Precal 911: Engaging Activities to Save the Day!

Presented by

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and

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CAMT

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Connecting Conceptual and Procedural Knowledge

*Spaghetti Trig: Use a hands-on approach to connect trigonometric exact values to the graphs of y = sin x and y = cos x. See handout for detailed instructions.

*Football Stadium Problem: In Texas, almost every high school math student experiences a football game in a stadium. This problem connects the mathematical formulas for ellipses and a real-world situation involving stadium seating. See handout for directions.

*How Can I Get There? : Students explore paths between two friends' houses as a way to discover and reinforce ideas related to permutations, combinations, and Pascal's triangle. See handout for activity details.

Procedural Fluency

*Exact Value Tricks: Hints to help students memorize these all-important values.

*BINGO games: Tailored to Precalculus topics, you can play "LOGO" for logarithm practice, "POLO" for practice with polar coordinates, and "TRIGO" for practice with trig exact values. Game boards and problems included.

Collaboration/Movement

*Scavenger Hunt: Write a problem towards the bottom of a half-sheet piece of colored paper, and the answer at the top of a different half-sheet of paper. Write another problem below that answer, and continue until there are 15-20 problems (write the answer to the last question on the top of the first page). To play, hang the problems RANDOMLY around the room, in your hallway, or around the school if you can, and put the students in groups. Students need paper to work on, and should pick a problem to start at. As they complete the problems, they search for their answer and then work the new problem on that page. Students follow the trail of questions until they have completed all questions.

*Trig Sticky Note Matchup: This is a fun way to start class and get the students thinking about math from the second they arrive. On small sticky notes, write an angle measure in degrees. On another, write an angle measure that is coterminal. Write enough pairs so that everyone in class will receive an angle. Stand at the door as students enter and hand them a sticky note (mix up the order first!). Once all are distributed, tell students to find their coterminal partner! You can ask students to take their homework paper with them to begin to compare answers once they find their match. Don't forget to use both positive and negative angles! This also works well for degrees and radians, or any other "matching" topic.

*Around the World: Write sets of problems on colored paper and post around the room, the hallway, etc. Students (in groups of 3-4) rotate around the room and work together on each posted problem. This is a great activity for review, or simply another way to provide additional practice. Music may be used to indicate when it is time to move to the next problem.

*Stacked Transparencies: Give groups of students a transparency and a marker, and ask them to create a graph together as a group. When all groups are done, stack the transparencies to show the entire class whether they agreed (or not) on the graph! Ensuing discussions could relate to good graphing practices, teamwork, accuracy, and neatness.

*Number Clothesline: Give students a card with a number, and ask them to clip their card on a prepared clothesline, working with their classmates to determine accurate placement in relation to other cards being clipped.

Fun and Different

*Magic Squares: Students work alone, in pairs, or in groups to work these puzzles. Cut out each of the individual squares and rearrange them so that equivalent sides are touching. When finished, students should have one big square. These puzzles are easy to make, and can be used on any topic you want. Just be sure to make a copy of the original before you start cutting! (**They are harder than they look!)

*Review Puzzles: Included are a crossmath puzzle as well as a coded message activity. Both review several concepts from Algebra and Precalculus and will boggle students' minds!

*Graphing Project: Students can be creative and use the graphing techniques learned in class to create amazing pictures. A couple of variations are included.

*Secret Message: This activity uses inverse functions to both encrypt and decode a secret message. See handout for details.

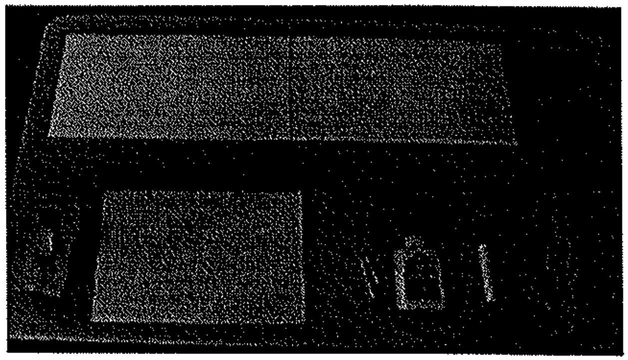
*Leaky Bottle Experiment: Activity to be completed in pairs or small groups, using a leaking water bottle to model an exponential function.

From M. Sturdivant, http://octm2009spaghetti.pbworks.com/f/Trig+Graphs+and+Laws.pdf

Spaghetti Trig: Developing the graphs of sine and cosine functions from the unit circle

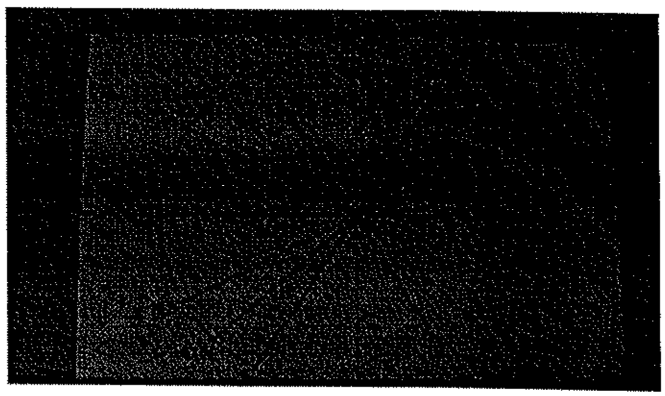
Objectives:
Students will develop a better understanding of the unit circle.
Students will make the connection between the unit circle and the graphs of the sine and cosine trig functions.

Materials Needed:
Poster board or two legal sheets for each student
Yarn (a light color preferably)
Tape or Glue
Protractors
Compasses
Spaghetti
Markers (a different color than the yarn)
One sheet of letter-size paper for each student



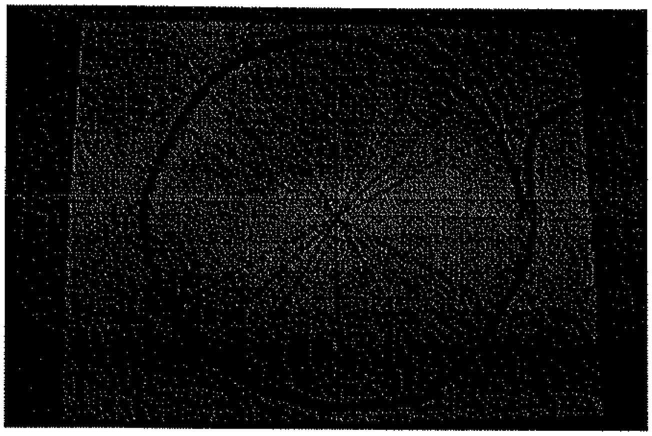
Prep before class begins:
You will need 50-65 minutes to complete the project, so the more you have prepared for the students, the better off you will be. The students will need the following items at their desks before class begins. They will need white poster board or two legal sheets taped together to make one long 8 ½" x 23" sheet, yarn (at least 3 feet for each student), tape or glue, protractor, compass, 7-10 noodles of spaghetti, and a marker.

Procedure:
The first part of this lesson could actually be done on a separate day. Students should create a unit circle on the letter-sized paper. Begin by having the students draw an x and y axis on the letter-sized paper. The origin should be as close to the center of the paper as possible. Use the compass to create a circle with a center at the origin and a radius of at least 3 ½". Use the protractor to measure angles of $\pi/6$, $\pi/4$, and $\pi/3$. The point where the terminating side of these angles intersect with the unit circle will have ordered pairs of $(\frac{\sqrt{3}}{2}, \frac{1}{2})$, $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ respectively.



Then have students measure the same reference angles in each of the remaining quadrants, marking the coordinates of the ordered pairs as they go (paying careful attention to sign). The students should also mark the ordered pairs for the points where the unit circle intersects the x and y axes as well.

Now have the students wrap the string around the unit circle. One end of the string should be placed at (1, 0). Use the marker to mark points on the string at each of the special angles.

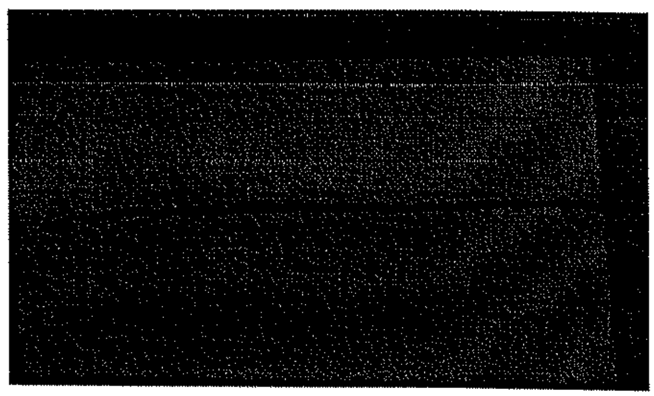


It is important to note to the students at this point that we are interested in finding a function in terms of the angle θ that will give us the value of the x coordinate and a function in terms of the angle θ that will give us the value of the y coordinate. From previous study, the students should know that the functions we are looking for are sine and cosine.

