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RESEARCH IN MATHEMATICS EDUCATION

Imagination Station (Istation): Universal Screener Instrument Development for Grades PK-1

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Spring 2015

Published by

Southern Methodist University

The Annette Caldwell Simmons School of Education & Human Development

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This research was supported by Imagination Station, Inc. Opinions expressed herein do not necessarily reflect those of Imagination Station or individuals within.

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Abstract

In this technical report, Research in Mathematics Education (RME) describes the development of the formative assessment item bank for pre-kindergarten (PK) through Grade 1 for Imagination Station (Istation). The formative assessment item bank will be used to deliver a computerized adaptive universal screening assessment to support teachers' instructional decision making. State and national mathematics content standards for PK–1 grades inform the mathematics topics underlying the items. In this technical report, we describe the process used to identify and sample the mathematics content and the levels of cognitive complexity assessed in the item bank. Next, we describe the item writing procedures. Finally, we describe how the external item review process and outcomes contribute to content-related evidence for validity.

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Imagination Station (Istation): Universal Screener Instrument Development for Grades PK–1

Introduction

The purpose of the pre-kindergarten (PK) through Grade 1 universal screening assessment for Imagination Station (Istation) is to support teachers' instructional decision-making. Items from the formative assessment item bank will be administered via a computerized adaptive universal screening assessment system to identify students' understanding of fundamental mathematics skills and grade level standards. A separate item bank was developed for each grade level. By administering this assessment system, teachers and administrators can use the results to answer two questions: (1) Are students at risk of mathematics failure in PK, kindergarten (K) and Grade 1 mathematics and (2) What level of instructional supports do students need to be successful in grades PK–1 mathematics? Multiple administrations of the universal screener (i.e., fall, winter, and early spring each year) will provide teachers with meaningful information about students' learning over time to support instructional decision-making over the course of grades PK–1. The universal screener is designed for administration to all students receiving grade-level instruction.

The purpose of this technical report is to describe the development of the formative assessment item bank. This description includes (a) the process used to identify and sample the mathematics content assessed in the item bank, (b) the item writing process, and (c) the external review process and results. The test development steps used to create the formative assessment item bank represent best practices in test development and align with the Test Standards published by the American Educational Research Association (AERA), American Psychological Association (APA), and National Council on Measurement in Education (NCME) (1999).

Construct Definition

The assessed construct for the Istation PK-1 universal screener consists of (a) mathematics content and (b) levels of cognitive engagement (National Research Council [NRC], 2001). The mathematics content of the grades PK-1 formative assessment item bank is based on the Curriculum Focal Points (CFP) (National Council of Teachers of Mathematics [NCTM], 2006), Common Core State Standards for Mathematics (CCSSM) for kindergarten (K)–Grade 1, and state mathematics content standards from California, Texas, Florida, New York, and Virginia. The referents for these content standards are reported in Appendix A. The CCSSM and state content standards were aligned to the CFPs put forth by the NCTM (2006) to ensure the most comprehensive representation of content across the multiple standards documents. Consequently, items for the Istation PK-1 universal screener were written to one of three NCTM CFPs: (1) Numbers and Operations, (2) Geometry and Spatial Reasoning, and (3) Measurement and Data. We created a fourth CFP (Algebra Standards) for each grade level to include state and CCSSM standards that were not represented in the NCTM focal points.

To develop the assessment blueprint, experienced mathematics educators and mathematics researchers engaged in discussions and worked iteratively to align the state content standards to the CFPs and to identify each content standard as *high-priority*, *secondary*, or *not assessable*. *High-priority* standards are skills that are critical to assess because they serve as the foundation for skills that are essential to a student’s future success with mathematics (e.g., composing and decomposing numbers from 11 to 19 in kindergarten). *Secondary* standards refer to skills that are important for grades PK–1 mathematics but are not as foundational for future mathematics understanding, may be more narrowly defined than *high-priority* skills, and/or support a high-priority standard (e.g., determining how many more to make 10 in kindergarten). *Not assessable* standards are skills that could not be assessed using an online multiple-choice assessment. These determinations were made based on expert opinion and were verified by at least two members of the RME staff, whom all have extensive mathematics education experience. We report the distribution of items by grade level, CFP and standard (e.g., high-priority, secondary, and not assessable) in Table 1.

The blueprinting team also consulted with an expert with extensive experience in assessment development for early grades mathematics and web-based learning environments to research and understand developmentally appropriate interactions for students in the web-based environment. For more information on the interfaces, please see the Imagination Station (Istation): Universal Screener and Inventory Instruments Interface Development for Grades PK-1 technical report (Hatfield, Perry, Basaraba, Miller, Simon, & Ketterlin-Geller, 2014).

Levels of Cognitive Engagement

The cognitive engagement dimension of the construct refers to the level of cognitive processing through which students are expected to engage with the content. The formative assessment item bank uses the taxonomy of cognitive engagement in mathematics published by the NRC (2001). The taxonomy consists of five interdependent strands that promote mathematical proficiency: (a) conceptual understanding, (b) procedural fluency, (c) strategic competence, (d) adaptive reasoning, and (e) productive disposition. Items in the formative assessment item bank assess student understanding of the content at four levels of cognitive engagement; therefore items were not written to assess a student’s productive disposition of mathematics. A brief description of each assessed level follows:

- *Conceptual understanding* refers to the functional grasp of mathematics that a student applies to concepts, operations, and relations. It involves being able to logically organize one’s knowledge to integrate and understand concepts as part of a coherent whole.
- *Procedural fluency* refers to students’ ability to accurately and appropriately carry out skills, including being able to select efficient and flexible approaches.
- *Strategic competence* involves one’s ability to formulate a problem in mathematical terms, to represent it strategically (verbally, symbolically, graphically, or numerically), as well as to solve it effectively. It is similar to problem solving and problem formation.

