Robust control of interval plants: A time domain method

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Abstract: A time-domain algorithm is proposed to control interval plants described by impulse response functions. The closed-loop control actions are determined based on the interval ranges of the model parameters. Robust steady-state performance in tracking a given set-point can be guaranteed if the sign of the static gain is certain despite possible open-loop overshooting, delay, and nonminimum phase of the interval plants. Simulation examples are given to illustrate the performance. An application example in welding process control is also included.

1 Introduction

Interval models are useful descriptions for many uncertain dynamic processes. Much of the present success in interval plant control is restricted to analysis issues [1–9]. However, limited progress has been made in achieving an effective systematic design method for the interval plant control [4]. Preliminary results on the regularity of the robust design problem with respect to the controller coefficients were obtained [10]. Recently, a class of interval plants with one interval parameter were addressed [10, 11]. However, due to the complexity of the polynomial based analysis, the issue of controller synthesis for uncertain systems with more independent interval parameters has not been solved. It still 'remains, to a large extent, an open and difficult problem' [4].

In this work, a prediction based algorithm is proposed to control interval plants. Robust steady-state performance in tracking a given set-point is guaranteed if the sign of the static gain of the interval plant remains fixed when the parameters change in their intervals. The authors observed that predictive controllers were traditionally designed primarily based on the nominal model without explicitly using the uncertainty of the controlled process [13, 14]. Predictive control algorithms for models with interval parameters have been developed [15, 16]. However, their efforts were towards the computational aspects and no performance results have been either given or proven.

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2 Problem description

2.1 Problem description

Consider the following single-input, single-output (SISO) discrete system:

$$y_k = \sum_{j=1}^{n} h(j) u_{k-j} \tag{1}$$

where k is the current instant, y_k is the output at k, u_{k-j} is the input at $(k - j)$ $(j > 0)$, while n and $h(j)$s are the order and the real parameters of the impulse response function:

$$H(z^{-1}) = \sum_{j=1}^{n} h(j) z^{-j} \tag{2}$$

Assume $h(j)$s $(1 \le j \le n)$ are time-invariant. They are unknown but bounded by the following intervals:

$$h_{\min}(j) \le h(j) \le h_{\max}(j) \quad (j = 1, \dots, n) \tag{3}$$

where $h_{\min}(j) \le h_{\max}(j)$ are known. Assume y_0 is the given set-point. The objective is to design a controller for determining the feedback control actions $\{u_k\}$s so that the closed-loop system achieves the following robust steady-state performance:

$$\lim_{k \to +\infty} y_k = y_0 \tag{4}$$

where y_k is the output of the closed-loop system.

2.2 System assumption

The unit step response function $s(i)$ and their upper and lower limits $s_{\max}(i)$ and $s_{\min}(i)$ are

$$\begin{cases} s_{\max}(i) = \sum_{j=1}^{i} h_{\max}(j) \ge s(i) \\ \qquad = \sum_{j=1}^{i} h(j) \ge s_{\min}(i) \\ \qquad = \sum_{j=1}^{i} h_{\min}(j) & (1 \le i \le n) \\ s_{\max}(i) = s_{\max}(n) \ge s(i) \\ \qquad = s(n) \ge s_{\min}(i) \\ \qquad = s_{\min}(n) & (i \ge n+1) \end{cases} \tag{5}$$

To achieve a negative feedback control, one should assume that the sign of the static gain of the addressed interval plant is certain, despite the interval model parameters, i.e.

$$s_{\max}(n) s_{\min}(n) > 0 \tag{6}$$

This is referred to as the sign certainty condition of the static gain in this study. Assume that eqn. 6 holds for the plant (eqn. 1) with intervals (eqn. 3).

For a given plant (eqn. 1) with intervals (eqn. 3), its $s_{\max}(n)$ and $s_{\min}(n)$ can be calculated. If they are negative and the set-point y_0 is also negative, one may redefine $-y$ and $-h(j)$s as the output and model parameters so that eqn. 1 still holds. Also, the following can be satisfied:

$$\begin{cases} s_{\max}(n) \geq s_{\min}(n) > 0 \\ y_0 \geq 0 \end{cases} \tag{7}$$

If $s_{\max}(n)$ and $s_{\min}(n)$ are positive and y_0 is negative, $-y$ and $-u$ can be defined as the new output and input so that eqns. 1 and 7 hold. When $s_{\max}(n)$ and $s_{\min}(n)$ are negative and y_0 is positive, if the new input and parameters are redefined as $-u$ and $-h(j)$s, eqns. 1 and 7 can still be employed. It is apparent that the intervals of the model parameters must be changed accordingly once the model parameters are redefined. Hence, assuming eqn. 6 guarantees eqn. 7. The objective is therefore to design a controller for the interval plant, which is described by eqns. 1 and 3 and satisfies eqn. 7, so that the output of the closed-loop control system satisfies eqn. 4.

