0 | - | All, On | - | - | - |
| 563 | 2 |  | 1-1 | - | - | 1 | - | 1 |
| 676 | 3 | 0-199 | $\begin{gathered} {[0-99]} \\ 1-0 \end{gathered}$ | - | - | 1 | - | 1 |
| Strategies Total by Skill Code |  |  |  | 4 | 3 | 3 | 1 | 2 |

Anticipated strategies: NRR.C.11.f

| Skill Code |  |  | C.11.f. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | C | NME | Counting | Relational |
| SID | Grade | Number Range |  |  |  |  |
| RK | K | 0-5 | S | S | - | - |
| 768 | K |  | 0 | 1 | - | - |
| 385 | K | 0-10 | $\begin{array}{r} {[0-5]} \\ 0 \end{array}$ | 1 | - | - |
| 495 | K |  | 0 | 1 | - | - |
| 223 | 1 | 0-10 | S | S | - | - |
| 946 | 1 |  | 0 | 1 | - | - |
| 152 | 1 | 0-19 | 1-1 | - | On | 1 |
| 793 | 1 |  | S | S | - | - |
| 993 | 2 | 0-50 | 0 | - | On | 1 |
| 284 | 2 | 0-99 | 0 | - | All | 1 |
| 563 | 2 |  | 1-0 | No Opp. | - | - |
| 676 | 3 | 0-199 | $\begin{gathered} {[0-99]} \\ 0 \end{gathered}$ | 1 | - | - |
| Strategies Total by Skill Code |  |  |  | 4 | 3 | 3 |

Anticipated strategies: NRR.C.11.g

|  | Skill Cod | de |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Anticipated Strategies |  |  | C | NME | Counting | NRR.B.5. | NRR.B.6. |
| SID | Grade | Number Range |  |  |  |  |  |
| RK | K | 0-5 | S | S | - | - | - |
| 768 | K |  | 0 | 1 | - | - | - |
| 385 | K | 0-10 | $\begin{gathered} {[0-5]} \\ 0 \end{gathered}$ | 1 | - | - | - |
| 495 | K |  | 0 | 1 | - | - | - |
| 223 | 1 | 0-10 | S | S | - | - | - |
| 946 | 1 |  | 0 | 1 | - | - | - |
| 152 | 1 | 0-19 | 1-0 | - | On | - | - |
| 793 | 1 |  | S | S | - | - | - |
| 993 | 2 | 0-50 | - | - | - | B.5.e. | B.6.f. |
| 284 | 2 | $0-99$ | 1-1 | - | - | - | B.6.f. |
| 563 | 2 |  | 1-0 | - | - | - | B.6.f. |
| 676 | 3 | 0-199 | $\begin{gathered} {[0-99]} \\ 0 \end{gathered}$ | - | - | - | B.6.f. |
| Strategies Total by Skill Code |  |  |  | 4 | 1 | 1 | 4 |

NRR.C.11.a. Summary

## Essentialized Skill Statement

| 11. Solving for Unknown Values |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.C.11.a. |  |  | Solve for an unknown value in a true equation using a relational definition of equal sign. |  |  |  |  |  |  |

## Student Expectation

Using relational understanding of equal sign, children were expected to find the unknown value in the given equation of format $\mathrm{a}+\mathrm{b}=---------+\mathrm{d}$.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| In the selected data, no evidence of <br> unanticipated strategies found. | $\mathrm{N} / \mathrm{A}$ |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - | - |  |  |  |  |
| $\mathbf{1}$ |  | - | - |  |  |  |
| $\mathbf{2}$ |  |  |  | - | - |  |

Unanticipated strategies by grade level and number range for NRR.C.11.a

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME <br> (RK,768) | NME (495) <br> NME (Guess385) |  |  |  |
| $\mathbf{1}$ |  | Count all (223) | Make Friendly No.10 <br> (152) <br> Count on (793) |  |  |
| $\mathbf{2}$ |  |  | Composition 5b <br> (993) | Count all (223) <br> Relational (563) Composition, 5d, 9d <br> (563) |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.11.a activity.