Remind the students that $x = \cos(\theta) = f(\theta)$ and $y = \sin(\theta) = g(\theta)$. This means that if we are to graph the sine function our new independent variable is the angle θ. The remainder of this lesson plan will discuss completing the project for graphing the sine function. It is easily adapted to graphing the cosine function.

Have the students draw a coordinate axis on their poster board or legal paper. Since θ is the independent variable, the horizontal axis should be labeled θ and the vertical axis should be labeled $g(\theta) = \sin\theta$.

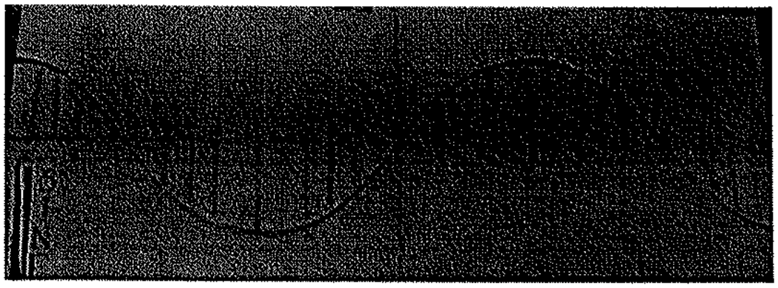
The yarn can now be placed along the horizontal axis with the end of the string that started at the point (1,0) now placed at the origin since this represents $\theta = 0$. The marks that were made on the yarn will give us the angle measures that we need. Thus the first mark can be transferred to the poster board and labeled π/6. The second mark can be transferred to the poster board and labeled π/4, etc. Once the students have labeled the positive horizontal axis, they can flip the string over and label the negative horizontal axis using the same marks. Note to the students that this is using negative angle measures.



We will now go back to our unit circle. Since we are interested in the sine function which corresponds to the y value, we are interested in the distance a given point is from the horizontal axis. This is where the spaghetti will help. At an angle measure of 0, the y value is 0, thus we will not need any spaghetti. However, at π/6, the y value is ½. Have the students place a noodle of spaghetti vertically with one end on the horizontal axis and the remainder of the noodle going through the point $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. They should break the noodle off at that ordered pair. Thus the noodle gives us a visual image of the y value (sin θ) at π/6. Take that piece of spaghetti and glue it on the poster board at π/6. Be sure to tell the students that since the y value was positive, then the spaghetti should be placed above the horizontal axis. Repeat this process at each special angle. When the students get to the third and fourth quadrants, remind them that they should now be placing their spaghetti below the horizontal axis.

Be sure to have the students also work with the negative angles as well, reminding them that -π/6 will be in the 4th quadrant and so the spaghetti will be placed below the horizontal axis. When the students finish they should see a perfect sine curve. You can fill in the gaps by having the students measure angles of π/12 and 5π/12 and their corresponding reference angles around the unit circle.

It may be a good idea to have half of your students graph the sine function and half graph the cosine function. The activity is identical with the exception that the spaghetti will be used to measure the ordered pairs distance from the vertical axis in the unit circle.



FOOTBALL STADIUM PROBLEM

(adapted from Paul Forester, *Algebra and Trigonometry: Functions and Applications*, 2nd edition, c.1998)

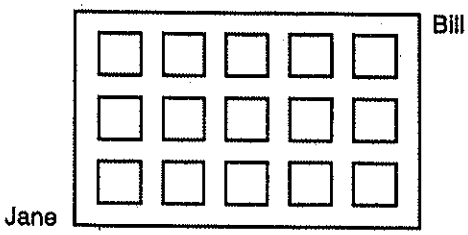
Ornery and Sly Construction Company needs your help. The company has a contract to build a new football stadium in the form of two concentric ellipses, with the playing field inside the inner ellipse and seats between the two ellipses. The seats are in the intersection of the graphs of $x^2 + 4y^2 \geq 100$ and $25x^2 + 36y^2 \leq 3600$ where each unit on the graph represents 10 meters.

1. Draw a graph of the stadium, shading the seating area.

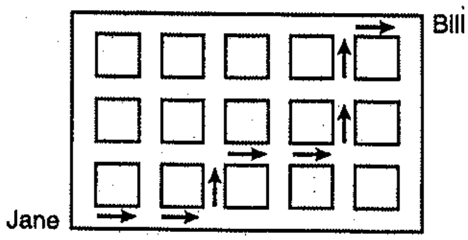
2. From a handbook, you find that the area of an elliptical region is πab, where $\pi \approx 3.14159$. The Engineering Department estimates that each seat occupies 0.8 sq. meters. What is the stadium's seating capacity?

How Can I Get There?

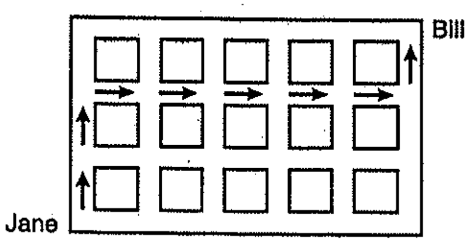
Jane visits her buddy Bill each day. Bill lives eight blocks away, and Jane likes to vary her routine to keep her interest, since the scenery is not especially exciting. Using a map of the neighborhood, Jane decides to figure out how many different paths she can take if she always travels either to the right or up. She chooses to take a different eight-block route each day. She wonders how many days will pass before she has taken every possible route.



Solid arrows are used to show two possible routes. Possible route 1's can be summarized by RRURRUUR, where R is "go right" and U is "go up." Possible route 2's can be summarized by UURRRRRU.

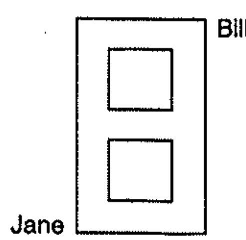


A possible route 1

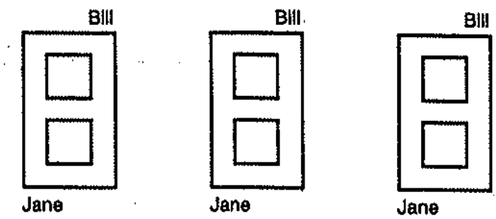


A possible route 2

Many paths are available. Jane wants to get organized so that she does not repeat any routes to Bill's house. Let us help her by first looking at the simpler example at the right.



1. Use solid arrows, one arrow for each block traveled, as shown in possible routes 1 and 2, to draw a different possible route on each diagram below. Remember, Jane has decided that she can go only either to the right or up.



2. Symbolize your three different routes using U and R as shown in the previous examples.

__________ __________ __________

In our original problem, when Jane leaves her house, she must travel either to the right or up. We can indicate this path by drawing solid arrows. Thus, Jane has only one way, indicated by [1] in **figure 1,** to get to either of the first two possible intersections by traveling either R or U.

If Jane first travels R, the two possible second blocks to travel are indicated by the clear arrows. If Jane first travels U, the two possible second blocks to travel are indicated by the dashed arrows. See **figure 1.**

In **figure 1,** Jane gets to intersection A by going RR; to get to intersection C, she must go UU. To get to intersection B, she could go either RU or UR. The total number of ways to get to B are indicated by [2] in **figure 2.** Using U's and R's, symbolize the paths that will get Jane from her house to these locations:

3. Intersection D: __________
   How many paths are possible? __________
4. Intersection E: __________
   How many paths are possible? __________
5. Intersection F: __________
   How many paths are possible? __________
6. Intersection G: __________
   How many paths are possible? __________

The editors wish to thank Bill Kring, The Savannah Country Day School, Savannah, GA 31419, for writing this issue of *NCTM Student Math Notes.*

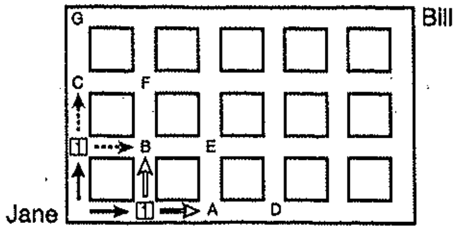


Fig. 1

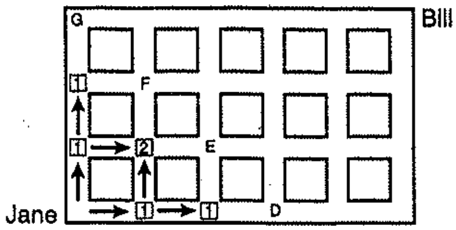


Fig. 2

Because Jane can go only to the right or up, the only way she can get to intersection D is from intersection A. Since only one way exists to get to intersection A, only one way exists to get to intersection D. Jane can get to intersection E from either intersection A or intersection B. Since only one way exists to get to intersection A and two ways exist to get to intersection B, 2 + 1, or 3, ways are available to get to intersection E.

