

THEMIS SIGNAL ANALYSIS STATISTICS RESEARCH PROGRAM

WAVE PROPAGATION IN AN INHOMOGENEOUS MEDIUM

by

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DEPARTMENT OF STATISTICS
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MATHEMATICAL SYMBOLS

Symbol	Meaning
∇^2	Laplacian operator
$\tilde{\mathbf{r}}$	position vector in space
t	time
p	pressure
c_0	wave speed of sound in water
p_0	incident pressure wave-spatial dependence
ω	circular frequency of incident wave
j	$\sqrt{-1}$
p_s	scattered field pressure
A_k	scatterer strength
a_k	radius of k^{th} scatterer
N	number of scatterers
p_k	external pressure field acting on k^{th} scatterer
$ $	absolute value
$\sum$	summation sign
p_T	total pressure field
$\langle \rangle$	configuration average
ψ	dummy function
λ	wave length

K_s	• • • • •	bulk modulus of scatterer
K_w	• • • • •	bulk modulus of water
ρ_s	• • • • •	mass density of scatterer
ρ_w	• • • • •	mass density of water
θ	• • • • •	azimuth angle measured from direction of incident wave
d	• • • • •	diameter of scatterer
ϕ	• • • • •	volume concentration of scatterers
C	• • • • •	average wave speed of phase velocity of mixture
V	• • • • •	amplitude variation
f	• • • • •	frequency of incident wave

"seeing" through such particles because of the lesser backscatter of lower frequency acoustic waves. Nevertheless, some backscatter, transmission scatter, and resolution problems do exist even for acoustic waves in such a mixture so that in order that something can be done to cause acoustic holograms to give good image resolution, the acoustic transmission properties of the water-particle mixtures must be known.

The problem of sound wave propagation in liquid-solid mixtures has

been studied by Urlick⁽¹⁾, Wood⁽²⁾, and Shumway⁽³⁾. The analysis of each is very nearly the same in that an equivalent density and compressibility for the mixture is derived from the densities and compressibilities of the constituents of the mixture. Good agreement with experiment is obtained by Urlick⁽¹⁾ for low concentrations of compressible particles. None predict any amplitude variation incoherence or are able to explain the large increase in wave speed at high concentrations.

In this paper, the theory of scattering of a sound wave from a single particle is applied for a large number of particles by assuming each particle to be an isotropic, flexible, spherical scattering source which has a scattering strength proportional to the external field acting on it, and by summing the contributions to the total sound field from each of these sources. The general theory of wave propagation in a discretely random inhomogeneous medium was developed by Foldy⁽⁴⁾. This paper describes the application of the general theory to the particular problem of acoustic wave transmission in water-solid particle mixtures of high concentrations.

ANALYSIS

We begin by assuming the medium (water) to be homogeneous so that a small acoustic pressure wave in the medium obeys the wave equation

$$\Delta^2 P(\tilde{r}, t) = \frac{1}{c_0^2} \frac{\partial^2 P(\tilde{r}, t)}{\partial t^2} \quad (1)$$

for a constant wave speed c_0 . Assuming a continuous harmonic wave to be

propagating in the medium we have

$$P(\tilde{r}, t) = P_0(\tilde{r}) e^{j\omega t} \quad (2)$$

Equation (1) then becomes

$$\Delta^2 P_0(\tilde{r}) + k_0^2 P_0(\tilde{r}) = 0 \quad (3)$$

where

$$k_0 = \frac{\omega}{c_0}.$$

If an isotropic scatterer is present at $\tilde{r} = \tilde{r}_k$ the scattered field

near the scatterer is

$$P_s(\tilde{r}, \tilde{r}_k) = A_k \frac{e^{-jk_0 R_k}}{R_k} \quad (4)$$

in which $R_k = |\tilde{r} - \tilde{r}_k|$ is the distance from the scatterer at $\tilde{r}_k$ to the

observation point at $\tilde{r}$. A_k is the strength of the scatterer and is

assumed to be proportional to the external field according to the equation

$$A_k = g(a_k, \omega) P_k(\tilde{r}_k) \quad (5)$$

in which $g(a_k, \omega)$ is the scattering coefficient, assumed to depend on

the radius a_k of the scatterer and the circular frequency ω . $P_k(\tilde{r}_k)$ is

the value of the external field acting on the scatterer. The external

field is the difference between the total field and the scattered field.

For a single scatterer the external field is just the incident $P_0(\tilde{r})$.

