

# Algebraic reasoning in K-5? Yes, we can!

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$$8 + 4 = \square + 5$$

12

17

12 and 17

7

(Falkner, Levi, & Carpenter, 1999)

$$3 + 8 = \square + 6$$

What number goes in the box to make this number sentence true?

- A. 17
- B. 11
- C. 7
- D. 5

Intl. Average:	39% correct
United States:	47%
Singapore:	85%

# Algebraic Reasoning in K-5?

Tell us one word to describe algebraic reasoning in K-5.

Go to [menti.com](https://menti.com)  
Use code 20 15 97

*Results*

# Algebraic Reasoning

Recognizing and describing patterns and relationships between quantities that may be *unknown*

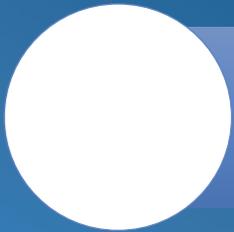
Recognizing and describing patterns and relationships between quantities that are *known*



# Strands of Algebraic Thinking



The study of structures in the number system, including those arising in arithmetic



The study of patterns, relations, and functions



The process of mathematical modeling, including the meaningful use of symbols

## Number Properties

- Allows students to manipulate and restructure expressions
- Commutative, associative, and the principle of inversion are frequently applied in K-5
- Develops between the ages of 4-6

## Additive Composition

- Any number can be composed or decomposed
- Allows students to manipulate values in expressions
- Develops between the ages of 4-7



# Why are Number Properties & Decomposition Useful?

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$$8 + 3 = \square + 2$$

$$8 + (1 + 2) = \square + 2$$

$$(8 + 1) + 2 = \square + 2$$

$$9 = \square$$

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$$44 \times 5 = \square$$

$$(40 + 4) \times 5 = \square$$

$$(40 \times 5) + (4 \times 5) = \square$$

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$$\frac{4}{5} + \frac{2}{5} = \square$$

$$\frac{4}{5} + \frac{1}{5} + \frac{1}{5} = \square$$

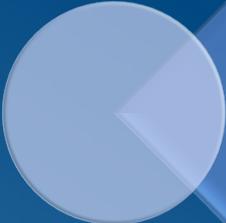
$$1 + \frac{1}{5} = \square$$

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# How can we promote Number Properties & Decomposition in the classroom?



Rich tasks



Routines



Stations



# Rich Tasks

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# What is a rich task?

- Multiple solutions or approaches
- Multiple entry points
  - Can be used with struggling students or advanced learners
- Utilize and broaden problem solving skills

Seven birds landed in your backyard,  
some landed on a tree,  
and some are at your feeder.

How many birds might be in the  
tree and how many might  
be at the feeder?

Multiple solutions

Multiple  
approaches

- Act it out
- Draw pictures
- Write equations



Sandra is bringing 4 pumpkins to school for the fall fair. She has small, medium, and large pumpkins. How many of each size could Sandra bring to school?

Are there multiple ways?

Can you prove you have found each possible solution?



# Routines

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# What is a routine?

- Whole class structured activity that gives students the opportunity to develop knowledge over time
- Encourages teacher-student discourse and student-student discourse
- Takes 8-10 minutes
- A way to open a lesson

# Goals of a routine

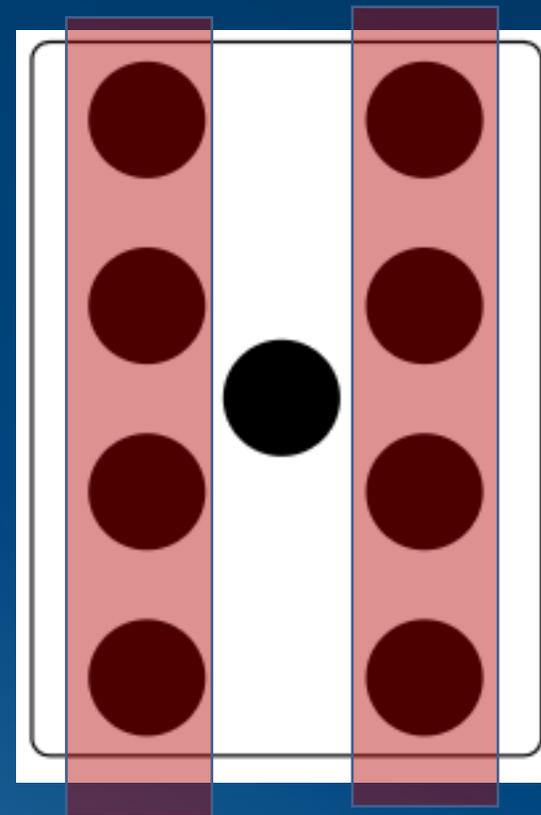
- Develops intuition about numbers and their relationships
- Supports fluency and flexibility
- Develops students' ability to engage in the Mathematical Process standards:
  - (D) Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.
  - (E) create and use representations to organize, record, and communicate mathematical ideas.
  - (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
  - (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