7. From which two intersections can Jane get to intersection F? ____________

8. How many paths can she take to get to intersection F? ____________

9. From which intersections can Jane get to intersection G? ____________

10. How many paths can she take to get to intersection G? ____________

11. Look for a pattern to find the number of paths to get to each intersection along the route from Jane's house to Bill's house. Record these numbers at the appropriate intersections in **figure 3.**

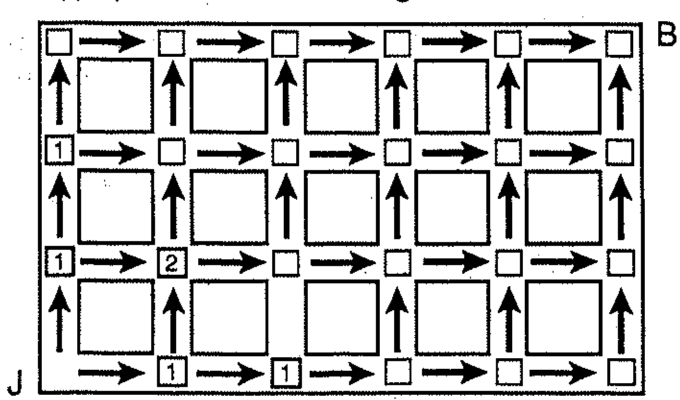


Fig. 3

12. Because Jane will travel a different path each day, how many days will pass before she has taken every possible route? ____________

13. If each possible path is symbolized with U's and R's, how many U's are in a path? ________ R's? ________

You should have determined from problem 13 that each of Jane's routes can be represented using a string of 5 R's and 3 U's in some order. The answer found in question 12 is the number of different ways to write 5 R's and 3 U's in a string. Each string is called a *permutation* of those eight letters. We can use this idea to help Jane when Bill moves.

In the situation shown in **figure 4,** Jane must travel four blocks to get to Bill. Let us look at this trip from a different perspective by considering the number of ways to arrange four different letters.

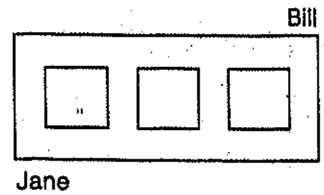


Fig. 4

14. List all the arrangements of A, B, C, and D by completing the following chart.

ABCD	BACD	CABD	DABC
ABDC	B____	_____	_____
ACBD	B____	_____	_____
ACDB	B____	_____	_____
ADBC	B____	_____	_____
ADCB	B____	_____	_____

How does this process relate to finding the number of ways to arrange the letters of the string URRR? Suppose that the string of letters is URrR. Those letters can be arranged in the following ways with the letter U first: URrR, URRr, UrRR, UrRR, URRr, and URrR. Since each R looks different from the others, these six arrangements do not look the same. The R's in our example are the same, so these arrangements can all be represented by URRR.

15. Of the twenty-four arrangements of the letters in the string ABCD, how many choices are available for the first letter? ____________

16. Once that first letter is chosen, how many choices are possible for the second letter? ____________

17. Once the first two letters are chosen, how many choices remain for the third letter? ____________

After the first three letters are in place, only one letter is left to put into the fourth spot. Thus, the total number of arrangements can be represented by the product 4 • 3 • 2 • 1. This result is the same as the one found in the chart in problem 14. A mathematical symbol called "4 factorial" represents this product:

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

18. Represent the number of arrangements of the letters in the string RRR using factorial notation. ________

Since each R in the arrangement RRR is the same, only one arrangement is possible: RRR. Thus, the number of arrangements of URRR is different from, but related to, the number of arrangements of ABCD. If four letters are different, 4! arrangements of them exist. If three of the four letters are identical, the number of arrangements is reduced by a factor of 3!. Thus,

$$\frac{4!}{3!} = 4$$

ways to arrange the letters in the string URRR are possible. Those four paths are shown in **figure 5.** They correspond to URRR (①), RURR (②), RRUR (③), and RRRU (④).

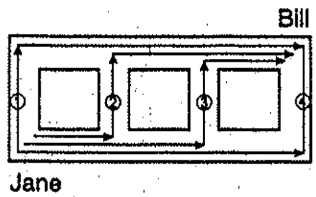


Fig. 5

Consider a different situation, shown in **figure 6.**

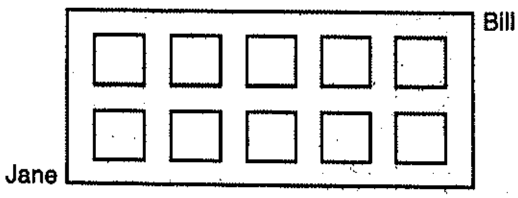


Fig. 6

19. How many blocks must Jane travel? ________
How many to the right? ________ Up? ________

This process is like finding the number of distinct arrangements of the string RRRRRUU.

20. How many arrangements of seven different letters are possible? ________
Of five different letters? ________
Of two different letters? ________

If we follow a similar procedure for RRRRRUU, we would find 7! arrangements of the seven letters, of which 5! are repeats of the R's and 2! are repeats of the U's. As a result,

$$\frac{7!}{5! \cdot 2!} = 21$$

distinct arrangements of RRRRRUU are possible. We can conclude that Jane has twenty-one different routes to follow to get to Bill's house.

We can use permutations to find the number of distinct paths in a different way. Remember that in the original problem, Jane had eight blocks to travel, five of which were to the right and three were up. Using these procedures for arranging letters, we see that Jane can take

$$\frac{8!}{5! \cdot 3!} = 56$$

distinct routes.

Looking back, we find that Jane can get to her final destination from one of two places: intersection K or intersection L (**fig. 7**). Both places are seven blocks from Jane's house. Intersection K requires her traveling five blocks to the right and two blocks up, as shown in question 19. Intersection L requires traveling four blocks to the right and three blocks up. By the reasoning used previously, we know that Jane can get to intersection K in

$$\frac{7!}{5! \cdot 2!} = 21$$

ways and to intersection L in

$$\frac{7!}{4! \cdot 3!} = 35$$

ways. Adding, we find that Jane can get to Bill's house in 21 + 35 = 56 distinct routes.

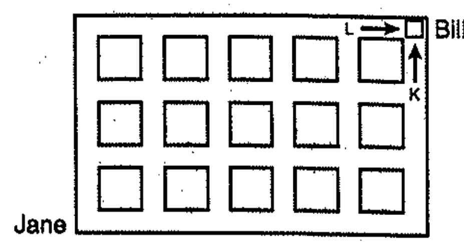


Fig. 7

Symbolizing this process, we find that

$$\frac{8!}{5! \cdot 3!}$$

can be written

$$\frac{7!}{5! \cdot 2!} + \frac{7!}{4! \cdot 3!}.$$

21. What do you notice in comparing the numerators of
$$\frac{7!}{5! \cdot 2!} \text{ and } \frac{7!}{4! \cdot 3!}$$
with the numerator of
$$\frac{8!}{5! \cdot 3!}?$$
____

22. What do you notice in comparing the denominators of
$$\frac{7!}{5! \cdot 2!} \text{ and } \frac{7!}{4! \cdot 3!}$$
with the denominator of
$$\frac{8!}{5! \cdot 3!}?$$
____

23. Use the observations in questions 21 and 22 to answer the following question. If

$$\frac{10!}{6! \cdot 4!}$$

represents the number of different routes for a complete trip, what two fractions represent the number of ways to get to the two places from which the final block can be traveled?__________ and __________

24. Verify that the sum of these two fractions is

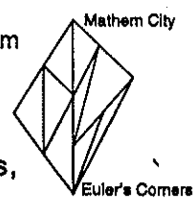
$$\frac{10!}{6! \cdot 4!}.$$

Did you know that ...

- Jane's original problem is analogous to moving from the point (0, 0) to the point (5, 3) on the coordinate grid if moves are restricted to one unit to the right or one unit up?
- this concept can be extended to three, four,..., n dimensions?
- this concept can be extended to finding the paths between two cities?
- this activity relates to the binomial expansion of $(R + U)^n$?

Can you ...

