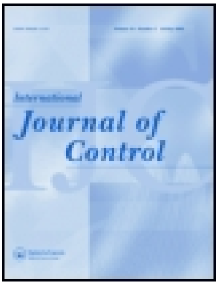

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Functional observer and state feedback for input time-delay systems

Y. X. YAO†, Y. M. ZHANG† and R. KOVACEVIC†

The design of state functional observer and state feedback for systems with input time delays is addressed using the factorization approach in the frequency domain. The design needs to achieve the loop transfer recovery of direct state feedback by using the functional observer-based feedback control. A necessary and sufficient condition is given for the existence of the state functional observers for such systems. A parametrization for all observers of time-delay systems is proposed. Based on the results of the parametrization, the state functional observer and state feedback design procedure is presented. The computation can be implemented in state-space form using standard algorithms. Design examples are given to illustrate the procedure.

1. Introduction

The functional observer and state feedback design for linear control systems has become matured through extensive studies (O'Reilly 1983, Zhang 1987). Different approaches such as state-space, transfer-function and polynomial-matrix system models (O'Reilly 1983) have been successfully used. The correlation between the state-space and transfer-function approaches, and between the state-space and polynomial-matrix approaches, have also been revealed (Zhang 1987, Hippe 1988).

Input time delays are frequently encountered in industrial processes. In general, the control of such processes is difficult. In fact, the time delay limits the achievable bandwidth and the allowed maximum gain. In addition, the time delay often significantly complicates the analysis and computation in system design. Hence, although significant results have been obtained by using the Smith predictor (Wang and Skogestad 1993), LQ regulators (Lee *et al.* 1988), internal model control (IMC) (Jones and Sbarbaro 1995), etc., many advanced design methods are still incapable of dealing with time-delay systems.

It has been shown that the factorization approach is a powerful tool in solving a variety of control system design problems (Vidyasagar 1985). Although the transfer function matrix of any finite-dimensional system admits a proper stable Bezout factorization, such a factorization does not in general exist for infinite-dimensional systems (Khargonekar and Sontag 1982). Recently, the proper stable Bezout factorizations of transfer function matrices have been studied for linear time-invariant systems with commensurate time delays (Nobuyama and Kitamori 1990, and Nobuyama 1992). An explicit procedure for computing proper stable Bezout factorizations has already been given (Nobuyama and Kitamori 1990). This progress provides necessary mathematics preparations to design functional

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observer and state feedback for input time-delay systems based on the factorization approach.

Our aims are to develop a procedure for designing the functional observer and state feedback, and to achieve the loop transfer recovery for the direct state feedback with desired properties by using the proposed functional observer and state feedback. It is noted that almost all the existing observer design schemes have, more or less, restricted the used observer structures to specific forms, for instance, the identity observer and the Luenberger type observer (O'Reilly 1983). This is also the case for the observer design of time delay systems such as shown by Lee *et al.* (1988). In this work the authors propose a general form of linear observers for time delay systems so that designers are provided with more degrees of freedom to improve the robustness of the resultant observers. Based on this general form, the parametrization of all functional observers can be achieved. Hence, a standard design procedure for the state functional observer and state feedback is obtained.

It should be pointed out that the functional observer proposed here is a state predictor. By employing the properties of the state predictor, the design can be converted to a delay-free problem. This not only simplifies the analysis and design but also avoids the rational time-delay approximation (Partington and Glove 1990). Furthermore, we can obtain new insights into the observer construction for time-delay systems.

2. Preliminaries

Assume that the system is linear and time-invariant. Let $\mathbb{R}$ denote real matrices, $\mathbb{R}(s)$ rational transfer function matrices, and H_∞ the set of all stable and proper transfer function matrices (Francis 1987). Let U denote the unit matrix (Vidyasagar 1985), I the unity matrix, and 0 the null matrix.

Consider a transfer function matrix $G(s) = G_0(s)\mathrm{e}^{-\tau s}$, where $G_0(s) \in \mathbb{R}(s)$ is a strictly proper rational $p \times m$ transfer matrix, with the state-space realization $G(s) = C(sI - A)^{-1}B\mathrm{e}^{-\tau s}$, and τ the delay time. It is assumed that (A, B) is stabilizable and (C, A) is detectable. The double coprime factorization of $G(s)$ can be written as (Nobuyama and Kitamori 1990):

$$G(s) = N(s)M^{-1}(s) = \tilde{M}^{-1}(s)\tilde{N}(s) \tag{1}$$

where $N(s)$ and $M(s)$, and $\tilde{N}(s)$ and $\tilde{M}(s)$, are right and left coprime H_∞ matrices, respectively. For this double coprime factorization, there exist H_∞ matrices $Y(s)$, $X(s)$ and $\tilde{Y}(s)$, $\tilde{X}(s)$ that satisfy

$$\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{M}(s) \end{bmatrix} \begin{bmatrix} M(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} = \begin{bmatrix} M(s) & \tilde{X}(s) \\ -N(s) & \tilde{Y}(s) \end{bmatrix} \begin{bmatrix} Y(s) & -X(s) \\ \tilde{N}(s) & \tilde{M}(s) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{2}$$

which is referred to as double Bezout factorization. The eight matrices above can be calculated by the standard algorithms in the state-space construction and are given below (Nobuyama and Kitamori 1990, Nobuyama 1992).