- *Adaptive reasoning* involves the student’s capacity to think logically about a problem, which requires reflecting on various approaches to solve a problem and deductively selecting an approach. Students who are able to do this are also able to rationalize and justify their strategy.

Each CFP was assessed by items targeting the four levels of cognitive engagement. Conceptual understanding and procedural fluency were oversampled to accurately reflect the relative emphasis in the content standards at grades PK–1. Easy, medium, and difficult items were written for each high-priority standard in each CFP. The level of difficulty of each item is a relative description that is subject to change with empirical analyses.

For PK, nine items were written for each high-priority standard, and 10 items were written for each secondary standard. For K, 10 items were written for each high-priority standard, and 2-7 items were written for each secondary standard. For Grade 1, 9–10 items were written for each high-priority standard, and six items were written for each secondary standard. The content sampling matrix is presented in Figure 1.

Item Writing

Item Specifications

Approximately 400 interactive items were written for each grade (PK, K, 1). Items were written for dichotomous scoring as either correct or incorrect. The majority of items and interfaces use visual representations and a variety of interactive features (e.g., drag and drop, multi-select) to minimize extraneous student reading in the item stem and response options, minimizing the possibility of construct-irrelevant variance introduced by inadvertently assessing students’ decoding and reading comprehension skills. Text in the item stem and response options was reviewed to ensure that the language was grade-level appropriate and decodable. Whenever possible, plain language and simple, straightforward statements were incorporated into the items. Graphics were used in instances where they explained the problem, provided a visual clue to clarify the context, or were integral to the stem or answer choices. The distractors for each item were designed to represent plausible misconceptions or errors in students’ conceptual or procedural understanding.

The assessment items were written according to the principles of universal design for assessment (see Ketterlin-Geller, 2005; 2008) and are amenable to accommodations. As delivered, the formative assessment system will include a read-aloud feature to support item readability. This ensures that mathematics knowledge and skills are tested, rather than students’ reading skills.

The computerized-adaptive test can be administered individually or in a group in an untimed setting.

Item Writers

Eight item writers contributed to the PK–1 formative assessment item bank. In the following paragraphs we describe each item writer’s qualifications and relevant previous experience.

Item Writer 1 holds a Master’s degree in Early Childhood Education, a Bachelor’s degree in Business Administration, and credentials in General Education (EC-4), Gifted and Talented, and English as a Second Language. She has five years of experience as a kindergarten teacher.

Item Writer 2 holds a Master’s degree in Education and Bachelor’s degrees in Sociology and Education. She holds a General Education certification for EC-4. She has 12 years of teaching experience and has taught students in Grades K-5. She also has experience developing assessments and editing and reviewing district level curriculum.

Item Writer 3 holds a Master’s degree in Special Education and a Bachelor’s degree in Environmental Horticultural Science. He has seven years of experience developing alternate assessments for students in Grades K-12 with significant cognitive disabilities. He is currently a research assistant on multiple mathematics assessment projects for Grades PK-8.

Item Writer 4 holds a Bachelor’s degree in Interdisciplinary Studies with emphases in Early Childhood and Mathematics. She holds credentials in General Education (PK-6), Mathematics (1-8), Technology Applications (EC-12), English as a Second Language (EC-4), and Gifted and Talented (EC-12). She has 15 years of experience in kindergarten, first, and fifth grade. She is currently a fourth and fifth grade mathematics teacher.

Item Writer 5 holds a Bachelor’s degree in Education and is currently pursuing her Master’s degree in Education, specializing in English as a Second Language (ESL) and Mathematics. She holds teaching credentials in General Education (EC-6), Social Studies (Grades 4-8), Mathematics (Grades 4-8), and ESL. She has 15 years of experience teaching EC-6 students as well as students in Grades 6-8 requiring intensive intervention and support.

Item Writer 6 holds a Bachelor’s of Science in General Education and holds certifications in General Education K-5, ESL, Special Education EC-12, and has a Special Education and diagnostician certification. She has eight years of experience as a first grade teacher. She is currently a special education teacher.

Item Writer 7 holds a Bachelor’s degree in Science. She holds certifications in Special Education, English as a Second Language and K-4. She has five years of experience teaching kindergarten and 4th grade. She currently teaches first grade students.

Item Writer 8 holds a Bachelor’s degree in Elementary Education and a General Education credential. She has seven years of experience in education as a private tutor, student teacher, and in her current role as a kindergarten teacher.