3 Uncertainty ranges

Predictive control [13, 17–19] is a widely accepted practical control method and has been applied to different areas [20–23]. The authors intend to control the interval plants using a prediction-based algorithm. Because of the uncertainty of the parameters in the interval model, no exact predictions can be made. Hence, the predictions can only be given in certain ranges.

Consider instant k. Assume the feedback y_k is available and u_k needs to be determined. From model (eqn. 1), the following can be obtained:

$$\Delta y_k = \sum_{j=1}^{n} h(j)\Delta u_{k-j} \tag{8}$$

where

$$\left.\begin{aligned} \Delta y_k &= y_k - y_{k-1} \\ \Delta u_{k-j} &= u_{k-j} - u_{k-j-1} \end{aligned}\right\}$$

3.1 One-step-ahead uncertainty range

Based on eqn. 8, the following equation can be used as the prediction equation to predict the output at instant $k + 1$:

$$\hat{y}_{k+1}(\Delta u_k/k) = y_k + \sum_{j=1}^{n} h(j)\Delta u_{k+1-j} \tag{9}$$

where k denotes the instant when the prediction is made, and Δu_k gives the condition under which the prediction is made. Here Δu_k implies that all the previous and current Δus are known (i.e. Δu_{k-j}s are known for $j \geq 0$, when the prediction is made).

Because of the uncertainties of the parameters, the predicted output is uncertain. However, both the upper and lower limits of the output can be predicted exactly using

$$\left.\begin{aligned} &\max \hat{y}_{k+1}(\Delta u_k/k) \\ &= y_k + \sum_{j=1}^{n} \max_{h_{\min}(j) \leq h(j) \leq h_{\max}(j)} (h(j)\Delta u_{k+1-j}) \\ &\min \hat{y}_{k+1}(\Delta u_k/k) \\ &= y_k + \sum_{j=1}^{n} \min_{h_{\min}(j) \leq h(j) \leq h_{\max}(j)} (h(j)\Delta u_{k+1-j}) \end{aligned}\right\} \tag{10}$$

Denote the one-step-ahead uncertain range of $\hat{y}_{k+1}(\Delta u_k/k)$ as

$$\begin{aligned} r(\hat{y}_{k+1}(\Delta u_k/k)) := &\max \hat{y}_{k+1}(\Delta u_k/k) \\ &- \min \hat{y}_{k+1}(\Delta u_k/k) \end{aligned} \tag{11}$$

It can be shown that

$$r(\hat{y}_{k+1}(\Delta u_k/k)) = \sum_{j=1}^{n} \Delta h(j)|\Delta u_{k+1-j}| \tag{12}$$

where

$$\Delta h(j) = h_{\max}(j) - h_{\min}(j) \geq 0 \quad (j = 1, \ldots, n) \tag{13}$$

It is apparent that $r(\hat{y}_{k+1}(\Delta u_k/k))$ is proportional to the amplitude of the control action increments Δu_{k+1-j}s. Assume the control increment constraint is $|\Delta u_k| \leq \Delta u_{max}$, where Δu_{max} is a positive real number. Then

$$\max r(\hat{y}_{k+1}(\Delta u_k/k)) \leq \Delta u_{\max} \sum_{j=1}^{n} \Delta h(j) \tag{14}$$

3.2 Multistep-ahead uncertainty ranges

Based on the one-step-ahead prediction eqn. 9, the following recursive multi-step-ahead prediction equation can be obtained:

$$\begin{aligned} &\hat{y}_{k+i}(\Delta u_{k+i-1}, \Delta u_{k+i-2}, \ldots, \Delta u_k/k) \\ &= \hat{y}_{k+i-1}(\Delta u_{k+i-2}, \Delta u_{k+i-3}, \ldots, \Delta u_k/k) \\ &+ \sum_{j=1}^{n} h(j)\Delta u_{k+i-j} \end{aligned} \tag{15}$$

where $\hat{y}_{k+i}(\Delta u_{k+i-1}, \Delta u_{k+i-2}, \ldots, \Delta u_k/k)$ denotes the prediction of y_{k+i} made at instant k for the known previous Δus and assumed output $\Delta u_{k+i-1}, \Delta u_{k+i-2}, \ldots, \Delta u_k$. Thus,

$$\begin{cases} \max \hat{y}_{k+i}(\Delta u_{k+i-1}, \Delta u_{k+i-2}, \ldots, \Delta u_k/k) \\ = \max \hat{y}_{k+i-1}(\Delta u_{k+i-2}, \Delta u_{k+i-3}, \ldots, \Delta u_k/k) \\ \quad + \sum_{j=1}^{n} \max_{h_{\min}(j) \leq h(j) \leq h_{\max}(j)} (h(j)\Delta u_{k+i-j}) \\ \min \hat{y}_{k+i}(\Delta u_{k+i-1}, \Delta u_{k+i-2}, \ldots, \Delta u_k/k) \\ = \min \hat{y}_{k+i-1}(\Delta u_{k+i-2}, \Delta u_{k+i-3}, \ldots, \Delta u_k/k) \\ \quad + \sum_{j=1}^{n} \min_{h_{\min}(j) \leq h(j) \leq h_{\max}(j)} (h(j)\Delta u_{k+i-j}) \\ r(\hat{y}_{k+i}(\Delta u_{k+i-1}, \Delta u_{k+i-2}, \ldots, \Delta u_k/k)) \\ = r(\hat{y}_{k+i-1}(\Delta u_{k+i-2}, \Delta u_{k+i-3}, \ldots, \Delta u_k/k)) \\ \quad + \sum_{j=1}^{n} \Delta h(j)|\Delta u_{k+i-j}| \\ (i \geq 2) \end{cases} \tag{16}$$

