Measurement-Error Model Multipole Variance Estimation

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Statistical models of cold multipole values can include information from many sources believed to affect the calculated multipole values. Among the ancillary variates that may affect multipoles are coil sizes, collar dimensions, and other physical properties of the magnets. Preliminary examination of warm-cold relationships also indicate a potentially strong association between warm and cold multipoles, by position along the magnet. Least squares regression estimation is the preferred estimation method when predictors used in the regression model are measured with little or no error, but not when one or more of the predictors are measured with substantive error. Not only are the warm multipole measurements subject to a variety of measurement errors, their variation at a fixed position exceeds that of the cold multipole values at the same position on a magnet.

Key to the satisfactory modeling of the cold multipoles when warm multipole values are subject to measurement errors is the estimation of the variability of the measurement errors for the cold and warm values. Such estimation is possible using data available on these multipoles for multiple rotations of the mole at each of the 24 positions along the lengths of prototype dipole magnets DCA 311, 314, 315, 318, and 320 for both cold and warm multipoles a1-a4, b1, b2, b4, and b6. Depending on the magnet and the multipole, there are 4, 5, or 10 rotations at each position. Two sets of analyses were performed on these data and are summarized in this report: (1) a diagnostic examination of the statistical control (stability) of the measurement variation across positions and magnets, and (2) estimation of the standard deviations of the multipole measurement variation after screening for outliers.

1. Statistical Control

Multipole measurement error variability is said to be in statistical control if variability estimates fluctuate within well defined limits about a center line across positions on a magnet. Both the limits and the center line can differ from multipole to multipole and from magnet to magnet. If measurement error variability changes substantially by position, variability estimates would be needed for statistical modeling purposes at each position along the length of the magnet. If this variability is reasonably stable, however, then the modeling of the multipole values is considerably less complicated.

The classical approach to an investigation of the stability of measurement error variation is to construct control charts for sample standard deviations. For the rotations at a position, the sample standard deviation, s, is the square root of the sample variance and is calculated from the formula:

$$s = \left\{ \sum_{i=1}^{n} (m_i - \overline{m})^2 / (n-1) \right\}^{1/2} ,$$

where m_i is the multipole coefficient for the ith of n rotations at a position on a magnet and $\overline{m}$ is the average of these n multipole values. Once the standard deviations are calculated for each position on a magnet, they are plotted versus the position and suitable control limits are superimposed on the plots.

Figures A.1 to A.5 in the appendix display the control charts for the four skew multipoles a_1 to a_4 from magnets DCA 311, 314, 315, 318, and 320. Figures B.1 to B.5 display the control charts for the four normal multipoles b_1, b_2, b_4, and b_6 from the same magnets. Only warm multipole values are displayed in these figures because none of the cold multipole control charts had any points that were out of control.

The horizontal lines in the figures identify the center lines and the upper limits for the estimated standard deviations. The center line (CL) on a plot is the average of the standard deviations across the 24 positions, $CL = \overline{s}$. It indicates the typical variability for a multipole on a magnet. The upper control limit (UCL) on a plot is a multiple of the average standard deviation, $UCL = B_4\overline{s}$, where B_4 is determined by the known natural variation of standard deviations and is based in part on the number n of rotations from which the standard deviations are calculated and on the assumption that the multipole values follow a normal probability distribution. Typical values for this quantity are $B_4 =$ 2.27 for n = 4, 2.09 for n = 5, and 1.72 for n = 10 (e.g., Grant, E. L. and Levenworth, R. S. (1988), *Statistical Quality Control*, McGraw-Hill Publishing Co.) Some of the figures show two upper control limits. These shifts in the UCL occur because the number of rotations from which s is calculated changed. For example, in Figure A.1 the larger UCL that occurs at 4.57 m indicates that the number of rotations from which the multipole standard deviation estimates were calculated decreased from five rotations to four rotations at position 4.57 m; B_4 correspondingly shifted from 2.09 to 2.27.