Item Writing Training

All item writers participated in a one-day, face-to-face, item-writing workshop held by the research team from RME at Southern Methodist University. During the workshop, item writers learned how to write items that aligned with the content expectations and item specifications for

grades PK–1, and assessed similar mathematical constructs across multiple interfaces within the Istation PK-1 assessment delivery system. Item writers received training from recognized experts in item design, including information about the elements of high-quality test design, how to write high-quality mathematics items, levels of cognitive engagement, and principles of universal design. The writers were also trained on each of the 21 Istation interfaces that could be used to test the content standards. The functionality of each interface was discussed, and videos were provided to assist the writers in understanding how to input items into the online system once they were written. Additionally, writers examined the content blueprint and identified which interfaces could be used to assess each standard. Item writers were provided an item writing guide that facilitated the training and was available for reference throughout the item writing process.

Item Writing Process

After participating in the item-writing training, item writers were given the item-writing spreadsheet matching the standards to the possible interfaces that could be used to write the items. Writers came to RME on different days to work in small groups; however, each writer was responsible for creating his/her unique items. Staff was available to assist the writers and to answer any questions they had.

Writers wrote in sets of 18 items. Upon completion of the items, item writers submitted items to researchers and project staff for review through the online system (i.e., the MIX). Each item submitted during the workshop was reviewed by at least one project team member to ensure that the item included all of the required components. Reviewers evaluated items for (a) mathematical accuracy, (b) alignment with the mathematics content standards, (c) age-appropriateness of language and graphics, and (d) compliance with principles of universal design. Items requiring revision with regard to these components were returned to the item writers with specific comments and suggestions for revising the item; once revised, item writers resubmitted their items for approval. All finalized items were cross-referenced to the test blueprint and specifically to the mathematics content standards to ensure that the content was adequately represented.

Next, Istation created the graphics for the items, and RME staff reviewed the items again for alignment with content standards, appropriateness of language and graphics, and compliance with principles of universal design. The number of items written by interface and the percentage of total items within each interface is reported in Table 2.

Content-Related Evidence for Validity

Mathematics education experts and mathematics teachers evaluated all items for accuracy and appropriateness of the content written for the formative assessment item bank for students in grades PK–1. In the sections that follow we present the results of the external review of the items first by person (e.g., Mathematics Education Expert 1, Mathematics Education Expert 2, etc.) to ensure that one reviewer was not more stringent or lenient in his or her ratings than another, and then by interface. The use of multiple interfaces introduced another source of potential variability in the mathematical accuracy and appropriateness of the methods for assessing students’

mathematical understanding. As such, we also chose to examine the results of the external review by interface to ensure that some interfaces did not receive consistently lower ratings for mathematical accuracy or appropriateness than others.

We present the results of the review by Mathematics Education Experts of the items first, followed by the results of the Teacher Review.

Mathematics Education Expert Review

Three mathematicians reviewed all items in Grades PK–1.

Mathematics Education Expert 1 holds a Doctoral degree in Curriculum and Instruction in Mathematics Education, a Master’s degree in Middle Level Education, and a Bachelor’s degree in Elementary Education. He has 13 years of experience as a classroom teacher and teacher educator. He also has experience developing assessments on quantitative literacy and mathematical knowledge for teaching Geometry. He is currently an assistant professor of mathematics.

Mathematics Education Expert 2 holds a Master’s degree in Special Education and a Bachelor’s degree in History. She has 11 years of combined experience in education as a resource specialist (K-12), teaching assistant, assessment evaluator, and program administrator intern. She has experience teaching special education students and struggling learners and developing and evaluating assessments. She is currently pursuing a doctoral degree.

Mathematics Education Expert 3 holds a Master’s degree in Mathematics and a Bachelor’s degree in Mathematics and Psychology. She has seven years experience in education teaching college level mathematics courses. She is currently an adjunct instructor of mathematics education.

The Mathematics Education Experts were each asked to review items and evaluate the accuracy of the content, precision of the vocabulary, and appropriateness (and plausibility) of distractors. Specifically, Mathematics Education Expert 1 reviewed 458 items, Mathematics Education Expert 2 reviewed 304 items, and Mathematics Education Expert 3 reviewed 416 items. The criteria used for item evaluation were as follows:

- *Mathematical accuracy of content*: Is the item mathematically accurate?
- *Precision of mathematical vocabulary*: Is the mathematical vocabulary used accurately? Is the mathematical vocabulary precise?
- *Appropriateness of the distractors*: Most students use a process of elimination to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions from choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Are the distractors mathematically plausible misconceptions?

Items were evaluated on a 4-point scale for each criterion. A rating of 1 indicated that the item was not at all accurate, precise, or appropriate; a rating of 2 indicated that the item was somewhat accurate, precise, or appropriate; a rating of 3 indicated that the item was mostly accurate, precise, or appropriate; and a rating of 4 indicated the item was extremely accurate, precise, or appropriate. In instances where the reviewer assigned a score of 1 or 2 for any criterion, recommendations were solicited that would aid in revision.

Overall, the Mathematics Education Experts rated the majority of items as being mostly or extremely mathematically accurate (93.3% of all items), having mostly or extremely appropriate mathematical vocabulary (94.1% of all items), and having mostly or extremely plausible distractors (90.8% of all items). Their ratings are reported in Table 3. Of the items that were rated as somewhat or not at all appropriate, 75% of these issues were with respect to language (e.g., lack of clarity, words missing in the item, etc.), 1.4% of the issues were with respect to the clarity of the graphics in the items, and 1.4% of issues were with respect to the plausibility of the distractors (mathematicians suggested other plausible distractors in lieu of the ones included in the items). In addition, 3.1% of the items were rated low with respect to mathematical accuracy and for 1.7% of those items ($n = 20$) the Mathematics Education Experts questioned whether the correct response was, indeed, correct (or if there was a correct response at all). RME staff reviewed the Mathematics Education Experts ratings and made changes where appropriate. In some instances, some of the changes with respect to language were applied across an entire interface for consistency.

