11

requirements, such as frequential ones and hence to derive new algorithms. Also, we can use the reduced-order formulation to consider the expansion of the positively invariant set when the saturations are applied [19] and to evaluate some performance requirements in the interior of the extended set. Investigations in these directions, as well as in the integration of reduced-order observers in the control schemes, are being developed.

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Observer Parameterization for Simultaneous Observation

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Abstract—**The stable inverse approach is used to obtain the observers for the simultaneous observation of a given set of plants. A parameterization in terms of a stable inverse and a stable null space is proposed for all simultaneous observers. To verify the effectiveness of the proposed method, a design example is also given.**

I. INTRODUCTION

The problem of simultaneous observation was first introduced in [1] and can be stated in the following way: given a set of plants $G_0(s), G_1(s), \cdots, G_r(s)$, find a common observer which can observe the states of each of these plants. The simultaneous observation can be regarded as a dual problem of the simultaneous stabilization in [2]–[4].

The simultaneous observation problem arises from the practical observation problem. An application of simultaneous observation is reliable observation, where the plant has known discrete perturbations which arise from sensor, actuator, or component faults. In this case, $G_0(s)$ can be regarded as a nominal plant model, representing the transfer matrix of the plant when no faults occur and $G_1(s), \cdots, G_r(s)$ as transfer matrices of the same plant in the presence of different faults. It is desired to observe the states of the nominal plant and the perturbed plants, that is, the states of $G_0(s), G_1(s), \cdots, G_r(s)$, using a single observer. The second application is the design of a fixed observer for a nonlinear plant having multiple operating conditions. The nonlinear plant can be linearized in these operating conditions. Thus $G_0(s), G_1(s), \cdots, G_r(s)$ represent linearized models of the plant at various operating points, and a common observer needs to be designed for all of the linearized models. Another important application is the robust observer design problem, where the objective is to design an observer for a given set of plants that represent all continuous perturbations of a nominal plant.

In the previous work [1], the mathematical formulation for the simultaneous observation problem was obtained. The coprime factorization technique was used to solve the proposed problem. Necessary and sufficient conditions for the existence of a simultaneous observation has been given. In particular, these conditions showed that simultaneously observing $r+1$ plants is equivalent to simultaneously observing r auxiliary plants using a common observer. The simultaneous observer can be computed for two plants using the proposed design method. However, the simultaneous observer design for $r > 2$ has not been solved yet.

In this paper, our main objective is to study the parameterization problem of all simultaneous observers for a given set of plants. The result can give the general form of simultaneous observers through a computational algorithm in the form of state space. The resultant parameterization can be used to obtain a simultaneous observer for r plants ($r > 2$) when certain conditions are satisfied.

When disturbances exist in a given set of plants, estimation errors are caused. With the aid of the simultaneous observer parameterization achieved here, the estimation error dynamics can also be

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parameterized. This will extend the result for the case of a single plant in [5] and [6].

The proposed approach is based on the fact that a nonsquare matrix of stable rational functions generally possesses a stable one-side inverse. The stable inverse approach [7] has been applied to obtain the controller parameterization for simultaneous stabilization [8]–[11]. In the present paper, motivated by the approach for the simultaneous stabilization which is developed recently in [9]–[11], we treat its dual problem using the same technique. It will be seen that the parameterization of a simultaneous observer presents a dual result to the parameterization of simultaneous stabilizing controllers.

The following notations will be used throughout this paper. $R(s)$ denotes real rational functions. RH_∞ denotes stable and proper rational functions with real coefficients. $X^{p\times m}$ denotes $p \times m$ matrices with their elements in X, where $X = R(s), RH_\infty$, etc. The state-space realizations of the transfer function matrix $G(s)$ are represented by

$$G(s) = C(sI - A)^{-1}B + D = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right].$$

