

SMU

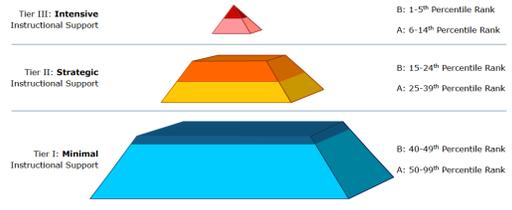
Based on your current level of understanding, what do you think the words **conceptual**, **procedural**, **strategic**, and **adaptive** mean?

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**Assessing Beyond the Algorithm**  
 Research in Mathematics Education Savannah Hill  
 Texas Middle School Association Conference Dawn Woods  
 February 28, 2013 Erica Simon  
 Beth Richardson  
 Cassandra Hatfield

SMU **Response to Intervention Model**



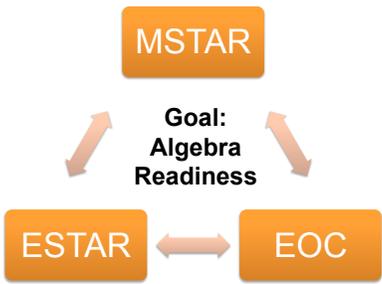
**Tier III: Intensive Instructional Support**  
 B: 1-5<sup>th</sup> Percentile Rank  
 A: 6-14<sup>th</sup> Percentile Rank

**Tier II: Strategic Instructional Support**  
 B: 15-24<sup>th</sup> Percentile Rank  
 A: 25-39<sup>th</sup> Percentile Rank

**Tier I: Minimal Instructional Support**  
 B: 40-49<sup>th</sup> Percentile Rank  
 A: 50-99<sup>th</sup> Percentile Rank

Performance Level	Instructional Need	Level Label	Range of Performance	Level of Additional Instructional Support
Tier III	Intensive Instructional Support	B	1-5 <sup>th</sup> Percentile Rank	Student needs urgent and intensive interventions that are highly specified to his/her individual needs. Additional instructional time is needed. Progress should be frequently and consistently monitored.
		A	6-14 <sup>th</sup> Percentile Rank	Student needs intensive interventions that are highly specified to his/her individual needs. Diagnostic assessments are needed to determine areas in need of improvement. Additional instructional time is needed. Progress should be frequently and consistently monitored.
Tier II	Strategic Instructional Support	B	15-24 <sup>th</sup> Percentile Rank	Student needs supplemental interventions that are targeted to his/her individual needs. Diagnostic assessments are needed to determine areas in need of improvement. Additional instructional time is needed. Progress should be frequently and consistently monitored.
		A	25-39 <sup>th</sup> Percentile Rank	Student needs targeted support including differentiated and scaffolded instruction, additional practice, corrective feedback. Additional instructional time may be warranted. Progress should be closely monitored to evaluate growth.
Tier I	Minimal to No Instructional Support	B	40-49 <sup>th</sup> Percentile Rank	Student needs minimal to no additional instructional support beyond the core instructional program. Student may benefit from differentiated instruction and strategic review to reinforce proficiency. Progress should be closely monitored to evaluate growth.
		A	50-99 <sup>th</sup> Percentile Rank	Student does not need additional instructional support beyond the core instructional program. Student may benefit from differentiated instruction and periodic review to reinforce proficiency.

