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Development of a Fuzzy Logic Method to Select Abrasive Water Jet Parameters for Notching Concrete Pavement Slabs

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The objective of this paper is to introduce a simple and effective method for selecting the optimal combination of the most significant abrasive water jet cutting parameters to achieve a predetermined depth of cut in a concrete slab. It is shown that the principles of fuzzy set theory could be effectively used to simulate human experience and the experimental information that is normally used for controlling the process. This approach will eliminate the need for extensive experimental investigation. Furthermore, it is possible to develop an expert system that can be used by control system engineers in devising a strategy for controlling the abrasive water jet cutting process.

Nomenclature

d	depth of cut
d_b	reference depth of cut
P	water jet pressure
Q	abrasive flow rate
R_i	ratio of actual depth of cut to reference depth
V	jet traverse speed
U	universe of discourse
μ	membership function
ε	accuracy of estimation

Introduction

The main objective in using the abrasive water jets in the rehabilitation of portland cement concrete (PCC) pavement slabs is to notch the slabs deep enough for propagation of a pattern of cracks below the surface subject to an appropriate compressive load. The depth of the notches will depend on the strength of the pavement, presence of the reinforcement and the procedure selected for imposing the compressive load (usually heavy tires). From a practical point of view, it is important to be able to select in advance an optimal combination of abrasive water jet cutting process variables for a given depth of cut.

To produce a cavity with controlled depth is a basic problem in slotting (notching) operation with an abrasive water jet (AWJ). The depth of penetration depends not only on the abrasive water jet parameters (water jet pressure, abrasive flow rate, etc.), but also on the mechanics of the jet-material interaction.

The mathematical modelling of the physics of the AWJ process is somewhat limited due to its inherent complexities and a less than full understanding of it. The fuzzy logic in such cases can be very useful. It can be looked upon as an interface between the qualitative world of human thinking and the quantitative world of engineering. It provides a translation of fuzzy (qualitative) variables such as big, shallow, fast, slow, etc., into a specific output using fuzzy relations **(1,2)**.

Since extensive analyses of the effects of various abrasive water jet parameters on depth of cut in portland cement concrete have already been reported by Kovacevic, Ram and Hirsher **(3)**, the objective in this paper is to develop a methodology for finding an optimal combination of the AWJ process variables to generate a required depth of cut in these concrete slabs using fuzzy logic. Fuzzimetric arcs are used for partitioning the universe of discourse and choosing the fuzzy set shapes for the control variables **(4)**. The study suggests that the methodology can lead to the development of an expert system, useful to control system engineers in establishing a strategy for controlling an AWJ cutting process.

Development of Fuzzy Algorithms

The main factors affecting the cutting performance of an abrasive water jet are: water jet pressure, abrasive flow rate, jet traverse speed, abrasive size, standoff distance, angle of impact, and jet diameter. Figure 1 illustrates the various factors that can affect the AWJ process and consequently the cutting performances. Efficiency and accuracy of slotting can be improved by a proper choice of the process parameters.

In order to use the fuzzy set theory, it was important to establish minimum/maximum input/output values. In this study, the water jet pressure ranged from 102 to 236 MPa, the abrasive flow rate from 4.5 to 9.07 g/s, and the jet traverse speed from 0.85 to 6.77 mm/s.

Several tests were conducted to determine the values of depth of cut for selected cutting conditions. During the tests, one of the three selected variables (water jet pressure, abrasive flow rate, or jet traverse speed) was varied from minimum, medium, to maximum levels while the other two were kept constant. The results are shown in Table 1. The depth of cut of 69.8 mm obtained at a water jet pressure of 172 Mpa, an abrasive flow rate of 6.78 g/s, a jet traverse speed of 2.4 mm/s and a standoff distance of 15 mm is taken as a reference point (d_b).

Now, varying any of these parameters will affect cutting depth. For instance, with reference to d_{basic}, increasing jet pressure and abrasive flow rate will increase the depth of cut while increasing traverse speed will have the reverse effect [see Figure 2 **(3)**].

