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MODELING OF THE INFLUENCE OF THE ABRASIVE WATERJET CUTTING PARAMETERS ON THE DEPTH OF CUT BASED ON FUZZY RULES

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Abstract—Currently, the abrasive waterjet cutting parameters for the milling operation have to be determined by a combination of prior experience and trial and error. It is shown that the selection of the abrasive waterjet cutting parameters for a required depth of cut in the given material can be effectively done by applying the principles of the fuzzy set theory. This approach will eliminate the need for extensive experimental work in order to select the magnitudes of the most influential abrasive waterjet parameters on the depth of cut. Fuzzy logic provides a methodology and imitation of a human's way of making decisions which is very useful in such applications where the mathematical model of the process does not exist, and one of such processes is indeed abrasive waterjet cutting. A number of case studies are performed to verify the validity of the proposed methodology for selecting the abrasive waterjet cutting parameters in order to achieve the predetermined depth of cut.

1. INTRODUCTION

THE INTRODUCTION of the abrasive waterjet (AWJ) as a machining method has opened a new way of machining difficult-to-machine materials. Originally, the AWJ machining technique was used only for linear cutting and shape cutting of difficult-to-machine materials, such as titanium, superalloys, glass, composites, metal matrix composites, and advanced ceramics. However, today this technology is used in such machining applications as turning, drilling of small diameter holes, and milling.

To produce a cavity with controlled depth is a basic problem of the AWJ milling operation. The depth of cut is determined by the mechanics of the jet–material interaction process [1]. In the milling operation by AWJ, the depth of cut is a function of a number of influence factors, including: waterjet pressure, abrasive flow rate, abrasive grain size, stand-off distance, jet traverse speed, angle of impact, AWJ nozzle wear, etc.

Currently, the AWJ cutting parameters for machining the given material have to be determined by a combination of prior experience and trial and error. Although there is a predictive model [2], an error will always exist between the calculated and the actual results because of the simplification and assumption of the modeling. If there is a change in the system set up or other operating factors, the existing methods cannot reliably predict the new parameters which will give accurate values of the depth of cut. As a result, the repeatability of the milled surface quality by abrasive waterjet cannot be guaranteed.

In the selection of the AWJ process variables for a desired depth of cut we are interested in the development of the relationship between inputs and outputs. It is evident that this relationship could be obtained by using a statistical approach. A statistical approach estimates functions, but it requires that we guess how outputs functionally depend on inputs. Fuzzy systems also estimate functions from sample data.

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However, a fuzzy system does not require that we articulate such a mathematical model. Thus, it is a model-free estimator.

The objective of this study is to implement a fuzzy set theory in the selection of the AWJ process variables in order to achieve the predetermined depth of cut in the given material. It is shown that with this approach one can simulate human experience and experimental information that is normally used for controlling the process. It is easy to introduce new rules and to modify the existing ones. By using this approach, experimentation and computation time can be reduced as compared to the conventional way of selecting the AWJ cutting parameters.

2. BASIC DESCRIPTION OF FUZZY SETS

The fuzzy set theory has proved to be successful in analyzing uncertain and complex systems which cannot be described mathematically. The fuzzy controller, based on the fuzzy set theory has been successfully applied to a number of industrial processes [3]. The idea of a fuzzy set allows imprecise and qualitative information to be expressed in an exact way and is a generalization of the concept of an ordinary set whose membership function only takes two values $\{0,1\}$. A fuzzy set has a membership function which takes all values between 0 and 1.

The fuzzy control system consists of three stages: fuzzification, decision making and defuzzification (Fig. 1). The fuzzification stage converts real number input values into fuzzy values. The decision making stage processes the input data and computes the controller outputs. The outputs which are fuzzy values, are converted into real numbers by the defuzzification stage, for example by using the center of gravity method [4].

A fuzzy system associates output fuzzy set with input fuzzy set and is characterized by a set of linguistic statements based on expert knowledge. The expert knowledge is usually in the form of "if–then" rules, which are easily implemented by fuzzy conditional statements in fuzzy logic. A human operator employs a set of fuzzy if–then rules to control the process.

The key concept throughout fuzzy set theory is the definition of a fuzzy set. A fuzzy set F in a universe of discourse U is characterized by a membership function $\mu_F : U \rightarrow [0,1]$. The primary fuzzy sets (linguistic terms) usually have a meaning, such as negative big, negative medium, . . ., positive big. The membership function of a fuzzy set in the continuous universe of discourse is expressed in a functional form, typically a bell-shaped function, triangle-shaped function, trapezoid-shaped function, etc. The grade of membership values are assigned subjectively to define the meaning of the labels of the fuzzy sets [5].

The essential problems in designing the fuzzy logic controller that have to be addressed are:

- definition of input/output variables
- design of rules/fuzzy labels
- design of the inference mechanism
- selection of fuzzification/defuzzification methods.

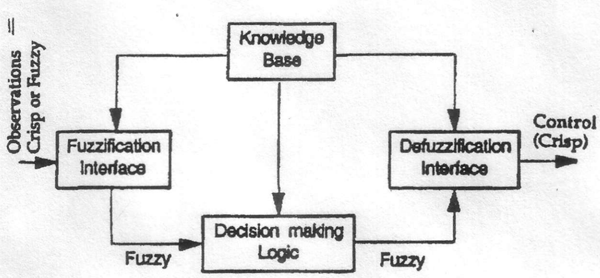

FIG. 1. Functions of fuzzy logic controller.