SMU **Texas Algebra Ready Initiative**

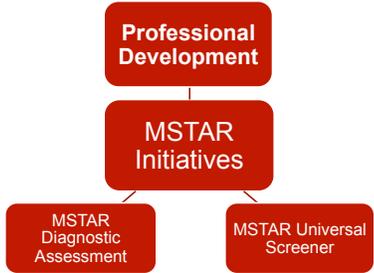


**MSTAR**

**Goal: Algebra Readiness**

**ESTAR** ↔ **EOC**

SMU **MSTAR Initiatives**

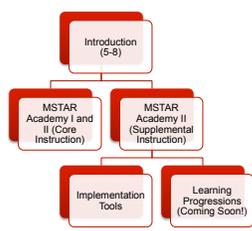


**Professional Development**

**MSTAR Initiatives**

**MSTAR Diagnostic Assessment**      **MSTAR Universal Screener**

### SMU MSTAR Professional Development

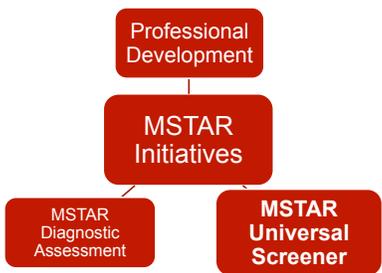


- Provide connections to and strengthen participants' knowledge of CCRS, ELPS, RtI; and
- Available via face-to-face and online professional development courses.

### SMU Additional Online MSTAR Courses

- MSTAR Introduction: An Executive Summary
- MSTAR Academy I: Fraction/Decimal Relationships and Operations
- MSTAR Academy I: Review and Needs Assessment
- MSTAR Academy I: Lesson Study Model Implementation
- More about Tier II for the Math Learner
- Addressing the G/T Math Learner through RtI
- Addressing the College and Career Readiness Standards in Math

### SMU MSTAR Initiatives

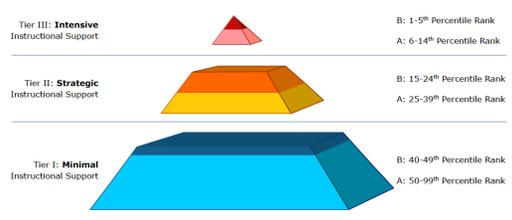


### SMU Purpose of the MSTAR Universal Screener

- Identify students who are at-risk for struggling with algebra-related core instruction
  - Determine **IF** interventions are needed
  - Determine **DEGREE OF INTENSITY** of the intervention needed
  - Monitor students' **RISK STATUS**
- Not intended to provide diagnostic information
- Guides instructional decisions
- Designed to be administered in fall, winter, and spring

Course: MSTAR Universal Screener Overview

### SMU Response to Intervention Model



**Tier III: Intensive Instructional Support**  
B: 1-5<sup>th</sup> Percentile Rank  
A: 6-14<sup>th</sup> Percentile Rank

**Tier II: Strategic Instructional Support**  
B: 15-24<sup>th</sup> Percentile Rank  
A: 25-39<sup>th</sup> Percentile Rank

**Tier I: Minimal Instructional Support**  
B: 40-49<sup>th</sup> Percentile Rank  
A: 50-99<sup>th</sup> Percentile Rank

### SMU Class Summ

• **Tier 3B: Intensive Instructional Support Recommended ( Students)**

Student	Scaled Score	Measurement Error
JONATHAN STUDENT	155	29
JONATHAN STUDENT	155	29
LETICIA STUDENT	155	28
JONATHAN STUDENT	155	29
JONATHAN STUDENT2	14	32
JOSHUA STUDENT	112	32
JOSHUA STUDENT	112	38
JOSHUA STUDENT1	112	38

• **Tier 2B: Strategic Instructional Support Recommended ( Students)**

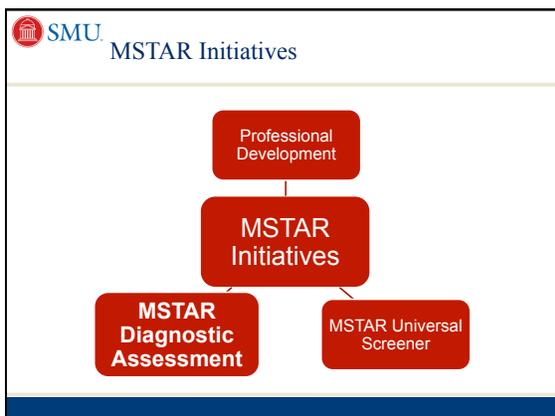
Student	Scaled Score	Measurement Error
JESSIE STUDENT	208	23
JONATHAN STUDENT	208	23
JONATHAN STUDENT	208	23
JONATHAN STUDENT	208	23
JONATHAN STUDENT	208	23