- find the number of distinct ways to move from the point (–4, 2) to the point (5, –3) by moving either to the right or down?
- extend this procedure to count the ways a king can move on a chessboard? Remember that a king can also move on a diagonal.
- determine the number of paths for Jane to travel to Bill's house if she must walk around a park that covers two adjacent blocks?
- find the number of distinct minimal paths to travel, on edges of the unit cubes, from one corner point to the opposite corner of a rectangular solid 5 units by 4 units by 3 units built from unit cubes?
- find the number of paths traveling south from Mathem City to Euler's Corners?



Mathematical Content

Graph theory, counting, permutations, combinations, Pascal's triangle

Bibliography

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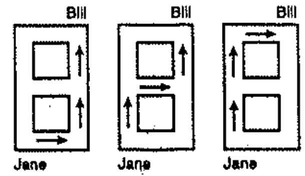
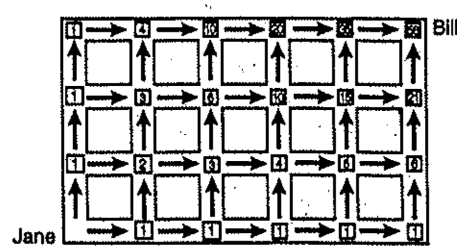
Yunker, Lee. "Random Walks." *NCTM Student Math Notes.* March 1985.

Answers:

1.



2. RUU, URU, UUR
3. RRR, 1
4. RRU, RUR, URR, 3
5. RUU, URU, UUR, 3
6. UUU, 1
7. B and C
8. 1 + 2 = 3
9. C
10. 1
11.



12. 56 days
13. 3, 5
14.

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

15. 4
16. 3
17. 2
18. 3!
19. 7, 5, 2
20. 7!, 5!, 2!
21. In each addend, the number before the factorial symbol is 1 less than the number before the factorial symbol in the sum.
22. In one addend, the first number before the factorial symbol is 1 less than the first number before the factorial symbol in the sum; in the other addend, the second number before the factorial symbol is 1 less than the second number before the factorial symbol in the sum.
23. $\frac{9!}{6!3!}, \frac{9!}{5!4!}$
24. $\frac{9!}{6!3!} + \frac{9!}{5!4!} = \frac{4}{4} \cdot \frac{9!}{6!3!} + \frac{6}{6} \cdot \frac{9!}{5!4!} = \frac{4 \cdot 9! + 6 \cdot 9!}{6!4!} = \frac{10 \cdot 9!}{6!4!} = \frac{10!}{6!4!}$

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Tricks to Remember Trigonometric Values

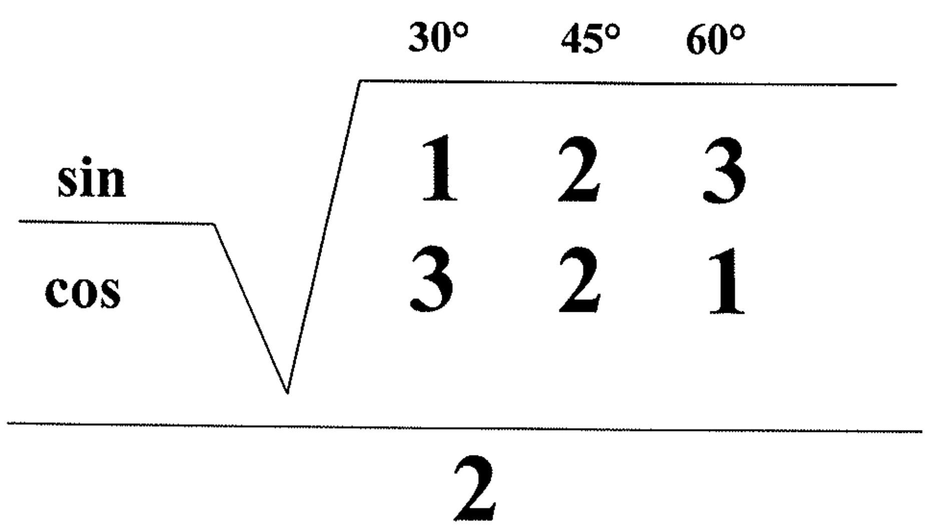
The O I O chart

	sinθ	cosθ	tanθ
0°	0	1	0
90°	1	0	U
180°	0	-1	0
270°	-1	0	U

Each line reads as follows:

Oh, I owe
I owe you
Oh, I don't owe
I don't owe you

The Square Root Chart



TRIGO – Basic Trig

Choose 24 of the following 28 answers to randomly fill in your Trigo card below. Notice some of the answers repeat, but you can only cover one space at a time when you are playing!!

135°	$\frac{\pi}{2}$	$\frac{5\pi}{4}$	$-\frac{2\pi}{3}$	$\frac{x}{r}$	$\frac{r}{x}$
-150°	$\frac{8\pi}{3}$	$-\frac{9\pi}{4}$	$-\frac{\pi}{6}$	$\frac{y}{r}$	$\frac{r}{y}$
-330°	315°	$\frac{5\pi}{3}$	$\frac{5\pi}{6}$	$\frac{x}{y}$	$\frac{y}{x}$
0	0	1	1	-1	-1
undefined	undefined	negative	positive		

T	R	I	G	O
		FREE		

TRIGO – Basic Trig

Problems:

$\tan 270°$ $\tan 90°$ $\sin 180°$ $\sin\frac{\pi}{2}$ $\cos\pi$ $\sin\frac{3\pi}{2}$

$\cos\frac{3\pi}{2}$ $\tan\frac{\pi}{4}$ $\cos\theta$ in Quadrant II $\tan\theta$ in Quadrant III

$\cos\theta$ $\sin\theta$ $\tan\theta$ $\sec\theta$ $\csc\theta$ $\cot\theta$

change $90°$ to radians change $(-120°)$ to radians change $150°$ to radians

change $\frac{3\pi}{4}$ to degrees change $\frac{7\pi}{4}$ to degrees change $-\frac{5\pi}{6}$ to degrees

change $-\frac{11\pi}{6}$ to degrees angle coterminal to $\frac{2\pi}{3}$ angle coterminal to $-\frac{7\pi}{3}$

angle coterminal to $-\frac{\pi}{4}$ angle coterminal to $-\frac{3\pi}{4}$ angle coterminal to $\frac{11\pi}{6}$

POLO – Polar Coordinates

Choose 24 of the following 28 answers to randomly fill in your Polo card below.

$(4,-150°)$ $(4,-30°)$ $(4,240°)$ $(2,-60°)$ $(2,225°)$ $(2,330°)$

$(1,\sqrt{3})$ $(3\sqrt{2},-3\sqrt{2})$ $\left(-\frac{5\sqrt{3}}{2},-\frac{5}{2}\right)$ $5+2i$ $-2+14i$ $-2\sqrt{10}$

$\frac{1+7i}{10}$ $\frac{-7-11i}{17}$ $3-15i$ $2\sqrt{3}+2i$ $-\frac{3\sqrt{3}}{2}-\frac{3}{2}i$ $4\sqrt{2}+4\sqrt{2}i$

$2cis330°$ $2cis180°$ $3cis90°$ $4cis40°$ $5cis62°$ $30cis73°$

$72cis42°$ $2cis(-135°)$ $8cis35°$ $88cis56°$

P	O	☺	L	O
		FREE		

POLO

Problems:

$\sqrt{-8}\cdot\sqrt{-5}$

$(2-5i)-(-3-7i)$

$(4-7i)-(1+8i)$

$(2-4i)(-3+i)$

$\dfrac{2-4i}{3+5i}$

$\dfrac{1+2i}{3-i}$

$\dfrac{8cis120°}{2cis80°}$

$\dfrac{35cis98°}{7cis36°}$

$\dfrac{2-4i}{3+5i}$

$\dfrac{56cis57°}{7cis22°}$

$(9cis17°)(8cis25°)$

$(11cis31°)(8cis25°)$

$(5cis32°)(6cis41°)$

another name for $(4,-120°)$

another name for $(-4,30°)$

another name for $(-4,-210°)$

change to rectangular form: $(-6,135°)$

change to polar form: $(\sqrt{3},-1)$

change to rectangular form: $(2,60°)$

change to polar form: $(-\sqrt{2},-\sqrt{2})$

change to polar form: $(1,-\sqrt{3})$

change to polar form: -2

change to polar form: $\sqrt{3}-i$

change to polar form: $-\sqrt{2}-\sqrt{2}i$

change to polar form: $3i$

change to complex form: $8cis45°$

change to complex form: $4cis30°$

change to complex form: $3cis(-150°)$

L	O	☺	G	O
		FREE		

Choose 24 of the following 28 answers and fill in your LOGO card at random. Mark out each answer as you go so you know which answers have been used.