3. DEVELOPMENT OF THE FUZZY ALGORITHMS FOR THE MODELING OF THE INFLUENCE OF THE AWJ CUTTING PARAMETERS ON THE DEPTH OF CUT

Abrasive waterjet cutting, like other material processing methods, involves many factors, as shown in Fig. 2. The output of the process, such as metal removal rate, surface quality and dimensional accuracy (depth of cut, width of cut) depends on the interrelationships of the abrasive waterjet variables. Most of these variables could be used as control values. However, the limitations of today's available abrasive waterjet cutting systems reduce on-line control variables to the following four: jet traverse speed, direction of motion, angle of impingement, and nozzle stand-off distance. Waterjet pressure, abrasive flow rate, abrasive grain size, abrasive material, abrasive waterjet nozzle diameter, and waterjet nozzle diameter could be changed and used as the control values between individual operations.

The efficiency and accuracy of applying abrasive waterjets in the milling operation can be improved considerably by an adequate choice of the main process parameters. In the following sections it will be shown that the selection of the AWJ cutting parameters for the given depth of cut in the selected material can be effectively done by performing just a few experiments and applying the principles of the fuzzy set theory.

In order to use the fuzzy set theory in the selection of the AWJ cutting parameters, it is important to know the boundaries of the universes of the input and the output of the fuzzy variables, i.e. minimum/maximum values expected of the input and output. In this study several experiments were conducted in order to determine the value of the depth of cut for selected cutting conditions. Namely, while the experiments were performed, one of the four selected process variables (waterjet pressure, abrasive flow rate, jet traverse speed and AWJ nozzle inside diameter) was varied amongst three levels (minimum, medium and maximum), while the other three variables were kept constant. The depth of cuts in the block (150 × 150 × 50 mm) made of AISI 1020 were measured by means of a depth gauge at 10 mm intervals along the slot (four measurements), starting at 10 mm from the beginning of the slot. The average of the measurement values was taken for further analysis. The experimental results are shown in Table 1. The depth of cut obtained under the medium magnitudes of the AWJ cutting parameters ($P = 250$ MPa, $Q = 13.58$ g/sec, $V = 0.63$ mm/sec and

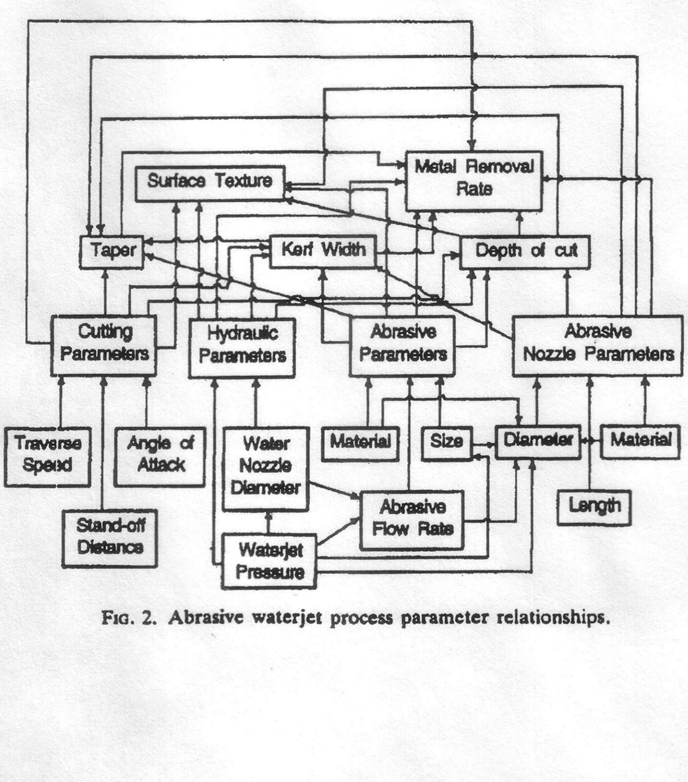

FIG. 2. Abrasive waterjet process parameter relationships.

TABLE 1. THE EXPERIMENTAL RESULTS

Cutting conditions			Depth of cut (mm)
Pressure (MPa)	300	V = 0.63 mm/sec	17.6
	250	Q = 13.58 g/sec	13.7
	200	*i.d.* = 1.65 mm	9.5
Abrasive flow rate (g/sec)	18.8	P = 250 MPa	15.5
	13.58	V = 0.63 mm/sec	13.7
	3.7	*i.d.* = 1.65 mm	10.9
Traverse speed (mm/sec)	1.00	P = 250 MPa	10.2
	0.63	*i.d.* = 1.65 mm	13.7
	0.33	Q = 13.58 g/sec	20.5
Nozzle inside diameter (mm)	1.2	P = 250 MPa	12.0
	1.65	D = 13.58 g/sec	13.7
	2.20	V = 0.63 mm/sec	18.1

i.d. = 1.65 mm) was taken as a reference magnitude (d_b). The other factors were kept constant, such as diameter of the waterjet orifice (0.33 mm), length of abrasive waterjet nozzle (63.5 mm), angle of impact (90°), and the type of the abrasive and its grain size (garnet with Mesh No. 120).

The effect of the AWJ cutting parameters on the depth of cut is expressed with respect to the reference depth value. From the experience and obtained experimental data it is evident that the depth of cut will increase with increasing waterjet pressure, abrasive flow rate and AWJ nozzle inside diameter, and will decrease with increasing the jet traverse speed (Fig. 3). In the AWJ cutting system an operator has to adjust the waterjet pressure, abrasive flow rate, jet traverse speed and stand-off distance based upon the type of material to be cut, depth of cut, surface quality, AWJ nozzle wear, etc.