• **Tier 1A: Strategic Instructional Support Recommended ( Students)**

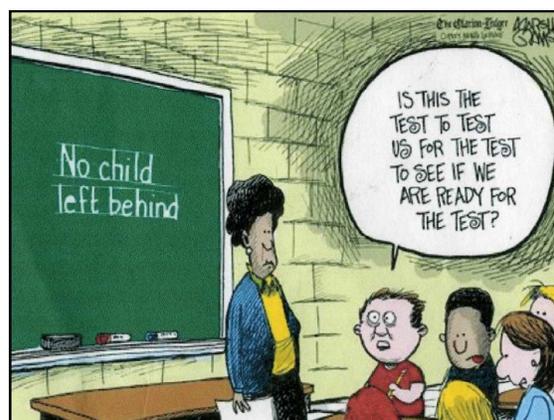
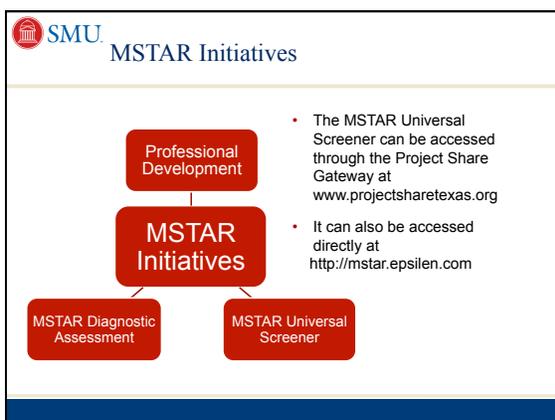
Student	Scaled Score	Measurement Error
JALEN STUDENT1	229	22
JALEN STUDENT	229	22
JAKOB STUDENT	218	23
JASMINE STUDENT	218	23
JAMES STUDENT	218	23
JAYSON STUDENT	218	23
JESSE STUDENT	218	23
JESSIE STUDENT	218	23
JESSICA STUDENT	218	23

• **Tier 1B: Minimal to No Instructional Support Recommended ( Students)**

Student	Scaled Score	Measurement Error
JALEN STUDENT	228	22



- SMU Purpose of the MSTAR Diagnostic Assessment
- Given as needed to address struggling students in Tiers 2 and 3 **after** the MSTAR Screener
  - Identify **WHY** students are struggling with algebra-related core instruction
    - Identify students' current level of understanding in key algebra-related content
    - Identify students' persistent misconceptions in key algebra-related content
  - Provides information that can be used to plan supplemental instruction
  - Not intended to provide screening information



SMU Focus on Assessment

Discussion Points	Outcome
<ul style="list-style-type: none"> <li>Universal Design</li> <li>Levels of Mathematics Proficiency</li> <li>Appropriate Question Stems</li> <li>Answer Choices: Including Student Misconceptions</li> </ul>	<ul style="list-style-type: none"> <li>Understand the anatomy of a test item</li> <li>Write test items at different proficiency levels</li> </ul>

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Assessing Beyond the Algorithm

Universal Design

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"It's not my fault. If a decimal point is so important, they should make it BIGGER!"

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## Universal Design

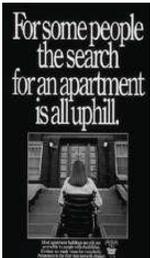
- Originated in the field of architecture
- Features of the environment influence the level of interaction by the user



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## Universal Design

- Characteristics of the users should be considered during the design stages
- Allow for maximum accessibility by the greatest percentage of the population



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## Universal Design for Assessment

- Universal design is a process for ensuring that tests will be developed and administered to provide the widest range of students with the opportunity to demonstrate their construct-relevant skills, knowledge, and abilities, using techniques that do not compromise the validity of inferences drawn from test results.