| NRR.C.11.a. | Solve for an unknown value in a true equation using a relational definition of equal sign. |  |
| :---: | :---: | :---: |
| Content Question | Here is a number sentence with a missing number. What is the missing number? |  |
| Reasoning Question | Please show me using words, pictures, or numbers how you figured out that____ is the missing number |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | - | No tools were embedded for this activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C11a_A_General tool | Child uses a general tool as listed above. |
|  | C11a_A_Add all | Add all numbers to find answer |
|  | C11a_A_Identify symbols | Identify numbers and symbols in the equation |
|  | C11a_A_ Left side | Write out total number from left side of the equation |
|  | C11a_A_ Operational | - Child uses the equal sign as an operation (i.e., equal sign means something "to do"). |
|  | C11a_A_ Relational | - Child uses the equal sign to show both sides are the same. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill Statements <br> [Also consider ESSs not specifically listed here; <br> refer to learning progressions.] | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | 05 - Composition 06 - Decomposition | - Find total then subtract part to find missing number <br> - Decompose number (add or subtract from either side) to find the missing value <br> - Select skill code in Composition that best fits child's approach. |

NRR.C.11.b. Summary
Essentialized Skill Statement


## Student Expectation

Using properties of operations (additive inverse), children were expected to solve a mathematical equation of the given contextual situation ( $a+b-b=--------)$.

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| In the selected data, no evidence of <br> unanticipated strategies found. | $\mathrm{N} / \mathrm{A}$ |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - | - |  |  |  |  |
| $\mathbf{1}$ |  | NME (223, <br> $946)$ | NME (793_no <br> interview data) |  |  |  |
| $\mathbf{2}$ |  |  | - | - |  |  |

Unanticipated strategies by grade level and number range for NRR.C.11.b

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME <br> $(768, \mathrm{RK})$ | 8c (385) <br> Pictorial <br> Reference <br> $(495)$ |  |  |  |  |
| $\mathbf{1}$ |  |  | Act Out (152) <br> 8c (152) |  |  |  |
| $\mathbf{2}$ |  |  |  | 8c (993) |  | 8c (284) |
| Compute (563) |  |  |  |  |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.11.b activity.

| NRR.C.11.b. | Given a contextual situation modeling a true equation, apply one or two properties of operations or property of equality to solve for an unknown value using concrete objects. |  |
| :---: | :---: | :---: |
| Content Question | There are $\qquad$ bees in this beehive. $\qquad$ more bees fly into the beehive. Then, $\qquad$ bees leave the beehive. How many bees are in the beehive now? |  |
| Reasoning Question | How do you know that there are____ bees in the hive now? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | - | While concrete objects were intended to be used in this activity, not all children used concrete objects. For this activity, when a child uses a concrete object, assign as a general mathematical tool. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C11b_A_General tool | Child uses a general tool as listed above. |
|  | C11b_A_Compute | Add and then subtract following the operations in the equation. |
|  | C11b_A_Count on | Count on from first number [counting] |
|  | C11b_A_Immediate recognition | Immediately identify that adding and then subtracting the same number leaves the original addend. Also assign additive inverse skill code. |
|  | C11b_A_ Operational | Child uses the equal sign as an operation (i.e., equal sign means something "to do"). |
|  | C11b_A_ Relational | Child uses the equal sign to show both sides are the same. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
| Essentialized Skill Statements <br> [Also consider ESSs not specifically listed here; refer to learning progressions.] | NRR.C.8.c. | - Additive inverse |

NRR.C.11.c. Summary

## Essentialized Skill Statement

| 11. Solving for Unknown Values |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten | Grade 1 | Grade 2 |  |  |  |  |  |
|  | F | B | $\mathbf{T}$ | F | B | T | F | B |
| NRR.C.11.c. |  | Given a contextual situation modeling a true equation, apply one or <br> two properties of operations <br> or property of equality to solve for an unknown value in a true <br> equation. |  |  |  |  |  |  |

## Student Expectation

Using properties of operations (additive inverse), children were expected to solve a mathematical equation of the given contextual situation ( $\mathrm{a}+\mathrm{b}+\mathrm{c}=-------+\mathrm{e}$ ).