$$M(s) = K(sI - A - BK)^{-1}B + I, \quad N(s) = C(sI - A - BK)^{-1}B\mathrm{e}^{-\tau s} \tag{3}$$

$$\tilde{M}(s) = C(sI - A - LC)^{-1}L + I, \quad \tilde{N}(s) = C(sI - A - LC)^{-1}B\mathrm{e}^{-\tau s} \tag{4}$$

$$\begin{aligned} Y(s) &= -K\mathrm{e}^{A\tau}(sI - A - LC)^{-1}B\mathrm{e}^{-\tau s} - KV(s) + I, \\ X(s) &= K\mathrm{e}^{A\tau}(sI - A - LC)^{-1}L \end{aligned} \tag{5}$$

$$\tilde{Y}(s) = -C(I - V(s)K)(sI - A - BK)^{-1}L + I, \quad \tilde{X}(s) = K(sI - A - BK)^{-1}\mathrm{e}^{A\tau}L \tag{6}$$

where K and L can be chosen such that $\det(sI - A - BK)$ and $\det(sI - A - LC)$ are stable. It is noted that the stabilizability and detectability assumptions on $G(s)$ ensure the existence of K and L such that (3)–(6) hold. The matrix $V(s)$ in $Y(s)$ and $\tilde{Y}(s)$ is given by

$$V(s) = (sI - A)^{-1}(I - e^{-\tau s}e^{A\tau})B \tag{7}$$

3. Problem description and formulation

Consider the following linear system with an input time delay:

$$\dot{x}(t) = Ax(t) + Bu(t - \tau) \tag{8}$$

$$y(t) = Cx(t) \tag{9}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^p$ is the measured output vector, and A, B, C are constant matrices with appropriate dimensions. Taking Laplace transforms of (8) and (9), the transfer function matrix description of the system is given by

$$y(s) = G(s)u(s) \tag{10}$$

and

$$G(s) = C(sI - A)^{-1}Be^{-\tau s} = G_0(s)e^{-\tau s} \tag{11}$$

In the case of state feedback control where the state vector $x(t)$ can be measured directly, the system (10) can be stabilized by the following feedback control law (Kwon and Pearson 1980):

$$u(t) = r(t) + v(t) \tag{12}$$

$$r(t) = P\left[e^{A\tau}x(t) + \int_{-\tau}^{0} e^{-A\theta} Br(t + \theta)\, d\theta\right] \tag{13}$$

with $P \in \mathbb{R}^{m\times n}$, which is chosen so that $\det(sI - A - BP)$ is stable, where $v(t)$ represents all input signals of the system. As shown by Nobuyama and Kitamori (1990), the following can be obtained from (8) for $v(t) = 0$:

$$x(t + \tau) = e^{A\tau} x(t) + \int_{-\tau}^{0} e^{-A\theta}Br(t + \theta)\, d\theta \tag{14}$$

Hence, (13) can also be regarded as a reconstruction of the feedback

$$r(t) = Px(t + \tau) \tag{15}$$

In the frequency domain, we can obtain

$$r(s) = P e^{\tau s}x(s) \tag{16}$$

from (15), and

$$r(s) = P[e^{A\tau}x(s) + V(s)r(s)] \tag{17}$$

from (13), where

$$V(s) = (sI - A)^{-1}(I - e^{-\tau s}e^{A\tau})B \tag{18}$$

The direct state feedback design is to find a matrix $P \in \mathbb{R}^{m\times n}$ such that the given plant is stabilized by feedback law (12) and (13), and a certain design performance index is achieved. As a result, the desired performance and robustness of the corresponding closed loop-system can be satisfied.

By substituting (8) into (15), we obtain the open loop output from $u(t)$ to $r(t)$. It is expressed in the frequency domain as follows:

$$r(s) = L_s(s)u(s) \tag{19}$$

where the desired state feedback loop transfer function matrix is

$$L_s(s) = P(sI - A)^{-1}B \in \mathbb{R}(s)^{m\times m} \tag{20}$$

The output of the closed-loop system is

$$y(s) = H_s(s)v_s(s) \tag{21}$$

with the closed-loop transfer function

$$H_s(s) = G_0(s)e^{-\tau s}[I - L_s(s)]^{-1} \in \mathbb{R}(s)^{p\times m} \tag{22}$$

The block diagram of the system under the state feedback control is shown in Fig. 1.

In most practical cases, the state of the system is not available. Thus, the following output feedback control law is considered:

$$u(s) = \hat{r}(s) + v(s) \tag{23}$$

$$\hat{r}(s) = F(s)u(s) + H(s)y(s) \tag{24}$$

where $F(s) \in H_\infty$ and $H(s) \in H_\infty$ are $m \times m$ and $m \times p$ stable transfer function matrices, respectively. The system scheme using the control law (23) and (24) is expressed in Fig. 2.