Finally, 4.7% of items received lower ratings due to issues with the interface. We present results of the Mathematics Education Experts' reviews of the items by interface with respect to mathematical accuracy and appropriateness in Table 4. Mathematics Education Experts rated the majority of items as extremely or mostly mathematically accurate and as having extremely or mostly appropriate mathematical vocabulary and distractors for 20 of the 21 interfaces. More items in the Image Multiple Choice, Media with Response, and Multi-Select interfaces received ratings of somewhat or not at all accurate/appropriate; however, this is not surprising given that there were significantly more items in these interfaces (96 – 451 items) than there were in the other interfaces. The one interface that received consistently low (i.e. not at all accurate or appropriate) ratings was the Measurement interface. The concerns with the Measurement interface were either validated or invalidated through usability testing. A list of recommendations to address the graphics and functionality of the interfaces was also provided to Istation based on the results of the review (Hatfield et al., 2014).

Mathematics Teacher Review

Three teachers with elementary mathematics experience reviewed the items.

Mathematics Teacher Reviewer 1 holds a Master's degree in Special Education, a Bachelor's degree in Human Development and Family Sciences, and Initial II Teaching Licenses in Special Education and Elementary Education. She has 13 years of experience as a district math coordinator and Special Education teacher. She has co-authored a first grade math intervention curriculum.

Mathematics Teacher Reviewer 2 holds a Bachelor's degree in Applied Learning and Development. She holds certifications in Early Childhood (PK-K), Elementary History (Grades 1-8) and Elementary Self-Contained (Grades 1-8). She has 15 years of experience as an elementary teacher. She currently teaches first grade.

Mathematics Teacher Reviewer 3 holds a Bachelor's degree in Education and has six years of experience teaching Grades 1-5. She has also served as a member of a mathematics curriculum alignment committee and worked as a Title 1 reading tutor for four years.

The Mathematics Teacher Reviewers were asked to review and analyze each item for appropriate (a) language, (b) mathematical vocabulary, (c) content or concepts, and (d) visual representations. Specifically, Teacher Reviewer 1 reviewed 299 items, Teacher Reviewer 2 reviewed 592 items, and Teacher Reviewer 3 reviewed 300 items. Teacher Reviewer 2 agreed to review more items since another reviewer had to leave the project for personal reasons. The criteria presented for item evaluation were as follows:

- *Appropriateness of language:* Is the language used in the item appropriate for students in your grade level?
- *Appropriateness of mathematical vocabulary:* Is the mathematical vocabulary representative of pre-requisite or instructional expectations in your grade level? Does the item use mathematical vocabulary students in your grade level will be familiar with?
- *Appropriateness of content or concepts:* Is the task representative of prerequisite or instructional expectations in your grade level?
- *Appropriateness of visual representations:* Is the visual representation (i.e., graphic, table, image) used in the item appropriate for students in your grade level? Can students in your grade level understand the meaning of the visual representation? Is the visual representation of the item clear?

The items were rated on a scale of 1 to 4 for each criterion. A rating of 1 indicated that the item/distractors were *not at all* appropriate based on the criterion; a rating of 2 indicated that the item/distractors were *somewhat* appropriate based on the criterion; a rating of 3 indicated that the item/distractors were *mostly* appropriate based on the criterion; and a rating of 4 indicated that the item/distractors were *extremely* appropriate based on the criterion. In instances where the teachers provided a rating of 2 or lower, they were asked to provide additional suggestions and comments to improve the item.

Mathematics Teacher Reviewers also analyzed each item for potential bias in language and/or content. The criterion for potential bias were as follows:

- *Bias in language or content:* Does the item require background knowledge unrelated to the concept being tested that would differ for students with different backgrounds? Is the language sensitive to students from diverse backgrounds, students with limited English proficiency and students with special needs? Example: "What is the most appropriate

measurement unit for the length of a **sub** or **hoagie**?” may be unfair for students in certain geographic regions and students with diverse background who are unfamiliar with these terms.

Mathematics Teacher Reviewers were asked to rate each item as *not biased*, *somewhat biased*, or *biased*. In instances where the teachers identified items as *biased*, they were asked to provide additional suggestions and comments to improve the item.

Overall, the Teacher Reviewers rated the majority of items as having *mostly* or *extremely* appropriate mathematical language (99.7% of all items), *mostly* or *extremely* appropriate mathematical vocabulary (99.4% of all items), *mostly* or *extremely* appropriate mathematical content (99.4% of all items), and as having *mostly* or *extremely* appropriate visual representation (93.1%). Their ratings are reported in Table 5. Of the items that were rated as *somewhat* or *not at all* appropriate, 0.3% of these issues were with respect to language (e.g., lack of clarity, words missing in the item, etc.), 0.6% were with respect to the appropriateness and clarity of the mathematical vocabulary, and 0.6% were with respect to the appropriateness of the mathematical content. In addition, 6.8% of the items were rated low with respect to the appropriateness of the visual representations; of these 81 items, reviewers observed that an incorrect graphic was pictured for the item (4.9%), the graphic was unclear (e.g., the shopping cart pictured was not clearly a shopping cart) (8.9%), items pictured were misrepresented with respect to how they would appear in the real-world (e.g, size of objects in relation to one another) (11.1%), graphics were mathematically inaccurate (2.5%), or no graphics were visible (2.5%).