That is, the maximum, minimum and uncertain range of the multi-step-ahead prediction can be calculated recursively.

3.3 Step response prediction

In the proposed algorithm, the control variable u_k will be determined, based on the output behaviour if the control variable remains at the current level (i.e. $u_{k+j} = u_k$ ($\forall\ j > 0$). Hence, the step response of the output needs to be predicted. Denote the prediction of the step response as $z_{k+i}(\Delta u_k/k)$:

$$\begin{cases} z_{k+i}(\Delta u_k/k) := \hat{y}_{k+i}(\Delta u_{k+i-1} = 0, \Delta u_{k+i-2} = 0, \\ \qquad \ldots, \Delta u_{k+1} = 0, \Delta u_k/k) \quad (i \geq 1) \\ z_k(\Delta u_k/k) \equiv y_k \end{cases} \tag{17}$$

Thus,

$$\begin{cases} z_{k+i}(\Delta u_k/k) \\ = z_{k+i-1}(\Delta u_k/k) \\ + \sum_{j=i}^{n} h(j)\Delta u_{k+i-j} \\ \max z_{k+i}(\Delta u_k/k) \\ = \max z_{k+i-1}(\Delta u_k/k) \\ + \sum_{j=i}^{n} \max_{h_{\min}(j) \leq h(j) \leq h_{\max}(j)} [h(j)\Delta u_{k+i-j}] \\ \min z_{k+i}(\Delta u_k/k) \\ = \min z_{k+i-1}(\Delta u_k/k) \\ + \sum_{j=i}^{n} \min_{h_{\min}(j) \leq h(j) \leq h_{\max}(j)} [h(j)\Delta u_{k+i-j}] \\ r(z_{k+i}(\Delta u_k/k)) \\ = r(z_{k+i-1}(\Delta u_k/k)) \\ + \sum_{j=i}^{n} \Delta h(j)|\Delta u_{k+i-j}| \end{cases} \quad (i \geq 1) \tag{18}$$

Hence, for a given Δu_k, the uncertain range of the step response can be recursively calculated as i increases.

From the recursive eqn. 18, the following correlations can be obtained:

$$\begin{cases} \frac{\partial z_{k+i}(\Delta u_k/k)}{\partial z_{k+i-l}(\Delta u_k/k)} = 1 \\ \frac{\partial \max z_{k+i}(\Delta u_k/k)}{\partial \max z_{k+i-l}(\Delta u_k/k)} = 1 \quad (n \geq i \geq 1, i \geq l \geq 0) \\ \frac{\partial \min z_{k+i}(\Delta u_k/k)}{\partial \min z_{k+i-l}(\Delta u_k/k)} = 1 \end{cases} \tag{19}$$

It can be seen that the prediction error $y_{k+1} - z_{k+1}(\Delta u_k/k)$ will contribute to the further prediction errors $y_{k+i} - z_{k+i}(\Delta u_k/k)$, $y_{k+i} - \min z_{k+i}(\Delta u_k/k)$, and $y_{k+i} - \max z_{k+i}(\Delta u_k/k)$, where $i \geq 2$. Thus, once the new feedback y_{k+1} is acquired, the predictions ($z_{k+i}(\Delta u_k/k)$s, min $z_{k+i}(\Delta u_k/k)$s, and max $z_{k+i}(\Delta u_k/k)$s ($i \geq 2$)) can be replaced by more precise innovative predictions ($z_{k+i}(\Delta u_k/k+1)$s, min $z_{k+i}(\Delta u_k/k+1)$s, and max $z_{k+i}(\Delta u_k/k+1)$s ($i \geq 2$)), so that the control action can be adjusted based on the new feedback, where

$$\begin{cases} z_{k+i}(\Delta u_k/k+1) \\ := z_{k+i}(\Delta u_k/k)|_{z_{k+1}(\Delta u_k/k)=y_{k+1}} \\ \max z_{k+i}(\Delta u_k/k+1) \\ := \max z_{k+i}(\Delta u_k/k)|_{z_{k+1}(\Delta u_k/k)=y_{k+1}} \\ \min z_{k+i}(\Delta u_k/k+1) \\ := \min z_{k+i}(\Delta u_k/k)|_{z_{k+1}(\Delta u_k/k)=y_{k+1}} \end{cases} \quad (i \geq 2) \tag{20}$$