II. Problem Description and Preliminaries

Consider a set of linear time-invariant multi-input–multi-output (MIMO) plants described by

$$\dot{x}(t) = A_i x(t) + B_i u(t) \tag{1}$$
$$y(t) = C_i x(t) + D_i u(t) \tag{2}$$
$$z(t) = E_i x(t), \qquad i = 1, 2, \cdots, r \tag{3}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $y(t) \in R^p$ is the measured output vector, $z(t) \in R^k$ is the state to be estimated, and A_i, B_i, C_i, D_i, and E_i are constant matrices of the ith system with appropriate dimensions. The transfer function description of (1)–(3) is given by

$$y(s) = G_i(s)u(s) \tag{4}$$
$$z(s) = E_i x(s) \tag{5}$$

with

$$G_i(s) = C_i(sI - A_i)^{-1}B_i + D_i \in R(s)^{p\times m}. \tag{6}$$

The simultaneous observer for (1)–(3) can be described by

$$r(s) = F(s)u(s) + H(s)y(s) \tag{7}$$

where $F(s) \in RH_\infty^{k\times m}$ and $H(s) \in RH_\infty^{k\times p}$. The estimation error for $z(t)$ using the observer (7) should satisfy

$$\lim_{t\to\infty} (E_i x(t) - r(t)) = 0 \tag{8}$$

for all $u(t)$. If (7) exists, it is said that $G_0(s), G_1(s), \cdots, G_r(s)$ are simultaneously observable [1]. The goal of this paper is to seek the set of all simultaneous observers, e.g., to parameterize all simultaneous observers.

A stable right coprime factorization of the ith plant $G_i(s)$ can be written as

$$G_i(s) = N_i(s)M_i^{-1}(s) \tag{9}$$

where $N_i(s) \in RH_\infty^{p\times m}, M_i(s) \in RH_\infty^{m\times m}$. The state-space realization of the factors $M_i(s)$ and $N_i(s)$ are [2]

$$M_i(s) = \left[\begin{array}{c|c} A_i + B_iK_i & B_i \\ \hline K_i & I \end{array}\right]$$
$$N_i(s) = \left[\begin{array}{c|c} A_i + B_iK_i & B_i \\ \hline C_i + D_iK_i & D_i \end{array}\right] \tag{10}$$

where the matrices K_i's $(i = 1, 2, \cdots, r)$ are chosen such that $(A_i + B_iK_i)$'s are stable.

Introducing the partial state $\xi(s)$, we can rewrite (4) with the factorization (9) as

$$M_i(s)\xi(s) = u(s) \tag{11}$$
$$N_i(s)\xi(s) = y(s). \tag{12}$$

Correspondingly, the variable $z(s) = E_i x(s)$ in (5) can be expressed by [1]

$$z(s) = P_i(s)\xi(s) \tag{13}$$

with

$$P_i(s) = E_i(sI - A_i - B_iK_i)^{-1}B_i \in RH_\infty^{k\times m}. \tag{14}$$

It is known that $E_i x$ can be observed using (7) if and only if the following condition holds [5], [12]:

$$F(s)M_i(s) + H(s)N_i(s) = P_i(s). \tag{15}$$

That is

$$[F(s) \quad H(s)]\begin{bmatrix} M_i(s) \\ N_i(s) \end{bmatrix} = P_i(s). \tag{16}$$

Define

$$M(s) = \begin{bmatrix} M_1(s) & M_2(s) & \cdots & M_r(s) \\ N_1(s) & N_2(s) & \cdots & N_r(s) \end{bmatrix} \in RH_\infty^{(m+p)\times mr} \tag{17}$$

$$P(s) = [P_1(s) \quad P_2(s) \quad \cdots \quad P_r(s)] \in RH_\infty^{k\times mr}. \tag{18}$$

From (9), the state-space realization of $M(s)$ can be given by

$$M(s) = \left[\begin{array}{c|c} A_m & B_m \\ \hline C_m & D_m \end{array}\right] = \left[\begin{array}{ccc|ccc} A_1 + B_1K_1 & \cdots & 0 & B_1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & A_r + B_rK_r & 0 & \cdots & B_r \\ \hline K_1 & \cdots & K_r & I & \cdots & I \\ C_1 + D_1K_1 & \cdots & C_r + D_rK_r & D_1 & \cdots & D_r \end{array}\right]. \tag{19}$$

Then (16) can be written as

$$[F(s) \quad H(s)]M(s) = P(s). \tag{20}$$

The objective is to find the set of all simultaneous observers which satisfies (20). The stable inverse approach will be used to solve this problem.