Overall, all ten of the control charts indicate that, with a small number of exceptions, the measurement variation of multipole values exhibits good statistical control; i.e., the standard deviations fluctuate around the center line and are well within the upper control limit. Because of this stability, a single standard deviation can be used to estimate the multipole measurement variability for a magnet. Thus, the control charts do not suggest the need to estimate separate standard deviations for each position.

The few sample standard deviations that exceed the UCL indicate that the measurement variability at those positions exceeds those of the other positions by an amount that cannot be reasonably attributed to chance variation of a stable system. There are several standard deviations plotted in Figures A.1 to A.5 and B.1 to B.5 that clearly exceed the upper control limits.

2. Outlier Screening

In addition to the graphical indication of out-of-control standard deviations, there are statistical tests that can be used to quantify the degree to which multipole values for specific rotations are discrepant when compared to the multipoles for the other rotations at the same position. Suppose for a particular position, the multipole value for the ith rotation appears to be substantially different from the multipoles for the other rotations. Suppose further that there are a total of k suspect multipole values for a particular skew or normal multipole. A test for statistical significance of the suspect multipole value can be based on the following t statistic:

$$t_i = (m_i - \overline{m}_{(i)}) / [s_{(k)} \{n / (n-1)\}^{1/2}],$$

where m_i is the suspect multipole for the ith rotation at a specified position which has n rotations, $\overline{m}_{(i)}$ is the average of the n - 1 multipoles at the position, excluding the ith one, and $s_{(k)}$ is the standard deviation of the multipole values for all rotations and at all positions with the exclusion of the k suspect multipoles. The standard deviation estimate $s_{(k)}$ is calculated from a weighted (by the degrees of freedom) average of the sample variances at each position, with all the suspect outliers eliminated. The pooled standard deviations are calculated for each multipole and each magnet. The consistency of the estimates across magnets is discussed in Section 3.

In general, a t value that is larger in absolute value than 3 should be regarded as identifying a discrepant multipole value. Values of the t statistic larger in magnitude than 5 provide very strong statistical evidence that a multipole value is different from the rest of the values at a position.

Tables 1 and 2 list all multipoles that have t values larger than 2.5 in magnitude. Of particular note is the fact that for most of the magnets, several multipole values at the same rotation have large t statistics. This suggests that erratic single rotational effects more than position effects may be the cause of the outliers. For this reason, for each outlier listed in Tables 1 and 2, the entire rotation was deleted and the control charts recomputed. Figures A.6 to A.9 and B.6 to B.9 (recall that DCA 318 had no outliers) are the resulting control charts. These control charts indicate that all the standard deviations are now in statistical control.

Table 1. Statistically Significant Warm Skew Multipole Values.

Magnet	Rotation	Multipole	Position	t-Statistic
311	2	a_1	0.91	11.91
	2	a_2	0.91	23.48
	2	a_3	0.91	34.27
	2	a_4	0.91	26.83
	2	a_4	2.74	7.79
314	3	a_1	-5.79	-2.92
	3	a_2	-5.79	-4.30
	3	a_3	-5.79	-5.87
	3	a_4	-5.79	8.89
	1	a_1	-4.57	7.02
	1	a_3	-4.57	-4.76
	1	a_4	-4.57	13.87
	1	a_2	4.57	-3.25
315	1	a_3	1.52	-8.50
	1	a_4	1.52	5.35
320	2	a_2	4.57	-7.95
	2	a_3	4.57	6.47

Table 2. Statistically Significant Warm Normal Multipole Values.