Also, it can be shown that

$$\begin{cases} z_{k+i}(\Delta u_k/k) = z_{k+i}(\Delta u_{k-1}/k) \\ \qquad + s(i)\Delta u_k \\ \max z_{k+i}(\Delta u_k/k) = \max z_{k+i}(\Delta u_{k-1}/k) \\ \qquad + \max(s(i)\Delta u_k) \\ \min z_{k+i}(\Delta u_k/k) = \min z_{k+i}(\Delta u_{k-1}/k) \\ \qquad + \min(s(i)\Delta u_k) \end{cases} \quad (i \geq 1) \tag{21}$$

Eqn. 21 gives the correlation between $z_{k+i}(\Delta u_k/k)$ and $z_{k+i}(\Delta u_{k-1}/k)$. Here, $z_{k+i}(\Delta u_{k-1}/k)$ predicts what the output will be if the control variable is not changed. Based on the required output, the ideal $z_{k+i}(\Delta u_k/k)$, which needs to be achieved by adjusting the control variable, can be known. Thus, the error in the output that the closed-loop control algorithm needs to eliminate can be known and used to determine Δu_k. A control criterion and algorithm can therefore be proposed.

4 Control algorithm

The following criterion is proposed to determine Δu_k:

$$\max z_{k+n}(\Delta u_k/k) = y_0 \tag{22}$$

This criterion can be realised by the following steps:

(i) Calculate

$$\max z_{k+i}(\Delta u_{k-1}/k) \quad (n \geq i \geq 1) \tag{23}$$

based on eqns. 18 and 20.

(ii) Because of the correlation in eqn. 21, calculate

$$d(n) = y_0 - \max z_{k+n}(\Delta u_{k-1}/k) \tag{24}$$

(iii) Then

$$\Delta u_k = \begin{cases} d(n)/s_{\max}(n) & (d(n) \geq 0) \\ d(n)/s_{\min}(n) & (d(n) < 0) \end{cases} \tag{25}$$

5 Performance

Theorem 1: For the given interval plant control problem eqns. 1, 3 and 7,

$$\lim_{k \to +\infty} y_k = y_0 \tag{26}$$

when algorithm (eqns. 23–25) is used.

Proof: When the upper limit of the prediction is used to predict the output y_{k+1} at instant k, the one-step-ahead prediction error defined by

$$e_{k+1} := \max z_{k+1}(\Delta u_k/k) - y_{k+1} \tag{27}$$

is larger than, or equal to, zero (i.e. $e_{k+1} \geq 0$). Based on eqns. 19 and 20, the following can be yielded:

$$\max z_{k+1}(\Delta u_k/k+1) = \max z_{k+i}(\Delta u_k/k) - e_{k+1} \quad (i = 2, 3, \ldots) \tag{28}$$

It is known that max $z_{k+i}(\Delta u_k/k + 1) \geq y_{k+i}$ and max $z_{k+i}(\Delta u_k/k) \geq y_{k+i}$. Hence, eqn. 28 and $e_{k+1} \geq 0$ imply that max $z_{k+i}(\Delta u_k/k + 1)$ gives a more accurate prediction than max $z_{k+i}(\Delta u_k/k)$, and $e_{k+1} \geq 0$ is a measure of the prediction accuracy improvement when the new feedback y_{k+1} is used for prediction.

In the following proving process, the correlation between Δu_{k+1} and e_{k+1} will be first established. Then $\lim_{k \to +\infty} \Delta u_k = 0$ will be shown based on $e_{k+1} \geq 0$. As given by eqn. 25, Δu_ks are proportional to the differences between the set-point and predictions. Hence, $\lim_{k \to +\infty} \Delta u_k = 0$ has actually implied the correctness of eqn. 26.

Eqn. 28 has the following form for $i = n + 1$:

$$\max z_{k+1+n}(\Delta u_k/k+1) = \max z_{k+1+n}(\Delta u_k/k) - e_{k+1} \tag{29}$$

Since

$$\max z_{k+i+n}(\Delta u_k/k) = \max z_{k+n}(\Delta u_k/k) \quad (i \geq 1) \tag{30}$$

then

$$\max z_{k+1+n}(\Delta u_k/k+1) = \max z_{k+n}(\Delta u_k/k) - e_{k+1} = y_0 - e_{k+1} \tag{31}$$

From eqns. 21 and 22, we have

$$\max z_{k+1+n}(\Delta u_{k+1}/k+1) = \max z_{k+1+n}(\Delta u_k/k+1) + s_{\max}(n)\Delta u_{k+1} = y_0 \tag{32}$$