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| In the selected data, no evidence of <br> unanticipated strategies found. | $\mathrm{N} / \mathrm{A}$ |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $0-19$ | $0-50$ | $0-99$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - | - |  |  |  |  |
| $\mathbf{1}$ |  | NME (223) | NME (793_no <br> interview data) |  |  |  |
| $\mathbf{2}$ |  |  | - | - |  |  |

Unanticipated strategies by grade level and number range for NRR.C.11.c

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME (RK) NME <br> (Guess768) | NME (385, 495) |  |  |  |  |
| $\mathbf{1}$ |  | Pictorial <br> Reference <br> (946) | Count on <br> (152) |  | 6f (993) <br> $5 c(993)$ | Count on (284) <br> Count by 5s (284) <br> Left Side (563) <br> $5 c-t h r e e ~ p a r t s ~(563) ~$ |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.11.c activity.

| NRR.C.11.c. | Given a contextual situation modeling a true equation, apply one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |
| :---: | :---: | :---: |
| Content Question | Heather knows that she has the same total amount of fish as Max. How many fish does she have in her other pond? |  |
| Reasoning Question | How do you know that there are___ fish in this pond? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | - | No tools were embedded for this activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C11c_A_General tool | Child uses a general tool as listed above. |
|  | C11c_A_Add all numbers across | Child adds all numbers across all ponds to determine an answer. |
|  | C11c_A_Compute | Child finds the sum for Max's ponds then uses the sum to decompose the number using Heather's known value. Differs from left side because in left child simply adds Max's ponds to determine an answer. |
|  | C11c_A_Count on | Count on from first number [counting] |
|  | C11c_A_Left side | Child adds all fish in Max's ponds to determine an answer. |
|  | C11c_A_Operational | Child uses the equal sign as an operation (i.e., equal sign means something "to do"). |
|  | C11c_A_ Relational | Child uses the equal sign to show both sides are the same. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. Ifa different strategy was used, create an unanticipated node for the child's strategy. |
| NRR Essentialized Skill | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | NRR.C.8.c. | - Additive inverse |
| [Also consider ESSs not specifically listed here; refer to learning progressions.] | 05 - Composition <br> 06 - Decomposition | - Add all to find the missing number <br> - Subtract parts from whole |

## Essentialized Skill Statement

| 11. Solving for Unknown Values |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  |  |  | Grade 2 |  |  |
|  | F | B | T | F |  | B | B | T | F | B |  |
| NRR.C.11.d. |  |  | Apply one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |  |  |  |  |  |  |  |

## Student Expectation

Using properties of operations or property of equality, children were expected to solve a mathematical equation of the given contextual situation ( $a+b=------+a$ ).

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| Pattern: For the given equation $50+55=-----+50$, child tried to <br> switch addends to form a pattern of increasing numbers and find <br> the missing number (like $50,55,60,65, \ldots . . . .).$. In this case, student <br> intended to swap the given numbers but it is not coded as | If you swap them, they would be |
| Commutative Property because child intended to form increasing | switched them, then it would be |
| pattern instead of changing the order of addends to make both |  |
| sides of the equation similar. | on. But you switch them, so I |
| thought that 55 is the answer |  |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - | - |  |  |  |  |
| $\mathbf{1}$ |  | NME (223) | NME (793_no interview data) |  |  |  |
| $\mathbf{2}$ |  |  |  | - | - | Pattern (676) |

Unanticipated strategies by grade level and number range for NRR.C.11.d

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | NME <br> (RK,768) | Confusing (385) <br> NME (495) |  |  |  |  |
| $\mathbf{1}$ |  | Count All (946) | Left Side <br> (152) |  |  |  |
| $\mathbf{2}$ |  |  | Operational (993) <br> Left Side (993) | Count On (284) <br> $8 b$ <br> 8b (563) |  |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.11.d activity.