– UDA Summit Partners (2006)

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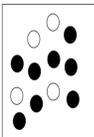
## Principles of Universal Design for Assessment

- Accurately measured construct
- Respect for diversity
- Concise and readable text
- Clear and understandable format
- Visuals support and enhance content; clear and relevant
- Supports accommodations without changing the construct

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## Universally Designed Assessment Items

Traditional Item	Universally Designed Item
<ul style="list-style-type: none"> <li>• Sue picks a ball without looking. What is the probability she will pick a white ball if there are 8 black balls and 4 white balls in a box?</li> </ul>	<ul style="list-style-type: none"> <li>• Sue picks a ball without looking. What is the probability of picking a white ball?</li> </ul>



**SMU. Universally Designed Assessment Items**

Traditional Item	Universally Designed Item
<ul style="list-style-type: none"> <li>Jim and Bob workout at the same gym. Jim wants to bench press more than Bob. The sum of weight Jim and Bob bench press is 180 pounds, and if you subtract what Jim bench presses from what Bob bench presses, you get half of the weight Bob bench presses. How many pounds does Bob bench press?</li> </ul>	<ul style="list-style-type: none"> <li>The sum of Jim and Bob's weight is 180 pounds. If you subtract Jim's weight from Bob's weight, you get half of Bob's weight. How many pounds does Bob weigh?               <ul style="list-style-type: none"> <li>- <math>J + B = 180</math></li> <li>- <math>B - J = \frac{1}{2} B</math></li> <li>- <math>B = ?</math></li> </ul> </li> </ul>

**SMU.**



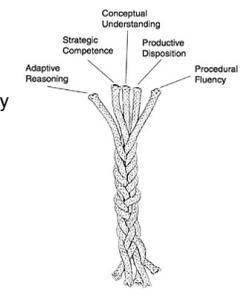
**Assessing Beyond the Algorithm**

Strands of Mathematical Proficiency

**SMU. Item Writing Template**

Item Writing Template			
Course/Grade Level:			
TEKS:			
Cognitive Engagement (Circle One):		Strategic Competence	
Procedural	Conceptual	Adaptive Reasoning	
Relative Difficulty (Circle One):		Difficult	
Easy	Medium		
Question Stem	Response Choices		
	Content/Graphic	Student Misconception(s)	
Stem Graphic	Answer		
	Distractor 1		
	Distractor 2		
	Distractor 3		

**SMU. Strands of Mathematical Proficiency**



- Intertwined Strands of Mathematical Proficiency
- "Interwoven and interdependent"

National Research Council (2001)

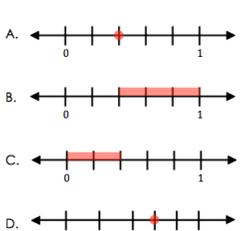
**SMU. Conceptual Understanding**

- Demonstrate an integrated and functional grasp of mathematical ideas
- Understand specific task as it relates to a whole concept
- Find relationships between pieces of information
- Make connections to similar representations
- Use models and multiple representations (e.g. pictures, numbers, real-life situations, words)

**SMU. Conceptual Understanding**

**TEKS 3.3D**  
 The student applies mathematical process standards to represent and explain fractional units.  
 The student is expected to:  
 (D) **Compose and decompose a fraction  $a/b$  with a numerator greater than zero and less than or equal to  $b$  as a sum of parts  $1/b$ .**

Which number line shows  $\frac{3}{5}$ ?