Hence, also from eqn. 8,

$$\Delta u_{k+1} = e_{k+1}/s_{\max}(n) \geq 0 \tag{33}$$

The control sequence satisfies:

$$u_k \leq u_{k+1} \leq u_{k+2} \leq \cdots \tag{34}$$

Since the plant is stable and $y_{k+i} \leq y_0$, $\lim_{i\to+\infty} \Delta u_{k+i} = 0$. In general, this can be written as

$$\lim_{k\to+\infty} \Delta u_k = 0 \tag{35}$$

Thus, from eqn. 16 it can be seen when $k \to +\infty$

$$\max \hat{y}_{k+i} = \min \hat{y}_{k+i} = y_k \quad (i \geq 1) \tag{36}$$

Also, since $\Delta u_k = 0$ when $k \to +\infty$,

$$y_{k+1+n} = \max \hat{y}_{k+1+n} = y_0 \quad (k \to +\infty) \tag{37}$$

That is,

$$\lim_{k\to+\infty} y_k = y_0$$

Remark 1: Theorem 1 shows that if eqn. 7 is satisfied, the proposed control algorithm can guarantee that the required closed-loop system performance (eqn. 4) is achieved. That is, the resultant closed-loop system is robust with respect to the uncertainty of the interval plants in achieving the closed-loop performance (eqn. 4).

Remark 2: It can be seen from the above proof that the robust performance achieved by the proposed algorithm is not affected by the dynamics of the controlled process such as open-loop overshooting, delay, non-minimum phase, and large intervals once the sign condition is satisfied.

Remark 3: For the convenience of derivation, the algorithm has been developed using impulse response function models. In general, a SISO interval plant can be described using an autoregressive moving-average interval model:

$$y_k = \sum_{j=1}^{p} a(j)y_{k-j} + \sum_{j=1}^{q} b(j)u_{k-j} \tag{38}$$

where (p, q) are the orders, and $a(j)$s $(j = 1, ..., p)$ and $b(j)$s $(j = 1, ..., q)$ are the real coefficients of the model and satisfy:

$$\begin{cases} a_{\min}(j) \leq a(j) \leq a_{\max}(j) \\ b_{\min}(j) \leq b(j) \leq b_{\max}(j) \end{cases} \tag{39}$$

To describe the interval plant (eqn. 38) using the impulse response function model (eqn. 1), one can compute the responses of eqn. 38 to an impulse input $u_k = \delta_k$ where $\delta_0 = 1$ and $\delta_k = 0$ $(k \neq 0)$ under zero initial state condition: $y_k = 0$ $(k \leq 0)$. For convenience of notation, denote

$$b_{\min}(j) = b(j) = b_{\max}(j) = 0 \quad (\forall j > q)$$

Thus, from eqn. 38, the following can be shown:

$$\begin{cases} h_{\max}(k) = \max y_k \\ = \sum_{j=1}^{p} \max_{\substack{a_{\min}(j)\leq a(j)\leq a_{\max}(j) \\ \min y_{k-j}\leq \max y_{k-j}}} a(j)y_{k-j} \\ + \max_{b_{\min}(k)\leq b(k)\leq b_{\max}(k)} b(k) \\ h_{\min}(k) = \min y_k \\ = \sum_{j=1}^{p} \min_{\substack{a_{\min}(j)\leq a(j)\leq a_{\max}(j) \\ \min y_{k-j}\leq \max y_{k-j}}} a(j)y_{k-j} \\ + \min_{b_{\min}(k)\leq b(k)\leq b_{\max}(k)} b(k) \end{cases} \quad (k \geq 1) \tag{40}$$

Hence, $\{h_{min}(k), h_{max}(k)\}$ $(k \geq 1)$ can be recursively calculated. We assume that the plant (eqn. 38) with interval parameters given in eqn. 39 is stable, and that the maximum and minimum of the impulse responses approach to zero, i.e.

$$\begin{cases} \lim_{k\to\infty} h_{\max}(k) = 0 \\ \lim_{k\to\infty} h_{\min}(k) = 0 \end{cases}$$

so that the plant (eqn. 38) can be described at any required accuracy by the interval impulse model with a sufficient n. In this case, the interval (eqn. 38) can be controlled using the proposed algorithm.

Remark 4: Consider the case with disturbance:

$$y_k = \sum_{j=1}^{n} h(k)u_{k-j} + \xi_k \tag{41}$$

where ξ_k is the disturbance at instant k. It can be shown that, if $\xi_l \equiv c(\forall l \geq 1)$ where c is an unknown (real) constant, then

$$\lim_{k\to+\infty} y_k = y_0 \tag{42}$$

when algorithm (eqns. 23–25) is used. If $k > 1$, all the recursive equations in Section 3 still hold so that the derivation in the proof of Theorem 1 can be exactly repeated. This implies that the robust performance for tracking a given set-point can also be obtained when the disturbance is present.

