Key Words or Not: Effective Instruction on Word Problems

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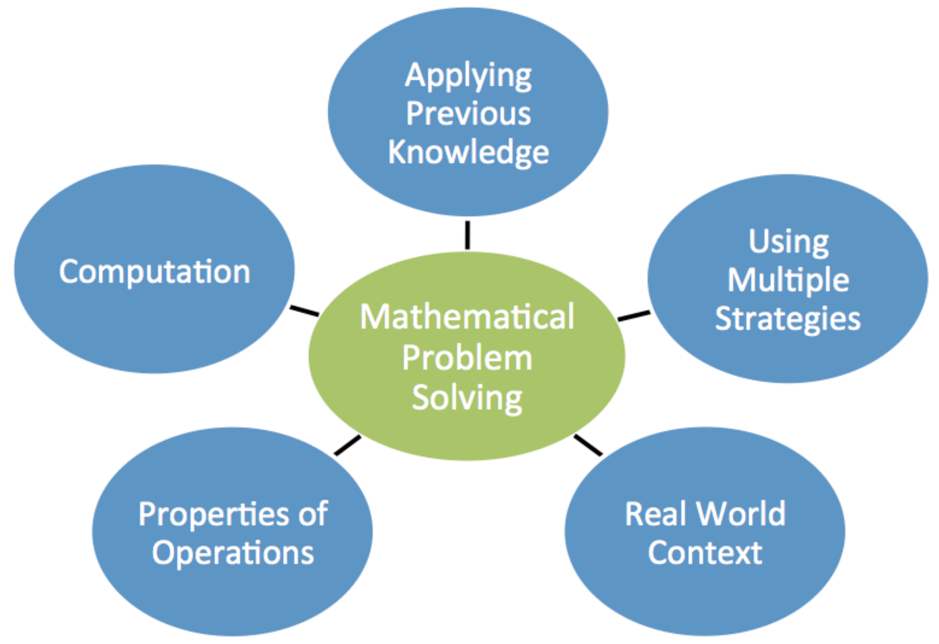
What is mathematical problem solving?

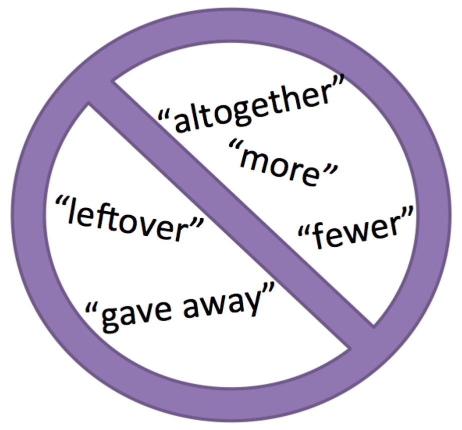
- Mathematical problem solving involves applying previous knowledge, skills, and strategies to story problems (Hudson & Miller, 2006). According to Van de Walle (2013) it is "the vehicle through which student develop mathematical ideas" (p. 4).



- Problem solving is a central theme in the *Principles and Standards for school Mathematics* (National Council of Teachers of Mathematics, 2000) for students in prekindergarten through twelfth grade.

Traditional Models with Key Words

Teachers frequently teach students to associate key words with mathematical operations.



- Elizabeth earned $100 on Friday and Ben earned $75 on Friday. How much **more** does Ben need to have Elizabeth's total?
- Sharri's dog **lost** 15 pounds. Now, he weights 32 pounds. How much did he weigh before?

When students rely on key words rather than reading and thinking, their understanding is hindered and may prompt them to use the wrong operation (Hudson and Miller, 2006).

Concerns:

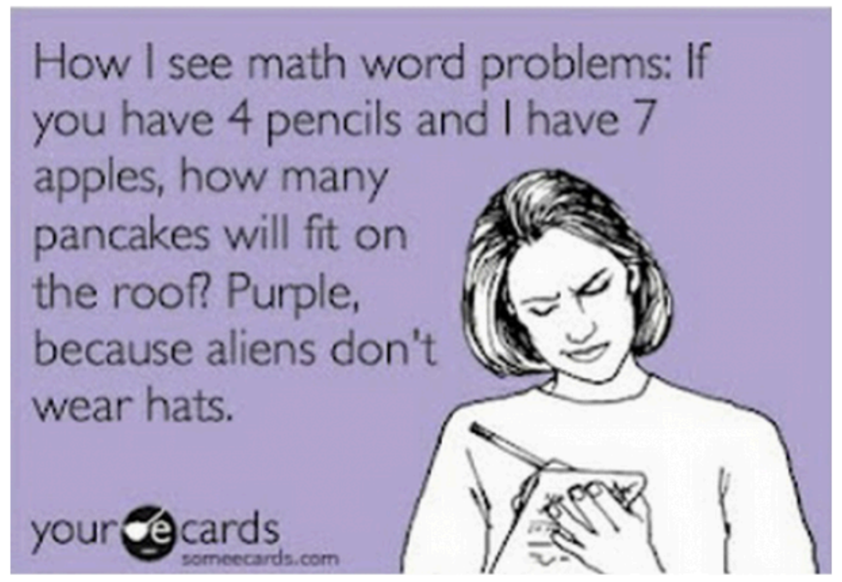
- Key word strategy sends a wrong message about doing mathematics.
- Key words are often misleading.
- Many problems have no key words.
- Key words don't work in two-step or more advanced problems.

Van de Walle (2013, p. 167)

A Research Based Alternative … Explicit Instruction of Problem Structures

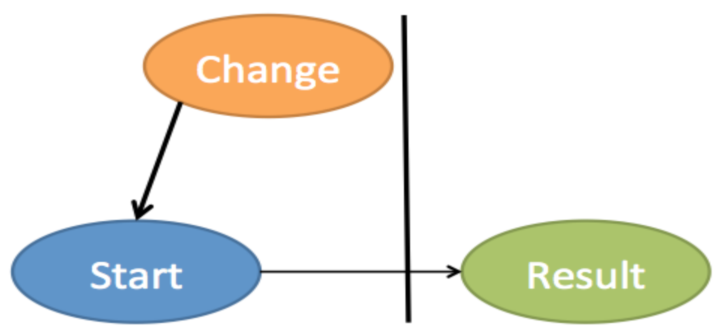
Research (Fuchs et al., 2006; Xin, Jitendra, & Deatline-Buchman, 2005) indicates that providing students with explicit instruction of problem structures can result in increased success in solving problems similar in structure and that those effects are generalizable and can be maintained over time.

Supporting teachers understanding of the underlying structures of problems is also useful in that teachers can incorporate that information into their classroom instruction by (a) focusing instruction on one problem type until students have demonstrated understanding of its underlying structure, and (b) then incorporating other, previously learned problem types so students learn to discriminate between the types and know which strategies they can use to solve different types of word problems.

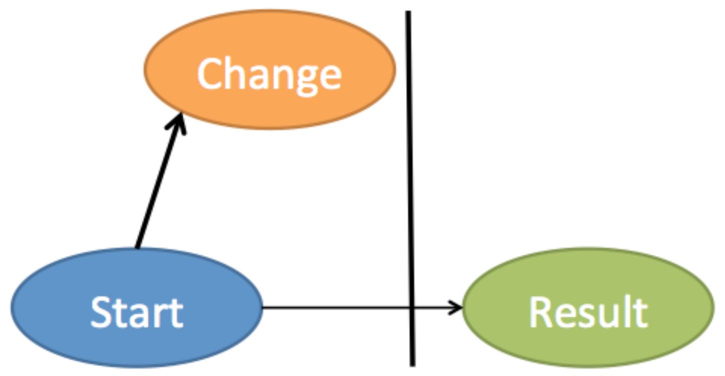


Addition and Subtraction Problem Structures

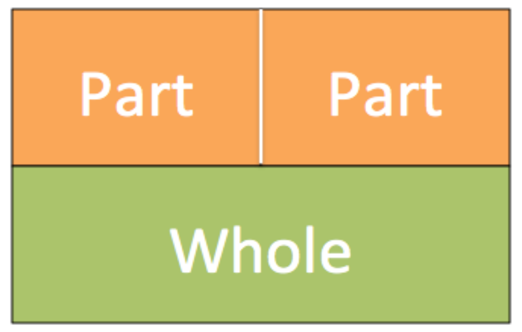
Change - Join



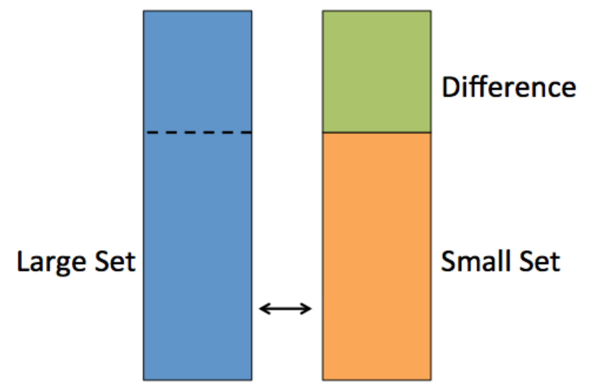
Change – Separate



Part-Part-Whole



Compare



(Van de Walle, 2013, p. 149)

Why Problem Structures?