III. Stable Left-Inverse of Matrix

Consider a class of plants with $M(s) \in RH_\infty^{(m+p)\times mr}$ having a stable left inverse, that is, there exists a matrix $L(s) \in RH_\infty^{mr\times(m+p)}$ such that

$$L(s)M(s) = I_{mr}. \tag{21}$$

It is known [7] that for the existence of such a $L(s)$, it is necessary that

$$(m + p) > mr \tag{22}$$
$$\operatorname{rank}(D_m) = mr. \tag{23}$$

When (23) is satisfied, one can always find a nonsingular matrix T such that

$$TM(s) = \left[\begin{array}{c|c} A_m & B_m \\ \hline C_{m1} & I_{mr} \\ C_{m2} & 0 \end{array}\right] = \begin{bmatrix} M_m(s) \\ N_m(s) \end{bmatrix}. \tag{24}$$

The stable left null space of $M(s)$ is also used in the study of parameterization. It is known that the condition of $M(s)$ having a stable inverse and stable null space is equivalent to $M(s)$ having no right half-plane zeros [11]. We will assume that this condition is satisfied. The following lemma solves the computation problem of stable left inverse and stable left null-space for matrix $M(s)$.

Lemma 1: For every $M(s) \in RH_\infty^{(m+p)\times mr}$ satisfying (22) and (23) and having no right half-plane transmission zeros, there exists a stable left inverse $L(s) \in RH_\infty^{mr\times(m+p)}$ and a left null space $S(s) \in RH_\infty^{(m+p-mr)\times(m+p)}$ such that

$$L(s)M(s) = I_{mr} \tag{25}$$

$$S(s)M(s) = 0_{(m+p-mr)\times mr}. \tag{26}$$

Furthermore, let $M(s)$ have the state-space realization (24). Suppose that $(A, C) = (A_m - B_m C_{m1}, C_{m1})$ is a detectable pair and choose J such that $(A - JC)$ is stable. Then, the above stable left inverse and left null space are given by

$$L(s) = \left[\begin{array}{c|cc} A - JC & B_m & J \\ \hline -C_{m1} & I & 0 \end{array}\right] T \tag{27}$$

$$S(s) = \left[\begin{array}{c|cc} A - JC & B_m & J \\ \hline -C_{m2} & 0 & I \end{array}\right] T. \tag{28}$$

Proof: Consider an auxiliary plant $G_p(s) = C(sI - A)^{-1}B + D \in RH_\infty^{(m+p)\times mr}$ with the right and left coprime factorization $(N_p(s), M_p(s)), (\hat{N}_p(s), \hat{M}_p(s))$, respectively. Then there exist $X_p(s)$ and $Y_p(s)$ such that [2]

$$\begin{bmatrix} Y_p(s) & X_p(s) \\ -\hat{N}_p(s) & \hat{M}_p(s) \end{bmatrix} \begin{bmatrix} M_p(s) \\ N_p(s) \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}. \tag{29}$$

The state-space construction of each factor above is given [2] as

$$\begin{bmatrix} M_p(s) \\ N_p(s) \end{bmatrix} = \left[\begin{array}{c|c} A + BK & B \\ \hline K & I \\ C + DK & 0 \end{array}\right] \tag{30}$$

$$\begin{bmatrix} Y_p(s) & X_p(s) \\ -\hat{N}_p(s) & \hat{M}_p(s) \end{bmatrix} = \left[\begin{array}{c|cc} A - JC & B - JD & J \\ \hline -K & I & 0 \\ -C & 0 & I \end{array}\right] \tag{31}$$

where K and J are matrices such that $A - JC$ and $A + BK$ are stable.