Remark 5: The proposed control criterion is

$$\max z_{k+n}(\Delta u_k/k) = y_0 \tag{43}$$

if the criterion were

$$\max z_{k+1}(\Delta u_k/k) = y_0 \tag{44}$$

the resultant control would be similar to the one-step-ahead prediction based control. In this case, the robustness of the resultant closed-loop performance is not guaranteed. In general, for many interval plants, criterion

$$\max z_{k+m}(\Delta u_k/k) = y_0 \tag{45}$$

may give the performance (eqn. 4) with $1 \leq m < n$. However, theoretical work which can be used to judge whether an m $(1 \leq m < n)$ exists for guaranteeing the performance (eqn. 4) for a given interval plant has not been established in this paper. When an m $(1 \leq m < n)$ is used, the regulation speed would improve when m decreases, whereas the robustness of the performance would tend to be poorer.

6 Simulation

Example 1: Consider an interval plant family described by

$$\left.\begin{aligned} H_{\min} &= [0,0,0,0.9,0.5,0,-0.3,-0.5]^T \\ H_{\max} &= [0.2,0.2,0.2,1.3,0.8,0.2,-0.1,-0.3]^T \end{aligned}\right\}$$

Thus,

$$\left.\begin{aligned} S_{\min} &= [0,0,0,0.9,1.4,1.4,1.1,0.6]^T \\ S_{\max} &= [0.2,0.4,0.6,1.9,2.7,2.9,2.8,2.5]^T \end{aligned}\right\}$$

Let $y_0 = 1$ and $\xi_k = 0$. When $H = H_{\min}$, $H = (H_{\min} + H_{\max})/2$, and $H = H_{\min} + 0.8(H_{\max} - H_{\min})$, the resultant closed-loop responses and control actions are plotted in Figs. 1–3, respectively. It can be seen that both open-loop delay and overshooting exist in the plant. Despite the significant uncertainties in the model parameters, stabilising closed-loop control has been achieved in all the cases.

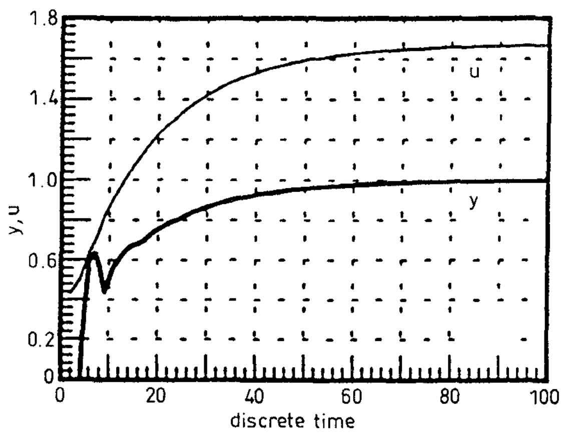

Fig. 1 *Control of delay interval plants*
$H = H_{\min}$

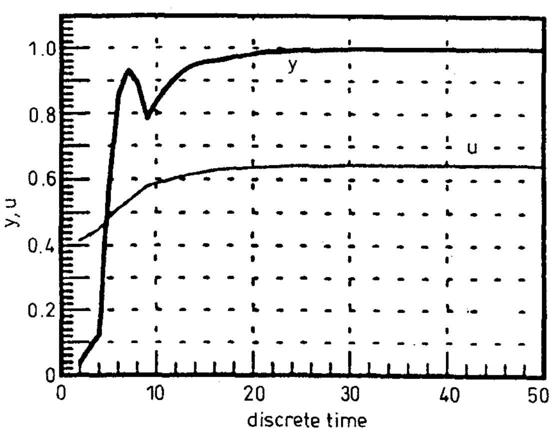

Fig. 2 *Control of delay interval plants*
$H = (H_{\min} + H_{\max})/2$

Example 2: In this example, all the parameters are the same as in Example 1 except for the disturbance. In this example, $\xi_k \equiv 0.5$. The results are shown in Figs. 4–6.

Example 3: Consider a nonminimum phase interval plant family described by

$$\left.\begin{aligned} H_{\min} &= [-0.8,-0.4,0,0.3,0.9,0.5,0.3,0.1,-0.2]^T \\ H_{\max} &= [-0.6,-0.2,0.2,0.5,1.3,0.8,0.5,0.2,0]^T \end{aligned}\right\}$$

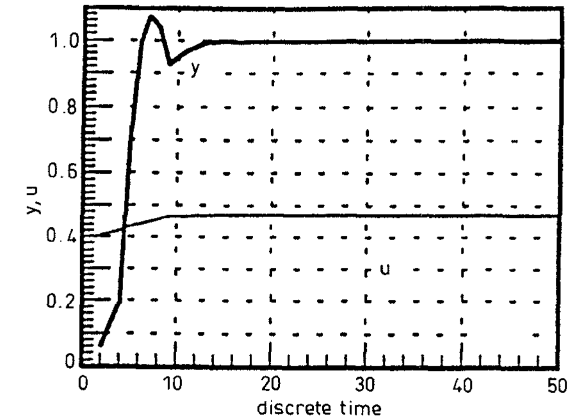

Fig. 3 *Control of delay interval plants*
$H = H_{\min} + 0.8(H_{\max} - H_{\min})$