# Routines

- Number of the Day
- Number Talks
- Comparing expressions

# Number of the Day

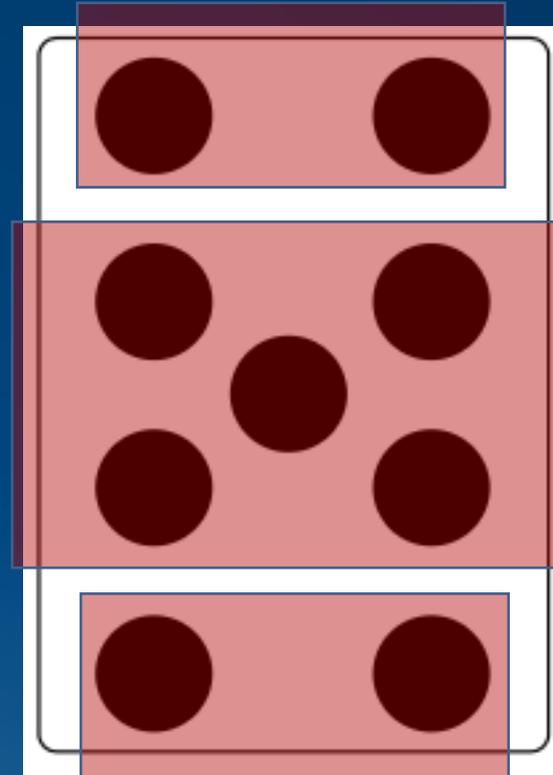
- Choose a number or representation
- Possible Questions
  - How many do you see?
  - How do you know how many dots there are?
  - “Tell me something about \_\_\_\_\_”
- Encourage discourse and multiple strategies



$$9 = 4 + 4 + 1$$

# Number of the Day

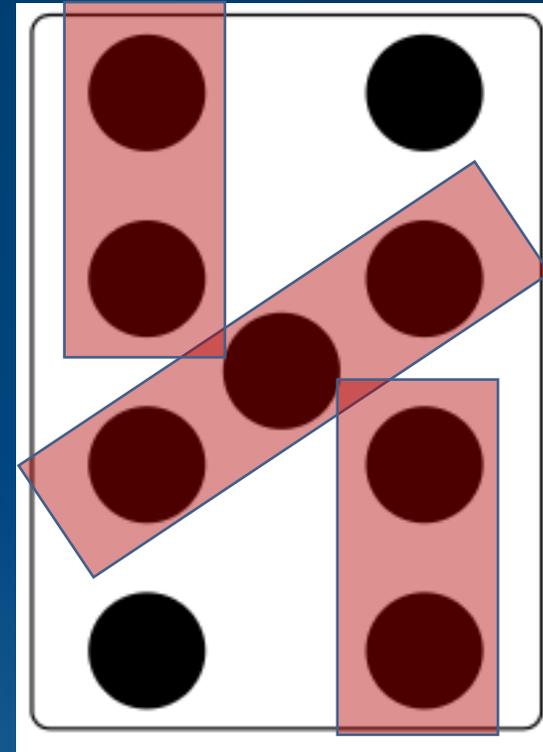
- Choose a number or representation
- “Tell me something about \_\_\_\_\_”
- Encourage discourse and multiple strategies



$$9 = 5 + 2 + 2$$

# Number of the Day

- Choose a number or representation
- “Tell me something about \_\_\_\_\_”
- Encourage discourse and multiple strategies