Since the main objective of this work is to illustrate the use of fuzzy logic in selecting the AWJ parameters to achieve a given depth of cut, for simplicity, a single input - single output approach is adapted. The changes in the depth of cut with respect to the reference depth, caused by variations in the water jet pressure, traverse speed and the abrasive flow rate are selected as the universe of the input and the corresponding abrasive water jet cutting variables are selected as the universe of the output. The total depth of cut is then determined by:

$$d = d_b \prod_{i=1}^{n} R_i$$

where R_i is the ratio between the actual and the basic reference depth of cut ($R_i = d/d_b$), and i is the number of selected cutting parameters (table 2).

The fuzzy set theory has been quite successful in analyzing complex or uncertain systems which cannot be described mathematically. A fuzzy system associates an output fuzzy set with an input fuzzy set and is characterized by a set of linguistic statements based on expert knowledge. The expert knowledge is usually in the form of "if-then" rules, which are easily implemented by fuzzy conditional statements in fuzzy logic. A human operator employs a set of fuzzy "if-then" rules to control the process under consideration **(1, 2, 4)**.

The key concept throughout fuzzy set theory is the definition of a fuzzy set. A fuzzy set in a universe of discourse U is characterized by a membership function $\mu_F : U \rightarrow \{0,1\}$. The primary fuzzy sets (linguistic terms) usually have a meaning such as big, negative medium, ... , positive big, etc. The membership function of a fuzzy set in the continuous universe of discourse is expressed in a functional form, typically bell-shaped, triangular, trapezoid, trigonometric circle, etc.

The trigonometric circles were adopted for partitioning the universe of discourse. It has been shown by Kouatli and Jones **(4)** that an analogy may be made between partitioning the universe of discourse and the structure of a trigonometric circle, that is, three quarters of a circular arc with a unit radius could be used as a partitioning scale. The positive fuzzy variables: positive zero (PZ), positive small (PS), positive medium (PM) and positive big (PB) can be defined from the positive (counter-clockwise direction) trigonometric circle as follows **(4)**:

PZ	$= \lvert \sin(\bar{u}/2 - x) \rvert$	for $0 < x < \bar{u}/2$
	$= 0$	otherwise
PS	$= \sin(x)$	for $0 < x < \bar{u}$
	$= 0$	otherwise
PM	$= \lvert \sin(\bar{u}/2 - x) \rvert$	for $\bar{u}/2 < x < 3\bar{u}/2$
	$= 0$	otherwise
PB	$= \lvert \sin(x) \rvert$	for $\bar{u} < x < 3\bar{u}/2$
	$= 0$	otherwise

Any value that exceeds the limit of the universe is considered as an infinity.

The negative fuzzy variables appear in a clockwise direction in Figure 3. The zero level is shared between the two.

The cutting depth changes for pressure and traverse speed are divided into 18 units by using positive and negative trigonometric circles with each unit corresponding to 30°. The zero value on the circle corresponds to the mean value of pressure and traverse speed, and the unit value corresponds to the changes in ratios of the cutting depth, R_1 and R_2.

The contribution of the abrasive flow rate to the depth of cut is less significant compared to the water jet pressure and the traverse speed. Consequently, only three levels of abrasive flow rates (4.5, 6.87 and 9.07 g/s) with the corresponding depth of cut ratios (0.93, 1.0 and 1.06) were considered in the analysis. The membership for the abrasive flow rate is defined as follows:

$$\mu_Q(r) = \begin{cases} 1 \text{ for } r = Q_i \\ i = 1,2,3 \\ 0 \text{ otherwise} \end{cases}$$

This type of fuzzy set is also known as a fuzzy singleton.

The results of discretization of the inputs and the outputs are given in Tables 3 and 4. From experience it is known that while the dependence of the depth of cut on water jet pressure is linear, it is non-linear with respect to the traverse speed. This has been taken into account in the discretization of the universe. In other words, while the universe of water pressure is partitioned at regular intervals of 8 MPa in the range from 172 to 236 Mpa, the universe of changes in depth of cut is partitioned by taking the arithmetic mean value for the intervals from 0.66 to 1.0 and from 1.0 to 1.28 (the difference between each of the two units is 0.035 and 0.042 respectively). The universe of the traverse speed, on the other hand, is partitioned by taking the geometric mean value for the interval from 0.85 to 6.77 mm/s, keeping a constant ratio between the two adjacent speeds, with a similar procedure for the depth of cut ratios (R_2) in the range from 1.8 to 0.55.