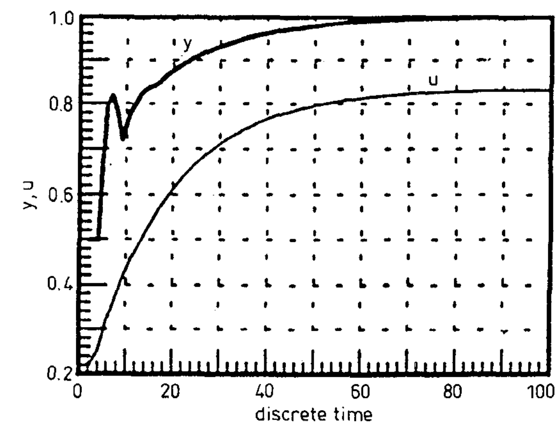

Fig. 4 *Control of delay interval plants under constant disturbances*
$H = H_{\min}$

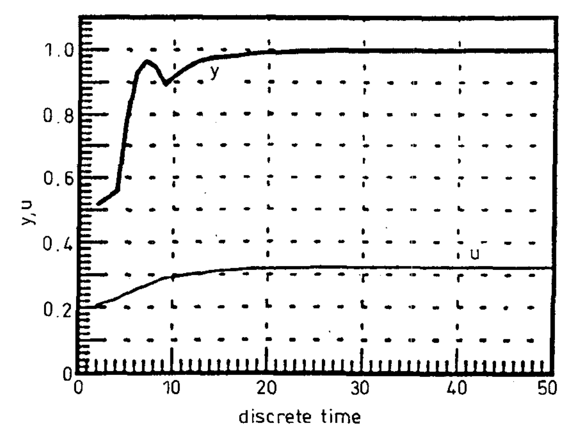

Fig. 5 *Control of delay interval plants under constant disturbances*
$H = (H_{\min} + H_{\max})/2$

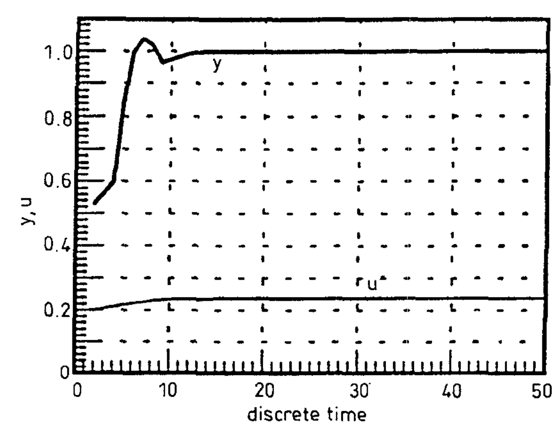

Fig. 6 *Control of delay interval plants under constant disturbances*
$H = H_{\min} + 0.8(H_{\max} - H_{\min})$

Thus,

$$\left.\begin{array}{l} S_{\min} = [-0.8, -1.2, -1.2, -0.9, 0, 0.5, 0.8, 0.9, 0.7]^T \\ S_{\max} = [-0.6, -0.8, -0.6, -0.1, 1.2, 2, 2.5, 2.7, 2.7]^T \end{array}\right\}$$

Let $y_0 = 1$ and $\xi_k = 0$. When $H = H_{\min}$, $H = (H_{\min} + H_{\max})/2$, and $H = H_{\min} + 0.8(H_{\max} - H_{\min})$, the resultant closed-loop responses and control actions are plotted in Figs. 7–9, respectively. It can be seen that the plants are nonminimum phase and stabilising closed-loop controls have been obtained.

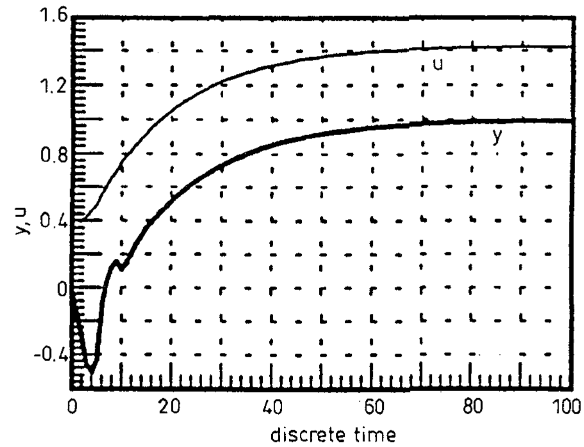

Fig. 7 *Control of nonminimum phase interval plants*
$H = H_{\min}$

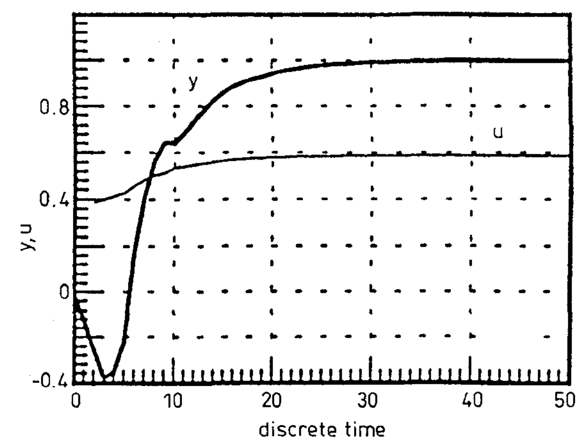

Fig. 8 *Control of nonminimum phase interval plants*
$H = (H_{\min} + H_{\max})/2$