$$9 = 3 + 2 + 2 + 1 + 1$$

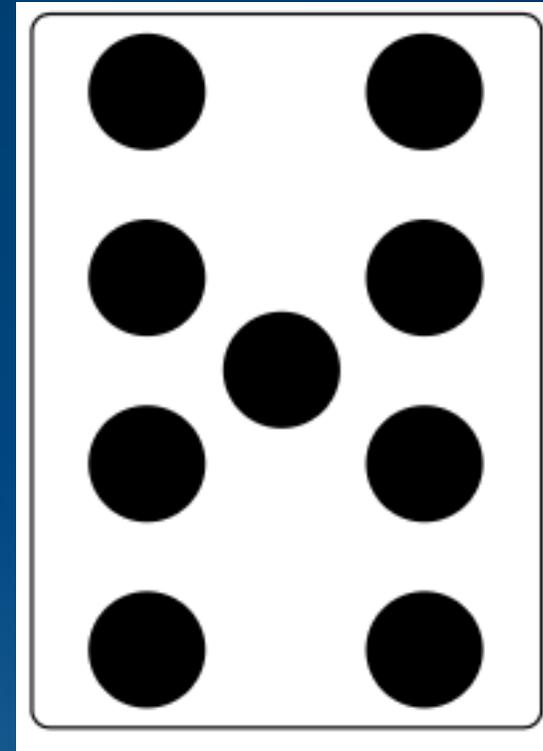
# Number of the Day

- Choose a number or representation
- “Tell me something about \_\_\_\_\_”
- Encourage discourse and multiple strategies

$$9 = 4 + 4 + 1$$

$$9 = 5 + 2 + 2$$

$$9 = 3 + 2 + 2 + 1 + 1$$



# Number of the Day

- Choose a number or representation
- “Tell me something about \_\_\_\_\_”
- “Write as many ways as you can to make \_\_\_\_\_”
- Encourage discourse and multiple strategies

120



# Number of the Day

## Add constraints:

- Describe 120 using addition
- Describe 120 using multiplication
- Use representations to show 120
- Write as many expressions as you can using 10s and 1s

120

# Number of the Day

- Choose a number or representation
- “Tell me something about \_\_\_\_\_”
- Encourage discourse and multiple strategies

3  
—  
4

# Routines

- Number of the Day
- Number Talks
- Comparing expressions

# Number Talks

- Select a number string of related problems
- Show one problem and have students solve mentally
- Discuss
- Continue showing problems individually and discussing

1003 - 7

# Number Talks

$$1003 - 3 =$$

# Number Talks

$$1003 - 3 = 1000$$



# Number Talks

$$1003 - 3 = 1000$$

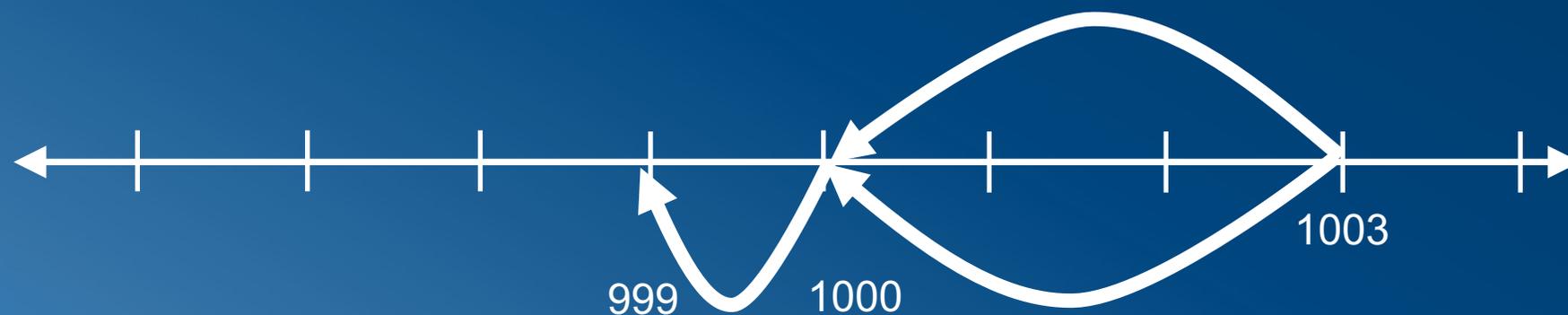
$$1003 - 4 =$$



# Number Talks

$$1003 - 3 = 1000$$

$$1003 - 4 = 999$$



# Number Talks

$$1003 - 3 = 1000$$

$$1003 - 4 = 999$$

$$1003 - 7 =$$



# Number Talks

$$1003 - 3 = 1000$$

$$1003 - 4 = 999$$

$$1003 - 7 = 996$$



# Number Talks

- Select a number string of related problems
- Show one problem and have students solve mentally.
- Discuss
- Continue showing problems individually and discussing