For a set of N scatterers the external field at a given scatterer is the

a neighborhood $d\tilde{r}_1$ of $\tilde{r}_1$, another scatterer of scattering coefficient which a scatterer of scattering coefficient between a_1 and $a_1 + da_1$ is in

the N scatterers in a configuration of an ensemble of configurations in $da_1 da_2 \dots da_N$ which defines the probability of finding probability distribution function $P(\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_N; a_1, a_2, \dots, a_N)$

At this point we introduce the assumption of the existence of a

positions and scattering coefficients of the scatterers are known.

in a liquid medium, so that at best only statistical descriptions of the are not known exactly for a large number of slowly moving, suspended particles

the positions and scattering coefficients for the individual scatterers and (7) simultaneously for P^T and P^S yields the desired results. However,

each scatterer the problem is deterministic and solution of Equations (6)

If $g(a_k, \omega)$ and the positions $\tilde{r}_k$, $k = 1, 2, \dots, N$ are known for

$$\begin{aligned} P^T(\tilde{r}) &= P^0(\tilde{r}) + \sum_{m=1}^N g(a_m, \omega) P^m(\tilde{r}_m) \exp(-jk_0 |\tilde{r} - \tilde{r}_m|) \\ P^S(\tilde{r}, \tilde{r}_m) &= P^0(\tilde{r}) + \sum_{m=1}^N g(a_m, \omega) P^m(\tilde{r}_m) \exp(-jk_0 |\tilde{r} - \tilde{r}_m|) \end{aligned} \quad (7)$$

that is,

the scattered waves from the N scatterers located at $\tilde{r}_k$, $k = 1, 2, \dots, N$,

observation point at $\tilde{r}$ is then the sum of the incident wave plus all of

which is represented by Figure (1). The total pressure $P^T(\tilde{r})$ at any

$$P^k(\tilde{r}_k) = P^0(\tilde{r}_k) + \sum_{m=1, m \neq k}^N g(a_m, \omega) P^m(\tilde{r}_m) \exp(-jk_0 |\tilde{r}_k - \tilde{r}_m|) \quad (6)$$

scatterers, that is

sum of the incident and the scattered waves from all of the other N-1

between a_2 and $a_2 + da_2$ is in a neighborhood dr_2 of $\tilde{r}_2$, and so on for

N scatterers. We assume also that each scatterer is described by the same probability distribution function and has a scattering coefficient and

position independent of the scattering coefficients and positions of the

other scatterers. Thus the joint probability distribution function above

can be written as a product of N identical probability distribution functions,

i.e.,

$$p(r_1, \dots, r_N; a_1, \dots, a_N) = \frac{N}{n(\tilde{r}_1, a_1)} \cdot \frac{N}{n(\tilde{r}_2, a_2)} \cdot \dots \cdot \frac{N}{n(\tilde{r}_N, a_N)} \quad (8)$$

where $n(r, a)$ represents the average number of scatterers per unit volume

in a neighborhood dr about $\tilde{r}$ having scattering parameters between a and

$a + da$. Thus the total number of scatterers per unit volume in a neighbor-

hood dr about $\tilde{r}$ is

$$n(\tilde{r}) = \int_a n(\tilde{r}, a) da \quad (9)$$

In order for the probability to be unity for a summation over the

volume we must have

$$N = \int n(\tilde{r}) d\tilde{r} \quad (10)$$

We define the configuration average of the total pressure as

$$\langle p_T(\tilde{r}) \rangle = \iiint_V \dots \iiint_V p_T(\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_N; a_1, a_2, \dots, a_N) \times$$

$$dr_1 dr_2 \dots dr_N da_1 da_2 \dots da_N \quad (11)$$

Taking the configuration average of both sides of Equation (7)

we obtain

$$\langle P_T^P(\tilde{x}) \rangle = \langle P^0(\tilde{x}) \rangle + \iiint \dots \iiint P(\tilde{x}_1, \dots, \tilde{x}_N; a_1, \dots, a_N) \quad (12)$$

$$\langle P_T^P(\tilde{x}) \rangle = \langle P^0(\tilde{x}) \rangle + \iiint \dots \iiint \frac{N^N}{u(\tilde{x}_1, a_1) \dots u(\tilde{x}_N, a_N)} \quad (13)$$

$$\times \sum_{m=1}^M g(a_m, w) P_m^P(\tilde{x}) \frac{| \tilde{x} - \tilde{x}^m |}{\exp(-jk_0 | \tilde{x} - \tilde{x}^m |)} d\tilde{x}_1 \dots d\tilde{x}_N da_1 \dots da_N$$

which can be written as

$$\langle P_T^P(\tilde{x}) \rangle = \langle P^0(\tilde{x}) \rangle + \sum_{m=1}^M \iiint \dots \iiint \frac{N^N}{u(\tilde{x}_1, a_1) \dots u(\tilde{x}_N, a_N)} \dots$$

$$\times P_m^P(\tilde{x}) d\tilde{x}_1 \dots d\tilde{x}_N da_1 \dots da_N \dots da_{m-1} da_{m+1} \dots da_N \left[\int da_m \frac{| \tilde{x} - \tilde{x}^m |}{\exp(-jk_0 | \tilde{x} - \tilde{x}^m |)} \right] \times$$

which by virtue of Equations (9) and (10) becomes

$$\langle P_T^P(\tilde{x}) \rangle = \langle P^0(\tilde{x}) \rangle + \sum_{m=1}^M \iiint \dots \iiint g(a_m, w) \dots \exp(-jk_0 | \tilde{x} - \tilde{x}^m |) u(\tilde{x}^m, a_m) \dots$$

$$\times d\tilde{x} da_m \quad (15)$$

in which $\langle P_m^P(\tilde{x}) \rangle$ is the configuration average of the external field with the m th scatterer left out. Equation (15) can be rewritten as

$$\Delta^2 \langle P_T \rangle (\tilde{x}) \rangle + k_0^2 \langle P_T \rangle (\tilde{x}) \rangle = \int_V G(x_m) \langle P_T \rangle (x_m) \langle P_T \rangle (\tilde{x}) \rangle \left\{ \frac{\exp(-jk_0 |\tilde{x} - x_m|)}{\exp(-jk_0 |\tilde{x} - x_m|)} \right\} d\tilde{x}_m \quad (21)$$