Items that received low ratings for visual representations (60.5% of the 81 items) were due to issues observed in some of the interfaces (e.g., highlighting of selected items was difficult to see, unclear what to do with the numeric keypad when it appeared, etc.). We present results of the Teacher Reviewers’ review of the items for the appropriateness of mathematical language, vocabulary, content, and visual representations by interface in Table 6. Similar to the Mathematics Education Experts, teacher reviewers rated the majority of items as having *extremely* or *mostly* appropriate mathematical language, vocabulary, content or concepts, and visual representations for all interfaces. Items within the Multi-Select interface received the greatest number of *somewhat* or *not at all* appropriate visual representations (38% of items); these ratings were due to the Teacher Reviewers’ observations that the highlighting that appeared when selecting objects within the interface was difficult to see. To address this concern, the assessment coordinator and principal investigator met with Istation to inform them of reviewers’ concerns. During the meeting recommendations were made and Istation agreed to modify the Multi-Select interface to improve the interaction students would have.

With respect to bias, Teacher Reviewers indicated they felt the majority of items (93.2%) were *not biased*. However, they did rate 4.6% of items as having *some bias* and 1.2% of items as being *biased*. Some issues that prompted teachers to rate the items as having *some bias* included the visual representation of the shopping cart and the use of some mathematical language that might not be familiar to all students. We made efforts to address the comments in cases where teachers thought the item was *biased* or had *some bias*.

Following reconciliation of the feedback from the Mathematics Education Experts and Teacher Reviewers, RME staff conducted one final review of the items. During this review, we examined

the items for appropriateness of mathematical language and vocabulary, conciseness, decodability of text, and clarity of visual representations. We also examined all items by interface to ensure that items assessing similar content were consistent in their phrasing and representation. Finally, we reviewed all items for mathematical accuracy to ensure there was one correct response for each item.

Conclusion

The purpose of this technical report was to describe the development of the formative assessment item bank for the Istation PK-1 Universal Screener Assessment. We described the components of the assessed construct—content standards and levels of cognitive complexity—and the process for sampling the content assessed in the formative assessment item bank. Next, we described the item writing procedures and provided the qualifications for the item writers. Finally, we documented the process and outcomes of an external item review by mathematicians and mathematics teachers to contribute to content-related evidence for validity.

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Table 1*Content Sampling Matrix*

| CFP | Prekindergarten | | | Kindergarten | | | Grade 1 | | |
|-------|-----------------|-----------|-------|---------------|-----------|-------|---------------|-----------|-------|
| | High Priority | Secondary | Total | High Priority | Secondary | Total | High Priority | Secondary | Total |
| 1 | 140 | 22 | 162 | 119 | 28 | 147 | 99 | 16 | 115 |
| 2 | 44 | 30 | 74 | 85 | 31 | 116 | 80 | 16 | 96 |
| 3 | 67 | 56 | 123 | 65 | 42 | 107 | 57 | 0 | 57 |
| 4 | 36 | 0 | 36 | 30 | 0 | 30 | 65 | 64 | 129 |
| Total | 287 | 108 | 395 | 299 | 101 | 400 | 301 | 96 | 397 |

Table 2*Number and Percent of Items in Istation PK-1 Screener Item Bank, by Interface*

| Item Type (Interface) | Number of Items | Percent of Total Items |
|------------------------------|------------------------|-------------------------------|
| Addition | 22 | 1.8% |
| Counting | 50 | 4.2% |
| Dial Response, Dynamic | 28 | 2.3% |
| Dial Response, Static | 40 | 3.4% |
| Graphing: Horizontal | 4 | 0.3% |
| Graphing: Vertical | 8 | 0.7% |
| Image Multiple Choice | 217 | 18.2% |
| Measurement | 16 | 1.3% |
| Media with Response | 458 | 38.4% |
| Multi-Select | 97 | 8.1% |
| Number Line | 7 | 0.6% |
| Ordering Length | 23 | 1.9% |
| Ordering Numbers | 11 | 0.9% |
| Ordering Pictures | 37 | 3.1% |
| Pattern Completion | 10 | 0.8% |
| Pattern Construction | 16 | 1.3% |
| Rotation | 11 | 0.9% |
| Shapes | 27 | 2.3% |
| Sorting | 54 | 4.5% |
| Tens and Ones | 15 | 1.3% |
| Total | 1,192 | 100% |