After partitioning the universe, the next step in the procedure is to establish the fuzzy algorithms, which is simply a set of linguistic statements, based on the expert knowledge. These statements, usually made by the water jet operator, could be:

IF the depth of cut is deep THEN the traverse speed is slow, OR
IF the depth of cut is shallow THEN the traverse speed is fast, etc.

Seven rules are employed, as shown in Table 5, for the two process variables. The inputs to the system are the changes in the depth of cut with respect to the reference depth (69.86 mm) and the outputs are the magnitudes of the water jet pressure and the traverse speed. The fuzzy variables in the form of positive and negative big, medium and small are specified with respect to the reference points that are described as positive and negative normal. Each fuzzy control rule is represented by a fuzzy relation and the behaviour of a fuzzy system is characterized by the fuzzy relations. The relation between the input and the output can be found using Cartesian expressions of the two sets as follows:

R = Input data * output data

where * represents the Cartesian product.
A membership function of this relationship is

$$\mu_R = \min\{\mu_{input}, \mu_{output}\}$$

By combining the fuzzy relationships, an input can be any of the selected linguistic values. The fuzzy algorithms for the modelling of the influence of the water jet pressure and the traverse speed on the depth of cut are obtained by combining all seven relationships with the "OR" operator as follows:

IF R_1 = NB THEN P = VL, OR
IF R_1 = NM THEN P = PL, OR
IF R_1 = NS THEN P = RL, OR
IF R_1 = NN THEN P = NN, OR
IF R_1 = PN THEN P = PN, OR
IF R_1 = PS THEN P = RH, OR
IF R_1 = PM THEN P = PH, OR
IF R_1 = PB THEN P = VH

IF R_2 = NB THEN V - VF, OR
IF R_2 = NM THEN V = VF, OR (R_2)
IF R_2 = NS THEN V = SF, OR
IF R_2 = NN THEN V = NN, OR
IF R_2 = PN THEN V = PN, OR
IF R_2 = PS THEN V = SS, OR
IF R_2 = PM THEN V = RS, OR
IF R_2 = PB THEN V = VS

The membership functions (μ_F) of these combined relationships are:

$$\mu_{FR}(P) = \max\{\mu_{FR1}(P), \mu_{FR2}(P), \ldots, \mu_{FR7}(P)\}$$
$$\mu_{FR}(S) = \max\{\mu_{FR1}(V), \mu_{FR2}(V). \ldots, \mu_{FR7}(V)\}$$

Tables 6 and 7 are combined fuzzy relationships representing respectively the influence of the water jet pressure and the jet traverse speed on the depth of cut. An operator using these relationships would find it possible to determine the magnitude of water jet pressure and jet traverse speed required to achieve a desired cutting depth. However, before transmitting fuzzy expressions to machines, it will be necessary to defuzzify them by using, for example, the centre of gravity method

$$x = \sum_{j=1}^{n} \mu(y_j)y_j \Big/ \sum_{j=1}^{n} \mu(y_j)$$

where x is the control action, y_j is the support value at which the membership function reaches the maximum value $\mu(y_j)$, and n is the number of quantization levels of the output.

The defuzzified relationship between the changes in the depth of cut (R_1 and R_2) and the water jet pressure and traverse speed can be presented graphically as shown in Figures 4 and 5 respectively.

Discussion

The fuzzy algorithms presented in Figures 4 and 5 can be used effectively for the off-line selection of AWJ cutting parameters. It is also possible to determine a combination of cutting parameters that is best suited for a given situation. In addition, fuzzy algorithms can be easily interpreted by computers thus simplifying the process even more.

The procedure for selecting the AWJ cutting parameters for a required depth of cut is shown in Figure 6. The first step is to determine the required depth of cut and then, by a series of iterative steps, select the most suitable AWJ cutting parameters. With this approach, the operator can simulate all possible combinations before the cutting starts.

In Figure 6, three parameters are considered, namely, water jet pressure, jet traverse speed and the abrasive flow rate. As the abrasive flow rate and its effect on depth of cut (R_3) is known in advance, the magnitudes of the combination of the remaining two parameters can be adjusted continually until a pre-determined accuracy, ε, is achieved where ε is the difference between the required and the estimated depths of cut. An example is given below to illustrate the procedure.