Because

$$\left\{ \frac{\exp(-jk_0 |\tilde{x} - x_m|)}{\exp(-jk_0 |\tilde{x} - x_m|)} \right\} = -4\pi \delta(\tilde{x} - x_m) \quad (22)$$

where $\delta(\tilde{x} - x_m)$ is the three dimensional Dirac delta function Equation (21)

becomes

$$\Delta^2 \langle P_T \rangle (\tilde{x}) \rangle + k_0^2 \langle P_T \rangle (\tilde{x}) \rangle = \int_V G(x_m) \langle P_T \rangle (x_m) \langle P_T \rangle (\tilde{x}) \rangle (-4\pi \delta(\tilde{x} - x_m)) d\tilde{x}_m \quad (23)$$

$$= -4\pi G(\tilde{x}) \langle P_T \rangle (\tilde{x}) \rangle$$

or

$$\Delta^2 \langle P_T \rangle (\tilde{x}) \rangle + k_0^2 \langle P_T \rangle (\tilde{x}) \rangle = 0 \quad (24)$$

where

$$k_0^2 = k_2^2 + 4\pi G(\tilde{x}) \quad (25)$$

Equations (24) and (25) indicate that the configuration average of

the total pressure obeys the wave equation but has a different phase velocity (or average wave speed) which is different from C_0 . If $g(a, \omega)$ and $n(r, a)$ are known then $G(\tilde{x})$ can be determined from Equation (19) and the wave speed

computed.

SOLID PARTICLE SCATTERER

In the determination of $g(a, \omega)$ the suspended solid particle scatterer is assumed to be a flexible sphere of radius a , as a first approximation.

A harmonic plane wave of wavelength $\lambda \gg a$, the radius of the scatterer, is assumed to impinge on a single scatterer and the ratio of the strength of the scattered wave to the incident wave can be shown (5) to be

$$(26) \quad \left(\frac{P_s}{P_0} \right)_{r=a} = g(a, \omega) = - \frac{3}{k_a^2} \frac{k_a^3}{K_s - K_w} + \frac{K_s}{3(p_s - p_w)} \cos \theta \quad \frac{2p_s + p_w}{K_s} \quad \frac{2p_s + p_w}{K_s} \quad \frac{2p_s + p_w}{K_s}$$

which is a combination of an isotropic source and a dipole source (see

Figure 2). In the analysis it is assumed the source is isotropic. One

may reject the dipole part of $g(a, \omega)$ in Equation (26) for a high concentration of scatterers by means of an intuitive physical argument. Let d be

the diameter of a sphere enclosing a sufficiently large enough number of

scatterers to completely surround a given scatterer (see Figure 3). If the

wavelength $\lambda \gg d$ then the radiation of the surrounding scatterers on the

m^{th} scatterer will be essentially isotropic. The result is that the directivity of the strength dipole source when integrated over the sphere of the

isotropic incident radiation reduces to an equivalent isotropic source of

zero strength. Thus the dipole part of Equation (26) can be rejected for

high concentrations of particles and the scattering coefficient is then

$$(27) \quad g(a, \omega) = - \frac{3}{k_a^2} \frac{k_a^3}{K_s - K_w} \left[\frac{K_w}{K_s} \right]$$

If one assumes the scatterers are all of equal size and have the same

scattering coefficient, $G(\tilde{r})$ in Equation (19) can be evaluated by assuming

the number of particles per unit volume can be represented by a concentration

function $\phi(\tilde{r}, a)$ such that

$$(28) \quad \int n \, da = \frac{4/3\pi a^3}{\phi(\tilde{r}, a)}$$

where ϕ represents the ratio of the volume of scatterers to the total volume over any arbitrary volume.

If the concentration ϕ is assumed to be uniform in space then Equation

(19) can be written as

$$(29) \quad G = - \frac{3}{k_0^2 a^3} \left(\frac{k_w^w}{k_s^w - k_s} \right) \left(\frac{4/3\pi a^3}{\phi} \right)$$

or

$$(30) \quad G = - \frac{k_0^2}{4} \left[\frac{k_w^w}{k_s^w - k_s} \right] \phi$$

and thus Equation (25) becomes

$$(31) \quad \frac{1}{1} = \frac{C_2}{1} \left[1 - \frac{k_w^w}{k_s^w - k_s} \right] \phi$$

or

$$(32) \quad C = \frac{C_0}{1/2} \left[1 - \frac{k_w^w}{k_s^w - k_s} \right] \phi$$

a quantity which we call the average wave speed in the composite medium.