Magnet	Rotation	Multipole	Position	t-Statistic
311	1	b_6	-4.57	3.92
	2	b_4	2.74	-6.12
	2	b_6	2.74	-28.55
	1	b_2	3.35	-3.26
	1	b_6	3.35	-21.91
	2	b_6	4.57	-14.76
	1	b_6	7.01	-3.81
314	3	b_1	-5.79	3.91
	3	b_2	-5.79	-7.23
	3	b_4	-5.79	7.25
	3	b_6	-5.79	16.86
	1	b_4	-4.57	-5.48
	1	b_6	-4.57	-5.25
	3	b_6	-0.91	3.82
315	1	b_1	-4.57	3.00
	1	b_6	-2.74	-6.85
	1	b_2	1.52	-10.16
	1	b_4	1.52	4.28
	1	b_6	1.52	-33.64
	2	b_1	4.57	-14.22
	2	b_2	4.57	7.18
	2	b_4	4.57	24.86
	2	b_6	4.57	39.79

3. Estimated Standard Deviations

Removal of rotational data corresponding to the outliers listed in Tables 1 and 2 produces estimated standard deviations that exhibit statistical control. From these estimated standard deviations, cold and warm magnet standard deviations can be estimated for each multipole. These estimates are the square roots of weighted averages of the sample variances at each position. The weight used for a position equals the degrees of freedom for the position, n - k - 1, where n is the number of rotations at the position and k is the number of outliers (0 or 1) removed at that position. Tables 3 and 4 display the estimated standard deviations for each magnet and each multipole. All magnets for which rotational information is available are included in the tables, not just those magnets for which rotations are available on both cold and warm multipoles.

An important question that arises from an examination of Tables 3 and 4 is whether the standard deviations for each magnet are sufficiently similar that a single standard deviation can be used to represent the variability of a multipole. A cursory examination of the standard deviations in the tables suggests considerable variability across magnets. For the cold multipoles, DCA 318 consistently has the smallest standard deviations and DCA 316 has the largest. For the warm multipoles, DCA 318 again consistently has the smallest standard deviations and DCA 311 has the largest (note that there are no standard deviation estimates for warm multipoles on DCA 316). In a future report, these standard deviation estimates will be examined in greater detail and a determination of whether a single standard deviation estimate can be used for all the magnets will be addressed.

Table 3. Estimated Cold Magnet Measurement Error Standard Deviations.

Multi-pole	DCA 311	DCA 314	DCA 315	DCA 316	DCA 317	DCA 318	DCA 319	DCA 320
a_1	.00534	.00724	.00612	.01143	.00537	.00316	.00570	.00845
a_2	.00179	.00235	.00198	.00275	.00179	.00114	.00203	.00244
a_3	.00202	.00233	.00211	.00356	.00229	.00106	.00225	.00293
a_4	.00124	.00149	.00142	.00200	.00135	.00063	.00171	.00123
b_1	.00471	.00635	.00543	.01017	.00516	.00234	.00532	.00747
b_2	.00176	.00213	.00211	.00238	.00196	.00109	.00211	.00133
b_4	.00124	.00154	.00148	.00153	.00143	.00057	.00152	.00117
b_6	.00050	.00059	.00049	.00079	.00048	.00022	.00053	.00052

Table 4. Estimated Warm Magnet Measurement Error Standard Deviations.

Multipole	DCA 311	DCA 314	DCA 315	DCA 318	DCA 320
a_1	0.4494	0.4739	0.4225	0.2869	0.3436
a_2	0.1330	0.1006	0.0833	0.0719	0.0782
a_3	0.1470	0.0974	0.0681	0.0386	0.0768
a_4	0.0974	0.0539	0.0426	0.0202	0.0412
b_1	0.4159	0.3214	0.2432	0.2296	0.2340
b_2	0.1150	0.0975	0.0802	0.0686	0.0737
b_4	0.0741	0.0580	0.0433	0.0231	0.0397
b_6	0.0294	0.0279	0.0153	0.0082	0.0154