Establish the following equalities using (30) and (24):

$$A + BK = A_m, B = B_m, K = C_{m1}, C = C_{m2}, D = 0.$$

Then

$$B = B_m, K = C_{m1}, C = C_{m2}, A = A_m - B_m C_{m1} \tag{32}$$

and $\begin{bmatrix} M_m(s) \\ N_m(s) \end{bmatrix}$ in (24) corresponding to $\begin{bmatrix} M_p(s) \\ N_p(s) \end{bmatrix}$ in (30). Using (29) and (30), substitute (32) into (30), then (27) is obtained with

$$L(s)[Y(s) \quad X(s)] = [Y_p(s) \quad X_p(s)]T \tag{33}$$

and (28) is obtained with

$$S(s) = [-\hat{N}(s) \quad \hat{M}(s)] = [-\hat{N}_p(s) \quad \hat{M}_p(s)]T. \tag{34}$$

Thus

$$\begin{bmatrix} L(s) \\ S(s) \end{bmatrix} M(s) = \begin{bmatrix} I_{mr} \\ 0_{(m+p-mr)\times mr} \end{bmatrix}.$$

□

The above proof of Lemma 1 closely followed the corresponding proof of the stable right inverse in [8] and [9].

If (23) is satisfied, we can find the left inverse of $M(s)$ directly using the result of Lemma 1. In general, for strictly proper plants the full-rank condition is not satisfied and $\text{rank}(D_m) < mr$. In this case, we can apply a stable derivative procedure to solve this problem [8], [11]. A nonsingular $mr \times mr$ polynomial matrix $V(s)$ with stable zeros of $\det(V(s))$ can be chosen such that $M(s)V(s)$ is proper and has the following realization

$$M(s)V(s) = \left[\begin{array}{c|c} A_{mv} & B_{mv} \\ \hline C_{mv} & D_{mv} \end{array}\right] \tag{35}$$

where rank $(D_{mv}) = mr$. Then Lemma 1 can be used to find a stable left inverse matrix and a left null space of $M(s)V(s)$. Note that the generalized derivative matrix $V(s)$ is nonunique.

IV. Main Result

Theorem 1: Given (1)–(3), the set of all simultaneous observers for $z(t)$ is parameterized by

$$F(s) = [P(s)Y(s) - Q(s)\hat{N}(s)] \tag{36}$$

$$H(s) = [P(s)X(s) + Q(s)\hat{M}(s)] \tag{37}$$

$$Q(s) \in RH_\infty^{k\times(m+p-mr)} \tag{38}$$

where

$$P(s) = [P_1(s) \quad P_2(s) \quad \cdots \quad P_r(s)] \in RH_\infty^{k\times mr} \tag{39}$$

$$L(s) = [Y(s) \quad X(s)] \text{ is a stable left inverse of } M(s) \tag{40}$$

$$S(s) = [-\hat{N}(s) \quad \hat{M}(s)] \text{ is null space of } M(s) \tag{41}$$

$$Y(s) \in RH_\infty^{mr\times m}, \quad X(s) \in RH_\infty^{mr\times p}$$
$$\hat{N}(s) \in RH_\infty^{(m+p-mr)\times m}, \quad \hat{M}(s) \in RH_\infty^{(m+p-mr)\times p}.$$

Proof: Prove necessity first. Select a $Q(s)$ satisfying (38). There exists a simultaneous observer

$$\begin{aligned} r(s) &= F(s)u(s) + H(s)y(s) \\ &= [P(s)Y(s) - Q(s)\hat{N}(s)]u(s) \\ &\quad + [P(s)X(s) + Q(s)\hat{M}(s)]y(s) \end{aligned}$$

or

$$\begin{aligned} &[F(s) \quad H(s)] \\ &= [P(s)Y(s) - Q(s)\hat{N}(s) \quad P(s)X(s) + Q(s)\hat{M}(s)] \\ &= [P(s) \quad Q(s)] \begin{bmatrix} Y(s) & X(s) \\ -\hat{N}(s) & \hat{M}(s) \end{bmatrix} \\ &= [P(s) \quad Q(s)] \begin{bmatrix} L(s) \\ S(s) \end{bmatrix} = P(s)L(s) + Q(s)S(s). \end{aligned} \tag{42}$$

Thus

$$\begin{aligned} &[F(s) \quad H(s)]M(s) \\ &= [P(s)L(s) + Q(s)S(s)]M(s) \\ &= P(s)L(s)M(s) + Q(s)S(s)M(s) = P(s). \end{aligned}$$

It can be seen that this satisfies (20) of the simultaneous observation.