4	27	$-\frac{2}{13}$	$\frac{1}{3}$	2	8
$\frac{2}{3}$	3	$\frac{17}{3}$	$\frac{1}{32}$	-4	$\frac{3}{5}$
0	-1	$\frac{3}{2}$	$\frac{2}{7}$	-3	$\frac{11}{5}$
1	$\frac{5}{6}$	$\frac{1}{2}$	-5	15	-2
$\frac{1}{36}$	$\frac{1}{8}$	$-\frac{1}{4}$	64		

LOGO

Problems:

$\log_3 81$ $\log_5 125$ $\log_{15}\sqrt{15^3}$ $\log_2 \frac{1}{32}$

$\log \frac{1}{100}$ $\log_{\frac{1}{2}} 16$ $\log \sqrt[3]{10}$ $\log_2 4 \cdot \log_2 16$

$\log_9(\log_3 27)$ $\log_3(\log_2 8)$ $\log_5 1 - \log_5 125$ $\left(\frac{125}{27}\right)^{-\frac{1}{3}}$

$\left(\frac{9}{4}\right)^{-\frac{1}{2}}$ $9^{\frac{3}{2}}$ $32^{-\frac{3}{5}}$ $16^{-\frac{5}{4}}$

$\log_8 x = 2$ $\log_6 x = -2$ $\log_x \frac{36}{25} = -2$ $4^{x-2} = 16^{7x}$

$25^{2x} = \frac{1}{5}$ $9^{3x} = \left(\frac{1}{3}\right)^{x-2}$ $\log_4 16 = 2x - 2$ $\log_3 27 = 3x + 6$

$\log_7 \frac{1}{49} = -x - 2$ $\log_4(x+1) = 2$ $\log_4(x+1) + \log_4 5 = 2$

$\log_2(x-3) + \log_2 3 = 3$

Around the World Sample Problem Sets

Trig Sum/Difference Identities

1) Find the exact value of $\cos(-15°)$.

2) Find $\sin(\alpha-\beta)$ if $\cos\alpha=-\frac{8}{17}$ for $\frac{\pi}{2}<\alpha<\pi$, and $\sin\beta=-\frac{7}{25}$ for $\pi<\beta<\frac{3\pi}{2}$.

3) Find the exact value of $\tan 75°$.

4) Find $\cos(A-B)$ if $\sin A=\frac{4}{5}$ for $0<A<\frac{\pi}{2}$, and $\cos B=-\frac{24}{25}$ for $\frac{\pi}{2}<B<\pi$.

5) Prove $\sin(\pi+\theta)=-\sin\theta$.

6) Find the exact value of $\sin 105°$.

7) Prove $\tan(\alpha-180°)=\tan\alpha$.

8) Find $\tan(\alpha-\beta)$ if $\sin\alpha=-\frac{4}{5}$ for $\pi<\alpha<\frac{3\pi}{2}$, and $\sin\beta=-\frac{7}{25}$ for $\frac{3\pi}{2}<\beta<2\pi$.

9) Prove $\tan\left(\frac{\pi}{4}-\alpha\right)=\frac{1-\tan\alpha}{1+\tan\alpha}$.

10) Prove $\cos\left(\frac{3\pi}{2}+A\right)=\sin A$.

Domain/Range/Functions

1) Graph $y=|x-1|$ for $-2\le x\le 3$.

2) What is the domain for $f(x)=\frac{3}{x^2-x-6}$?

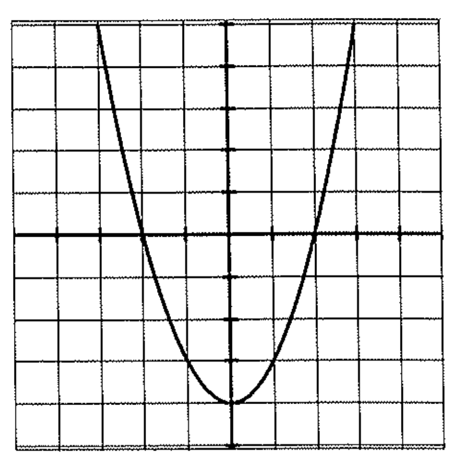
3) If $f(x)=x^2-1$ and $g(x)=x+1$, find $f(g(x))$.

4) Find the domain and range of $f(x)=\sqrt{x}+3$.

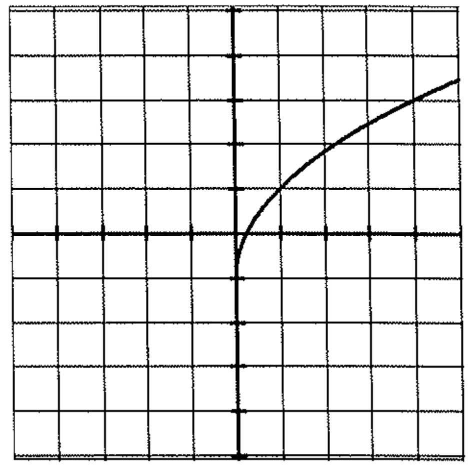
5) What is the domain and range of $g(x)=|x+4|$.

6) What is the range of $y=x^2+5$?

7) Name the domain and range of the following graph:



8) Name the domain and range of the following graph:



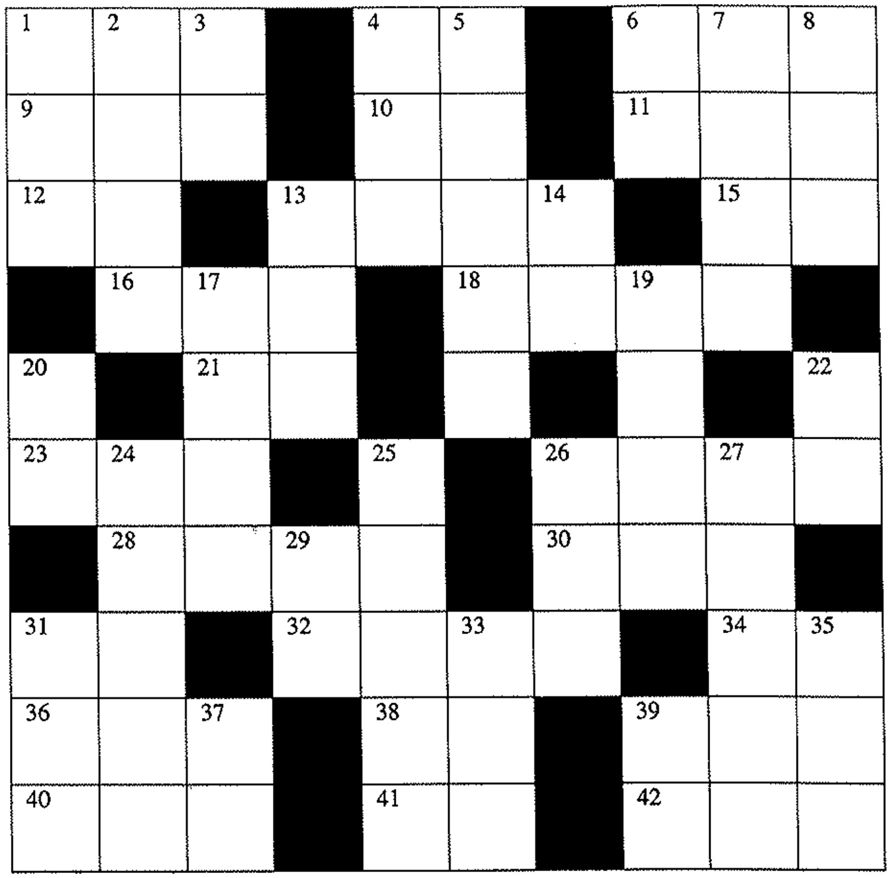
9) If $f(x) = 2x^2 - x$ and $g(x) = 5x + 1$, find $(f - g)(x)$.

10) What is the domain of $y = \sqrt{x-3}$?