Appendix

Multipole Standard Deviation Control Charts

Figure A.1 Control Charts for Warm Multipole Standard Deviations, Magnet DCA311

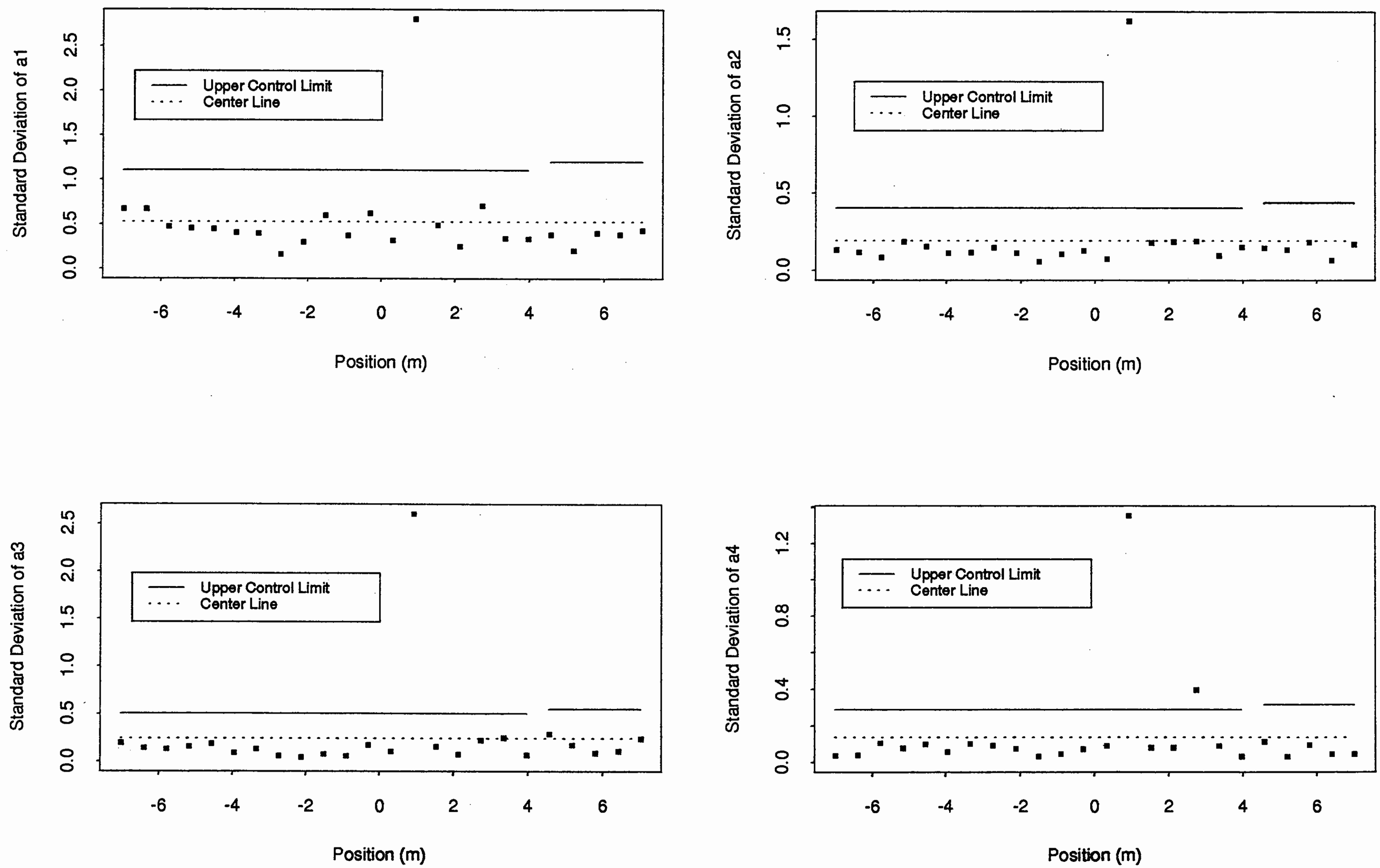