Using physical models or visual representations to solve problems is beneficial.

When students learn to solve problems based on problem structure, as they are exposed to new problems, they have greater success in problem solving, and will develop a deeper understanding to the mathematical ideas behind word problems (Peterson, Fennema, & Carpenter, 1989).

This knowledge can be transferred to unfamiliar problems with the same underlying structure, providing students an opportunity to practice solving problems of a specific type and promoting their ability to discriminate when and when not to apply a given problem structure to solve a problem (Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004).

Computational vs. Semantic Forms

Students need to understand that there are several ways to represent a problem as well as understand the equivalence between different forms of an equation.

Frank has 54 marbles. He gives some to a friend and now he has 32. How many marbles did he give to his friend?

Semantic – Numbers are listed in the order that follows the meaning of the word problem.

- Example: 54 – ☐ = 32

Computational – Isolates the unknown to one side of the equation.

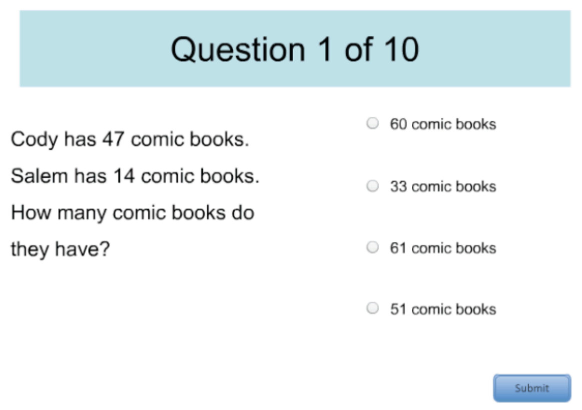
- Example: 54 – 32 = ☐

Van de Walle (2013, p. 167)

Illustrative Examples

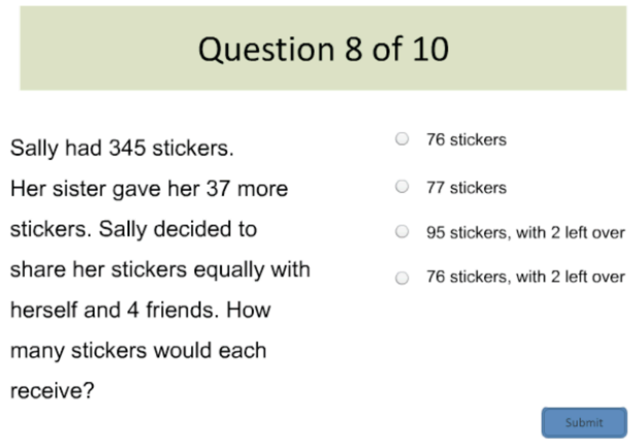
During the development of a universal screening assessment for Grades 2-4 we collected think-aloud data with 30 students in Grades 2-4. Example 1 illustrates the deleterious effect relying solely on key words can have on student performance while Example 2 illustrates a case where explicit instruction for the problem structure may have mitigated the confusion created by a poorly written item.

Example 1



- Only 67% of students selected the correct response when presented with this item in both a paper-pencil and computer based format
- Multiple students said they wished there was "a plus or minus" in the stem to indicate how they were supposed to solve the problem

Example 2



- Only 63% of students selected the correct response
- Multiple students told us it was unclear what was meant by "share her stickers" and who "her" was
- It's possible that the confusion created by the lack of clarity in the language could be mitigated by knowledge of problem structures

Sequence for Teaching

1. Problem Identification: (a) read the problem and (b) verbalize your thought processes, including asking yourself what type of this problem this is (e.g., Is this a comparison problem?)
2. Problem Representation: (a) use a graphic organizer or schematic diagram to provide a visual representation of the problem, and (b) "Think aloud" for students as you identify which numbers go where in the graphic organizer
3. Problem Solution: (a) model proficient problem-solving for students and (b) as problem types become more complex, model for students how to represent problems with unknowns *and* how to translate the information in the graphic organizer into mathematical symbols to solve

Jitendra (2008); Jungjohann (2010)

Questions? For further information about our research, please contact Research in Mathematics Education at Southern Methodist University (rme@smu.edu).