Then, prove sufficiency. The simultaneous observer is given by

$$r(s) = F(s)u(s) + H(s)y(s).$$

It is desired to find a $Q(s) \in RH_\infty^{k\times(m+p-mr)}$ such that the observer can be expressed as (36) and (37), that is

$$[P(s) \quad Q(s)] \begin{bmatrix} L(s) \\ S(s) \end{bmatrix} = [F(s) \quad H(s)]. \tag{43}$$

It is known that a simultaneous observer satisfies

$$[F(s) \quad H(s)]M(s) = P(s).$$

A stable left matrix $L(s)$ and a stable left null-space matrix $S(s)$ of $M(s)$ can be computed. From Lemma 1, $\begin{bmatrix} L(s) \\ S(s) \end{bmatrix}$ is nonsingular. Let

$$\begin{bmatrix} L(s) \\ S(s) \end{bmatrix}^{-1} = [M(s) \quad G(s)]. \tag{44}$$

This is a stable matrix [9]. From (42), we obtain

$$\begin{aligned} [P(s) \quad Q(s)] &= [F(s) \quad H(s)] \begin{bmatrix} L(s) \\ S(s) \end{bmatrix}^{-1} \\ &= [F(s) \quad H(s)][M(s) \quad G(s)]. \end{aligned} \tag{45}$$

That is

$$P(s) = [F(s) \quad H(s)]M(s), \quad Q(s) = [F(s) \quad H(s)]G(s). \tag{46}$$

Thus (43) is satisfied. □

Theorem 1 gives the result of parameterization of all simultaneous observers. According to this parameterization, the simultaneous observer design reduces to searching for a suitable parameterization matrix in RH_∞ set. This provides us with a systematic procedure to design simultaneous observers.

Theorem 1 also shows that the simultaneous observer can be designed for r plants. The following corollary is a more intuitive explanation for Theorem 1.

Corollary 1: Given any plant $G_i(s) = N_i(s)M_i^{-1}(s)$ in (1)–(3). Its states can be observed using observer (7) with the form of (36) and (37), and satisfying

$$F(s)M_i(s) + H(s)N_i(s) = P_i(s). \tag{47}$$

Proof: This is a direct result from Theorem 1, and

$$\begin{aligned} F(s)M_i(s) &+ H(s)N_i(s) \\ &= [P(s)Y(s) - Q(s)\hat{N}(s)]M_i(s) \\ &\quad + [P(s)X(s) + Q(s)\hat{M}(s)]N_i(s) \\ &= [P(s) \quad Q(s)]\begin{bmatrix} Y(s) & X(s) \\ -\hat{N}(s) & \hat{M}(s) \end{bmatrix}\begin{bmatrix} M_i(s) \\ N_i(s) \end{bmatrix} \\ &= [P(s) \quad Q(s)]\begin{bmatrix} L(s) \\ S(s) \end{bmatrix}\begin{bmatrix} M_i(s) \\ N_i(s) \end{bmatrix} \\ &= [P(s)L(s) + Q(s)S(s)]\begin{bmatrix} M_i(s) \\ N_i(s) \end{bmatrix}. \end{aligned} \tag{48}$$

From (25) and (26) in Lemma 1, we get

$$L(s)\begin{bmatrix} M_i(s) \\ N_i(s) \end{bmatrix} = [0 \quad \cdots \quad 0 \quad \underbrace{I}_{i\text{th row}} \quad 0 \quad \cdots \quad 0]^T \tag{49}$$

$$S(s)\begin{bmatrix} M_i(s) \\ N_i(s) \end{bmatrix} = [0 \quad \cdots \quad 0 \quad \underbrace{0}_{i\text{th row}} \quad 0 \quad \cdots \quad 0]^T. \tag{50}$$

Then we have

$$P(s)L(s)M_i(s) = P_i(s)$$
$$Q(s)S(s)M_i(s) = 0$$

in (48). Thus (47) is obtained.