Since the main objective is to illustrate the use of the fuzzy set theory in selecting

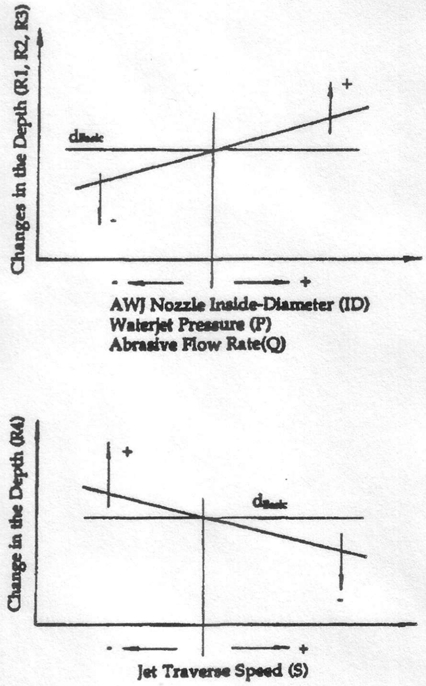

FIG. 3. The trends of the changes of the depth of cut with respect to the AWJ cutting parameters.

the AWJ cutting parameters for the given depth of cut, only the ***single input–single output cases*** will be considered in the interest of simplicity. The changes in the depth of cut with respect to the reference depth value caused by the change in the AWJ nozzle inside diameter, waterjet pressure, abrasive flow rate and jet traverse speed were selected as the universe of the input, and the corresponding abrasive waterjet cutting variables were selected as the universe of the output.

The total depth of cut for the given AWJ cutting variables will be

$$d = d_b \prod_{i=1}^{4} R_i ,$$

where d_b is the reference depth (d = 13.7 mm) obtained under the following cutting conditions: P = 250 MPa, Q = 13.58 g/sec, $i.d.$ = 1.65 mm and V = 0.63 mm/sec; R_i are the changes in the depth of cut with respect to the reference depth caused by the changes in the cutting conditions $R_i = d_i/d_b$, i = 1,2,3,4; and n is number of the selected cutting variables; in this study four variables were selected.

Universes of discourse for AWJ nozzle inside diameter, waterjet pressure, abrasive flow rate, jet traverse speed and the change of the depth of cut are discretized into 17 levels with 5 terms (primary fuzzy sets). The selected primary fuzzy sets are named as follows: *NB*—negative big, *NS*—negative small, *ZO*—zero, *PS*—positive small, and *PB*—positive big. The results of the discretization of the inputs and outputs are given in Tables 2–9. It should be noticed that due to discretization, the performance of a fuzzy logic controller is less sensitive to small deviations in the values of the process state variables.

A triangular shape which was found to be the best shape for this application was selected for the membership function (Fig. 4). Overlapping was set only between any two adjacent variables. It should be noticed that the correct choice of the membership functions of a term set which is associated with a linguistic variable, plays an essential role in the success of an application.

The knowledge extracted from the operator may be organized into a logic control rules format which describes the behavior of the skilled operator. Suppose that the AWJ operator has made a manual control of the process according to his own hypothetical verbal descriptions:

IF depth of cut is high THEN waterjet pressure is high OR

TABLE 2. QUANTIZATION AND PRIMARY FUZZY SETS OF *i.d.*

		Fuzzy sets				
Level No.	Range	*NB*	*NS*	*ZO*	*PS*	*PB*
−8	≤ 0.76	1	0	0	0	0
−7	0.76–1.09	0.75	0.25	0	0	0
−6	1.09–1.17	0.5	0.5	0	0	0
−5	1.17–1.26	0.25	0.75	0	0	0
−4	1.26–1.35	0	1	0	0	0
−3	1.35–1.44	0	0.75	0.25	0	0
−2	1.44–1.52	0	0.5	0.5	0	0
−1	1.52–1.60	0	0.25	0.75	0	0
0	1.60–1.69	0	0	1	0	0
1	1.69–1.77	0	0	0.75	0.25	0
2	1.77–1.84	0	0	0.5	0.5	0
3	1.84–1.92	0	0	0.25	0.75	0
4	1.92–1.99	0	0	0	1	0
5	1.99–2.07	0	0	0	0.75	0.25
6	2.07–2.14	0	0	0	0.5	0.5
7	2.14–2.3	0	0	0	0.25	0.75
8	≥ 2.3	0	0	0	0	1

TABLE 3. QUANTIZATION AND PRIMARY FUZZY SETS OF $R1$

Level No.	Range (mm)	Fuzzy sets				
		NB	*NS*	*ZO*	*PS*	*PB*
−8	≤ 0.68	1	0	0	0	0
−7	0.68–0.74	0.75	0.25	0	0	0
−6	0.74–0.78	0.5	0.5	0	0	0
−5	0.78–0.82	0.25	0.75	0	0	0
−4	0.82–0.86	0	1	0	0	0
−3	0.86–0.90	0	0.75	0.25	0	0
−2	0.90–0.94	0	0.5	0.5	0	0
−1	0.94–0.98	0	0.25	0.75	0	0
0	0.98–1.03	0	0	1	0	0
1	1.03–1.07	0	0	0.75	0.25	0
2	1.07–1.12	0	0	0.5	0.5	0
3	1.12–1.16	0	0	0.25	0.75	0
4	1.16–1.20	0	0	0	1	0
5	1.20–1.25	0	0	0	0.75	0.25
6	1.25–1.30	0	0	0	0.5	0.5
7	1.30–1.36	0	0	0	0.25	0.75
8	≥ 1.36	0	0	0	0	1

TABLE 4. QUANTIZATION AND PRIMARY FUZZY SETS OF P

Level No.	Range (MPa)	Fuzzy sets				
		NB	*NS*	*ZO*	*PS*	*PB*
−8	< 160	1	0	0	0	0
−7	160–191	0.75	0.25	0	0	0
−6	191–200	0.5	0.5	0	0	0
−5	200–209	0.25	0.75	0	0	0
−4	209–218	0	1	0	0	0
−3	218–227	0	0.75	0.25	0	0
−2	227–237	0	0.5	0.5	0	0
−1	237–245	0	0.25	0.75	0	0
0	245–254	0	0	1	0	0
1	254–262	0	0	0.75	0.25	0
2	262–270	0	0	0.5	0.5	0
3	270–278	0	0	0.25	0.75	0
4	278–286	0	0	0	1	0
5	286–294	0	0	0	0.75	0.25
6	294–302	0	0	0	0.5	0.5
7	302–330	0	0	0	0.25	0.75
8	≥ 330	0	0	0	0	1

IF depth of cut is small THEN waterjet pressure is small, etc.