It has been pointed out that the state feedback control law (13) actually reconstructs the state $Px(t+\tau)$. To maximally approximate the state feedback control (13), it is desired that the output feedback control law (24) reconstruct $Px(t+\tau)$.

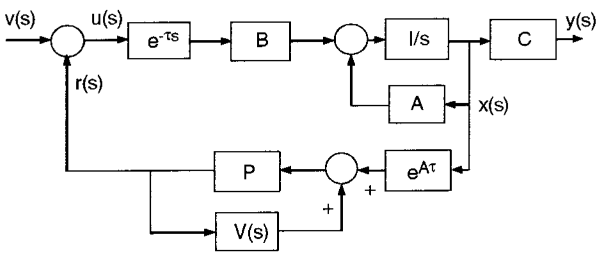

Figure 1. System using state feedback control.

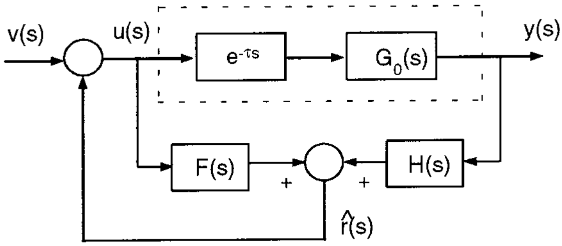

Figure 2. System using functional observer-based control.

Thus, the feedback control law in (24) is supposed to be a state functional observer which gives the function of system state. The expression of (24) describes a very general class of observers. Its structure is not restricted to any specific form.

If the functional observer (24) exists, the observation vector $\hat{r}(t)$ will be used instead of $Px(t+\tau)$ in the state feedback control law (5). From the above analysis, (23) and (24) can be regarded as the functional observer-based feedback control law.

From (10), (23) and (24), we obtain the open-loop output from $u(t)$ to $\hat{r}(t)$

$$\hat{r}(s) = (F(s) + H(s)G_0(s)\mathrm{e}^{-\tau s})u(s) \tag{25}$$

and the output of the closed-loop system is

$$y(s) = G_0(s)\mathrm{e}^{-\tau s}(I - L(s))^{-1}v(s) \tag{26}$$

where

$$L(s) = F(s) + H(s)G_0(s)\mathrm{e}^{-\tau s} \in \mathbb{R}(s)^{m\times m} \tag{27}$$

is the associated loop transfer function matrix.

It is required to determine $F(s)$ and $H(s)$ such that the corresponding loop transfer function matrix $L(s)$ of the dynamic output feedback control is either exactly or approximately equal to the loop transfer function $L_s(s)$ of the direct state feedback control. As a result, the closed-loop system will recover the performance and robustness of the direct state feedback design. This is just the basic idea behind our design. It is apparent that this design idea is the same as in the delay-free systems (Fu 1990). To reach this objective, make $L(s) = L_s(s)$ directly; that is

$$F(s) + H(s)G_0(s)\mathrm{e}^{-\tau s} = L_s(s) \tag{28}$$

That is

$$F(s)M(s) + H(s)N(s) = L_s(s)M(s) \tag{29}$$

where $M(s)$ and $N(s)$ are the right coprime H_∞ factors of $G(s)$ in (1).

It is seen that the functional observer (24) is also a predictor for the function $Px(t+\tau)$. By employing the properties of the predictor, we are able to convert our problem to a delay-free problem. This simplifies the analysis and design, and also avoids the problem of the rational time-delay approximation.

4. Necessary and sufficient condition of observation

From the point of view of observer design, the functional observer (24) for the system (8) and (9) is a dynamic system with the property that the observation error for $Px(t+\tau)$ satisfies

$$\lim_{t\to\infty} (Px(t+\tau) - \hat{r}(t)) = 0 \tag{30}$$

for all $u(t)$ and initial states.

A necessary and sufficient condition for the existence of a functional observer that satisfies (30) will be addressed. The interconnection between the loop transfer recovery of direct state feedback and the functional observation problem is also discussed below.

Suppose that a right coprime factorization of $G(s)$ in (11) can be written as

$$G(s) = N(s)M^{-1}(s) \tag{31}$$

where $N(s)$ and $M(s)$ are H_∞ matrices.

By introducing the partial state $\xi(s)$, we can rewrite (10) with the factorization (31) as

$$u(s) = M(s)\xi(s) \tag{32}$$

$$y(s) = N(s)\xi(s) \tag{33}$$

and

$$r(s) = P\mathrm{e}^{\tau s}x(s) = P(s)\xi(s) \tag{34}$$

with

$$P(s) = P(sI - A - BK)^{-1}B \tag{35}$$

Equation (34) can be proved as follows.