It is assumed that the required depth of cut in the concrete slab is 30 mm and ε = 1.0 mm. A value of 4.5 g/s for the abrasive flow rate is selected in advance. From Table 2, for this abrasive flow rate, the contribution ratio R_3 = 0.93. Since the reference depth of cut is 69.86 mm (see Table 2), the product of the contribution ratios (R_i) should be equal to 0.429. The values of the water jet pressure and the traverse speed can be determined using the iteration procedure depicted in Figure 6. Using the universe listed in Tables 6 and 7, it is easy to see that the magnitude of water jet pressure is 104 MPa (R_1 = 0.69) and the traverse speed is 4.65 mm/s (R_2 = 0.67). The estimated depth of cut is therefore:

$$d = d_b \prod_{i=1}^{3} R_i = 69.8 \times (0.69 \times 0.67 \times 0.93) = 30.03\text{mm}$$

Experiments were conducted in the laboratory to confirm the accuracy of the estimated AWJ cutting parameters. Three cuts of about 200 mm in length were made using the estimated parameters in portland cement pavement slabs. The depths of cut were measured at 50 mm intervals starting at 25 mm from the beginning of the slot. Average depths of cut for the example cited above and other two cases are listed in Table 8. It is evident from this table that the agreement between the estimated and measured depths of cut is reasonably good. Any deviation between the measured and estimated values can be attributed to such factors as the inhomogeneity of the slabs, fluctuations in the water jet pressure, etc.

These results confirm that the proposed methodology, based on the fuzzy set theory, for selecting the AWJ cutting parameters to achieve a given depth of cut is very effective. The methodology, as shown in a block diagram in Figure 7, can be used to adjust on-line the AWJ cutting parameters in the new generation of AWJ cutting systems. The block diagram also incorporates a nozzle wear sensing device to compensate for the influence of the nozzle wear on the depth of cut.

Conclusions

Previous tests have clearly indicated that abrasive water jets can be effectively used for slotting of reinforced concrete slabs. However, extensive experimental work will be required to determine the best combination of abrasive water jet parameters to achieve a given depth of cut. The methodology based on the fuzzy concepts explained in this paper shows that the need for the experimental data can be reduced significantly. The required knowledge base can be built by specifying the boundaries of the universe of the input and output of the fuzzy variables with respect to a reference point. Fuzzy logic provides a simple and effective imitation of human way of making decisions in situations where mathematical model of the process is either crude or does not even exist.

Acknowledgement

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Table 1. Test results for developing basic data.

Cutting Conditions			Depth of Cut (mm)
Water Jet Pressure (MPa)	102	Q = 6.78 g/s	46.10
	172	S = 15 mm	69.86
	236	V = 2.4 mm/s	89.42
Jet Traverse Speed (mm/s)	0.85	P = 172 MPa	125.74
	2.4	Q = 6.78 g/s	69.86
	6.77	S = 15 mm	38.42
Abrasive Flow Rate (g/s)	4.5	S = 15 mm	64.96
	6.78	P = 172 MPa	69.86
	9.07	V = 2.4 mm/s	74.05
Fixed parameters:	· Diameter of the AWJ nozzle - 1.27 mm · Diameter of the water jet nozzle - 0.45mm · Angle of jet impact - 90 degrees · Abrasive material - garnet #36		

Table 2. The changes in the depth of cut with respect to the reference depth.

	Water Jet Pressure (MPa)			Jet Traverse Speed (mm/s)			Abrasive Flow Rate (g/s)		
	102	172	236	0.85	2.4	6.77	4.5	6.78	9.07
Depth of cut (mm)	46.10	69.86	89.42	125.74	69.86	38.42	64.96	69.86	74.05
Contribution ratio (d/d_b)	0.66	1	1.28	1.79	1	0.55	0.93	1	1.06

Table 3. Quantization and primary fuzzy sets of water jet pressure and R_1.

R_1		<0.66	0.664	0.706	0.748	0.79	0.832	0.874	0.916	0.958	1	1.035	1.07	1.105	1.14	1.175	1.21	1.245	1.28	>1.28
	P	<102	102	110	118	126	136	148	156	164	172	180	188	196	204	212	220	228	236	>236
		-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
NB	VL	1	0.86	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NM	PL	0	0.5	0.86	1	0.86	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0
NS	RL	0	0	0	0	0.5	0.86	1	0.86	0.5	0	0	0	0	0	0	0	0	0	0
NN	NN	0	0	0	0	0	0	0	0.5	0.86	1	0	0	0	0	0	0	0	0	0
NP	NP	0	0	0	0	0	0	0	0	0	1	0.86	0.5	0	0	0	0	0	0	0
PS	RH	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1	0.86	0.5	0	0	0	0
PM	PH	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1	0.86	0.5	0
PB	VH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1

Table 4. Quantization and primary fuzzy sets of jet traverse speed and R_2.