A set of linguistic statements based on expert knowledge will form a fuzzy algorithm. In this study, five rules are employed for four process variables (AWJ nozzle inside diameter, waterjet pressure, abrasive flow rate and jet traverse speed), as shown in Table 10.

After establishing the control rules and partitioning the universe of the inputs and outputs using the triangle-shaped membership functions it is necessary to establish the relationship between the input and the output of the analyzed process by applying the fuzzy relationships. Each fuzzy control rule is represented by a fuzzy relation and the behavior of a fuzzy system is characterized by the fuzzy relations. The relationship between input and output can be found using Cartesian product expressions of the two sets [3] as follows:

R input* output

TABLE 5. QUANTIZATION AND PRIMARY FUZZY SETS OF *R2*

		Fuzzy sets				
Level No.	Range	*NB*	*NS*	*ZO*	*PS*	*PB*
−8	0.59	1	0	0	0	0
−7	0.59–0.67	0.75	0.25	0	0	0
−6	0.67–0.72	0.5	0.5	0	0	0
−5	0.72–0.77	0.25	0.75	0	0	0
−4	0.77–0.82	0	1	0	0	0
−3	0.82–0.87	0	0.75	0.25	0	0
−2	0.87–0.92	0	0.5	0.5	0	0
−1	0.92–0.97	0	0.25	0.75	0	0
0	0.97–1.02	0	0	1	0	0
1	1.02–1.07	0	0	0.75	0.25	0
2	1.07–1.12	0	0	0.5	0.5	0
3	1.12–1.17	0	0	0.25	0.75	0
4	1.17–1.22	0	0	0	1	0
5	1.22–1.27	0	0	0	0.75	0.25
6	1.27–1.32	0	0	0	0.5	0.5
7	1.32–1.39	0	0	0	0.25	0.75
8	≥ 1.39	0	0	0	0	1

TABLE 6. QUANTIZATION AND PRIMARY FUZZY SETS OF *Q*

		Fuzzy sets				
Level No.	Range (g/sec)	*NB*	*NS*	*ZO*	*PS*	*PB*
−8	≤ 1.04	1	0	0	0	0
−7	1.04–5.45	0.75	0.25	0	0	0
−6	5.45–6.72	0.5	0.5	0	0	0
−5	6.72–8.0	0.25	0.75	0	0	0
−4	8.0–9.2	0	1	0	0	0
−3	9.2–10.42	0	0.75	0.25	0	0
−2	10.42–11.71	0	0.5	0.5	0	0
−1	11.71–12.96	0	0.25	0.75	0	0
0	12.96–14.08	0	0	1	0	0
1	14.08–15.08	0	0	0.75	0.25	0
2	15.08–16.08	0	0	0.5	0.5	0
3	16.08–17.08	0	0	0.25	0.75	0
4	17.08–18.08	0	0	0	1	0
5	18.08–19.08	0	0	0	0.75	0.25
6	19.08–20.08	0	0	0	0.5	0.5
7	20.08–23.58	0	0	0	0.25	0.75
8	≥ 23.58	0	0	0	0	1

where * represents the Cartesian product.

A membership function of this relationship is

$$\mu_R = \min\{\mu_{input}, \mu_{output}\}.$$

In the following analysis, the procedure of applying the fuzzy set theory in calculating the necessary AWJ cutting parameters in order to achieve the desired depth of cut by abrasive waterjet will be illustrated in detail. Namely, in this analysis the desired depth of cut is the input variable and the outputs are AWJ cutting variables.

As was already stated (Table 10), for every AWJ variable five rules were developed, based on the observation of the AWJ milling process. The AWJ nozzle inside diameter is taken as the first variable to show the procedure for determining the magnitude of the AWJ cutting variable for the given depth of cut. The input to the system is the change in the depth of cut with respect to the reference depth (13.7 mm) and the output is the magnitude of the AWJ nozzle inside diameter. The fuzzy variables in the

TABLE 7. QUANTIZATION AND PRIMARY FUZZY SET OF *R3*

Level No.	Range	Fuzzy sets *NB*	*NS*	*ZO*	*PS*	*PB*
−8	≤ 0.79	1	0	0	0	0
−7	0.79–0.83	0.75	0.25	0	0	0
−6	0.83–0.85	0.5	0.5	0	0	0
−5	0.85–0.87	0.25	0.75	0	0	0
−4	0.87–0.91	0	1	0	0	0
−3	0.91–0.94	0	0.75	0.25	0	0
−2	0.94–0.96	0	0.5	0.5	0	0
−1	0.96–0.99	0	0.25	0.75	0	0
0	0.99–1.01	0	0	1	0	0
1	1.01–1.04	0	0	0.75	0.25	0
2	1.04–1.07	0	0	0.5	0.5	0
3	1.07–1.09	0	0	0.25	0.75	0
4	1.09–1.11	0	0	0	1	0
5	1.11–1.14	0	0	0	0.75	0.25
6	1.14–1.17	0	0	0	0.5	0.5
7	1.17–1.2	0	0	0	0.25	0.75
8	≥ 1.2	0	0	0	0	1