The solution for $\langle P^T(\vec{r}) \rangle$ can now be obtained since Equation (24)

is just the ordinary wave equation and the solution in spherical coordinates

is

$$(33) \quad \langle P^T(r) \rangle = \frac{r}{\exp(-jkr)}$$

where $k = \omega/C$

for a diverging wave.

We define the amplitude variation in the pressure or the incoherence

as

$$V^2 = \frac{\langle |P_T(\tilde{x})|^2 \rangle - \langle |P_T(\tilde{x})| \rangle^2}{\langle |P_T(\tilde{x})|^2 \rangle} = \frac{\langle |P_T(\tilde{x})|^2 \rangle - \langle |P_T(\tilde{x})| \rangle^2}{1} \quad (34)$$

which may be found from taking the configuration average of Equation (7)

and using Equation (33). One finds $\bar{Q}_2$ to be negligible compared to $\bar{Q}_1$ which

is

$$\bar{Q}_1(\tilde{x}) = \int_V H(\tilde{x}_m) | \langle P_T(r_m) \rangle |^2 \cdot | K(\tilde{x}, \tilde{x}_m) |^2 d\tilde{x}_m \quad (35)$$

where

$$H(\tilde{x}_m) = \int |g(a, \omega)|^2 n(r_m, a) da \quad (36)$$

and

$$K(\tilde{x}, \tilde{x}_m) = \frac{\exp(jk|\tilde{x} - \tilde{x}_m|)}{|\tilde{x} - \tilde{x}_m|} \quad (37)$$

For long transmitter-to-receiver distances in a large scattering

medium $|\tilde{x}| \gg |\tilde{x}_m|$ so that an order of magnitude analysis on Equation (35)

is possible, making use of Equation (27) we obtain

$$V \approx B \left(\frac{K_w}{K_s - K_w} \right)^{\phi} r^{1/2} F^{1/2} \quad (38)$$

where B is a constant.

COMPARISONS WITH EXPERIMENTS

(6) McIeroy and DeLoach measured the speed of sound in saturated sand

water mixtures between frequencies 15 to 1500 KC and found the wave speed of the mixture to be independent of frequency and 1.19 times the speed of sound in water. Shumway⁽³⁾ obtained the same values for quartz sand in water at concentration $\phi = 0.64$. Inserting this value for ϕ and the values of $\frac{K}{K_s} = 15.0$ we obtain a ratio of sound speed in the mixture to sound speed in water of 1.47, some 42% greater than the experimental values.

An interesting comparison can be made by accepting the experimental values for C from the previously mentioned experiments of Shumway and comparing a multiplying factor for ϕ in Equation (39) below

$$C = \frac{C_0}{[1 - K\phi]^{1/2}} \quad (39)$$

we find $K = 0.33$ and then using Equation (39) to predict sound speeds in another quartz-water concentration. Using Shumway's⁽³⁾ data for sound speed of a mixture of continental shelf silt (quartz) and water at a concentration of $\phi = 0.38$ we obtain a value of $C = 4,900$ ft/sec as compared to the experimental value of 4,850 ft/sec which differ by only 1%.

LOW CONCENTRATIONS

The decrease in the wave speed for low concentration mixtures of solid particles and water can be explained using the scattering theory of the previous section by considering each scatterer to be rigid, i.e., $K_s \gg K_w$ so that Equation (26) becomes

$$g(a, w) = -\frac{3}{2} \frac{K_a^2}{K_w^3} \left(1 + \frac{2}{3} \cos \theta \right) \quad (40)$$

and assuming the scatterers occupy a configuration as that shown in

Figure 4.

Consider that a plan wave impinges on scatterer A, giving use to both a scattered isotropic pressure field (dashed line) and a scattered dipole pressure field (solid line). The amount of scattered pressure arriving at D from A can be determined from the relationship

$$P_s = P_0 \frac{e^{j(\omega t - kr)}}{r} g(a, \omega) \quad (41)$$

by setting $r = d$, $\theta = \pi$ in Equation (41) which yields

$$P_s \approx P_0 \frac{e^{j(\omega t - kd)}}{d} \frac{k_a^2}{3} \left(\frac{1}{2} \right) \quad (42)$$

which is also the scattered pressure at E due to the scatterer B. From

Figure 4 it can be seen that the contribution to the pressure at E caused by the scattered pressure fields of A and C are negligible since

$$1 + \frac{2}{3} \cos \theta \approx 0 \text{ for } \theta = \pm 135^\circ \text{ and the contribution to the pressure field}$$

at E due to multiple scattering from F or D is a second order effect and therefore negligible. Therefore, if one measures the scattered pressure

at points E, D, or F it would appear that points A, B, and C have scattering

coefficients of

$$g(a, \omega) = \frac{k_a^2}{3} \frac{1}{2} \quad (43)$$

Using the above expression the wave speed is then

$$C = \frac{C_0}{[1 + 0.5\phi]^{1/2}} \quad (44)$$

which indicates that as the volume concentration of the particles increases as

the average wave speed decreases. As a comparison, Equation (44) above

yields $\frac{C}{C_0} = 0.965$ for $\phi = 0.15$ whereas Urlick (1) found from experiments that

$$\frac{C}{C_0} = 0.973 \text{ for } \phi = 0.15 \text{ for kaolin and water mixtures.}$$