From (11) and (31)

$$C(sI - A)^{-1}B\mathrm{e}^{-\tau s} = N(s)M^{-1}(s) \tag{36}$$

By using the state-space description of $N(s)$ in (3), we have

$$(sI - A)^{-1}B\mathrm{e}^{-\tau s} = (sI - A - BK)^{-1}B\mathrm{e}^{-\tau s}M^{-1}(s) \tag{37}$$

Thus

$$\begin{aligned} x(s) &= (sI - A)^{-1}B\mathrm{e}^{-\tau s}u(s) \\ &= (sI - A - BK)^{-1}B\mathrm{e}^{-\tau s}M^{-1}(s)u(s) \end{aligned} \tag{38}$$

Then

$$r(s) = Px(s)\mathrm{e}^{\tau s} = P(sI - A - BK)^{-1}B\xi(s) \tag{39}$$

Hence, (34) and (35) are obtained.

It is noted that $P(s)$ and $M(s)$ are the right-coprime factors of $L_s(s)$, which is known to be the desired transfer function in the state feedback. That is

$$L_s(s) = P(s)M^{-1}(s) \tag{40}$$

This equation plays an important role in revealing the correlation between the loop transfer recovery of direct state feedback and the functional observation of time-delay systems.

By using the factorization expression, a new system description (31)–(33) is introduced. According to this description, the existence condition of functional observers that satisfy (30) can be given below.

Theorem 1: *For the given system* (8) *and* (9), *the variable* $Px(t+\tau)$ *can be observed by using the observer* (24) *if and only if the following condition holds*:

$$F(s)M(s) + H(s)N(s) = P(s) \tag{41}$$

Proof: For the proof, see the Appendix. □

It can be seen that to observe the function $Px(t+\tau)$ using the observer (24), (41) must be satisfied. This is the necessary and sufficient condition for constituting the functional observer and state feedback. From (41), the functional observer-based state feedback law (23) can be rewritten as

$$\begin{aligned} u(s) &= \hat{r}(s) + v(s) \\ &= \left[F(s)M(s) + H(s)N(s)\right]\xi(s) + v(s) \\ &= P(s)\xi(s) + v(s) \end{aligned} \tag{42}$$

Hence, (42) and (32) produce

$$v(s) = [M(s) - P(s)]\xi(s)$$

Along with (33), the following can be obtained:

$$y(s) = N(s)[M(s) - P(s)]^{-1} v(s) \tag{43}$$

This is just the functional observer and state feedback. From (43), the closed-loop system is stable if and only if

$$T(s) = [M(s) - P(s)]^{-1} \in U$$

On the other hand, to achieve the loop transfer recovery of the direct state feedback by using observer-based feedback (23) and (24), it is necessary to satisfy the condition of (29). By comparing (41) with (29) in the sense of (40), it is seen that the two equations are the same. Thus, it can be concluded that the two problems, the loop transfer recovery of the direct state feedback satisfying (29) by using the observer-based feedback and the functional observation problem satisfying (41), are exactly the same.

According to (40), (43) can be rewritten as

$$y(s) = G(s)[I - L_s(s)]^{-1} v(s) \tag{44}$$

It is seen that (44) is the same as (22) in the state-feedback control.

Thus, we can use the results from the functional observer design to study the loop transfer recovery of the direct state feedback. The objective in the next section is to find the set of all observers that satisfy condition (41), for example, to parametrize all observers. The factorization approach for time-delay systems will be used to solve this problem.

5. Parametrization of observers for input time-delay systems

Theorem 2: *Given the system* (8)–(10), *the set of all functional observers for* $Px(t+\tau)$ *is parametrized by*

$$F(s) = [P(s)Y(s) - Q(s)\tilde{N}(s)] \tag{45}$$

$$H(s) = [P(s)X(s) + Q(s)\tilde{M}(s)] \tag{46}$$

$$Q(s) \in H_\infty \tag{47}$$

where $\tilde{N}(s)$, $\tilde{M}(s)$, $Y(s)$ *and* $X(s)$ *satisfy the Bezout identity* (2) *corresponding to the coprime factorization of* $G(s)$.

Proof: For the proof, see the Appendix. □

Theorem 2 gives the result of parametrization of all functional observers for the system (8)–(10). According to this parametrization, the observer design is reduced to searching for a suitable parametrization matrix $Q(s)$ in an H_∞ set. It can be seen that the distinction between different functional observers is only caused by the selection of parametrization $Q(s)$ as well as the coprime factorization of $G(s)$. The selection of the parametrization matrix $Q(s)$ depends on the required performance specification in system design. When a certain design specification such as H_∞ norm specification (Ding *et al.* 1994) is given, one can solve the corresponding functional observer design in the feedback law (23) and (24) by finding an optimal $Q(s)$. Therefore, the parametrization of the observer provides a basic tool for this kind of optimal problem.