| NRR.C.11.d. | Apply one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |
| :---: | :---: | :---: |
| Content Question | Here is a number sentence with a missing number. What is the missing number? |  |
| Reasoning Question | How do you know that ____ is the missing number? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | - | No tools were embedded for this activity. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C11d_A_General tool | Child uses a general tool as listed above. |
|  | C11d_A_Add all numbers across | Child adds all numbers across the equation to determine an answer. |
|  | C11d_A_Compute | Child finds the sum for the left side the decomposes the sum using the number on the right side to determine the answer. |
|  | C11d_A_Count on | Count on from first number [counting] |
|  | C11d_A_Immediate recognition | - Immediately identify the ordering of the numbers are "switched" or "swapped". Also assign commutative property skill code. |
|  | C11d_A_Left side | - Add all numbers on the left-side of the equation |
|  | C11d_A_ Operational | - Child uses the equal sign as an operation (i.e., equal sign means something "to do"). |
|  | C11d_A_ Relational | - Child uses the equal sign to show both sides are the same. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | NRR.C.8.b. | - Commutative property |
| [Also consider ESSs not specifically listed here; refer to learning progressions.] | 05 - Composition 06 - Decomposition | - Add all to find the missing number <br> - Subtract parts from whole |

## Essentialized Skill Statement

| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F | B | T | F | B | T | F | B | T |
| NRR.C.11.e. |  |  |  |  | Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value using concrete objects. |  |  |  |  |

## Student Expectation

For this essentialized skill statement, the student workbook displayed four rectangular fields, two each for Max and Heather. Concrete objects were intended to be used for this skill statement; however, they were not consistently available during the interviews. Numbers were written into the fields to demonstrate an equation involving decomposition and the associative property.

| Unanticipated Strategy. Justification/Description | Examples |
| :---: | :---: |
| Unfounded. In the number range 0-5, the situation given was associated with the equation $2+3=1+$. In the example provided, the child did get the correct answer of four; however, the child followed a counting pattern to determine the missing number. <br> A second-grade student used a different unfounded strategy by attempting to decrease both addends by one. | [The answer is four] because you could do one, and then two, and then three, and then four (385, 39:42). <br> Since this one's [36 to 35] is going backwards, there will be 16 in this....because I feel like you're going backwards, like 36, so it will be 35. And 17, behind it it's 16 (284, 52:28, 52:42). |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - | Unfounded (385) |  |  |  |  |
| $\mathbf{1}$ |  | - | - |  |  |  |
| $\mathbf{2}$ |  |  |  | - |  |  |

Unanticipated strategies by grade level and number range for NRR.C.11.e.

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (768, RK) | 5e (495) |  |  |  |  |
| $\mathbf{1}$ |  | NME (946) | Skipped (793) <br> Add All Numbers, <br> Count <br> On (152) |  |  <br> NME (223) |  |
| $\mathbf{2}$ |  |  |  | Compute, <br> Relational (993) <br> Decomposition, <br> Relational (563) | Decomposition, <br> Relational (676) |  |

Anticipated strategies by grade level and number range for NRR.C.11.e.

| NRR.C.11.e. | Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value using concrete objects. |  |
| :---: | :---: | :---: |
| Content Question | If Max and Heather have the same number of cows all together, how many cows should Heather have in her other field if she has $\qquad$ cow(s) in one of her fields? |  |
| Reasoning Question | How do you know that ___ is the missing number? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | - | While concrete objects were intended to be used in this activity, not all children used concrete objects. For this activity, when a child uses a concrete object, assign as a general mathematical tool. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C11e_A_General tool | Child uses a general tool as listed above. |
|  | C11e_A_Add all numbers across | Child adds all numbers in each of the fields to determine an answer. |
|  | C11e_A_Compute | Add total then subtract to find the missing value. |
|  | C11e_A_Count on | Count on from first number to find total [counting] |
|  | C11e_A_Left side | Add all numbers on the left-side of the equation |
|  | C11e_A_ Operational | - Child uses the equal sign as an operation (i.e., equal sign means something "to do"). |
|  | C11e_A_ Relational | - Child uses the equal sign to show both sides are the same. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
| Statements <br> [Also consider ESSs not specifically listed here; <br> refer to learning progressions.] | 05 - Composition 06 - Decomposition | - Add all to find the missing number <br> - Decompose first addend on the left to equal the first addend on the right plus 1. <br> - Assign the appropriate skill code associated with decomposition. |