TABLE 8. QUANTIZATION AND PRIMARY FUZZY SETS OF *S*

Level No.	Range (mm/sec)	Fuzzy sets *NB*	*NS*	*ZO*	*PS*	*PB*
−8	≤ 0.18	1	0	0	0	0
−7	0.18–0.34	0.75	0.25	0	0	0
−6	0.34–0.38	0.5	0.5	0	0	0
−5	0.38–0.43	0.25	0.75	0	0	0
−4	0.43–0.47	0	1	0	0	0
−3	0.47–0.52	0	0.75	0.25	0	0
−2	0.52–0.57	0	0.5	0.5	0	0
−1	0.57–0.61	0	0.25	0.75	0	0
0	0.61–0.66	0	0	1	0	0
1	0.66–0.72	0	0	0.75	0.25	0
2	0.72–0.78	0	0	0.5	0.5	0
3	0.78–0.84	0	0	0.25	0.75	0
4	0.84–0.90	0	0	0	1	0
5	0.90–0.97	0	0	0	0.75	0.25
6	0.97–1.03	0	0	0	0.5	0.5
7	1.03–1.23	0	0	0	0.25	0.75
8	≥ 1.23	0	0	0	0	1

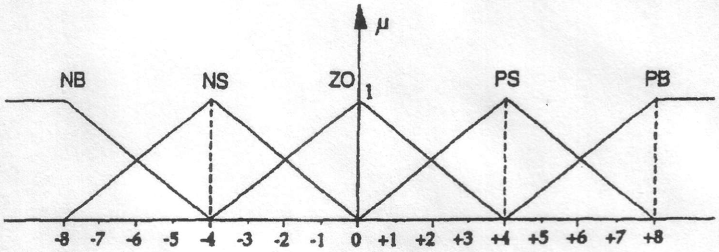

FIG. 4. The membership function of fuzzy sets in universe of discourse.

TABLE 9. QUANTIZATION AND PRIMARY FUZZY SETS OF $R4$

		Fuzzy sets				
Level No.	Range	*PB*	*PS*	*ZO*	*NS*	*NB*
−8	≤ 0.71	1	0	0	0	0
−7	0.71–0.76	0.75	0.25	0	0	0
−6	0.76–0.80	0.5	0.5	0	0	0
−5	0.80–0.84	0.25	0.75	0	0	0
−4	0.84–0.87	0	1	0	0	0
−3	0.87–0.91	0	0.75	0.25	0	0
−2	0.91–0.94	0	0.5	0.5	0	0
−1	0.94–0.98	0	0.25	0.75	0	0
0	0.98–1.03	0	0	1	0	0
1	1.03–1.10	0	0	0.75	0.25	0
2	1.10–1.17	0	0	0.5	0.5	0
3	1.17–1.25	0	0	0.25	0.75	0
4	1.25–1.31	0	0	0	1	0
5	1.31–1.39	0	0	0	0.75	0.25
6	1.39–1.45	0	0	0	0.5	0.5
7	1.45–1.56	0	0	0	0.25	0.75
8	≥ 1.56	0	0	0	0	1

TABLE 10. 'IF–THEN' RULES

	For the AWJ nozzle inside diameter (i.d.)
Rule 1:	IF $R1 = NB$ THEN $i.d. = NB$
Rule 2:	IF $R1 = NS$ THEN $i.d. = NS$
Rule 3:	IF $R1 = ZO$ THEN $i.d. = ZO$
Rule 4:	IF $R1 = PS$ THEN $i.d. = PS$
Rule 5:	IF $R1 = PB$ THEN $i.d. = PB$
	For the water jet pressure (P)
Rule 1:	IF $R2 = NB$ THEN $P = NB$
Rule 2:	IF $R2 = NS$ THEN $P = NS$
Rule 3:	IF $R2 = ZO$ THEN $P = ZO$
Rule 4:	IF $R2 = PS$ THEN $P = PS$
Rule 5:	IF $R2 = PB$ THEN $P = PB$
	For the abrasive flow rate (Q)
Rule 1:	IF $R3 = NB$ THEN $Q = NB$
Rule 2:	IF $R3 = NS$ THEN $Q = NS$
Rule 3:	IF $R3 = ZO$ THEN $Q = ZO$
Rule 4:	IF $R3 = PS$ THEN $Q = PS$
Rule 5:	IF $R3 = PB$ THEN $Q = PB$
	For the jet traverse speed (S)
Rule 1:	IF $R4 = NB$ THEN $S = PB$
Rule 2:	IF $R4 = NS$ THEN $S = PS$
Rule 3:	IF $R4 = ZO$ THEN $S = ZO$
Rule 4:	IF $R4 = PS$ THEN $S = NS$
Rule 5:	IF $R4 = PB$ THEN $S = NB$

form of negative big, negative small, and positive small and positive big are specified with respect to the reference points that are described as zero points.

In the case of rule 1 (IF $R1 = NB$ THEN $i.d. = NB$), the fuzzy relation will be

$$FR1 = (R1)NB^* \, (i.d.)NB$$

which has a membership function of

$$\mu_{FR1} = \min\{\mu_{NB}(R1), \mu_{NB}(i.d.)\} .$$

The fuzzy set for a negative big change in the depth of cut $\{R1(NB)\}$ is defined from Table 3 as

TABLE 11. THE RELATIONSHIP OF THE TWO FUZZY SETS $\{R1(NB)$ and $i.d.(NB)\}$

		i.d. universe						
	*R*1 universe	−8	−7	−6	−5	−4	...	8
	−8	1	0.75	0.5	0.25	0	...	0
	−7	0.75	0.75	0.5	0.25	0	...	0
	−6	0.5	0.5	0.5	0.25	0	...	0
	−5	0.25	0.25	0.25	0.25	0	...	0
*FR*1	−4	0	0	0	0	0	...	0
	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.
	8	0	0	0	0	0	...	0

$$R1(NB) = 1/-8 + 0.75/-7 + 0.5/-6 + 0.25/-5 + 0/-4 + \ldots + 0/7 + 0/8\,.$$

The fuzzy set for the negative big AWJ nozzle inside diameter $\{i.d.(NB)\}$ is defined from Table 2 as

$$i.d.(NB) = 1/-8 + 0.75/-7 + 0.5/-6 + 0.25/-5 + 0/-4 + \ldots + 0/7 + 0/8\,.$$

The relationship between these two fuzzy sets $\{R1(NB)$ and $i.d.(NB)\}$ in the first rule is shown in Table 11.