In most cases the plants are stable; this means that $L_s(s)$ is stable. In this case by using (40) and $M(s)Y(s) = I - \tilde{X}(s)\tilde{N}(s)$ in (2), (45) becomes

$$\begin{aligned} F(s) &= L_s(s)M(s)Y(s) - Q(s)\tilde{N}(s) \\ &= L_s(s)(I - \tilde{X}(s)\tilde{N}(s)) - Q(s)\tilde{N}(s) \\ &= L_s(s) - R(s)\tilde{N}(s) \end{aligned} \tag{48}$$

where

$$R(s) = L_s(s)\tilde{X}(s) + Q(s) \tag{49}$$

Accordingly, by using (40), (49) and $M(s)X(s) = \tilde{X}(s)\tilde{M}(s)$, (46) can be rewritten as

$$\begin{aligned} H(s) &= P(s)X(s) + (R(s) - L_s(s)\tilde{X}(s))\tilde{M}(s) \\ &= L_s(s)M(s)X(s) - L_s(s)\tilde{X}(s)\tilde{M}(s) + R(s)\tilde{M}(s) \\ &= R(s)\tilde{M}(s) \end{aligned} \tag{50}$$

Thus, the following theorem is proven.

Theore 3: *When $L_s(s)$ is stable, the parametrization of all functional observers is given by*

$$F(s) = L_s(s) - R(s)\tilde{N}(s) \tag{48}$$

$$H(s) = R(s)\tilde{M}(s) \tag{50}$$

with

$$R(s) = L_s(s)\tilde{X}(s) + Q(s) \in H_\infty \tag{49}$$

The parametrization of all observers for time delay-free systems was recently given by Ding *et al.* (1994). In this section it has been shown that the parametrization of time-delay systems can be regarded as a generalization of those recent results of time-delay-free systems.

6. Observer construction for input time-delay systems

Based on the parametrization of the functional observer, we can obtain a clear observer construction. From (24), (45) and (46) we have

$$\begin{aligned} \hat{r}(s) &= F(s)u(s) + H(s)y(s) \\ &= \left[P(s)Y(s) - Q(s)\tilde{N}(s)\right]u(s) + \left[P(s)X(s) + Q(s)\tilde{M}(s)\right]y(s) \end{aligned} \tag{51}$$

From (2), $Y(s) = M^{-1}(s) - X(s)N(s)M^{-1}(s)$. Thus, (51) becomes

$$\hat{r}(s) = P(s)M^{-1}(s)u(s) + (P(s)X(s) + Q(s)\tilde{M}(s))\left[y(s) - G(s)u(s)\right] \tag{52}$$

Therefore

$$\hat{r}(s) = P(s)M^{-1}(s)u(s) + L_g(s)\left[y(s) - G(s)u(s)\right] \tag{53}$$

where

$$L_g(s) = P(s)X(s) + Q(s)\tilde{M}(s) \tag{54}$$

It can be seen that the observer (53) consists of two terms. The first term $r(s) = P(s)M^{-1}(s)u(s) = P(s)\xi(s)$ is an estimate of the dynamics of the state function $Px(t+\tau)$. The second term reflects the mismatch between the measurable output and its estimate. The mismatch is generally caused by the disturbance signals and plant perturbations in the process. If no mismatch exists between the measurable output $y(s)$ and its estimate, (53) becomes

$$\hat{r}(s) = P(s)M^{-1}(s)u(s) = L_s(s)\xi(s) \tag{55}$$

By properly selecting the gain matrix $L_g(s)$, the estimation error for $r(t)$ can be bounded to a prescribed range.

The observer design is to select the gain matrix. In our observer structure (53), this gain matrix is dynamic and therefore provides designers with more degrees of freedom for improving the robustness of observers. It is necessary to find an optimal $Q(s)$ such that a certain specification which represents the effect of plant perturbations and disturbances is satisfied. Thus, the design can be regarded as a model-based frequency-domain technique.

When the given plant is stable, substituting (48) and (50) into (24) gives

$$\hat{r}(s) = L_s(s)u(s) + R'(s)(y(s) - G(s)u(s)) \tag{56}$$

with $R'(s) = R(s)\tilde{M}(s)$. The corresponding control system scheme is shown in Fig. 3.

Figure 3 can be rearranged as Fig. 4. It is known that the IMC (Wang and Skogestad 1993, Jones and Sbarbaro 1995) and Smith predictor have been successfully used in time-delay systems. In general, the Smith predictor can only be used for stable systems, whereas the IMC can be used for unstable and nonminimum-phase systems. It is found that the controller structure in Fig. 4 is the same as the IMC

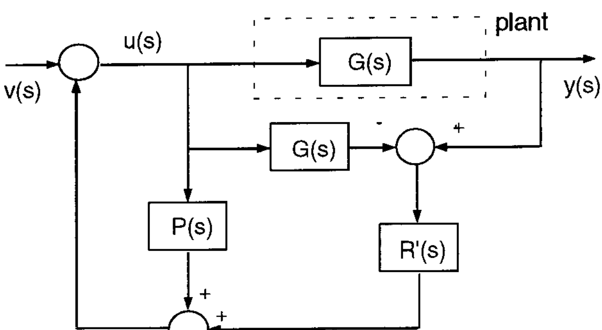

Figure 3. System using functional observer-based feedback with stable plant.