Correct answer: B

 **SMU** Procedural Fluency

- Use formal language or symbolic representations
- Carry out accurate computations
- Follow multiple steps sequentially
- Make proper use of algorithm and properties

 **SMU** Procedural Fluency

TEKS 7.11A  
The student applies mathematical process standards to solve one-variable equations and inequalities. The student is expected to:

**(A) Model and solve one-variable, two-step equations and inequalities.**

Solve for w:

$$\frac{w + 12}{3} = 20$$

A. 72  
B. 58  
C. 48  
D. 16

Correct answer: C

 **SMU** Strategic Competence

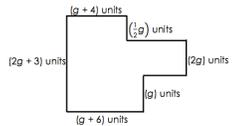
- Ability to formulate a problem in mathematical terms
- Represent problem solving strategically (verbally, symbolically, graphically, or numerically)
- Identify and use strategy necessary to solve problems effectively (e.g. use the distributive property to solve)

 **SMU** Strategic Competence

TEKS 7.11C  
The student applies mathematical process standards to solve one-variable equations and inequalities. The student is expected to:

**(C) Write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationship.**

The perimeter of the figure below is 43 units.



Which equation can be used to solve for the variable, g?

A.  $7.5g + 13 = 43$   
B.  $7g + 13 = 43$   
C.  $5.5g + 13 = 43$   
D.  $5g + 13 = 43$

Correct answer: A

 **SMU** Adaptive Reasoning

- Think logically about a problem, which requires reflecting on various approaches to solve a problem and deductively selecting an approach
- Rationalize and justify strategies
- Appropriately explain a procedure or concept

 **SMU** Adaptive Reasoning

TEKS 3.3H  
The student applies mathematical process standards to represent and explain fractional units. The student is expected to:

**(H) Compare two fractions having the same numerator or denominator in problems by reasoning about their sizes and justifying the conclusion using symbols, words, objects, and pictorial models.**

Why is  $\frac{2}{3} > \frac{2}{7}$ ?

A. 2 is equal to 2 and thirds are larger than sevenths.  
B. 2 is equal to 2 and thirds are smaller than sevenths.  
C. 2 is equal to 2 and 3 is smaller than 7.  
D. 2 is equal to 2 and 3 is greater than 7.

Correct answer: A

**SMU Gallery Walk**

- In your packet of materials is a 4 x 3 matrix with the 4 strands of mathematical proficiency along the top.
- Around the room are 12 problems written to align with the 6<sup>th</sup> grade TEKS.
- As you read each item, determine the strand of mathematical proficiency and relative level of difficulty.
- Write the number associated with the item in the appropriate cell.

**SMU Gallery Walk – Answer Key**

	Procedural	Conceptual	Strategic	Adaptive
	5	3	1	12
	10	6	4	7
	8	11	9	2

**SMU Assessing Beyond the Algorithm**



Difficulty Levels

**SMU Level of Difficulty**

- ◆ Easy
  - Basic Knowledge
  - Skills that are familiar to students
  - Sometimes conceptually based
- ◆ Medium
- ◆ Difficult
  - Skills that are peripheral to curriculum
  - Not all students will have mastered these

(Leong, 2006)

**SMU Procedural**

Medium	Simplify the expression: $11x + 5y - 2y + 4x$	$15x + 3y$
		$16x + 2y$
		$18xy$
		$11x + 7y$
Easy	Simplify the expression: $7x - 3x + 3$	$4x + 3$
		$7x$
		$-4x + 3$
		$13x$
Difficult	Simplify the expression: $4(3r + 2) + 5r$	$17r + 8$
		$32r + 8$
		$12r + 6$
		$17r + 2$