R_2		>1.80	1.8	1.67	1.55	1.44	1.34	1.24	1.15	1.07	1	0.92	0.86	0.79	0.74	0.68	0.63	0.59	0.55	<0.55
	V	<0.75	0.85	0.97	1.10	1.26	1.43	1.63	1.85	2.11	2.40	2.73	3.11	3.54	4.03	4.58	5.22	5.94	6.77	>7.5
		-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
PB	VS	1	0.86	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PM	RS	0	0.5	0.86	1	0.86	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0
PS	SS	0	0	0	0	0.5	0.86	1	0.86	0.5	0	0	0	0	0	0	0	0	0	0
PN	NN	0	0	0	0	0	0	0	0.5	0.86	1	0	0	0	0	0	0	0	0	0
NN	NP	0	0	0	0	0	0	0	0	0	1	0.86	0.5	0	0	0	0	0	0	0
NS	SF	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1	0.86	0.5	0	0	0	0
NM	RF	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1	0.86	0.5	0
NB	VF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1

Table 5. IF-THEN rules.

FOR WATER JET PRESSURE		
Rule 1	If R_1 = NB	then P = VL
Rule 2	If R_1 = NM	then P = PL
Rule 3	If R_1 = NS	then P = RL
Rule 4	If R_1 = NN	then P = NN
	If R_1 = PN	then P = NP
Rule 5	If R_1 = PS	then P = RH
Rule 6	If R_1 = PM	then P = PH
Rule 7	If R_1 = PB	then P = VH
FOR JET TRAVERSE SPEED		
Rule 8	If R_2 = NB	then V = VF
Rule 9	If R_2 = NM	then V = RF
Rule 10	If R_2 = NS	then V = SF
Rule 11	If R_2 = NN	then V = NN
	If R_2 = PN	then V = NP
Rule 12	If R_2 = PS	then V = SS
Rule 13	If R_2 = PM	then V = RS
Rule 14	If R_2 = PB	then V = VS

where:

PB / NB	positive / negative big	VH / VL	very high / low	VH / VS	very fast / slow
PM / NM	positive / negative medium	PH / PH	pretty high / low	RF / RS	regular fast / slow
PS / NS	positive / negative small	RH / RL	regular high / low	SF / SS	slightly fast / slow
PN / NN	positive / negative normal	NP / NN	positive / negative normal	NP / NN	positive / negative normal

Table 6. The combined fuzzy relationship representing the influence of the water jet pressure on the depth of cut.

R_1 Universe \ Water Jet Pressure Universe	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
-9	1 *A	0.86	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-8	0.86	0.86	0.5	0.5	0.5	0.5	0	0	0	0	0	0	0	0	0	*A	Rule 1		0
-7	0.5	0.5	0.86	0.86	0.86	0.5	0	0	0	0	0	0	0	0	0	*B	Rule 2		0
-6	0	0.5	0.86	1 *B	0.86	0.5	0	0	0	0	0	0	0	0	0	*C	Rule 3		0
-5	0	0.5	0.86	0.86	0.86	0.5	0.5	0.5	0.5	0	0	0	0	0	0	*D	Rule 4		0
-4	0	0.5	0.5	0.5	0.5	0.86	0.86	0.86	0.5	0	0	0	0	0	0	*E	Rule 5		0
-3	0	0	0	0	0.5	0.86	1 *C	0.86	0.5	0	0	0	0	0	0	*F	Rule 6		0
-2	0	0	0	0	0.5	0.86	0.86	0.86	0.5	0.5	0.5	0.5	0	0	0	*G	Rule 7		0
-1	0	0	0	0	0.5	0.5	0.5	0.5	0.86	0.86	0.86	0.5	0	0	0				0
0	0	0	0	0	0	0	0	0.5	0.86	1 *D	0.86	0.5	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0.5	0.86	0.86	0.86	0.5	0.5	0.5	0.5	0	0	0	0
2	0	0	0	0	0	0	0	0.5	0.5	0.5	0.5	0.86	0.86	0.86	0.5	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1 *E	0.86	0.5	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0.5	0.86	0.86	0.86	0.5	0.5	0.5	0.5	0
5	0	0	0	0	0	0	0	0	0	0	0.5	0.5	0.5	0.5	0.86	0.86	0.86	0.5	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1 *F	0.86	0.5	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	0.86	0.86	0.5	0.5 *G
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.5	0.5	0.5	0.86	0.86
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1

Table 7. The combined fuzzy relationship representing the influence of the jet traverse speed on the depth of cut.