Magic Squares – Basic Reciprocal Values

$-\frac{\sqrt{2}}{2}$ -1 $-\sqrt{2}$ $tan60°$	$\frac{\sqrt{3}}{3}$ $-\sqrt{3}$ U $-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{2}}{2}$ $sec60°$ $-\frac{2\sqrt{3}}{3}$ $csc270°$
$sec30°$ -2 $cot45°$ $-\frac{1}{2}$	$sin45°$ $\frac{1}{2}$ 0 $-\frac{\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$ $cos30°$ 1 $cot90°$
1 $-\frac{\sqrt{3}}{2}$ $csc45°$ $tan30°$	-1 $\sqrt{3}$ $tan90°$ $\frac{1}{2}$	$\sqrt{2}$ $cos60°$ $\frac{\sqrt{3}}{2}$ 2

A Precalculus Crossmath Puzzle by Larry Riddle
Agnes Scott College



DIRECTIONS

- A numeric answer is entered one digit per box. Ignore the decimal point when entering the digits. For a negative digit, enter the minus sign with the digit in its own box.
- If the answer is a polynomial then each coefficient is entered in a box starting with the term of highest degree.
- The number of significant places to round to is determined by the number of boxes available for the answer.
- For (*x*, *y*) enter *x* then *y*.
- For a line, enter *m* then *b*.
- For two numeric answers enter the smallest number first.
- For a fraction, enter numerator then denominator.

ACROSS

1. The base 10 equivalent of 1110101 in base 2.
4. The prime factors of 35
6. $(2x-3)^2+8x$
9. log 121
10. The numerator of $\frac{7}{5}+\frac{3}{4}$
11. Quotient and remainder of $\dfrac{5x^2-3x-6}{x+1}$
12. The antilog of $(\log 3a+\log 4a-2\log a)$
13. $\frac{5}{12} \div \frac{7}{10}$ in reduced form
15. The positive root of $x^2+4x-6=0$
16. $\dfrac{4956x^2}{12x^2}$ when $x=\sqrt{\pi}$
18. The ratio of the circumference of a circle to the diameter
20. The value of *c* if *y* = *c* is the horizontal asymptote of $y=\dfrac{3x-1}{x+2}$
21. The only two digit prime with each digit the same
22. The value of *c* if *x* = *c* is a vertical asymptote of the graph in 20 across
23. $\cos\theta$ x10 if θ is $5\pi/9$ radians
25. The maximum number of roots of a cubic polynomial
26. Two numbers that differ by 2 and whose product is 195
28. $(6x^3+2x^2+3x+1)-(2x^2+x-1)$
30. $\tan(-31^\circ)\times 10^3$
31. The exponent of *a* in $a^6(a^5)^2/a^4$
32. Quotient and remainder of $\dfrac{5x^3+x+13}{x^2-x+1}$
34. The line through (0,5) that is perpendicular to $y=-\frac{1}{3}x+2$
36. The digits of this number (which is larger than 500) multiply to 21
38. Sum of the divisors of 42
39. Numerator of $\dfrac{6x}{x-1}-\dfrac{1}{x}-3$ when all terms are combined
40. The maximum area of a rectangle with perimeter 80
41. $\log_2 4096$
42. The volume of a box of dimension $6\times 8\times 17\frac{3}{4}$

DOWN

1. The square of 11
2. The 10th power of 2
3. $f(3)$ if $f(x)=2x^3+5x^2-4x-9$
4. The vertex of $y=x^2-10x+70$
5. $5^{2.6695}$
6. Roots of $x^2-9x+20=0$
7. $(1-2x)(x+2)(1+2x)$
8. The length of the hypotenuse of a right triangle of sides 6 and 7
13. $\dfrac{x+3x^2}{x}+2x^2$
14. $f(g(x))$ when *x* = 2 if $f(x)=3x+6$ and $g(x)=x^2+1$
17. The diameter of a circle with area 102 sq. units
19. The area of an equilateral triangle of side length 10
20. A factor of $3x^2-4x+1$
22. The slope of the line through the points (1,3) and (6,1)
24. The height of a building with angle of elevation of 60° at a distance 44 feet from the base
25. 478 ft/sec when expressed in miles per hour
26. A monic polynomial with roots 2 and 4
27. The sum of the first 150 positive integers
29. The intersection of the lines $3y-2x=11$ and $3x+4y=26$
31. The volume of a right circular cone of radius 2 and height 4.15
33. Largest solution to $x-4\sqrt{x}=0.1$
35. When cubed, the sum of the digits of this number gives the original number
37. The perimeter of the triangle in 19 down
39. The area of a . ½ . inch wide border that surrounds a 17 x 20 inch picture

Answers to Crossmath Puzzle

(1) 1	(2) 1	(3) 7		(4) 5	(5) 7		(6) 4	(7) –4	(8) 9
(9) 2	0	8		(10) 4	3		(11) 5	–8	2
(12) 1	2		(13) 2	5	4	(14) 2		(15) 1	2
	(16) 4	(17) 1	3		(18) 3	1	(19) 4	2	
(20) 3		(21) 1	1		5		3		(22) –2
(23) –1	(24) 7	4		(25) 3		(26) 1	3	(27) 1	5
	(28) 6	0	(29) 2	2		(30) –6	0	1	
(31) 1	2		(32) 5	5	(33) 1	8		(34) 3	(35) 5
(36) 7	1	(37) 3		(38) 9	6		(39) 3	2	1
(40) 4	0	0		(41) 1	2		(42) 8	5	2

HOW MUCH DO YOU REMEMBER?

A	B	C	D	E	F
G	H	I	J	K	L
M	N	O	P	Q	R
S	T	U	V	W	X

1. Pick a whole number between 2 and 9. Write it in Box A.
2. Multiply the number in Box A by 9, and write the second digit in Box B.
3. Add the number in Box A to the number in Box B and write the sum in Box C.
4. Write your favorite two digit number in Box D.
5. Take the logarithm of the number in Box C and write it in Box E.
6. Using the number in Box A as the slope and the number in Box E as the y-intercept, write the equation of the line in Box F.
7. Find the x-intercept of the line in Box F and write it in Box G.
8. Using the number in Box G as the slope and the number in Box D as y-intercept, write the equation of the line in Box H.
9. Evaluate the function in Box H at the number in Box A. Subtract the result from the number in Box D and write the difference in Box I.
10. Find the absolute value of the number in Box G and write it in Box J.
11. Divide the natural logarithm of the number in Box J by the natural logarithm of the number in Box A, and write the result in Box K.
12. Find the letter of the box that contains the largest number in the gird so far. Think of a European country beginning with that letter and write it in Box L.
13. Take the second letter of the country name in Box L. Think of a one digit number beginning with that letter and write that number in Box M.
14. Find the sine and cosine of the number in Box M. Square these two numbers and write the sum of their squares in Box N.
15. Write the number in Box C as a word. Change a single letter of the word to get the name of a trig function. Write that function in Box O.
16. Find the period of the function in Box O and write it in Box P.
17. Multiply the number in Box A by the number in Box P. Plug the product into the function in Box O and write the function value in Box Q.
18. Find the product of all of the numbers in the grid so far and write that product in Box R.
19. Raise the number in Box A to the power of the number in Box R. Write the result in Box S.
20. Add the number in Box M to the number in Box S and multiply the sum by the number in Box D. Write this product in Box T.
21. Add the digits of the number in Box T together. If the result is a single digit, write it in Box U. If the result is two digits, add those two numbers together and write the sum in Box U.
22. There is one number that appears in every row of the grid. The letters that correspond to the boxes containing that number can be rearranged to spell a function. Write that function in Box V.
23. Find the number in the grid that rhymes with the function in Box V. Write the letter of the box containing that number in Box W.
24. Find the angle of intersection (in radians) of the two lines in Boxes F and H. Evaluate the function in Box V at that angle and write the answer in Box X.

If you have successfully filled in all of the lettered boxes, read the contents of the last two boxes aloud and hear the special message for you.

Precalculus preAP
Trig Graph Project

name: ______________________
date: ______________________

You should now have had a chance to play around on the graphing calculator with the 2 student samples I showed you. You should know how to restrict your domain (if not, talk to another student who knows how).

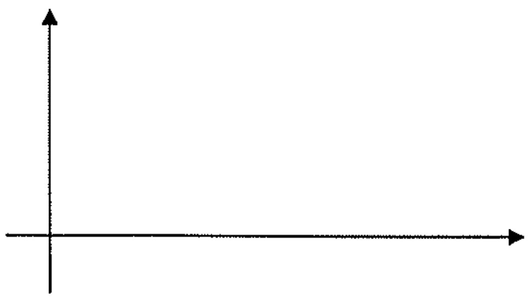
<u>Project:</u> 15 points .... (due 12/7/12)

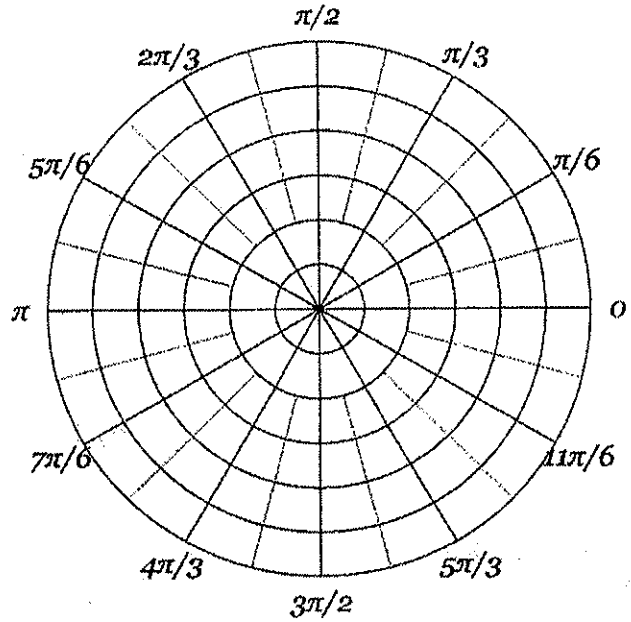
You will create a picture using each of the six trigonometric functions ONLY once each. You are allowed to transform each of the functions and restrict their domains. The specs follow.