NRR.C.11.f. Summary

## Essentialized Skill Statement

| 11. Solving for Unknown Values |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | B | T | F | B | T |
| NRR.C.11.f. |  |  |  |  | Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |  |  |  |  |

## Student Expectation

Using properties of operations or property of equality, children were expected to apply decomposition to solve the given mathematical equation of the form ( $a+b=c+---------)$. It was anticipated that students will decompose c in the given equation to balance the equation and finding missing the number ' $d$ ' in the given equation.

| Unanticipated Strategy. Justification/Description | Examples |
| :---: | :---: |
| Count All: For the given equation student started counting all numbers on RHS to make it equal to LHE (that student used counted on strategy to find sum of LHE). It was not anticipated for a $2^{\text {nd }}$ grade student to use Count On to find sum and balance both sides of the equation. | For the given equation $60+30=32+-------$, student used Count On from 60 to 100 to find $60+30$. Then student started Counting All from 1 onward to count to 32 (on RHS) (284, 55:25) |


| Grade | $0-5$ | $0-10$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| K |  |  |  |  |  |  |
| $\mathbf{1}$ |  |  |  |  |  |  |
| $\mathbf{2}$ |  |  |  | NME (793) | Count All (284) <br> No Opportunity Provided (563) | NME (676) |

Unanticipated strategies by grade level and number range for NRR.C.11.f

| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $\mathbf{0 - 1 9}$ | $\mathbf{0 - 5 0}$ | $\mathbf{0 - 9 9}$ | $\mathbf{0 - 1 9 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{K}$ | NME (768,RK) | NME (946, 223) |  |  |  |  |
| $\mathbf{1}$ |  | NME (385, 495) | Count On (152) <br> Relational (152) |  |  |  |
| $\mathbf{2}$ |  |  |  | Count On (993) <br> Relational (993) | Relational (284) |  |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.11.f activity.

| NRR.C.11.f. | Given a contextual situation modeling a true equation, apply decomposition with one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |
| :---: | :---: | :---: |
| Content Question | Roberta has \$___and \$__. Meganhas \$___and another amount. Roberta and Megan have the same amount. How much is this amount [point to missing value]? |  |
| Reasoning Question | Can you use the numbers on this side of the equation to help you figure out the missing number? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded Mathematical Tools | - | No tools were embedded for this activity. |
| General <br> Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C11f_A_General tool | Child uses a general tool as listed above. |
|  | C11f_A_Add all numbers across | Add all numbers across |
|  | C11f_A_Compute | Add total then subtract to find the missing value. |
|  | C11f_A_Count on | Count on from first number [counting] |
|  | C11f_A_Left side | Add all numbers on the left-side of the equation |
|  | C11f_A_ Operational | - Child uses the equal sign as an operation (i.e., equal sign means something "to do"). |
|  | C11f_A_ Relational | - Child uses the equal sign to show both sides are the same. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR <br> Essentialized Skill | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
| Statements <br> [Also consider ESSs not | NRR.C.8.b. | - Commutative property |
| [Also consider ESSs not specifically listed here; refer to learning progressions.] | 05 - Composition 06 - Decomposition | - Add all to find the missing number <br> - Decompose first addend by 2. <br> - Assign the appropriate skill code associated with decomposition. |

NRR.C.11.g. Summary

## Essentialized Skill Statement

| 11. Solving for Unknown Values |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Kindergarten |  |  | Grade 1 |  |  | Grade 2 |  |  |
|  | F | B | T | F | B | T | F | B | T |
| NRR.C.11.g. |  |  |  |  | Apply decomposition with one or two properties of operations or Property of equality to solve for an unknown value in a true equation. |  |  |  |  |