$$1003 - 3$$

$$1003 - 4$$

$$1003 - 7$$

# Routines

- Number of the Day
- Number Talks
- Comparing expressions

# Comparing Expressions

- Show two related expressions
- Ask students to describe the relationship between the expressions using comparative language ( $>$ ,  $<$ , and  $=$ )
- Encourage students to justify their answer

$$3 + 4 \quad \square \quad 4 + 3$$

$$1 + 4 \quad \square \quad 4 + 3$$

$$\frac{1}{5} + \frac{4}{9} \quad \square \quad \frac{4}{9} + \frac{1}{3}$$

# Stations

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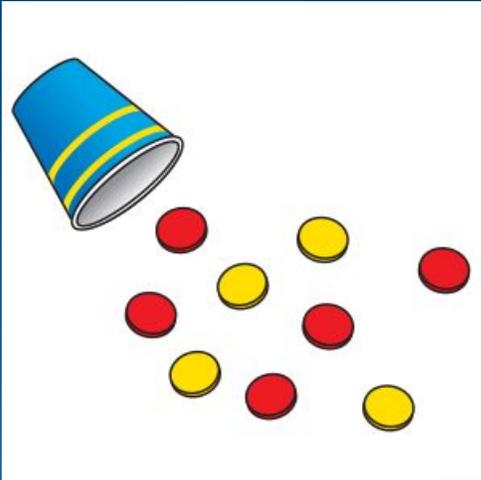
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# What is a station?

- Independent or partner activities around your room
- Focused on one central concept or multiple concepts to allow for spiraled practice
- Follow station activity time with mathematical community discussions to support generalizations

# Stations

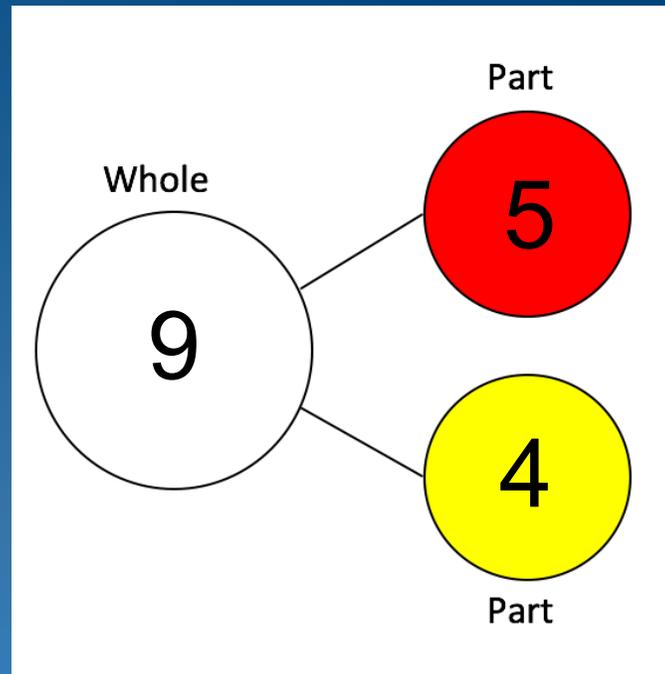
- **Dump It**
- **Bears in a Cave**
- **Missing Part Sentence Strips**
- **How Close to 20, 100, 1000, 1**
- **Four Strikes and You're Out**



# Dump It!

- Put a specified number of two colored counters in a cup
- Spill the counters onto the table

