TIME-SPACE FIXED-CHARGE NETWORK FLOW PROBLEMS

Approved by:

Dr. Jeffery Kennington
Professor, EMIS, SMU

Dr. Richard Barr
Chair and Associate Professor, EMIS, SMU

Dr. U. Narayan Bhat
Professor Emeritus, Statistical Science, SMU

Dr. Richard Helgason
Associate Professor, EMIS, SMU

Dr. Eli Olinick
Associate Professor, EMIS, SMU
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Charles Nicholson
(B.S. Physics, University of North Texas, 1999)
(B.S. Mathematics, University of North Texas, 1999)
(M.S. Decisions Technologies, University of North Texas, 2002)

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The nodes and arcs of a network configuration replicated over time is a common structure found in many applications, particularly in the area of logistics. A common cost structure for flows in arcs for such problems involves both a fixed and variable cost. Combining the two concepts results in the time-space fixed-charge network flow problem. These problems can be modeled as mixed binary linear programs and can be solved with commercial software. Creating these models requires determining implied arc capacities that are sufficiently large so that the solution space has not been altered, but are small enough that the linear programming relaxations are tight. In this investigation, we present multiple strategies for determining these implied arc capacities for the time-space fixed-charge network flow problem.

In particular, we present methods for solving to optimality the single commodity, generalized, and multicommodity versions of the time-space fixed-charge problem. In extensive empirical tests, we provide statistical evidence that the strategies introduced are superior to the usual techniques applied to these types of problems. For many of the most difficult single commodity problem sets, our method required only 5% of the computational time required by standard techniques. The most difficult generalized network problem subsets solved up to two orders of magnitude faster us-
ing our strategy; the multicommodity problems solved up to one order of magnitude faster.

We also introduce two novel approximate methods. The first method, tested on single commodity problems, resulted in speed-ups of up to a factor of 3,200 with an average optimality gap of about 1%. The second method, applicable only to the generalized version of the problem, found exact solutions in every completed test and resulted in speed-ups up to a factor of nearly 600.

Problems with this time-replicated, fixed-cost structure type have been used by large banks to solve cash management logistics problems. A set of real-world multi-commodity problems are acquired from a national bank and solved using the ideas presented in this investigation. While most of the real-world problems were easy, the empirical work provides more evidence that the techniques we present are valid approaches to this problem class.
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