Corollary 1 further indicates that the simultaneous observer in the form of (36) and (37) can observe the state of any one of the given plants such that the observation condition for every plant is satisfied. Thus, this observer can be used to simultaneously observe the states of r plants.

When $M(s)$ does not satisfy (23), we can apply the stable derivative procedure [10], [11] and ensure that $M(s)V(s)$ is proper with realization (35). Postmultiplying both sides of (17) by $V(s)$, we obtain

$$[F(s) \quad H(s)]M(s)V(s) = P(s)V(s). \tag{51}$$

Then the following result can be obtained.

Corollary 2: Assume that $M(s)V(s)$ has a stable left inverse and a stable left null space. Then (36) and (37) in Theorem 1 can be expressed by

$$F(s) = [P(s)V(s)Y(s) - Q(s)\hat{N}(s)] \tag{52}$$
$$H(s) = [P(s)V(s)X(s) + Q(s)\hat{M}(s)] \tag{53}$$

where

$L(s) = [Y(s) \quad X(s)]$ is a stable left inverse of $M(s)V(s)$ (54)

$S(s) = [-\hat{N}(s) \quad \hat{M}(s)]$ is null space of $M(s)V(s)$. (55)

Proof: The corollary can be proven by replacing $M(s)$ by $M(s)V(s)$ in the proof of Theorem 1.

When $M(s)$ has the closed right half-plane transmission zeros and its stable inverse does not exist, the observer design can be solved by using the similar procedure provided in [11]. By using the design freedom $Q(s)$ offered by the observer parameterization, a stable observer can be obtained.

The following result will give an application for the simultaneous observer parameterization. When there are disturbances in given plants, the description for the set of plants in (1) and (2) becomes

$$\dot{x}(t) = A_i x(t) + B_i u(t) + U_i d(t) \tag{56}$$
$$y(t) = C_i x(t) + D_i u(t) + W_i d(t) \tag{57}$$

where $d(t) \in R^q$ is the unknown disturbance vector, and W_i and U_i are constant matrices with appropriate dimensions. Then the transfer function description is rewritten by

$$y(s) = G_i(s)u(s) + G_{d_i}(s)\, d(s). \tag{58}$$

Furthermore, through introducing the partial state $\xi(s)$, we have

$$y(s) = N_i(s)\xi(s) + G_{d_i}(s)\, d(s) \tag{59}$$

and

$$z(s) = P_i(s)\xi(s) + F_{d_i}(s)\, d(s) \tag{60}$$

where [5]

$$F_{d_i}(s) = E_i(sI - A_i)^{-1}U_i. \tag{61}$$

The estimation error is described by

$$\Delta e(s) = z(s) - r(s). \tag{62}$$

Substituting $F(s)$ and $H(s)$ of (36) and (37) into (62), we get

$$\Delta e(s) = (T_1(s) - Q(s)T_2(s))\, d(s) \tag{63}$$

where

$$T_1(s) = (F_{d_i}(s) - P(s)X(s))G_{d_i}(s), \quad T_2(s) = \hat{M}(s)G_{d_i}(s). \tag{64}$$

Equation (63) is the parameterization of estimation error dynamics. It gives a straightforward relationship between the estimation error and disturbance vector. $Q(s)$ is the only unknown parameterization matrix in (63). Thus when a certain design specification such as H_∞ norm specification is used for the transfer matrices between the estimation error and disturbance vector, one can solve this optimal simultaneous observation problem. It can be seen that the parameterization of a simultaneous observer provides a basic tool for this kind of optimal problem.