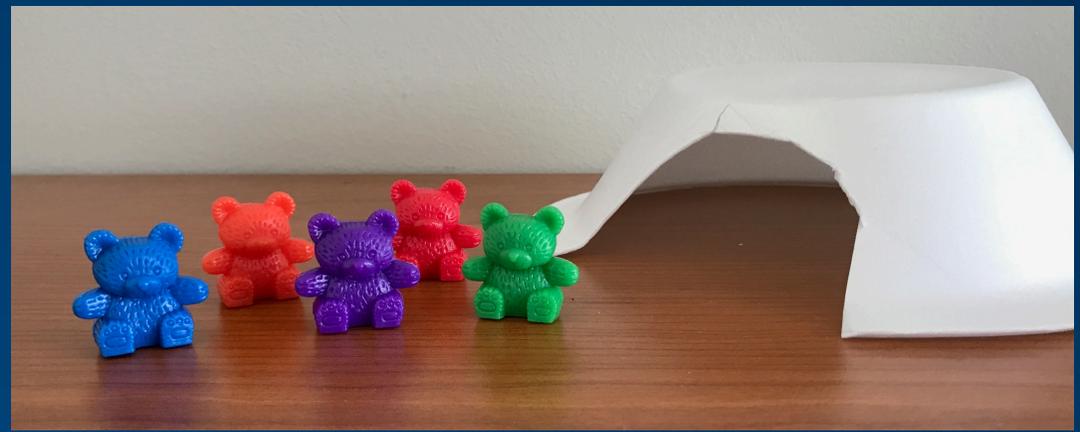
Red	Yellow	Total
5	4	9
3	6	9
2	7	9



$$\begin{array}{|c|} \hline \text{Red} \\ \hline 5 \\ \hline \end{array} + \begin{array}{|c|} \hline \text{Yellow} \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline \text{Total} \\ \hline 9 \\ \hline \end{array}$$

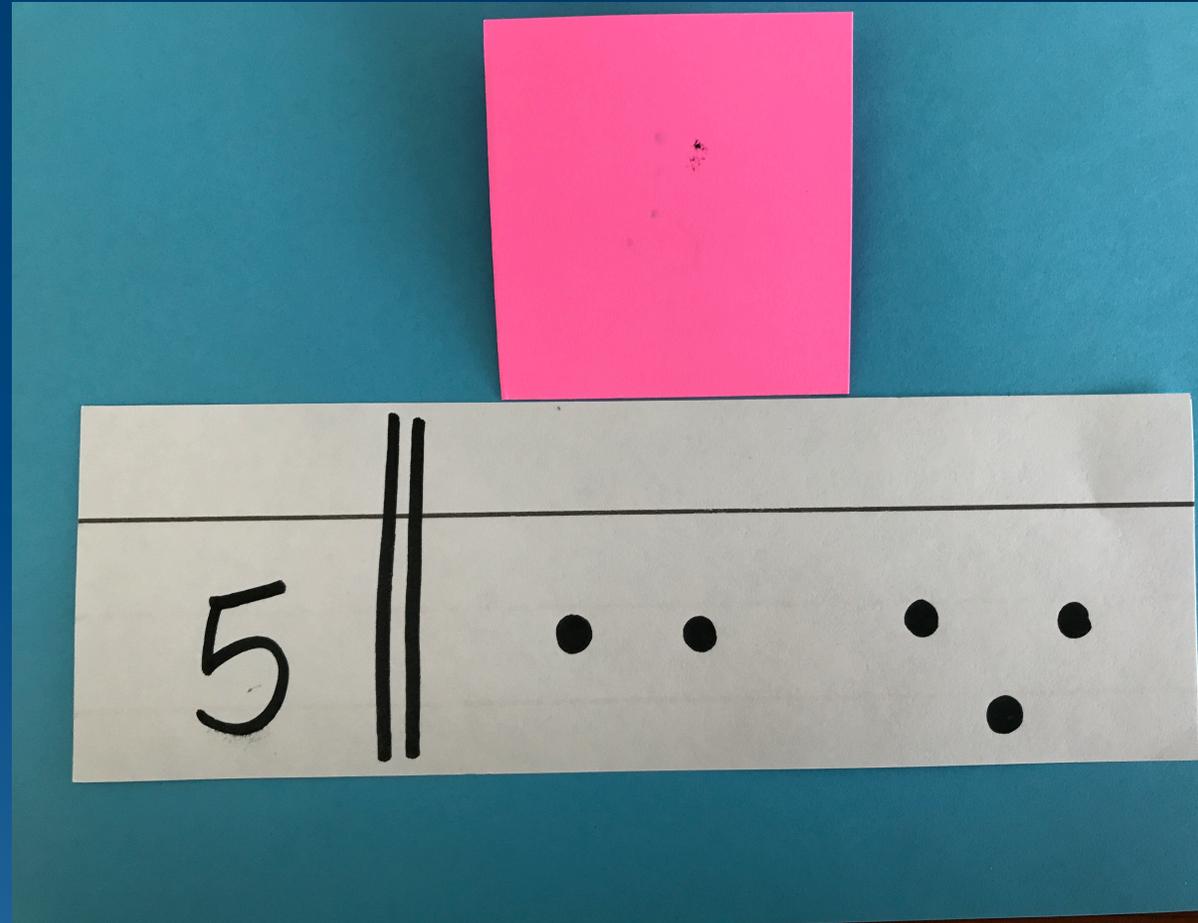
# Bears in a Cave

- Roll to determine how many bears are on a hike. Count out the number of bears.
- Close your eyes. Let your partner hide some of the bears in the cave.
- Determine how many bears are in the cave.
- Explain your thinking with pictures, number sentences, or words.