1. Use $y = A\sin B(x - C) + D$, $y = A\cos B(x - C) + D$, $y = A\tan B(x - C) + D$, $y = A\csc B(x - C) + D$, $y = A\sec B(x - C) + D$, $y = A\cot B(x - C) + D$ **only once each.** The values of A, B, C, and D can be different in each equation. (6 points)
2. You may restrict the domains correctly for all the functions. (3 points)
3. Create your graph on www.desmos.com (launch the calculator). (1 point)
4. Take a screen shot of your final graph and equations. (1 points)
5. Import your picture into paint and add color to your picture.
6. You may add a FEW details to enhance the picture, but the drawing should be mostly the trig. (2 points)
7. You MUST indicate for each equation, what drapwing part it created (i.e. this is the body ...) (2 points)
8. You will mail your final picture to me by **MIDNIGHT 12/7/12** to be considered on time.

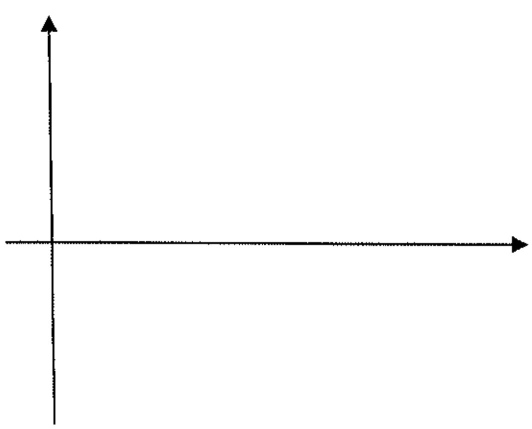
Practice with Polar Graphing

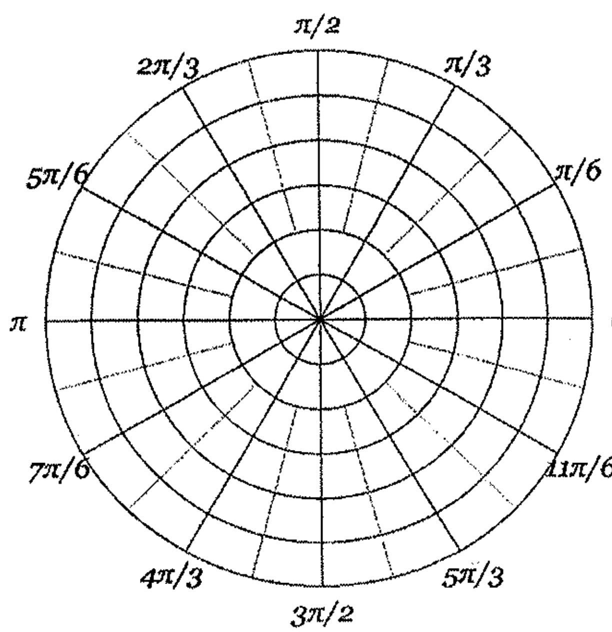
1.) $r = 3 + 3\cos\theta$



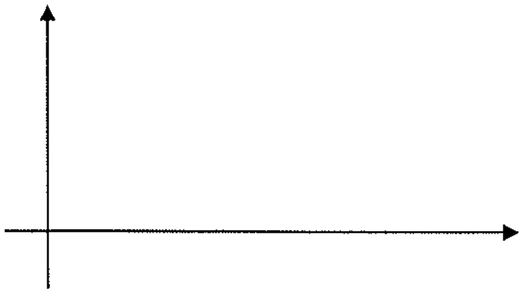


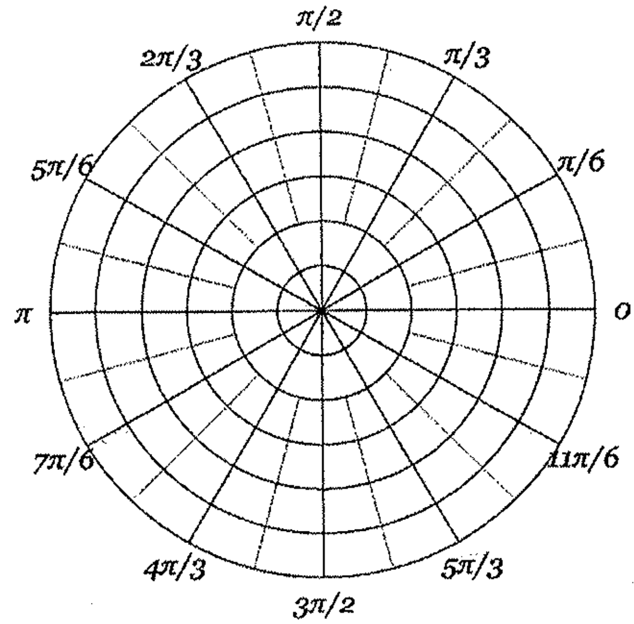
2.) $r = 1 + 3\sin\theta$



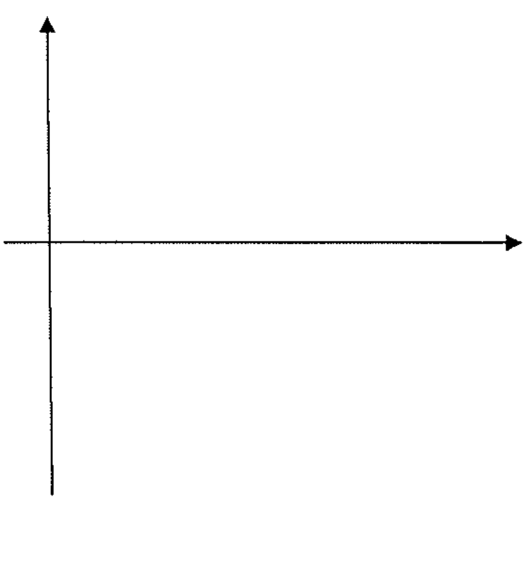


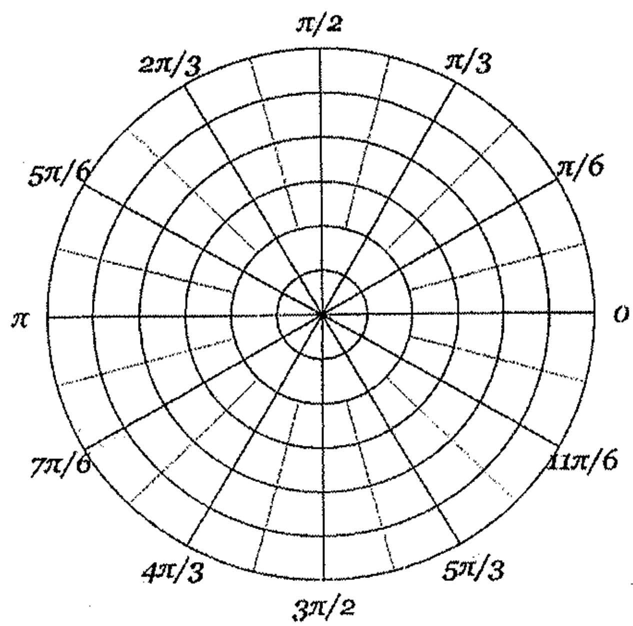
3.) $r = 2 - 3\cos\theta$





4.) $r = -1 - 4\sin\theta$





Fun with Polars Project

Use what you have learned about polar equations and anything you have derived from your own calculator explorations to produce a POLAR PICTURE.