**SMU Conceptual**

Difficult	Which expression is equivalent? $(x + 2)(x + 7)$	$x^2 + 14$
		$(x^2 + 7x) + (2x + 14)$
		$7x + 2x$
		$(x + x + 7) + (2 + x + 7)$
Easy	Which expression is equivalent? $(5 + 3) \times 7$	$(5 + 7) + (3 + 7)$
		$(5 \times 3) + (5 \times 7)$
		$(5 \times 3) + (3 \times 7)$
		$(5 \times 7) + (3 \times 7)$
Medium	Which expression is equivalent? $2.1 \times 3.5$	$(2 \times 3.5) + (0.1 \times 3.5)$
		$(2 + 3.5) + (0.1 + 3.5)$
		$(2 \times 3) + (0.1 \times 0.5)$
		$(2 \times 0.5) + (0.1 \times 3)$

**SMU Strategic**

Easy	The length of John's backyard is 50 feet. Which expression can be used to find the length of John's backyard in inches?	$50 \times 12$
		$50 \div 12$
		$50 + 12$
		$50 - 12$
Medium	Jake reads 3 pages in 1 minute. At this rate, which expression can be used to find how many pages Jake can read in 1 hour?	$\frac{3 \text{ pages}}{1 \text{ min}} \times \frac{1 \text{ hour}}{60 \text{ min}}$
		$\frac{1 \text{ min}}{3 \text{ pages}} \times \frac{1 \text{ hour}}{60 \text{ min}}$
		$\frac{3 \text{ pages}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hour}}$
		$\frac{1 \text{ min}}{3 \text{ pages}} \times \frac{60 \text{ min}}{1 \text{ hour}}$
		$\frac{1 \text{ min}}{3 \text{ pages}} \times \frac{1 \text{ hour}}{60 \text{ min}}$
Difficult	A class has 12 girls and 16 boys. Which expression can be used to find what percentage of the students in the class are boys?	$\frac{16}{28} \times 100\%$
		$\frac{12}{28} \times 100\%$
		$\frac{12}{16} \times 100\%$
		$\frac{16}{28} \div 100\%$

**SMU Strategic**

Easy	Jane wants to shade $\frac{3}{4}$ of the model. Which explanation describes why she multiplies $\frac{3}{4} \times \frac{1}{4}$ ?		She is finding an equivalent fraction.
			She is simplifying the fraction.
Difficult	Which explanation best describes why Model A represents a greater fraction?	The shaded portion of Model A covers more of the total area than Model B.	
		The total area of Model A is larger than the total area of Model B.	
		The squares are larger in Model A than the squares in Model B.	
Medium	Doug has 4 fish and 2 dogs. He buys another fish. How does the additional fish change the ratio of dogs to fish?	There are fewer un-shaded squares in Model A than in Model B.	
		The ratio gets smaller because only the denominator increases.	
		The ratio gets larger because the total number of pets increases.	

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**Assessing Beyond the Algorithm**  
Assessment Item Development

**SMU Guidelines for Item Development**

- Item writing requires careful consideration of:
  - general item-writing procedures
  - overall content of the items
  - response options in multiple choice items

**SMU General Item-Writing (Procedures)**

- Avoid the complex multiple-choice format. (i.e., A and D, B and C).
- Use plain language.
  - Avoid conditional phrases (if..., then...).
- Keep the language of the stem and response options at the appropriate grade level.
- Minimize examinee reading time.

(Haladyna, 2004)

**SMU Stem Development**

- State the stem in question form.
  - If completion format is necessary, do not leave a blank for completion in the beginning or middle of the stem.
- Avoid extraneous information.
- Word the stem positively.
  - If an item must be stated negatively, underline or capitalize the negative word.
- Keep all essential information in the stem.
  - Items that require students to read and evaluate each response option prior to selecting an answer increase the cognitive load required.

(Haladyna, 2004)

**SMU** General Item-Writing (Content)

- Base each item on important content to learn; avoid trivial content.
- Keep the content of each item independent from content of other items on the test.
- Avoid cuing one item with another; keep items independent of one another.
- Avoid items based on opinions.
- Develop items that measure higher-level thinking.
- Avoid potentially insensitive content or language.
- Use present tense.