# Missing Part Sentence Strips

- Identify the number you are trying to make.
- Look at the picture shown and determine how many more you need to make the number.
- Lift up the flap to check your answer.



# Missing Part Sentence Strips

$$465 \parallel \quad ? \quad 400$$

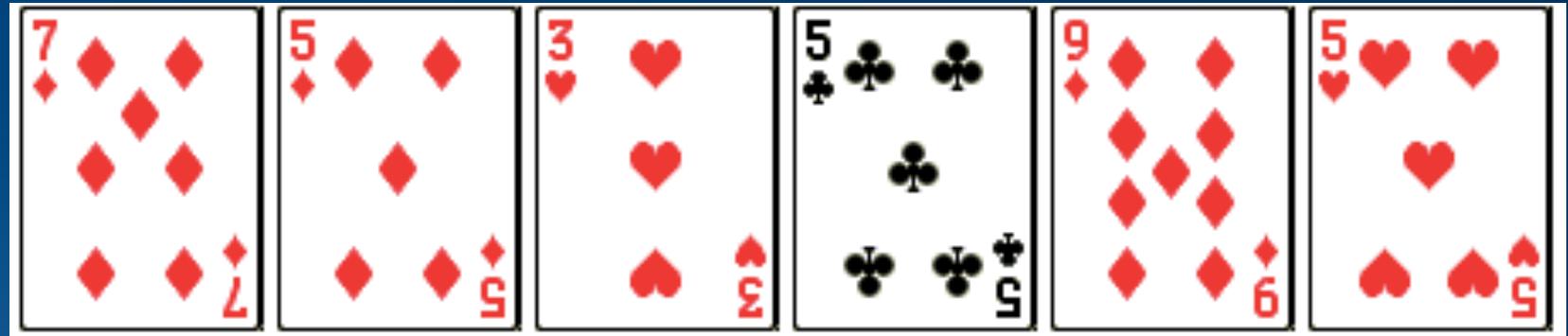
$$\frac{4}{5} = \quad ? \quad + \frac{1}{5}$$

$$0.64 = \quad ? \quad + 0.04$$

$$1 = \quad ? \quad + \frac{2}{3}$$

# Close to 100

- Deal out 6 cards
- Use any 4 cards to make two 2-digit numbers as close to 100 as possible.
- Your score is the difference between 100 and the sum of your two 2-digit numbers
- The person with the lowest score after 5 rounds wins



Score

\_\_\_\_ + \_\_\_\_ = \_\_\_\_

Close to...

20

1000

1

5 cards

8 cards

6 cards

\_\_\_ + \_\_\_ + \_\_\_

\_\_\_ \_\_\_ + \_\_\_ \_\_\_

0. \_\_\_ + 0. \_\_\_

# Four Strikes and You're Out

- Create a blank frame for a problem you know children can solve; one number goes in each blank
- Guess a number. If it is in my problem, I'll record it. If not, you get a strike.

$$\underline{\quad} \underline{\quad} + \underline{\quad} \underline{\quad} = \underline{\quad} \underline{\quad}$$

0 1 2 3 4 5 6 7 8 9

# Tips and Tools

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Concrete  
Objects

Visual  
Representations

Number  
Sentences

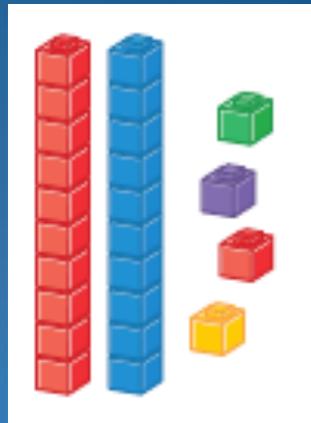


Concrete  
Objects

Visual  
Representations

Number  
Sentences

- Difference between groupable and pre-grouped objects



Concrete  
Objects

Visual  
Representations

Number  
Sentences

- Students aren't "too old" for models
- Possible models:
  - Pan balances
  - Number lines

Concrete  
Objects

Visual  
Representations

Number  
Sentences

$$8 + 4 = \square + 5$$

$$\square \times 6 = 42$$

$$\square = 1,908 - 10$$

- Encourage non-standard equations



# Algebraic Reasoning

Recognizing and describing patterns and relationships between quantities that may be *unknown*

Recognizing and describing patterns and relationships between quantities that are *known*



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