R_2 Universe \ Jet Traverse Speed Universe	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
-9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1 *H
-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.5	0.5	0.5	0.86	0.86
-7	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	0.86	0.86	0.5	0.5
-6	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1 *I	0.86	0.5	0
-5	0	0	0	0	0	0	0	0	0	0	0.5	0.5	0.5	0.5	0.86	0.86	0.86	0.5	0
-4	0	0	0	0	0	0	0	0	0	0	0.5	0.86	0.86	0.86	0.5	0.5	0.5	0.5	0
-3	0	0	0	0	0	0	0	0	0	0	0.5	0.86	1 *J	0.86	0.5	0	0	0	0
-2	0	0	0	0	0	0	0	0.5	0.5	0.5	0.5	0.86	0.86	0.86	0.5	0	0	0	0
-1	0	0	0	0	0	0	0	0.5	0.86	0.86	0.86	0.5	0.5	0.5	0.5	0	0	0	0
0	0	0	0	0	0	0	0	0.5	0.86	1 *K	0.86	0.5	0	0	0	0	0	0	0
1	0	0	0	0	0.5	0.5	0.5	0.5	0.86	0.86	0.86	0.5	0	0	0	*H	Rule 8		0
2	0	0	0	0	0.5	0.86	0.86	0.86	0.5	0.5	0.5	0.5	0	0	0	*I	Rule 9		0
3	0	0	0	0	0.5	0.86	1 *L	0.86	0.5	0	0	0	0	0	0	*J	Rule 10		0
4	0	0.5	0.5	0.5	0.5	0.86	0.86	0.86	0.5	0	0	0	0	0	0	*K	Rule 11		0
5	0	0.5	0.86	0.86	0.86	0.5	0.5	0.5	0.5	0	0	0	0	0	0	*L	Rule 12		0
6	0	0.5	0.86	1 *M	0.86	0.5	0	0	0	0	0	0	0	0	0	*M	Rule 13		0
7	0.5 *N	0.5	0.86	0.86	0.86	0.5	0	0	0	0	0	0	0	0	0	*N	Rule 14		0
8	0.86	0.86	0.5	0.5	0.5	0.5	0	0	0	0	0	0	0	0	0				0
9	1	0.86	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 8. Case studies.

Case No.	Reference Depth of Cut (mm)	Required Depth of Cut (mm)	AWJ Cutting Parameters with Corresponding Contribution Ratios, R_i						Estimated Depth of Cut (mm)	Actual Depth of Cut (mm)
			P(MPa)	R_1	V(mm/s)	R_2	Q(g/s)	R_3		
1	69.8	30	104	0.69	4.65	0.67	4.5	0.93	30.03	27.6
2	69.8	60	172	1	2.9	0.92	4.5	0.93	59.77	62.4
3	69.8	90	241	1.28	2.74	0.954	9.07	1.06	89.19	87.1

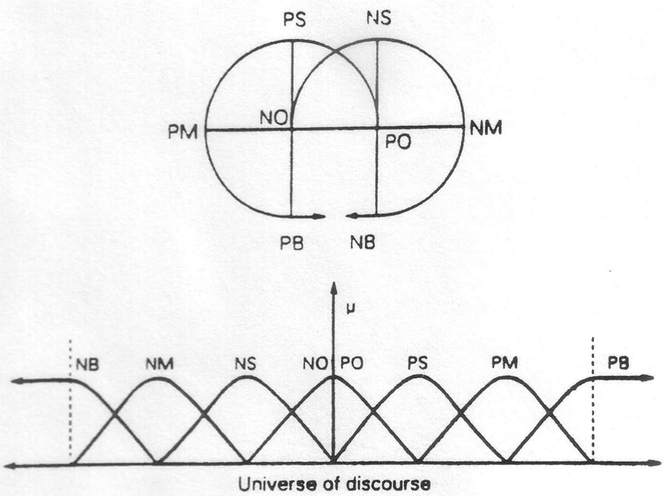

Figure 3. Trigonometric circles, positive and negative **(4)**.