Using the same procedure, the other four relationships of the second, third, fourth and fifth rules were developed and the results are shown in Tables 12–15.

An inference mechanism is used, in conjunction with the rules, to determine the proper action during the operation. By combining the fuzzy relationships an input will be allowed to be any of the selected linguistic values (either "negative big" OR "negative small" OR . . . OR "positive big"). The combination operator "OR" represents the maximum of the membership values of all of the involved fuzzy relations.

The fuzzy algorithm for the modeling of the influence of the AWJ nozzle inside diameter on the depth of cut is obtained by combining all five fuzzy relationships with the "OR" operator as follows:

IF $R1 = NB$ THEN $i.d. = NB$ OR

IF $R1 = NS$ THEN $i.d. = NS$ OR

IF $R1 = ZO$ THEN $i.d. = ZO$ OR

IF $R1 = PS$ THEN $i.d. = PS$ OR

IF $R1 = PB$ THEN $i.d. = PB$.

TABLE 12. THE RELATIONSHIP OF THE TWO FUZZY SETS $\{R1(NS)$ and $i.d.(NS)\}$

		i.d. universe										
	*R*1 universe	−8	−7	−6	−5	−4	−3	−2	−1	0	...	8
	−8	0	0	0	0	0	0	0	0	0	...	0
	−7	0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	...	0
	−6	0	0.25	0.5	0.5	0.5	0.5	0.5	0.25	0	...	0
	−5	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0	...	0
	−4	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0	...	0
	−3	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0	...	0
*FR*2	−2	0	0.25	0.5	0.5	0.5	0.5	0.5	0.25	0	...	0
	−1	0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	...	0
	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.
	8	0	0	0	0	0	0	0	0	0	...	0

Table 13. The relationship of the two fuzzy sets $\{R1(ZO)$ and $i.d.(ZO)\}$

	*R*1 universe	*i.d.* universe −8	...	−4	−3	−2	−1	0	1	2	3	4	...	8
	−8	0	...	0	0	0	0	0	0	0	0	0	...	0
	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	−4	0	...	0	0	0	0	0	0	0	0	0	...	0
	−3	0	...	0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	...	0
	−2	0	...	0	0.25	0.5	0.5	0.5	0.5	0.5	0.25	0	...	0
	−1	0	...	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0	...	0
*FR*3	0	0	...	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0	...	0
	1	0	...	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0	...	0
	2	0	...	0	0.25	0.5	0.5	0.5	0.5	0.5	0.5	0	...	0
	3	0	...	0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	...	0
	4	0	...	0	0	0	0	0	0	0	0	0	...	0
	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	8	0		0	0	0	0	0	0	0	0	0		0

Table 14. The relationship of the two fuzzy sets $\{R1(PS)$ and $i.d.(PS)\}$

	*R*1 universe	*i.d.* universe −8	...	0	1	2	3	4	5	6	7	8
	−8	0	...	0	0	0	0	0	0	0	0	0
	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.
	1	0		0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0
	2	0	...	0	0.25	0.5	0.5	0.5	0.5	0.5	0.25	0
*FR*4	3	0	...	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0
	4	0	...	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0
	5	0	...	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0
	6	0	...	0	0.25	0.5	0.5	0.5	0.5	0.5	0.25	0
	7	0	...	0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0
	8	0	...	0	0	0	0	0	0	0	0	0

Table 15. The relationship of the two fuzzy sets $\{R1(PB)$ and $i.d.(PB)\}$

	*R*1 universe	*i.d.* universe −8	...	4	5	6	7	8
	−8	0	...	0	0	0	0	0
	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.
*FR*5	4	0	...	0	0	0	0	0
	5	0	...	0	0.25	0.25	0.25	0.25
	6	0	...	0	0.25	0.5	0.5	0.5
	7	0	...	0	0.25	0.5	0.75	0.75
	8	0	...	0	0.25	0.5	0.75	1

The membership function of this combined fuzzy relationship is

$$\mu_{FR} = \max\{\mu_{FR1}, \mu_{FR2}, \mu_{FR3}, \mu_{FR4}, \mu_{FR5}\} .$$

The combined fuzzy relationship is given in Table 16. This relationship describes the action of the AWJ operator. Namely, using this relationship it is possible to determine the needed AWJ nozzle inside diameter in order to achieve the desired depth of cut.

TABLE 16. THE COMBINED FUZZY RELATIONSHIP THAT REPRESENTS THE INFLUENCE OF THE AWJ NOZZLE INSIDE DIAMETER ON THE DEPTH OF CUT

		i.d. universe																
	*R*1 universe	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8
	−8	1	0.75	0.5	0.25	0	0	0	0	0	0	0	0	0	0	0	0	0
	−7	0.75	0.75	0.5	0.25	0.25	0.25	0.25	0.25	0	0	0	0	0	0	0	0	0
	−6	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.25	0	0	0	0	0	0	0	0	0
	−5	0.25	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0	0	0	0	0	0	0	0	0
	−4	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0	0	0	0	0	0	0	0	0
	−3	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0.25	0.25	0.25	0.25	0	0	0	0	0
	−2	0	0.25	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.25	0	0	0	0	0
	−1	0	0.25	0.25	0.25	0.25	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0	0	0	0	0
FR	0	0	0	0	0	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0	0	0	0	0
	1	0	0	0	0	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0.25	0.25	0.25	0.25	0
	2	0	0	0	0	0	0.25	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0
	3	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0
	4	0	0	0	0	0	0	0	0	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0
	5	0	0	0	0	0	0	0	0	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0.25
	6	0	0	0	0	0	0	0	0	0	0.25	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	7	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25	0.5	0.75	0.75
	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0.25	0.5	0.75	1

Contrary to this, it is possible to predict the depth of cut based on the known AWJ nozzle inside diameter.