V. Design Example

Consider the following three plants:

$$\dot{x}(t) = -x(t) + u(t), \quad y(t) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$
$$z(t) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x(t) \tag{65}$$

$$\dot{x}(t) = -2x(t) + u(t), \quad y(t) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t)$$
$$z(t) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x(t) \tag{66}$$

$$F(s) = \frac{s^2+6s+10}{(s+3)(s+4)(s+2)}\begin{bmatrix}1\\1\\1\end{bmatrix} - Q(s),$$

$$H(s) = \frac{1}{(s+3)(s+4)(s+2)(s^3+10s^2+28s+22)}$$
$$\cdot \begin{bmatrix} 2s^4+26s^3+124s^2+264s+212 & s^4+10s^3+32s^2+34s-4 & -(s^4+16s^3+80s^2+162s+112) \\ 2s^4+26s^3+124s^2+264s+212 & s^4+10s^3+32s^2+34s-4 & -(s^4+16s^3+80s^2+162s+112) \\ 2s^4+26s^3+124s^2+264s+212 & s^4+10s^3+32s^2+34s-4 & -(s^4+16s^3+80s^2+162s+112) \end{bmatrix}$$
$$+ Q\frac{1}{s^3+10s^2+28s+22}[(s^3+7s^2+10s-2) \quad (s^3+7s^2+13s+10) \quad (s^3+7s^2+19s+16)].$$

$$\dot{x}(t) = -4x(t) + u(t), \quad y(t) = \begin{bmatrix}1\\1\\1\end{bmatrix} x(t) + \begin{bmatrix}0\\0\\1\end{bmatrix} u(t)$$
$$z(t) = \begin{bmatrix}1\\1\\1\end{bmatrix} x(t) \tag{67}$$

where $m = 1, p = 3, r = 3$, and $(m+p) > mr$. Based on the right coprime factorization for these three plants, the state-space realization of the $M(s)$ matrix in (19) is

$$M(s) = \left[\begin{array}{ccc|ccc} -3 & 0 & 0 & 1 & 0 & 0 \\ 0 & -4 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \\ \hline -2 & -2 & 2 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{array}\right]. \tag{68}$$

Using the results of Lemma 1, we get the stable left inverse $L(s)$ and left null space $S(s)$

$$L(s) = \left[\begin{array}{ccc|cccc} -2 & -1 & -1 & 0 & 1 & 0 & 0 \\ -1 & -3 & -1 & 0 & 0 & 1 & 0 \\ -1 & -1 & -5 & 0 & 0 & 0 & 1 \\ \hline -2 & -4 & -4 & -1 & 2 & 1 & 1 \\ 2 & 4 & 2 & 1 & -1 & 0 & -1 \\ 2 & 2 & 0 & 1 & -1 & -1 & 0 \end{array}\right] \tag{69}$$

$$S(s) = \left[\begin{array}{ccc|cccc} -2 & -1 & -1 & 0 & 1 & 0 & 0 \\ -1 & -3 & -1 & 0 & 0 & 1 & 0 \\ -1 & -1 & -5 & 0 & 0 & 0 & 1 \\ \hline -3 & -3 & -3 & -1 & 1 & 1 & 1 \end{array}\right]. \tag{70}$$

Thus the simultaneous observer is

$$r(s) = F(s)u + H(s)y$$

where the next equation is shown at the top of the page, and $Q(s) \in RH_\infty^{3\times 1}$.

VI. Conclusions

The simultaneous observer parameterization is achieved using the stable inverse approach. This provides a dual result to the parameterization of simultaneous stabilizing controllers [11] and extends the applications for the stable inverse approach developed recently in [8]–[11]. It is also an extension of the observer parameterization results for a single plant in [5] and [6].

The result of simultaneous observer parameterization here can also be applied to strictly proper plants in the most practical cases, where the full rank condition for stable inverse of the matrix is not satisfied. The required stable inverse can be obtained using the existing technique of stable derivative procedure. In systems where a stable inverse fails to exist, it is suggested to use the method in [11] to obtain a stable observer.

This parameterization also provides a useful tool for designing an optimal simultaneous observer in terms of a certain performance specification, such as H_∞ norm specification. It is also suitable for the development of other systematic simultaneous observer design methods.

It should be pointed out that the results presented in this paper are based on the stable inverse approach under state-space representation, and therefore, it can be easily realized and implemented with the aid of modern computer-aided control system design programs.

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