Table 3*Mathematics Education Expert Reviewers' Ratings of Items by Category*

| Mathematics Education Expert ID | Scale | Mathematical Accuracy | Mathematical Vocabulary | Distractors |
|--|---------------------------------------|------------------------------|--------------------------------|--------------------|
| 1 | 4 – Extremely Accurate / Appropriate | 436 | 436 | 417 |
| | 3 – Mostly Accurate / Appropriate | 8 | 18 | 28 |
| | 2 – Somewhat Accurate / Appropriate | 5 | 4 | 11 |
| | 1 – Not at All Accurate / Appropriate | 9 | 0 | 2 |
| 2 | 4 – Extremely Accurate / Appropriate | 272 | 296 | 247 |
| | 3 – Mostly Accurate / Appropriate | 14 | 2 | 27 |
| | 2 – Somewhat Accurate / Appropriate | 6 | 2 | 19 |
| | 1 – Not at All Accurate / Appropriate | 12 | 4 | 11 |
| 3* | 4 – Extremely Accurate / Appropriate | 344 | 278 | 296 |
| | 3 – Mostly Accurate / Appropriate | 25 | 78 | 55 |
| | 2 – Somewhat Accurate / Appropriate | 25 | 41 | 37 |
| | 1 – Not at All Accurate / Appropriate | 22 | 19 | 28 |
| Total | 4 – Extremely Accurate / Appropriate | 1052 | 1010 | 960 |
| | 3 – Mostly Accurate / Appropriate | 47 | 98 | 110 |
| | 2 – Somewhat Accurate / Appropriate | 36 | 47 | 67 |
| | 1 – Not at All Accurate / Appropriate | 43 | 23 | 41 |

* Scores for 13 items are missing from items assigned to Expert Reviewer 301 due to being unable to view the items ($n = 10$) or failing to provide scores for the items ($n = 3$)

Table 4*Mathematics Education Expert Review of Items by Interface*

| Interface | Scale | Mathematical Accuracy | Mathematical Vocabulary | Distractors |
|---------------------------|---------------------------------------|------------------------------|--------------------------------|--------------------|
| Addition | 4 – Extremely Accurate / Appropriate | 22 | 15 | 16 |
| | 3 – Mostly Accurate / Appropriate | 0 | 7 | 6 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 0 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Counting | 4 – Extremely Accurate / Appropriate | 50 | 49 | 46 |
| | 3 – Mostly Accurate / Appropriate | 0 | 1 | 2 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 2 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Dial Response, Dynamic | 4 – Extremely Accurate / Appropriate | 28 | 24 | 26 |
| | 3 – Mostly Accurate / Appropriate | 0 | 2 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 2 | 2 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Dial Response, Static | 4 – Extremely Accurate / Appropriate | 37 | 40 | 21 |
| | 3 – Mostly Accurate / Appropriate | 2 | 0 | 6 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 13 |
| | 1 – Not at all Accurate / Appropriate | 1 | 0 | 0 |
| Expressions | 4 – Extremely Accurate / Appropriate | 36 | 36 | 40 |
| | 3 – Mostly Accurate / Appropriate | 5 | 5 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 1 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Graphing: Horizontal | 4 – Extremely Accurate / Appropriate | 4 | 2 | 4 |
| | 3 – Mostly Accurate / Appropriate | 0 | 2 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 0 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Graphing: Vertical | 4 – Extremely Accurate / Appropriate | 8 | 5 | 8 |
| | 3 – Mostly Accurate / Appropriate | 0 | 3 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 0 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |

| | | | | |
|-----------------------|---------------------------------------|-----|-----|-----|
| Image Multiple Choice | 4 – Extremely Accurate / Appropriate | 193 | 200 | 180 |
| | 3 – Mostly Accurate / Appropriate | 6 | 14 | 19 |
| | 2 – Somewhat Accurate / Appropriate | 10 | 1 | 9 |
| | 1 – Not at all Accurate / Appropriate | 6 | 0 | 7 |
| Measurement | 4 – Extremely Accurate / Appropriate | 5 | 6 | 6 |
| | 3 – Mostly Accurate / Appropriate | 0 | 0 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 1 | 0 | 0 |
| | 1 – Not at all Accurate / Appropriate | 10 | 10 | 10 |
| Media with Response | 4 – Extremely Accurate / Appropriate | 404 | 389 | 368 |
| | 3 – Mostly Accurate / Appropriate | 10 | 34 | 54 |
| | 2 – Somewhat Accurate / Appropriate | 18 | 16 | 14 |
| | 1 – Not at all Accurate / Appropriate | 19 | 12 | 15 |
| Multi-Select | 4 – Extremely Accurate / Appropriate | 90 | 82 | 72 |
| | 3 – Mostly Accurate / Appropriate | 1 | 8 | 12 |
| | 2 – Somewhat Accurate / Appropriate | 4 | 6 | 10 |
| | 1 – Not at all Accurate / Appropriate | 1 | 0 | 2 |
| Number Line | 4 – Extremely Accurate / Appropriate | 4 | 4 | 6 |
| | 3 – Mostly Accurate / Appropriate | 3 | 3 | 1 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 0 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Ordering Length | 4 – Extremely Accurate / Appropriate | 23 | 19 | 22 |
| | 3 – Mostly Accurate / Appropriate | 0 | 0 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 4 | 1 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Ordering Numbers | 4 – Extremely Accurate / Appropriate | 7 | 7 | 11 |
| | 3 – Mostly Accurate / Appropriate | 4 | 4 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 0 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Ordering Pictures | 4 – Extremely Accurate / Appropriate | 31 | 23 | 30 |
| | 3 – Mostly Accurate / Appropriate | 5 | 4 | 1 |
| | 2 – Somewhat Accurate / Appropriate | 1 | 10 | 3 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 3 |
| Pattern Completion | 4 – Extremely Accurate / Appropriate | 10 | 7 | 10 |
| | 3 – Mostly Accurate / Appropriate | 0 | 0 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 3 | 0 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |

| | | | | |
|-------------------------|---------------------------------------|------|------|-----|
| Pattern Construction | 4 – Extremely Accurate / Appropriate | 11 | 11 | 13 |
| | 3 – Mostly Accurate / Appropriate | 0 | 0 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 2 | 2 | 0 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Rotation | 4 – Extremely Accurate / Appropriate | 9 | 8 | 11 |
| | 3 – Mostly Accurate / Appropriate | 0 | 3 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 0 |
| | 1 – Not at all Accurate / Appropriate | 2 | 0 | 0 |
| Shapes | 4 – Extremely Accurate / Appropriate | 18 | 27 | 24 |
| | 3 – Mostly Accurate / Appropriate | 8 | 0 | 0 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 0 | 1 |
| | 1 – Not at all Accurate / Appropriate | 1 | 0 | 2 |
| Sorting | 4 – Extremely Accurate / Appropriate | 47 | 43 | 42 |
| | 3 – Mostly Accurate / Appropriate | 3 | 8 | 5 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 1 | 4 |
| | 1 – Not at all Accurate / Appropriate | 3 | 1 | 2 |
| Tens and Ones | 4 – Extremely Accurate / Appropriate | 15 | 13 | 4 |
| | 3 – Mostly Accurate / Appropriate | 0 | 0 | 4 |
| | 2 – Somewhat Accurate / Appropriate | 0 | 2 | 7 |
| | 1 – Not at all Accurate / Appropriate | 0 | 0 | 0 |
| Total | 4 – Extremely Accurate / Appropriate | 1052 | 1010 | 960 |
| | 3 – Mostly Accurate / Appropriate | 47 | 98 | 110 |
| | 2 – Somewhat Accurate / Appropriate | 36 | 47 | 67 |
| | 1 – Not at all Accurate / Appropriate | 43 | 23 | 41 |

Table 5*Teacher Reviewer Ratings of Items by Category*

| Teacher Reviewer ID | Scale | Mathematical Language | Mathematical Vocabulary | Mathematical Content | Visual Representations |
|----------------------------|----------------------------|------------------------------|--------------------------------|-----------------------------|-------------------------------|
| 1 | 4 – Extremely Appropriate | 259 | 297 | 298 | 237 |
| | 3 – Mostly Appropriate | 37 | 2 | 1 | 46 |
| | 2 – Somewhat Appropriate | 1 | 0 | 0 | 13 |
| | 1 – Not at all Appropriate | 2 | 0 | 0 | 3 |
| 2 | 4 – Extremely Appropriate | 592 | 592 | 590 | 580 |
| | 3 – Mostly Appropriate | 0 | 0 | 1 | 6 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 2 |
| | 1 – Not at all Appropriate | 0 | 0 | 1 | 3 |
| 3 | 4 – Extremely Appropriate | 0 | 35 | 62 | 0 |
| | 3 – Mostly Appropriate | 299 | 258 | 232 | 240 |
| | 2 – Somewhat Appropriate | 0 | 6 | 6 | 20 |
| | 1 – Not at all Appropriate | 1 | 1 | 0 | 40 |
| Total | 4 – Extremely Appropriate | 851 | 924 | 950 | 817 |
| | 3 – Mostly Appropriate | 336 | 260 | 234 | 292 |
| | 2 – Somewhat Appropriate | 1 | 6 | 6 | 35 |
| | 1 – Not at all Appropriate | 3 | 1 | 1 | 46 |

Table 6*Teachers' Review of Items by Interface*

| Interface | Scale | Mathematical Language | Mathematical Vocabulary | Mathematical Content | Visual Representations |
|---------------------------|----------------------------|------------------------------|--------------------------------|-----------------------------|-------------------------------|
| Addition | 4 – Extremely Appropriate | 8 | 9 | 10 | 9 |
| | 3 – Mostly Appropriate | 14 | 13 | 12 | 0 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 7 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Counting | 4 – Extremely Appropriate | 37 | 39 | 41 | 35 |
| | 3 – Mostly Appropriate | 13 | 11 | 9 | 7 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 8 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Dial Response, Dynamic | 4 – Extremely Appropriate | 18 | 19 | 22 | 18 |
| | 3 – Mostly Appropriate | 10 | 9 | 6 | 10 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Dial Response, Static | 4 – Extremely Appropriate | 36 | 36 | 36 | 29 |
| | 3 – Mostly Appropriate | 4 | 4 | 4 | 11 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Expressions | 4 – Extremely Appropriate | 29 | 36 | 37 | 30 |
| | 3 – Mostly Appropriate | 11 | 5 | 4 | 11 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 1 | 0 | 0 | 0 |
| Graphing: Horizontal | 4 – Extremely Appropriate | 4 | 4 | 4 | 4 |
| | 3 – Mostly Appropriate | 0 | 0 | 0 | 0 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Graphing: Vertical | 4 – Extremely Appropriate | 8 | 8 | 8 | 8 |
| | 3 – Mostly Appropriate | 0 | 0 | 0 | 0 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Image Multiple Choice | 4 – Extremely Appropriate | 151 | 164 | 161 | 152 |
| | 3 – Mostly Appropriate | 66 | 48 | 51 | 61 |
| | 2 – Somewhat Appropriate | 0 | 5 | 5 | 2 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 2 |