As an example, assume that the required depth of cut is 12 mm. What will be the corresponding AWJ nozzle inside diameter knowing all other AWJ cutting parameters (waterjet pressure is 250 MPa, abrasive flow rate is 13.58 g/sec and jet traverse speed is 0.63 mm/sec)? The ratio of the required depth of cut with respect to the reference depth of cut is 0.88. From Table 16 the corresponding fuzzy set is

$$\mu\{R1_{(0.88)}\} = \{0\ 0.25\ 0.5\ 0.75\ 0.75\ 0.75\ 0.5\ 0.25\ 0.25\ 0.25\ 0.25\ 0.25\ 0\ 0\ 0\ 0\ 0\}\ .$$

Since the machine does not understand fuzzy variables, it is necessary to perform defuzzification by using, for example, the center of gravity method.

The required AWJ nozzle diameter will be

$$i.d._{\text{aver}} = \left\{\sum_{1}^{17} i.d. \times \mu(i.d.)\right\} \Big/ \sum_{1}^{17} \mu(i.d.)$$

$$\begin{aligned} i.d._{\text{aver}} = {} & \{0.25 \times (1.03 + 1.56 + 1.65 + 1.72 + 1.8 + 1.87) \\ & + 0.5 \times (1.12 + 1.47) + 0.75(1.21 + 1.3 + 1.34)\}/0.25 \times 6 \\ & + 0.5 \times 2 + 0.75 \times 3 = 1.38 \text{ mm}. \end{aligned}$$

Based on Table 16, the defuzzified relationship between the change in depth of cut

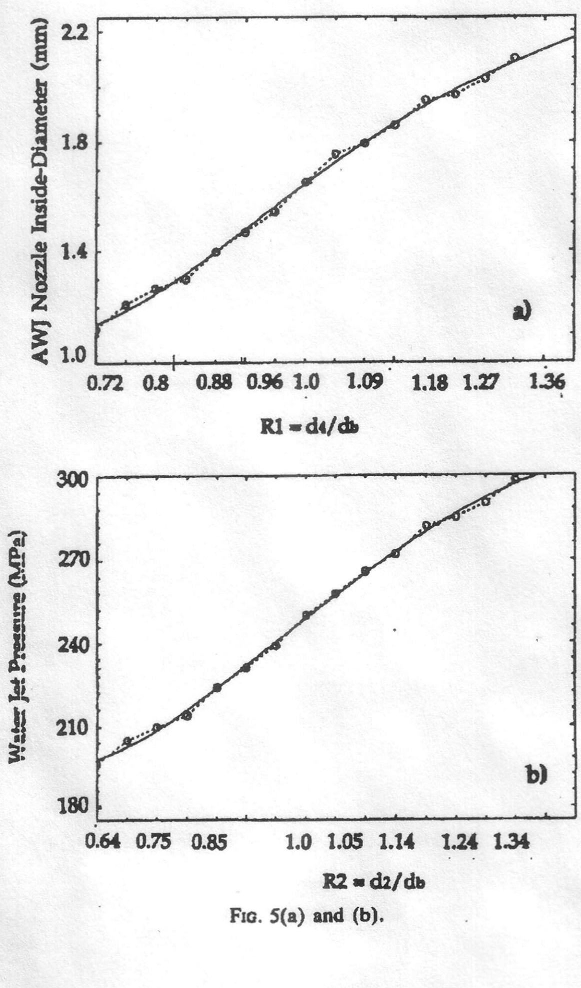

FIG. 5(a) and (b).

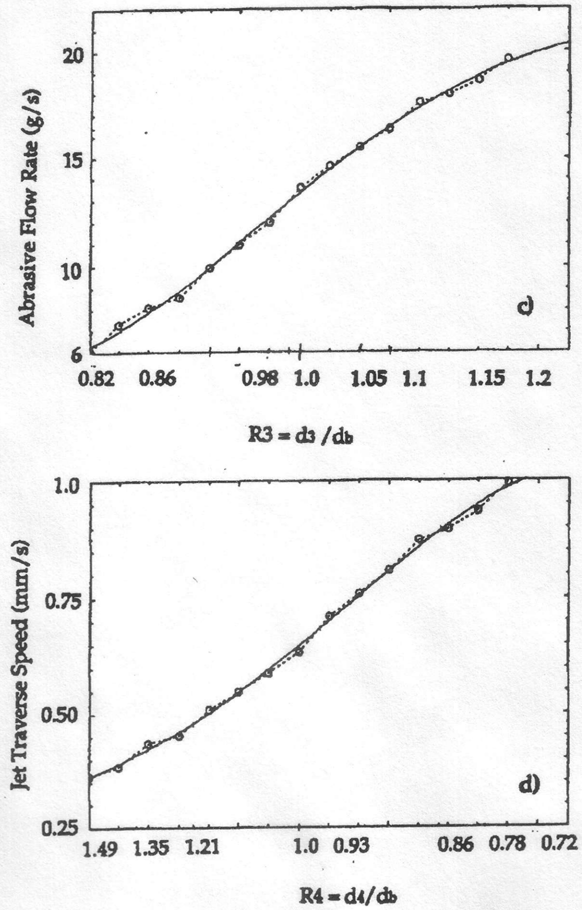

FIG. 5. The relations between the AWJ cutting parameters and their contribution ratios to the depth of cut.

and the AWJ nozzle inside diameter can be represented graphically as shown in Fig. 5(a). Using the same procedure, the defuzzified relationships between the change in depth of cut and waterjet pressure, abrasive flow rate and jet traverse speed are obtained and presented graphically in Fig. 5(b), (c) and (d).

