Grounding Justifications in Concrete Embodied Experience: The Link between Action and Cognition

Tangibility for the Teaching, Learning, and Communicating of Mathematics

MAGIC Research Group

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Brief Framing

• Theories of embodied cognition
  • Mental processes rooted in perceptual and motor systems (Wilson, 2002)
  • Mathematical objects experiential, perception-based, and multimodal in nature (Barsalou, 1999; Lakoff & Nunez, 2000; Landy, Brooks, & Smout, 2012)

• Importance of action and simulated action for learning mathematical ideas (Abrahamson & Howsin, 2010; Martin & Schwartz, 2005; Nathan et al., 1992)

• Gesture as an instructional scaffold (Alibali et al., 2011; Alibali & Nathan 2007)
Directed Movement

• Directed Action
  (Thomas & Lleras, 2007, 2009)

• Directing Gesture
  (Goldin-Meadow, Cook, & Mitchell, 2009)

• Directed action & gesture can implicitly influence cognition
Projection

- Observed high school geometry classes \( (N = 17) \)
- Mathematical justification difficult practice to learn
- Mathematical ideas instantiated in different contexts
  - Computer lab (GSP) → Classroom (Discussion)
- Produce **cohesion** of mathematical ideas using **projection** (reference past/future activity)
- **Gesture** and **action** critical to cohesion production

Nathan et al., under review
Viewpoint

- Gesturers express ideas with their bodies using different **viewpoints** (McNeill, 1992; Gerofsky, 2010)

- **Observer**: Spectator of situation, third-person

- **Character**: Agent in situation, first-person

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Srisurichan et al., under review
Research Questions

• How are action and gesture used spontaneously to support mathematical justification?
• Is there an implicit link between action and cognition that can support mathematical reasoning?
• Can explicitly linking actions to mathematical ideas using projection support mathematical reasoning?
• What is the effect of viewpoint condition? (character vs. observer)
Participants and Procedure

- **Undergraduate students** \((N = 107)\) enrolled in a psychology course at large Midwestern university
- **Think aloud** (Ericsson & Simon, 1993) with only scripted prompts by interviewer
- Provide **justifications** for 2 mathematical tasks
- Prior to being given task, directed to perform body-based actions **relevant** or **irrelevant** to solution
Environment

- Large interactive whiteboard
-Directed actions performed on images in GSP - scaled to body through initial measurements
Triangle Task

Mary came up with the following conjecture: “For any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side.” Provide a justification as to why Mary’s conjecture is true or false.

Actions

Character Viewpoint

Relevant Actions

Irrelevant Actions

Observer Viewpoint

Relevant Actions

Irrelevant Actions
An unknown number of gears are connected in a chain. **If you know what direction the first gear turns, how can you figure out what direction the last gear will turn?** Provide a justification for your answer.

<table>
<thead>
<tr>
<th>Gear Task</th>
<th>Actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>An unknown number of gears are connected in a chain. <strong>If you know what direction the first gear turns, how can you figure out what direction the last gear will turn?</strong> Provide a justification for your answer.</td>
<td><strong>Character Viewpoint</strong></td>
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<td>Relevant Actions</td>
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<td>![Image 1]</td>
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<td>![Image 2]</td>
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<td><strong>Observer Viewpoint</strong></td>
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<td>Relevant Actions</td>
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<td>![Image 3]</td>
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<td>![Image 4]</td>
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<tr>
<td></td>
<td><strong>Irrelevant Actions</strong></td>
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<td>![Image 5]</td>
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<td>![Image 6]</td>
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</tbody>
</table>
Design

• **Relevant** action for one conjecture, **irrelevant** action for other

• One set of actions from **character viewpoint**, other from **observer viewpoint**

• No participants reported being aware of connection

• Backwards **projection** at end of session
  • Participants told that there is a connection between actions and task, opportunity to solve again
Findings

• How are action and gesture used spontaneously to support mathematical justification?
  • Action and gesture used in formulating (ascertaining) and communicating (persuading) mathematical justifications (Harel & Sowder, 1998)
  • Participants “think with their bodies”
  • Use action as an essential modality for mathematical communication

“If one gear was turning this way, then the spokes on it would push...”

(Later)
“All the odd gears would be turning in the original direction.”
Findings

- Is there an *implicit* link between action and cognition that can support mathematical reasoning?

N = 40

Note: All participants included report not being consciously aware that there was a connection at this stage of the session.

![Graph showing justification accuracy for Gear and Triangle categories. The x-axis represents Gear and Triangle, and the y-axis represents justification accuracy. The graph shows two bars for each category, one for Irrelevant and one for Relevant, with error bars indicating variability.](image)
Findings

• Can explicitly linking actions to mathematical ideas using projection support mathematical reasoning?

“Oh! I see! If this was side A and this was side B...”

“They couldn’t reach anything greater than A + B”

N = 40
Findings

- Can explicitly linking action-based interventions to mathematical ideas support mathematical reasoning?

“Oh! I see! If this was side A and this was side B...”

“They couldn’t reach anything greater than A + B”

N = 40
Findings

- What is the effect of viewpoint condition? (character vs. observer)

![Character Observer Accuracy Graph]

- N = 40
Implications

- **Gesture** and **action** play critical role in formulating and communicating mathematical justifications.
- Directing students to perform **relevant actions** can support key mathematical insights.
- Having students **generate connections** can be powerful, although some actions may work implicitly.
- **Character viewpoint**, first-person embodied experience, especially effective support.
References

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