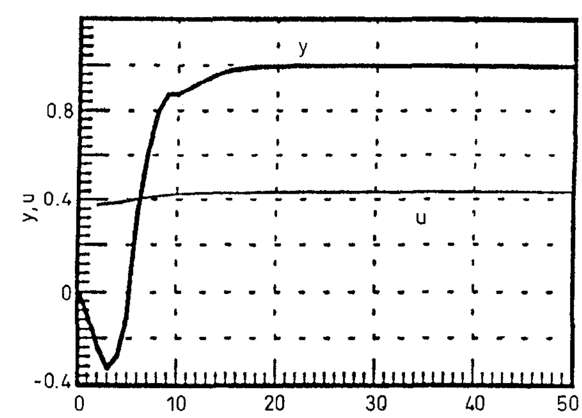

Fig. 9 *Control of non-minimum phase interval plants*
$H = H_{\min} + 0.8(H_{\max} - H_{\min})$

7 Application example

The proposed control algorithm has been applied to control the weld penetration. It is known that weld penetration control is a major research issue in automated welding. The difficulty arises from the invisibility of the weld penetration from the front-side. The present authors have proposed to estimate the weld penetration by processing the image of the weld pool [24, 25]. The input and output of the controlled system are the welding current and the weld penetration state, respectively. It is known that the process model varies with the welding conditions such as the thickness of the material etc. Hence, the interval model has been used for controller design. The resultant interval model can be illustrated by $h_{\min}$ and $h_{\max}$ as shown in Fig. 10. Using this interval model, a closed-loop system has been developed to control the weld penetration.

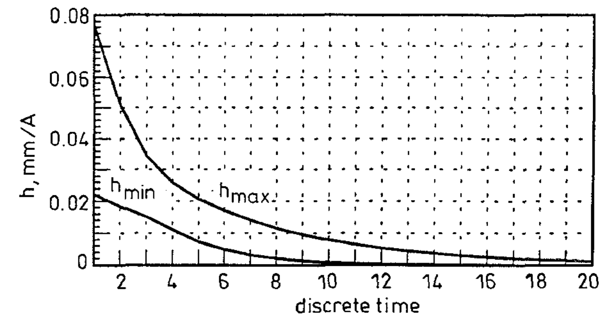

Fig. 10 *Identified interval model*

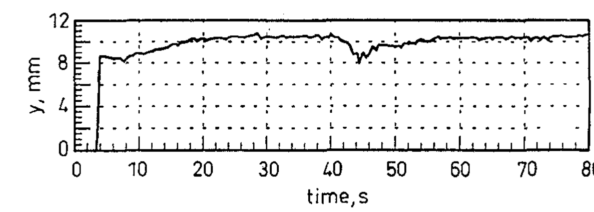

Fig. 11 *Closed-loop experiment for controlling weld penetration*
Output
Parametric perturbation is applied by increasing welding speed from 2 to 3mm/s at t = 40s

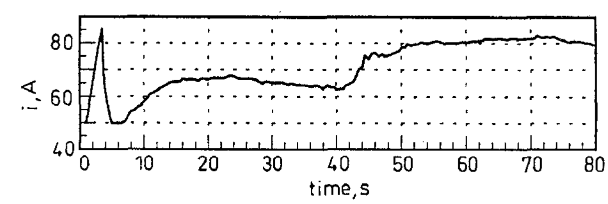

Fig. 12 *Closed-loop experiment for controlling weld penetration*
Control action
Parametric perturbation is applied by increasing welding speed from 2 to 3mm/s at t = 40s

Extensive experiments have been carried out. As an example, Figs. 11 and 12 shows an experiment where the travel speed changes from 2.0mm/s to 3.0mm/s. It can be seen that, when the speed increases, the output decreases (Fig. 11). However, the controller can increase the current (Fig. 12). As a result, the output is maintained at the desired level again (Fig. 11). In this case, no overshooting or fluctuation of the output occurs, so the geometrical regularity and appearance of the resultant welds are excellent.

8 Conclusions

The interval plants described by eqns. 1 and 3 can be controlled using the proposed algorithm. The closed-loop control actions are directly determined from uncertainty ranges (i.e. the intervals, of the model parameters). Robust performance (eqn. 4) is guaranteed if the sign certainty condition (eqn. 6) is satisfied, despite possible open-loop overshooting, delay, non-minimum phase and large uncertainty intervals.

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