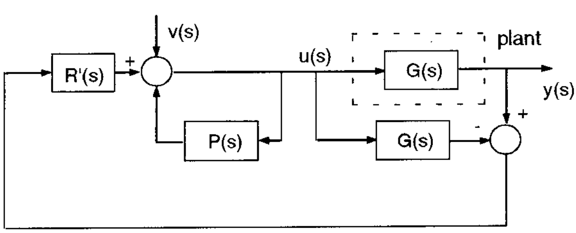

Figure 4. Rearranged system with stable plant.

structure when $K(s) = R(s)(I - P(s))^{-1}$. Thus, the proposed control scheme is a more general form for controlling time-delay systems and can be used in unstable and nonminimum-phase systems.

7. Design procedure

The results in this work can provide us with a systematic procedure to design the functional observer and state feedback. The objective of loop transfer recovery of direct state feedback can be achieved by using this design. In summary, the controller design can be performed using the following procedure.

Step 1. Choose the direct state feedback gain P such that the loop transfer function $L_s(s)$ is obtained.

Step 2. Find a parametrization matrix $Q(s) \in H_\infty$ of the observer such that the state feedback properties are recovered. More specifically

- *Step* (i) Obtain a coprime factorization of $G(s)$ for $\tilde{N}(s)$ and $\tilde{M}(s)$.
- *Step* (ii) Calculate $Y(s)$ and $X(s)$ which satisfy the Bezout identity (3) and $P(s)$ given in (33).
- *Step* (iii) Find $Q(s) \in H_\infty$ under the desired performance index.

The control law, including functional observers, is given in the form

$$u(t) = \hat{r}(t) + v(t) \tag{57}$$

$$\hat{r}(s) = \left[P(s)Y(s) - Q(s)\tilde{N}(s)\right]u(s) + \left[P(s)X(s) + Q(s)\tilde{M}(s)\right]y(s) \tag{58}$$

The functional observers are also expressed in the following form when the plant is stable.

$$\hat{r}(s) = \left[L_s(s) - R(s)\tilde{N}(s)\right]u(s) + R(s)\tilde{M}(s)y(s) \tag{59}$$

where $R(s) \in H_\infty$ is the parametrization matrix.

The matrix $Q(s)$ is a free parameter which can be chosen by the designer. When the disturbance and plant perturbation exist, the condition (30) may not necessarily hold. In this case, a performance index such as H_∞ norm can be given to evaluate the observation error, and $Q(s)$ will be determined to optimize the given performance index. It is known that such optimization has been studied for time-delay-free systems (Ding *et al.* 1994). The parametrization of all observers achieved in this work has made such optimization possible for the time-delay systems.

The above procedure is based on the proper stable Bezout factorization under state-space representation and standard matrix calculations. It can be directly realized and implemented through computer-aided control system design programs such as Matlab.

8. Design examples

Example 1: Consider the plant described in (8) and (9), where

$$A = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C = (2 \quad 1), \quad D = 0, \quad \tau = 0{\cdot}5 \tag{60}$$

Then the transfer function of the plant is

$$G(s) = \frac{s+2}{s^2+2s-3}\mathrm{e}^{-0\cdot 5s} \tag{61}$$

In the case of state feedback, the gain matrix in (13) is chosen to be

$$P = (-5 \quad -1)$$

such that

$$\det(sI - A - BP) = s^2 + 3s + 2 = (s+1)(s+2)$$

The corresponding loop transfer function is

$$L_s(s) = P(sI - A)^{-1}B = \frac{-s-5}{s^2+2s-3} \tag{62}$$

The closed-loop system transfer function is

$$H(s) = G(s)\left[I - L_s(s)\right]^{-1} = \frac{1}{s+1}\mathrm{e}^{-0\cdot 5s} \tag{63}$$

By choosing $K = P = (-5 \quad -1)$ and $L = (-1\cdot 3333 \quad 2\cdot 6666)^{\mathrm{T}}$ such that

$$\det(sI - A - BK) = s^2 + 3s + 2 = (s+1)(s+2)$$

$$\det(sI - A - LC) = s^2 + 2s + 1 = (s+1)(s+1)$$

we obtain

$$P(s) = \frac{-s-5}{s^2+3s+2} \tag{64}$$

$$\tilde{M}(s) = \frac{s^2+2s-3}{s^2+2s+1}, \tag{65}$$

$$\tilde{N}(s) = \frac{s+2}{s^2+2s+1}\mathrm{e}^{-0\cdot 5s} \tag{66}$$

$$X(s) = \frac{-10\cdot 3215s + 34\cdot 4451}{s^2+2s+1} = \frac{-10\cdot 3215(s - 3\cdot 3372)}{(s+1)(s+1)} \tag{67}$$

$$Y(s) = \frac{s^2 + 10\cdot 6115s + 20\cdot 8030}{s^2+2s+1} + KV(s) = \frac{(s+8\cdot 0164)(s+2\cdot 595)}{(s+1)(s+1)} + KV(s) \tag{68}$$

where $V(s)$ is given in (7). It is noted that $V(s)$ is a stable transfer function matrix. The functional observer-based control law described in (57) and (58) is obtained.