One can see from the above calculations that the scattering theory

does predict an increase in wave speed for high concentrations and a decrease in wave speed for low concentrations. One would expect that at

some intermediate concentrations that the wave speed would pass through a minimum value and that at some concentration beyond the concentration at

the minimum, the wave speed of the mixture would be the same as the speed

of sound in water alone. Urlick⁽¹⁾ observed this experimentally and found

the minimum wave speed $C = 0.97 C_0$ occurs at $\phi = 0.20$ and that $C = C_0$

at $\phi \approx 0.35$. One would expect this phenomenon of minimum wave speed to occur

at a concentration at which multiple scattering begins to come into effect.

This is usually associated with particle spacings of the order of one particle

radius a . It is interesting to note that if the particle configuration

can be regarded as a cubic packing arrangement (Figure 5) that for such an

arrangement

$$\phi = \frac{4}{3} \pi \frac{a^3}{d^3} \quad (45)$$

and that the separation distance between particles becomes less than a

particle radius for $\phi \approx 0.20$ the value measured by Urlick⁽¹⁾ as the concentration at minimum wave speed.

CONCLUSIONS

Acoustic wave transmission in water containing either a high or low

concentration of solid particulate matter in suspension has been discussed

mathematically using a multiple and single scattering model, respectively.

The results have been compared to existing experimental data. The comparisons

indicate that the mathematical model does predict some of the observed

physical phenomenon, namely the fact that the average wave speed in such a

mixture of solids and liquids does not depend significantly on frequency for wavelengths long with respect to particle size. Also, the model predicts a decrease in wave speed for an increase in particle concentration for low concentrations, and an increase in wave speed for an increase in particle concentration for high concentrations. Furthermore, this mathematical model is more satisfying, from a physical point of view, than some of the other theories in which equivalent density and compressibility are determined for the mixture according to the proportions of the constituents. Realistically speaking, the presence of the particles is certain to cause a complex scattering and absorption of acoustic energy. This attempt at modeling such a complex situation can only be regarded as a first approximation. Many physical assumptions were made on physical intuition alone, in order to reduce the mathematical expressions to a tractable form. Perhaps, nothing has been gained by using this model over one of the simpler models as far as predicting average wave speed vs. particle concentration is concerned. However, using this scattering theory one does learn something about the relation of the incoherence of the pressure field [Equation (38)] to the particle concentration and frequency of the incident-wave. This incoherence cannot be obtained from the simpler theory of Urlick⁽¹⁾. And since the resolution of a holographic image of an object depends on the incoherence of the pressure field, the incoherence must be taken into account in determining or improving resolution. It is quite possible that signal processing of the electrical signal analog of the pressure field, generated by the electroacoustic transducer, could be performed to improve the quality and resolution of the hologram, by incorporating the information from Equation (38) into the processor.