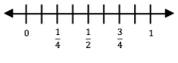
(Haladyna, 2004)

**SMU** Response Development

- Make all distractors plausible.
  - Create distractors that represent common misconceptions may have about the content being assessed
- Keep all options in an item homogenous in content and grammatical structure.
- Keep the length of options brief and fairly consistent.
- Phrase options positively, not negatively.

(Haladyna, 2004)

**SMU** Example of a Well-Written Item

Item Writing Template	
Course/Grade Level: 3 <sup>rd</sup> Grade	
TEKS: The student is expected to determine the corresponding fraction greater than zero and less than or equal to one with denominators of 2, 3, 4, 6, and 8 given a specified point on a number line.	
1	
Cognitive Engagement (Circle One): Procedural    Conceptual    Strategic Competence    Adaptive Reasoning	
Relative Difficulty (Circle One): Easy    Medium    Difficult	
Question Stem	Response Choices
Which fraction is greater than zero but less than $\frac{1}{2}$ ?	Answer 1 4
	Distractor 1 2 May not understand that $\frac{1}{2}$ is equal to $\frac{2}{4}$ .
Stem Graphic 	Distractor 3 4 Found fraction greater than zero but greater $\frac{1}{2}$ .
	Distractor 1 1 May not understand that $\frac{1}{1}$ names the whole as a fraction.

**SMU** Uh Oh ☹️  
Examples of Poorly Written Math Items

Dan enjoys going to the park. The park has lots of trees and a pond. Dan goes to the park at 10:15 a.m., plays on the swings, slides down the slide, and runs around the pond. He is hungry when he finally makes it home 1 hour 55 minutes later. What time did he arrive at home?

A. 12:00 p.m.  
B. 12:10 p.m.  
C. 8:20 a.m.  
D. 6:00 p.m.

**SMU** Uh Oh ☹️  
Examples of Poorly Written Math Items

Sue has a box of 2 stars and 2 circles. She wants to make Michelle's box of 6 stars and 3 circles proportional to her box. How many stars does she need to add to her box to make Michelle's box proportional to hers?

A. 6  
B. 0  
C. 2  
D. 20



**SMU** Item Writing Practice

- 6.b.4.F
  - The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to represent benchmark fractions and percents such as 1%, 10%, 25%, 33 1/3%, and multiples of these values using 10 by 10 grids, strip diagrams, number lines, and numbers
- 7.b.3.B
  - The student applies mathematical process standards to add, subtract, multiply, and divide while solving problems and justifying solutions. The student is expected to apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers.
- 8.b.6.C
  - The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas. The student is expected to use models and diagrams to explain the Pythagorean theorem

 **SMU** Some Final Thoughts & Take-Aways

- **Assessment and instruction should be considered together** – performance on assessments can inform instruction and assessments can be specifically designed to provide students with opportunities to demonstrate what they've learned during instruction.
- When designing tests or assessments for use in your classroom, be sure to include items that target **multiple levels of Strands of Mathematical Proficiency**.
- Revisit the **guidelines for item development** as often as needed to ensure that the items you write provide students with the best opportunity possible to demonstrate their knowledge and understanding of the content.

 **SMU** Get Involved With RME!

RME is always looking for qualified mathematicians, math teachers, and math coaches to partner with us in many ways.

- Item Writing
  - These items range all grades from pre-k to 8th grade, and are written to align with multiple mathematics content standards. RME provides training and writing can be done on-site or off campus.
- Item Reviewers
  - We review an item for language, visual representation, and mathematical content including vocabulary and concepts. In addition, reviewers examine each item for potential bias and to evaluate the effectiveness of the distractors.

 **SMU** Get Involved With RME!

[www.smu.edu/RME](http://www.smu.edu/RME)

Email: [RME@smu.edu](mailto:RME@smu.edu)

 [@RME\\_SMU](https://twitter.com/RME_SMU)

 **SMU Research in Mathematics**

Education- RME

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