Example 2: Consider the plant model of a high-purity distillation system described by Limebeer *et al.* (1993)

$$G(s) = \frac{\mathrm{e}^{-s}}{75s+1}\begin{bmatrix} 0\cdot 878 & -0\cdot 864 \\ 1\cdot 082 & -1\cdot 096 \end{bmatrix} \tag{69}$$

The state-space realization is given by

$$A = \begin{pmatrix} -0\cdot 0133 & 0 \\ 0 & -0\cdot 0133 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 0\cdot 0117 & -0\cdot 0115 \\ 0\cdot 0144 & -0\cdot 0146 \end{pmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \tau = 1 \tag{70}$$

If the gain matrix in state feedback is chosen as

$$P = \begin{pmatrix} 1 & 0{\cdot}4 \\ -3{\cdot}062 & -1{\cdot}2134 \end{pmatrix} \tag{71}$$

the corresponding loop transfer function is

$$L_s(s) = \frac{1}{s + 0{\cdot}0133}\begin{bmatrix} 1 & 0{\cdot}4 \\ -3{\cdot}062 & -1{\cdot}2134 \end{bmatrix} \tag{72}$$

Thus, the observer-based control law is given in (59), where

$$\tilde{M}(s) = \frac{1}{s^2 + 0{\cdot}24s + 0{\cdot}0144}\begin{bmatrix} s^2 + 1{\cdot}24s + 0{\cdot}0163 & 0{\cdot}4s + 0{\cdot}0053 \\ -3{\cdot}062s - 0{\cdot}0407 & s^2 - 0{\cdot}9734s - 0{\cdot}0131 \end{bmatrix} \tag{73}$$

$$\tilde{N}(s) = \frac{e^{-s}}{s^2 + 0{\cdot}24s + 0{\cdot}0144}\begin{bmatrix} 0{\cdot}0117s + 0{\cdot}0496 & -0{\cdot}0115s + 0{\cdot}0160 \\ 0{\cdot}0144s + 0{\cdot}0624 & -0{\cdot}0146s + 0{\cdot}0202 \end{bmatrix} \tag{74}$$

9. Conclusions

A novel approach for the design of the functional observer and state feedback control for time-delay systems is given via the factorization approach which has been found to be a useful tool in resolving various control design issues such as simultaneous stabilization (Vidyasagar 1985), simultaneous observation (Kovacevic *et al.* 1996), etc. The design can achieve the loop transfer recovery of the state feedback by using functional observer-based control. It gives a new insight into the observer-based control problem for such systems from the point of view of functional observer design. Based on the interconnection between the loop transfer recovery and the functional observation for the given system, the feedback design problem is solved by using the results of functional observation. As a result, a loop transfer recovery problem can be treated in a stable transfer function space so that the standard factorization approach can be used. This extends the existing results of loop transfer recovery designs for time-delay systems given by previous researchers, such as Lee *et al.* (1988). The design can be applied to unstable and nonminimum-phase systems.

Appendix

Proof of Theorem 1:

Necessity. The observation error in (30) is given as

$$r(t) - \hat{r}(t) = L^{-1}[r(t) - \hat{r}(s)] \tag{A 1}$$

where $L^{-1}[\cdot]$ denotes inverse Laplace transformation. Furthermore, let $\hat{r}(s) = \hat{r}_1(s) + \hat{r}_2(s)$ in the observer (24), where $\hat{r}_1(s) = F(s)u(s)$, $\hat{r}_2(s) = H(s)y(s)$ with stable vectors x_f and x_h and initial values $x_f(0)$ and $x_h(0)$, respectively. Then from (34) and (24), we have

$$L^{-1}[r(s)] = L^{-1}[P(s)\xi(s) + P(sI - A_k)^{-1}x(0)] \tag{A 2}$$

$$\begin{aligned} L^{-1}[\hat{r}(s)] &= L^{-1}[F(s)u(s) + C_f(sI - A_f)^{-1}x_f(0) + H(s)y(s) + C_h(sI - A_h)^{-1}x_h(0)] \\ &= L^{-1}[(F(s)M(s) + H(s)N(s))\xi(s) + C_f(sI - A_f)^{-1}x_f(0) \\ &\quad + C_h(sI - A_h)^{-1}x_h(0)] \end{aligned} \tag{A 3}$$

where $A_k = A + BK$. Substituting (A 2) and (A 3) into (A 1), yields

$$r(t) - \hat{r}(t) = L^{-1}\Big[(P(s) - F(s)M(s) + H(s)N(s))\xi(s) + P(sI - A_k)^{-1}x(0) - C_f(sI - A_f)^{-1}x_f(0) - C_h(sI - A_h)^{-1}x_h(0)\Big] \quad (\mathrm{A}\,4)$$

Because $C(sI - A_k)^{-1}$, $C_f(sI - A_f)^{-1}$ and $C_h(sI - A_h)^{-1} \in H_\infty$, we have

$$\lim_{t\to\infty}\left[P(sI - A_k)^{-1}x(0) - C_f(sI - A_f)^{-1}x_f(0) - C_h(sI - A_h)^{-1}x_h(0)\right] = 0 \quad (\mathrm{A}\,5)$$

Therefore the condition (41) is a necessary condition to satisfy condition (30).