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LIST OF FIGURES

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FIGURE 2

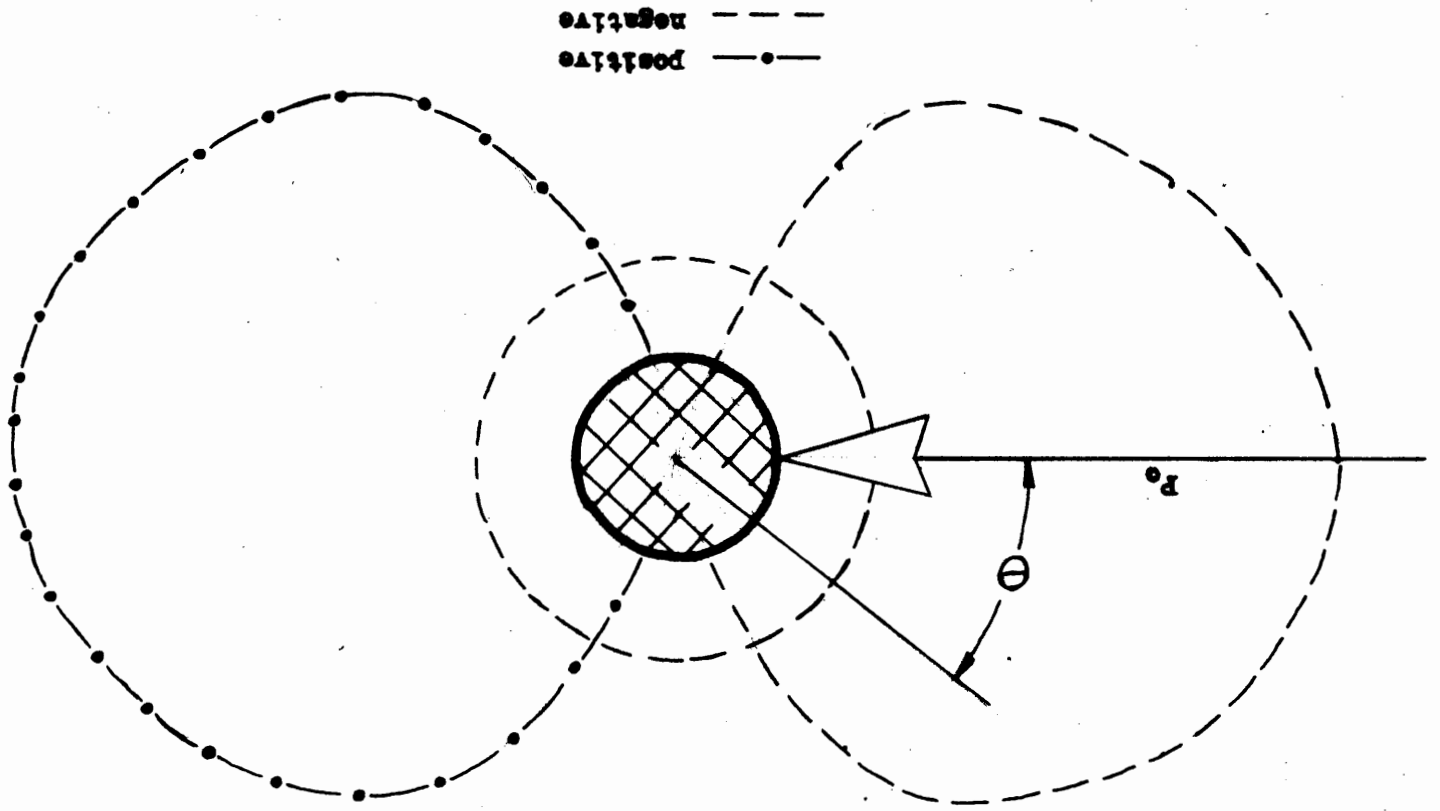


FIGURE 1

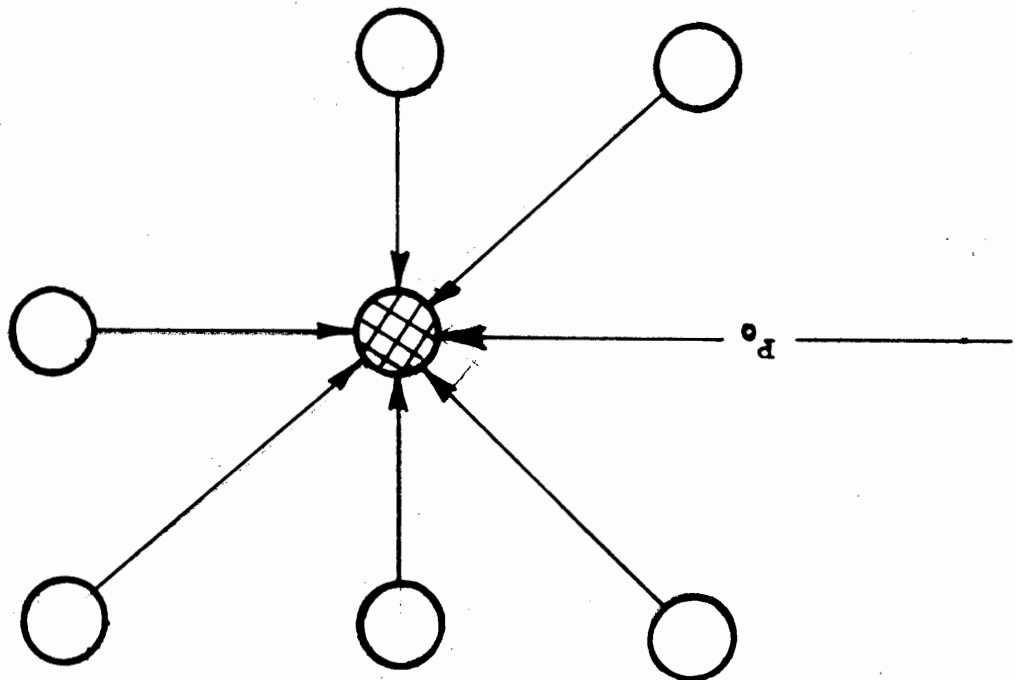


FIGURE 4

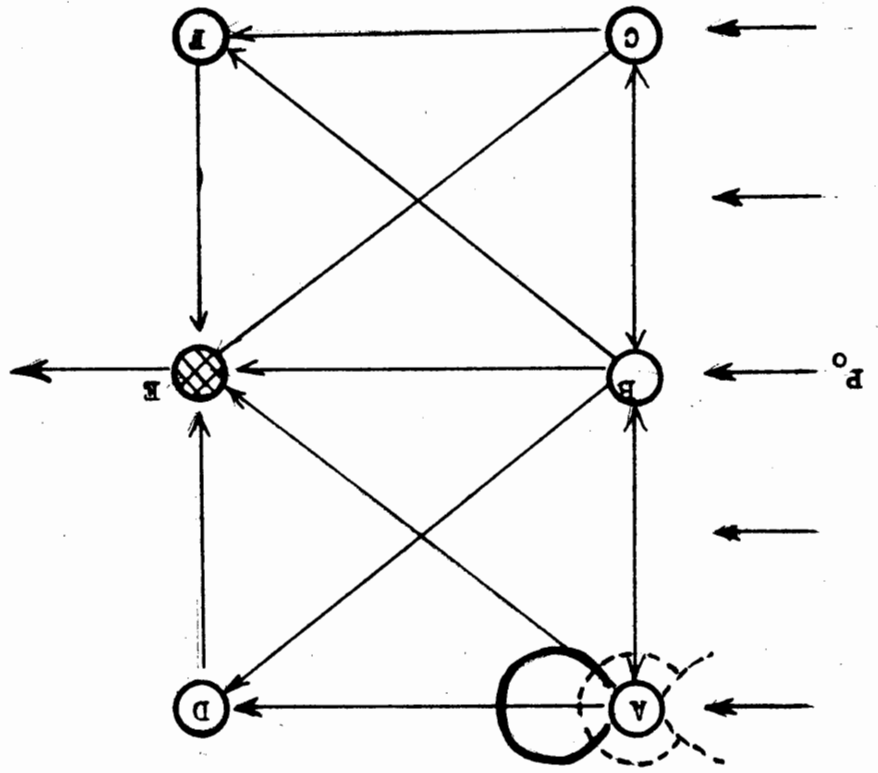


FIGURE 3

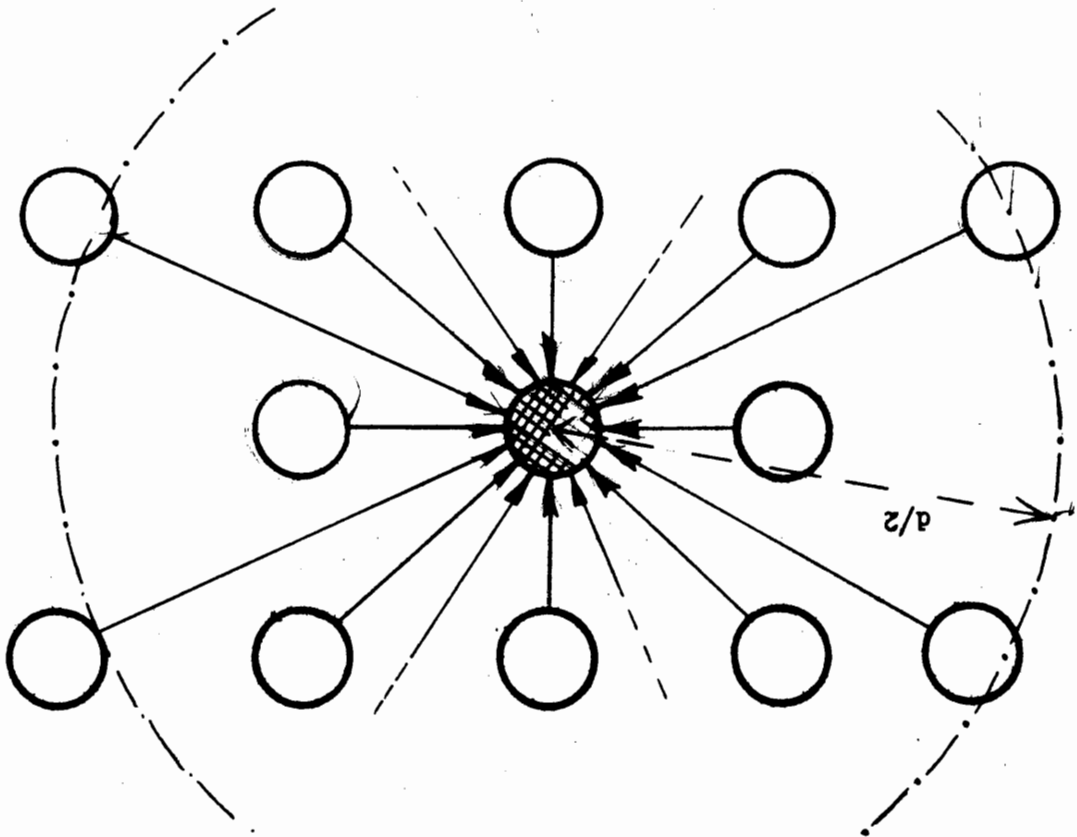
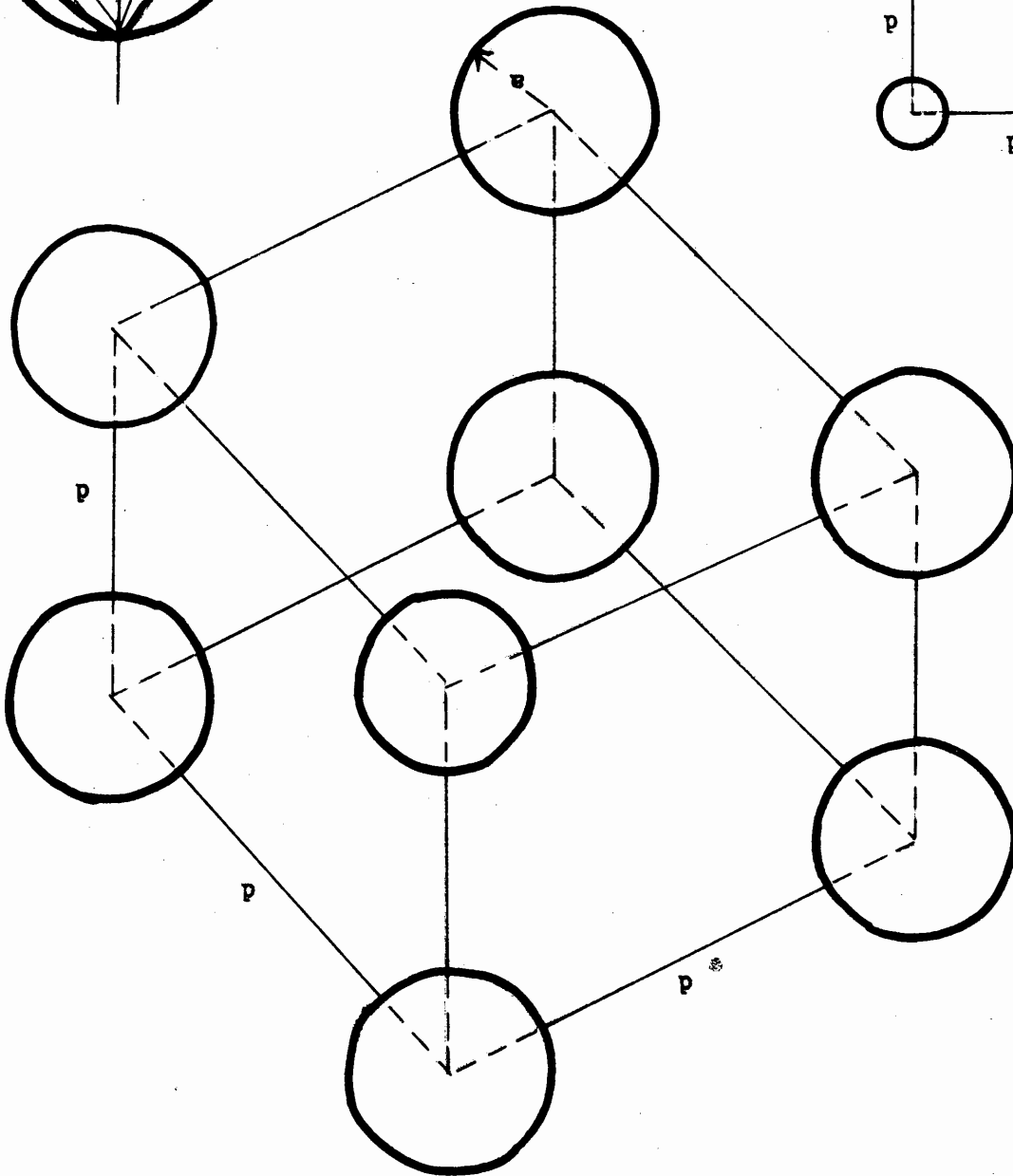
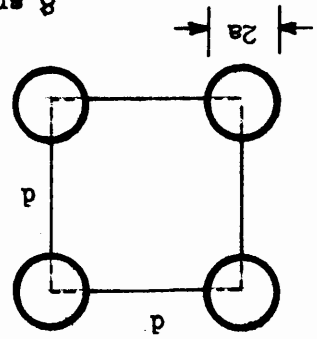
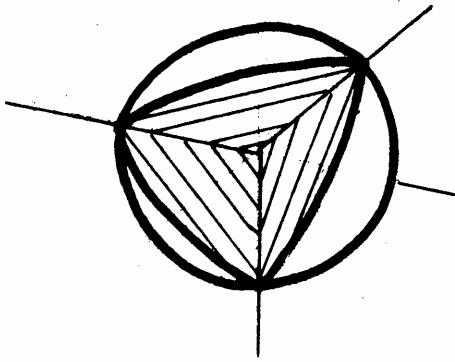


FIGURE 5

$$\phi = \frac{\text{sphere volume}}{\text{total volume}} = \frac{4/3 \pi a^3}{8 \text{ spheres} \times 1/8 \text{ sphere volume} = 1 \text{ sphere}}$$



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13. ABSTRACT

The effects of a high concentration of particulate matter held in suspension in water on the propagation of an acoustic wave in water is discussed. Expressions for the average wave speed, average pressure, and incoherence of the pressure field are obtained for an incident harmonic pressure wave on a large number of discrete, suspended scatterers in water. The wavelength of the wave is assumed to be large compared to a scatterer diameter. Results are compared with experiments of acoustic wave propagation in sand-water mixtures.