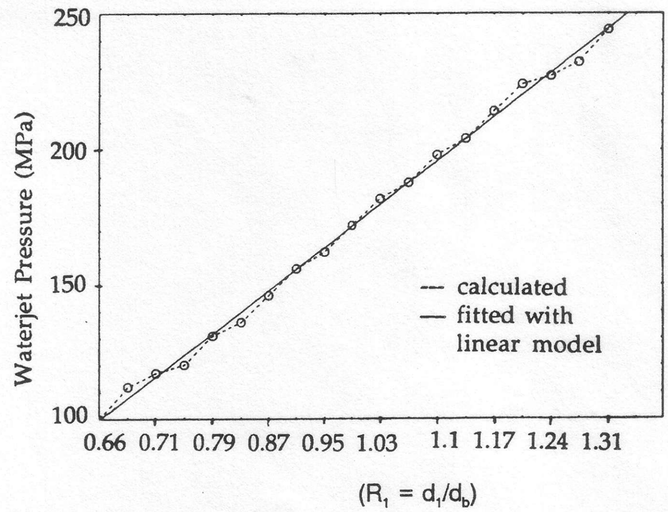

Figure 4. Relationship between water jet pressure and its contribution ratio to the depth of cut in a concrete slab.

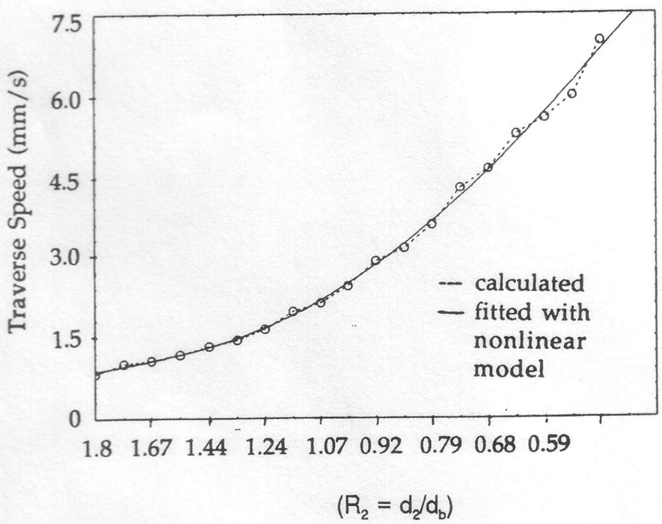

Figure 5. Relationship between jet traverse speed and its contribution ratio to the depth of cut in a concrete slab.