4. DISCUSSION

The fuzzy algorithms developed for modeling the influence of the AWJ cutting parameters on the depth of cut, presented graphically in Fig. 5, could be effectively used for the off-line selection of the magnitudes for the AWJ cutting parameters in order to achieve the desired depth of cut. It is evident that the fuzzy algorithms could be easily processed by computer and thereby make the AWJ cutting parameter selection procedure more convenient for the operator.

Figure 6 illustrates the procedure of selecting the AWJ cutting parameters for the required depth of cut. The first stage of the procedure is to determine the required depth of cut and then, through the iterations procedure, select the most suitable AWJ cutting parameters whose combination will produce the required depth of cut. With this approach the operator can simulate all possible combinations of the AWJ cutting parameters before the cutting starts.

Among the four selected parameters, the magnitude of the AWJ nozzle inside diameter is known in advance. The operator can measure it and input its value into the program in order to determine the ratio $R1$, which represents the contribution of the AWJ nozzle inside diameter to the required depth of cut. The combination of the

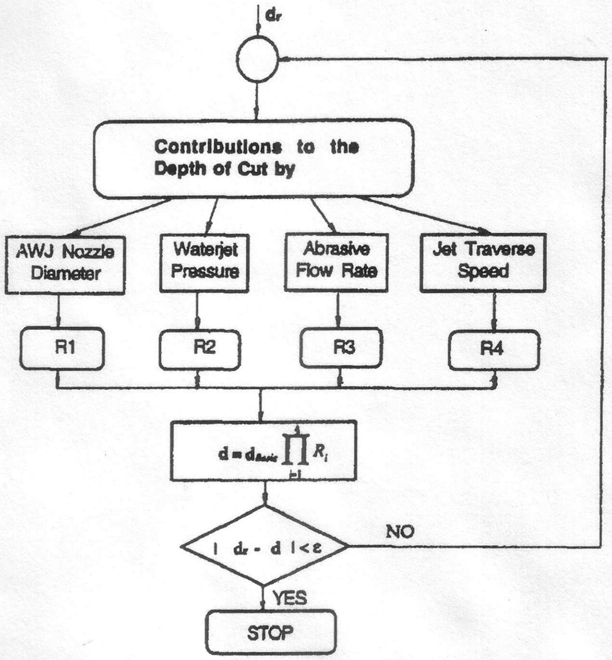

FIG. 6. The procedure of selecting the AWJ cutting parameters for the required depth of cut.

remaining three parameters—waterjet pressure, abrasive flow rate and jet traverse speed—will continue until the difference between the required depth of cut and estimated depth of cut is less than the predetermined value ε.

The following example shows how this proposed procedure could be used in the selection of the AWJ cutting parameters in order to achieve the required depth of cut. Assume that the required depth of cut in the milled steel AISI 1020 is 15 mm. What will be the combination of the AWJ cutting parameters in order to achieve this depth of cut? The operator will measure the inside diameter of the AWJ nozzle outlet. Assume that in this case this diameter is 1.3 mm. The contribution ratio $R1(0.83)$ of the AWJ nozzle inside diameter to the depth of cut is obtainable from Table 16 or from the corresponding diagram shown in Fig. 5. The values of the remaining three AWJ cutting parameters will be selected through the iterations satisfying the condition that the absolute difference between the required depth of cut and estimated one will be less than or equal to the selected magnitude. In this case the difference is selected to be 0.2 mm. The selected values of these three AWJ cutting parameters with their corresponding contribution ratios to the depth of cut are: waterjet pressure, 231 MPa with $R2 = 0.89$; abrasive flow rate, 16.34 g/sec with $R3 = 1.07$; and jet traverse speed, 0.42 mm/sec with $R4 = 1.37$. The reference depth of cut, as already mentioned is 13.7 mm. Then the estimated depth of cut will be

$$d = d_b \prod_{i=1}^{4} R_i = 14.8 \text{ mm} .$$

The experiment was performed in order to obtain the actual depth of cut and to verify the validity of the proposed methodology. The average actual depth of cut obtained under the following conditions—waterjet pressure, 231 MPa; abrasive flow rate, 16.34 g/sec; jet traverse speed, 0.42 mm/sec; and AWJ nozzle inside diameter of 1.3 mm—was equal to 14.3 mm. Table 17 shows the results for the other two examples.

From the obtained results, it is evident that the proposed methodology, based on the fuzzy rules, for selecting the AWJ cutting parameters in order to achieve the

TABLE 17. CASE STUDIES

Case No.	Reference depth of cut d_b (mm)	Required depth of cut d_r (mm)	AWJ cutting parameters with corresponding ratios R_i								Estimated depth of cut (mm)	Actual depth of cut (mm)
			i.d. (mm)	*R*1	*P* (MPa)	*R*2	*Q* (g/sec)	*R*3	*S* (mm/sec)	*R*4		
1	13.7	10	1.65	1	224	0.85	12.85	0.99	0.87	0.87	10.02	9.1
2	13.7	15	1.3	0.83	231	0.89	16.34	1.07	0.47	1.37	14.8	14.3
3	13.7	20	1.47	0.91	280	1.20	15.47	1.05	0.42	1.27	19.94	20.1

logic controller for the new generation of the AWJ cutting systems, which could insure the achievement of the uniform depth of cut.

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