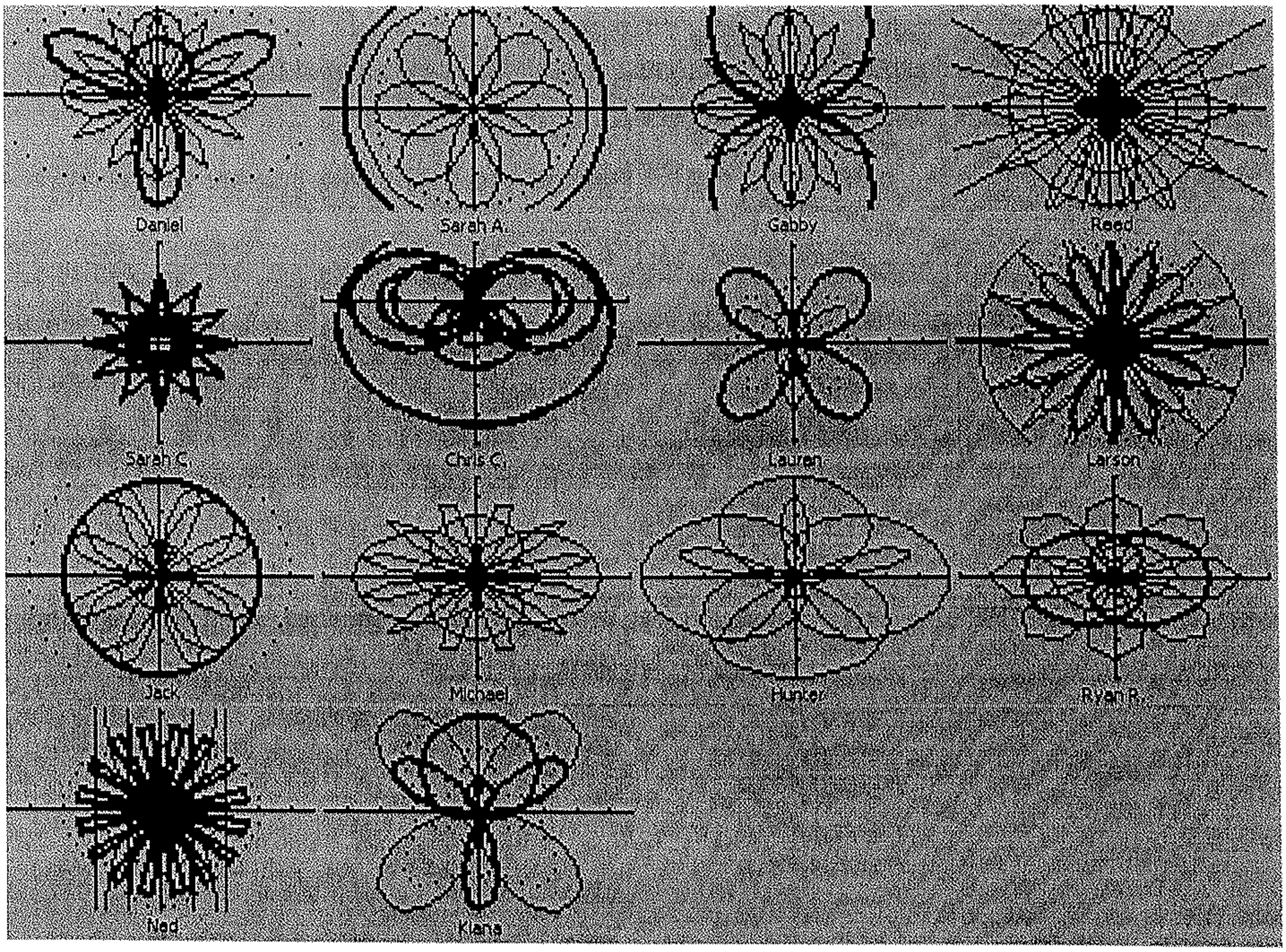
You must use at least 6 equations. If you choose to use more than 6 equations, you may store up to 6 of them by doing the following:

1) Graph equations
2) DRAW
3) STO
4) StorePic
5) 1
6) ENTER
7) Graph remaining equations
8) DRAW
9) STO
10) RecallPic
11) 1
12) ENTER

Hand in a neatly written page including:

MODE
WINDOW SETTINGS
STYLE OF GRAPH (connected, dotted, bold, etc)
EQUATIONS (including any stored equations)
ROUGH SKETCH OF PICTURE

Sample graphs from students:



Coding/Encryption Name_______________________

Objective: To use functions and inverse functions to demonstrate how written information (text) can be encrypted and decrypted.

Here is a CODING table for the alphabet. To code a message you would simply replace the letters of the message with the numbers from the table. Computers use a system like this to capture text as numeric data.

A	1
B	2
C	3
D	4
E	5
F	6
G	7
H	8
I	9
J	10
K	11
L	12
M	13
N	14
O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26
Space	27
!	28
?	29
@	30

Encryption changes the coding of the letters according to a particular rule; hopefully the rule is known only to the person intended to receive the message!

First ENCODE the letters of your first name by replacing the letters with numbers from the table:

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____

Choose a simple algebraic function to ENCRYPT your name. It is easiest to choose a linear function (in the form $y = mx + b$). Write it here:

____

ENCRYPT the letters of your name according to the rule of the function:

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____

To DECRYPT your name, you will need to use the INVERSE of your ENCRYPTING FUNCTION. Find the INVERSE FUNCTION and write it here:

____

See if you can successfully decrypt your name using the inverse function.

In real communications systems many messages are encrypted for security and privacy. This is not exclusively the domain of spies and the military; private messages of all kinds need to be secure, especially over a public medium like the internet or the cellular phone system. Financial information and personal data is particularly valuable and needs to be protected. Some modern data communication systems use an encryption system that is fundamentally the same as the one you are using although the encryption functions are **much** more complex.

Think of a message you would like to send to a friend; I recommend no more than six words. Then write a simple function you could use to encrypt it. Write them both in the space below.

On the next sheet write the encrypted message along with the function. Trade sheets with a friend to decode each other's messages.

Message Sender: ______________________________

Encrypted Message:

A	1
B	2
C	3
D	4
E	5
F	6
G	7
H	8
I	9
J	10
K	11
L	12
M	13
N	14
O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26
Space	27
!	28
?	29
@	30

Message Receiver: ______________________________

Decrypted Message:

ACTIVITY #1

Here is a secret message from me. This time see if you can "crack the code" without my giving you the encoding function.

Helpful tips: The encoding function is a linear function. Try looking at the differences between coded numbers to find the encoding function.

51 103 75 47 91 119 135 127 47 91

23 27 127 23 127 95 75 55 31 63

39 91 95 131

A	1
B	2
C	3
D	4
E	5
F	6
G	7
H	8
I	9
J	10
K	11
L	12
M	13
N	14
O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26
Space	27
!	28
?	29
@	30

ACTIVITY # 2

Here is a secret message from me. This time see if you can "crack the code" without my giving you the encoding function.

Helpful tips: The encoding function is a linear function. Try looking at the differences between coded numbers to find the encoding function.

142 92 152 102 122 62 42

107 92 152 117 22 32 92

152 27 42 77 77 157

A	1
B	2
C	3
D	4
E	5
F	6
G	7
H	8
I	9
J	10
K	11
L	12
M	13
N	14
O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26
Space	27
!	28
?	29
@	30

Answers to Coding/Encryption Activities

Activity #1:

The equation is $y = 4x + 19$. The message is: HUNGRY? GRAB A SNICKERS!

Activity #2:

The equation is $y = 5x + 17$. The message is: YO QUIERO TACO BELL!

A Leaky Bottle Experiment

- Each group needs a timekeeper, water-level reader/bottle holder, and recorder
- Fill the bottle so the water level is below the curve at the top
- When the timekeeper says "go," uncover the hole and let the water run freely
- The timekeeper calls out time every 10 seconds
- The water-level reader reads aloud the water level to the nearest millimeter
- The recorder records the data
- Stop measuring when the water level reaches about a centimeter from the hole

Time (seconds)	Height of water (cm)
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	
110	
120	
130	
140	
150	
160	
170	
180	
190	
200	
210	
220	
230	
240	
250	
260	
270	
280	
290	
300	

1.) Enter the data into your calculator and look at the scatterplot. Describe in words what your graph looks like and draw a quick sketch. Write a conjecture about what types of functions might fit the data.

2.) Find regression equations for 2 different types of functions that fit the data reasonably well.

3.) Decide which of your models is the best fit, and explain why.

4.) Use your best model to predict when the container would be empty.

TURN IN ONE REPORT WITH ALL THE MEMBERS OF YOUR GROUP WRITTEN ON THE TOP.

Be sure you have explained everything thoroughly – this will be a group grade, so be sure all members are satisfied with the finished product!

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