| Interface | Scale | Mathematical Language | Mathematical Vocabulary | Mathematical Content | Visual Representations |
|----------------------|----------------------------|------------------------------|--------------------------------|-----------------------------|-------------------------------|
| Measurement | 4 – Extremely Appropriate | 6 | 6 | 6 | 6 |
| | 3 – Mostly Appropriate | 10 | 10 | 10 | 10 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Media with Response | 4 – Extremely Appropriate | 347 | 376 | 385 | 347 |
| | 3 – Mostly Appropriate | 107 | 79 | 70 | 99 |
| | 2 – Somewhat Appropriate | 1 | 1 | 1 | 2 |
| | 1 – Not at all Appropriate | 2 | 1 | 1 | 8 |
| Multi-Select | 4 – Extremely Appropriate | 54 | 68 | 78 | 55 |
| | 3 – Mostly Appropriate | 43 | 29 | 19 | 5 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 25 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 12 |
| Number Line | 4 – Extremely Appropriate | 7 | 7 | 7 | 7 |
| | 3 – Mostly Appropriate | 0 | 0 | 0 | 0 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Ordering Length | 4 – Extremely Appropriate | 19 | 19 | 19 | 16 |
| | 3 – Mostly Appropriate | 4 | 4 | 4 | 5 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 2 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Ordering Numbers | 4 – Extremely Appropriate | 11 | 11 | 11 | 11 |
| | 3 – Mostly Appropriate | 0 | 0 | 0 | 0 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Ordering Pictures | 4 – Extremely Appropriate | 27 | 27 | 27 | 21 |
| | 3 – Mostly Appropriate | 10 | 10 | 10 | 13 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 2 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 1 |
| Pattern Completion | 4 – Extremely Appropriate | 0 | 1 | 1 | 1 |
| | 3 – Mostly Appropriate | 10 | 9 | 9 | 9 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Pattern Construction | 4 – Extremely Appropriate | 7 | 7 | 7 | 7 |
| | 3 – Mostly Appropriate | 9 | 9 | 9 | 9 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |

| Interface | Scale | Mathematical Language | Mathematical Vocabulary | Mathematical Content | Visual Representations |
|------------------|----------------------------|------------------------------|--------------------------------|-----------------------------|-------------------------------|
| Rotation | 4 – Extremely Appropriate | 8 | 8 | 8 | 6 |
| | 3 – Mostly Appropriate | 3 | 3 | 3 | 0 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 2 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 3 |
| Shapes | 4 – Extremely Appropriate | 23 | 23 | 27 | 15 |
| | 3 – Mostly Appropriate | 4 | 4 | 0 | 12 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 0 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Sorting | 4 – Extremely Appropriate | 36 | 41 | 40 | 31 |
| | 3 – Mostly Appropriate | 18 | 13 | 14 | 19 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 3 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 1 |
| Tens and Ones | 4 – Extremely Appropriate | 15 | 15 | 15 | 9 |
| | 3 – Mostly Appropriate | 0 | 0 | 0 | 4 |
| | 2 – Somewhat Appropriate | 0 | 0 | 0 | 2 |
| | 1 – Not at all Appropriate | 0 | 0 | 0 | 0 |
| Total | 4 – Extremely Appropriate | 851 | 924 | 950 | 817 |
| | 3 – Mostly Appropriate | 336 | 260 | 234 | 292 |
| | 2 – Somewhat Appropriate | 1 | 6 | 6 | 35 |
| | 1 – Not at all Appropriate | 3 | 1 | 1 | 46 |

Table 7

Teacher Reviewer Ratings of Items for Bias

| Teacher Reviewer ID | No Bias | Some Bias | Bias |
|----------------------------|----------------|------------------|-------------|
| 1 | 235 | 50 | 13 |
| 2 | 593 | 0 | 0 |
| 3 | 289 | 5 | 1 |
| Total | 1117 | 55 | 14 |

* Data were missing for 6 items

Appendix A – State Content Standards Referent Sources

National Council of Teachers of Mathematics (NCTM) Curricular Focal Points

The National Council of Teachers of Mathematics (NCTM) Curricular Focal Points were retrieved from http://www.nctmmedia.org/cfp/front_matter.pdf on 4/20/2010. Additional information was also retrieved on 4/20/2010 from: <http://www.nctm.org/standards/content.aspx?id=270>

Florida

Florida's Next Generation Sunshine State Math Standards (adopted 2010) were retrieved from <http://www.cpalms.org/Downloads.aspx>. These standards include the Common Core Standards in Mathematics.

California

California's Common Core Content Standards for Mathematics (adopted 2010) were retrieved from http://www.scoe.net/castandards/agenda/2010/math_ccs_recommendations.pdf. These standards include the Common Core Standards in Mathematics.

New York

The New York State P-12 Common Core Learning Standards for Mathematics were retrieved from: <http://engageny.org/sites/default/files/resource/attachments/nysp12ccclsmath.pdf>. These standards include the Common Core Standards in Mathematics plus additional state-specific standards.

Texas

The Texas Essential Knowledge and Skills (adoption 2012) were retrieved from: <http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html>.

Common Core Standards

The Common Core Standards in Mathematics were retrieved from www.corestandards.org/the-standards/mathematics. These standards were published in 2010. They were developed as part of an initiative led by National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

Virginia

Virginia's Standards for Learning Document for Mathematics (adopted 2009 for full implementation in 2011-12) were retrieved from http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/