Sufficiency. It is known that the observer satisfies

$$\begin{bmatrix} F(s) & H(s) \end{bmatrix}\begin{bmatrix} M(s) \\ N(s) \end{bmatrix} = P(s) \quad (\mathrm{A}\,6)$$

That is

$$\begin{bmatrix} F(s) & H(s) \end{bmatrix}\begin{bmatrix} M(s) \\ N(s) \end{bmatrix}\xi(s) = P(s)\xi(s)$$

then

$$\begin{bmatrix} F(s) & H(s) \end{bmatrix}\begin{bmatrix} u(s) \\ y(s) \end{bmatrix} = P(s)\xi(s) \quad (\mathrm{A}\,7)$$

This guarantees that $\hat{r}(s)$ in (24) is equivalent to $r(s)$. Therefore the observer can be expressed as the form of (24). Thus, the proof is complete. □

Proof of Theorem 2:

Necessity. Select a $Q(s)$ that satisfies (47). There exists an observer

$$\begin{aligned} r(s) &= F(s)u(s) + H(s)y(s) \\ &= \left[P(s)Y(s) - Q(s)\tilde{N}(s)\right]u(s) + \left[P(s)X(s) + Q(s)\tilde{M}(s)\right]y(s) \end{aligned} \quad (\mathrm{A}\,8)$$

or

$$\begin{aligned} \begin{bmatrix} F(s) & H(s) \end{bmatrix} &= \begin{bmatrix} P(s)Y(s) - Q(s)\tilde{N}(s) & P(s)X(s) + Q(s)\tilde{M}(s) \end{bmatrix} \\ &= \begin{bmatrix} P(s) & Q(s) \end{bmatrix}\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{M}(s) \end{bmatrix} \end{aligned} \quad (\mathrm{A}\,9)$$

By postmultiplying (83) by

$$\begin{bmatrix} M(s) \\ N(s) \end{bmatrix}$$

we have

$$\begin{aligned} \begin{bmatrix} F(s) & H(s) \end{bmatrix}\begin{bmatrix} M(s) \\ N(s) \end{bmatrix} &= \begin{bmatrix} P(s) & Q(s) \end{bmatrix}\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{M}(s) \end{bmatrix}\begin{bmatrix} M(s) \\ N(s) \end{bmatrix} \\ &= \begin{bmatrix} P(s) & Q(s) \end{bmatrix}\begin{bmatrix} I \\ 0 \end{bmatrix} = P(s) \end{aligned} \quad (\mathrm{A}\,10)$$

It can be seen that this satisfies the condition (41) of the observation.

Sufficiency. The observer is given by

$$r(s) = F(s)u(s) + H(s)y(s)$$

It is required to find a $Q(s) \in H_\infty$ such that the observer can be expressed as (45) and (46), that is

$$\begin{bmatrix} P(s) & Q(s) \end{bmatrix} \begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{M}(s) \end{bmatrix} = \begin{bmatrix} F(s) & H(s) \end{bmatrix} \tag{A 11}$$

It is known that the observer satisfies

$$\begin{bmatrix} F(s) & H(s) \end{bmatrix} \begin{bmatrix} M(s) \\ N(s) \end{bmatrix} = P(s) \tag{A 12}$$

From (2),

$$\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{M}(s) \end{bmatrix}$$

is non-singular. Let

$$\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{M}(s) \end{bmatrix}^{-1} = \begin{bmatrix} M(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} \tag{A 13}$$

This is a stable matrix (Vidyasagar 1985). From (A 11) we obtain

$$\begin{aligned} \begin{bmatrix} P(s) & Q(s) \end{bmatrix} &= \begin{bmatrix} F(s) & H(s) \end{bmatrix} \begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{M}(s) \end{bmatrix} \\ &= \begin{bmatrix} F(s) & H(s) \end{bmatrix} \begin{bmatrix} M(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} \end{aligned} \tag{A 14}$$

That is

$$P(s) = \begin{bmatrix} F(s) & H(s) \end{bmatrix} \begin{bmatrix} M(s) \\ N(s) \end{bmatrix}, \quad Q(s) = \begin{bmatrix} F(s) & H(s) \end{bmatrix} \begin{bmatrix} -\tilde{X}(s) \\ \tilde{Y}(s) \end{bmatrix} \tag{A 15}$$

Thus (41) is satisfied. □

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