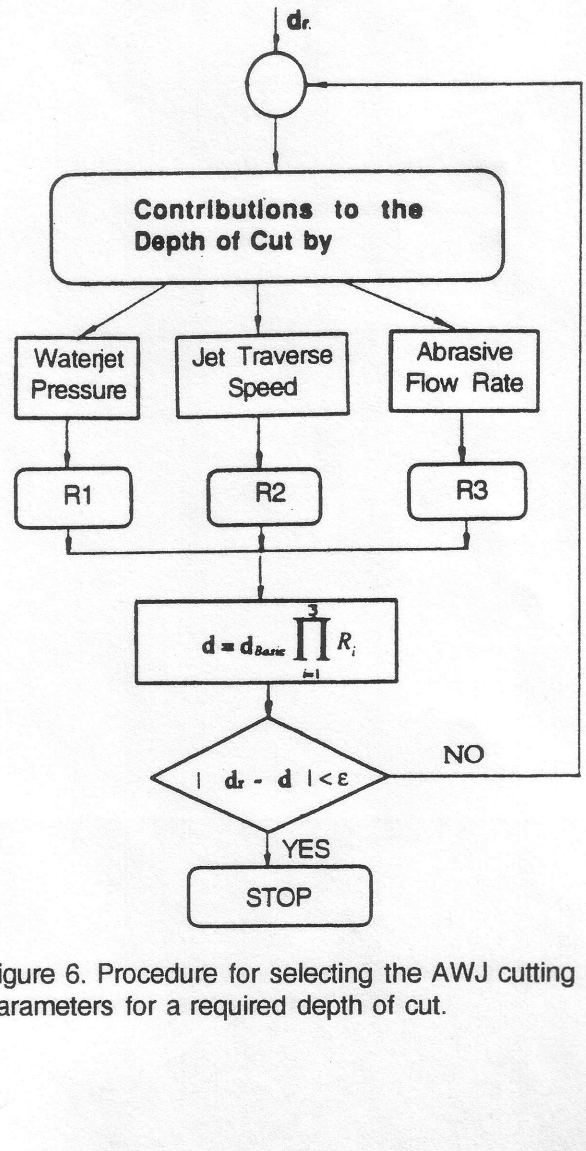

Figure 6. Procedure for selecting the AWJ cutting parameters for a required depth of cut.

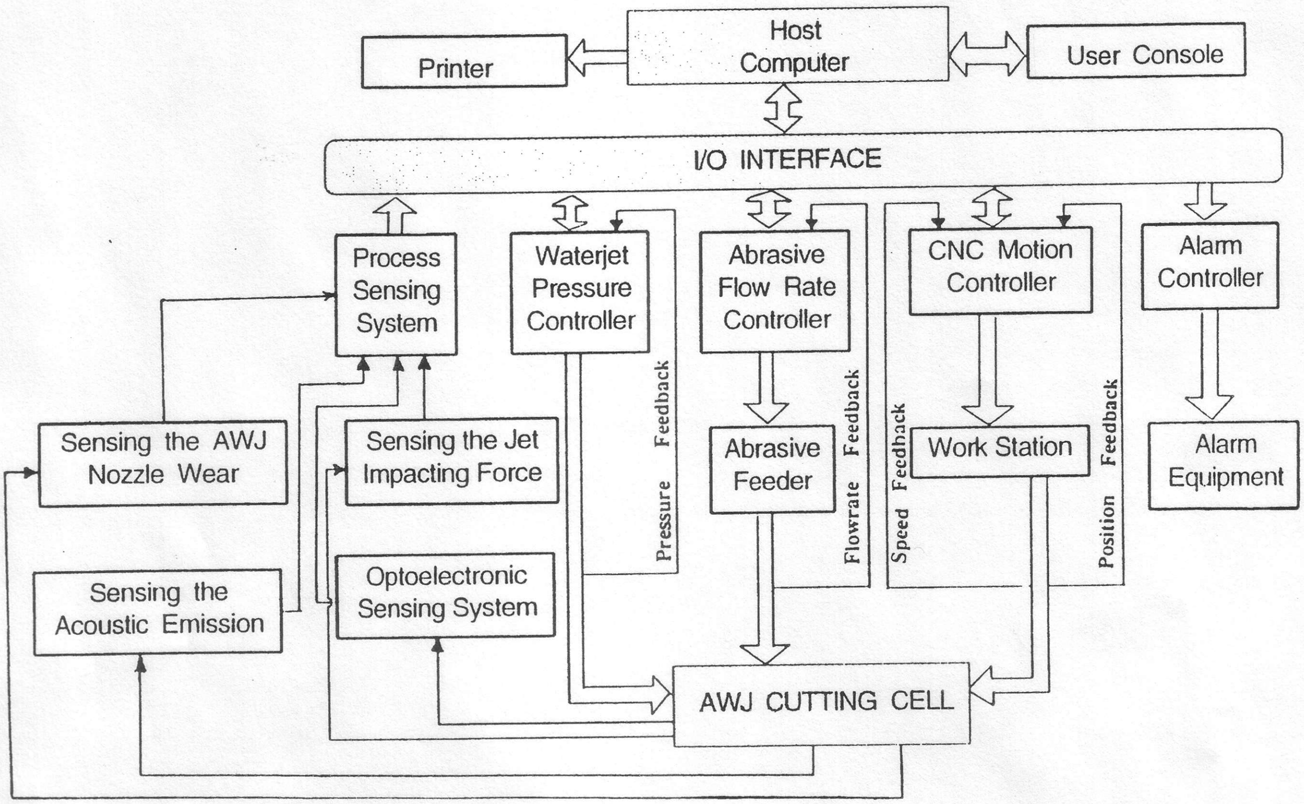

Figure 7. The block diagram of new generation of abrasive water jet cutting systems.