## Student Expectation

Using properties of operations or property of equality, children were expected to solve a mathematical equation by applying Decomposition to the given equation $[a+b-(b-1)=------$ ].

| Unanticipated Strategy. <br> Justification/Description | Examples |
| :--- | :--- |
| In the selected data, no evidence of <br> unanticipated strategies found. | $\mathrm{N} / \mathrm{A}$ |


| Grade | $\mathbf{0 - 5}$ | $\mathbf{0 - 1 0}$ | $0-19$ | $0-50$ | $0-99$ | $0-199$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | - | - |  |  |  |  |
| $\mathbf{1}$ |  |  | NME (793_no interview <br> data) |  |  |  |
| $\mathbf{2}$ |  |  | - | - | Pattern (676) |  |

Unanticipated strategies by grade level and number range for NRR.C.11.g

| Grade | 0-5 | 0-10 | 0-19 | 0-50 | 0-99 | 0-199 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $\begin{gathered} \text { NME(768) } \\ \text { NME (RK_no } \\ \text { interview data) } \end{gathered}$ | NME (385,495) |  |  |  |  |
| 1 |  | NME (946) <br> NME (223_no interview data) | $\begin{gathered} \text { Count on } \\ (152) \end{gathered}$ |  |  |  |
| 2 |  |  |  | $\begin{aligned} & 6 f(993) \\ & 5 e(993) \end{aligned}$ | $\begin{aligned} & \text { 6f (284) } \\ & \text { 6f (563) } \end{aligned}$ | 6 f (676) |

Anticipated strategies and skill codes used by grade level and number range for the NRR.C.11.g activity.

| NRR.C.11.g. | Apply decomposition with one or two properties of operations or property of equality to solve for an unknown value in a true equation. |  |
| :---: | :---: | :---: |
| Content Question | Here is a number sentence with a missing number. What is the missing number? |  |
| Reasoning Question | How do you know that the missing number is ___ ? |  |
| Anticipated |  |  |
|  | Code | Description |
| Embedded <br> Mathematical Tools | - | No tools were embedded for this activity. |
| General Mathematical Tools | These tools are located under Mathematical Tools > a_Type of Tool If any of these tools are used, the General tool code must also be coded under strategies. |  |
|  | Square tiles | Colored square tiles as counters |
|  | Linking cubes | Linking cubes as counters |
|  | Fingers | Fingers as counters |
| Strategies | Code | Description |
|  | C11f_A_General tool | Child uses a general tool as listed above. |
|  | C11g_A_operational | - Child uses the equal sign as an operation (i.e., equal sign means something "to do"). |
|  | C11g_A_relational | - Child uses the equal sign to show both sides are the same. |
|  | C11g_A_Count on | - Count on from first number [counting] |
|  | C11g_A_Left side | - Add all numbers on the left-side of the equation |
|  | C11g_A_Add all | - Add all numbers across |
|  | C11g_A_Compute | - Add total then subtract to find the missing value. |
| A different way | - | - A different strategy was not anticipated for this essentialized skill statement. If a different strategy was used, create an unanticipated node for the child's strategy. |
| NRR Essentialized Skill | Current Skill Code | - Only specific student talk-turn within activity that is attributed to the current skill code. |
|  | NRR.C.8.c. | - Additive inverse |
| [Also consider ESSs not specifically listed here; refer to learning progressions.] | 05 - Composition <br> 06 - Decomposition | - Add all to find the missing number <br> - Decompose second addend by subtracting 1. <br> - Assign the appropriate skill code associated with decomposition. |


[^0]:    ${ }^{1}$ A.1.a, and A.1.b paper coded copies are missing.

[^1]:    Anticipated strategies and skill codes used by grade level and number range for the NRR.B.6.d. activity.

[^2]:    Unanticipated strategies by grade level and number range for NRR.B.6.e.

[^3]:    ${ }^{2}$ C.8.b paper coded copies are missing.