Figure A.2 Control Charts for Warm Multipole Standard Deviations, Magnet DCA314

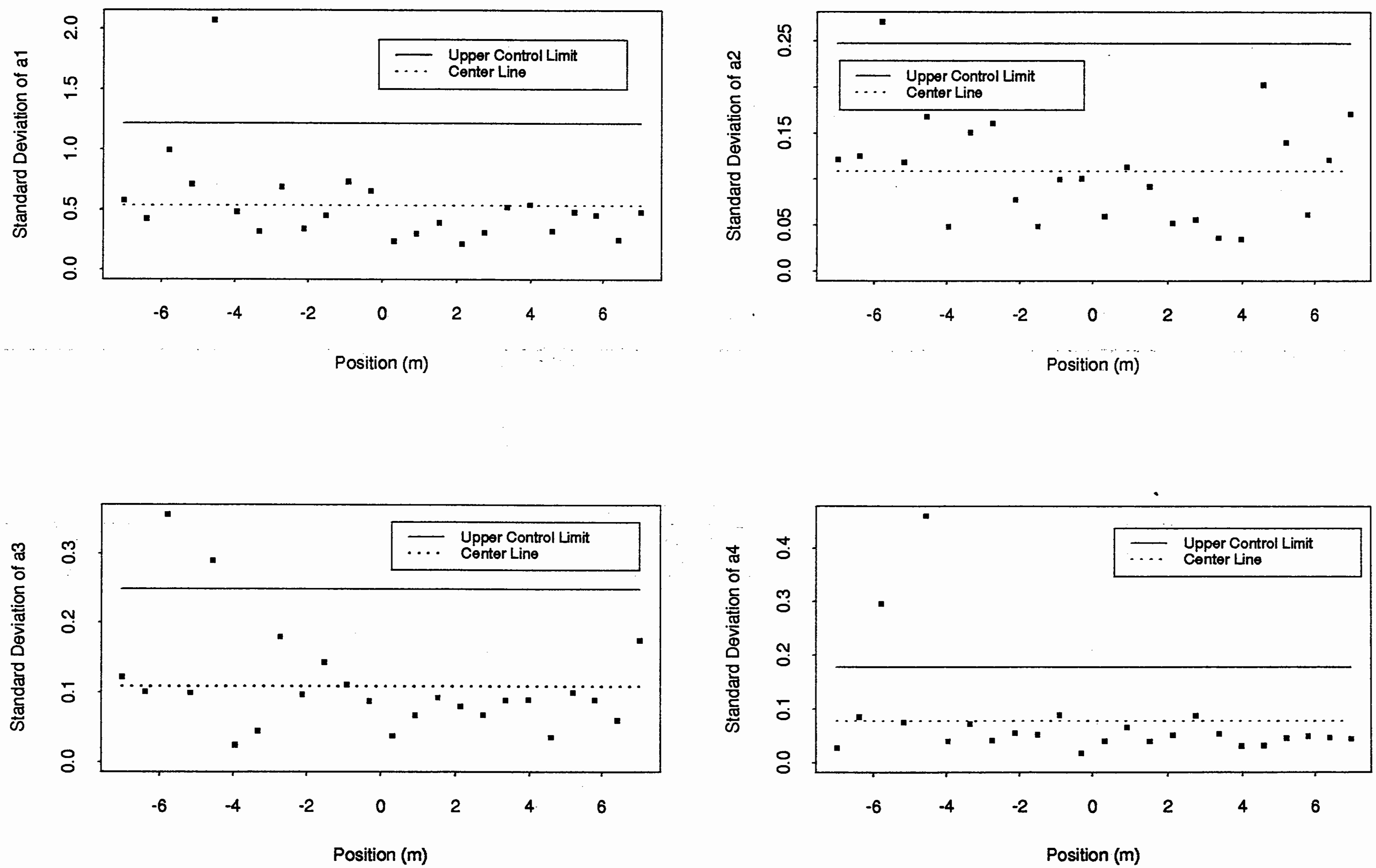


Figure A.3 Control Charts for Warm Multipole Standard Deviations, Magnet DCA315

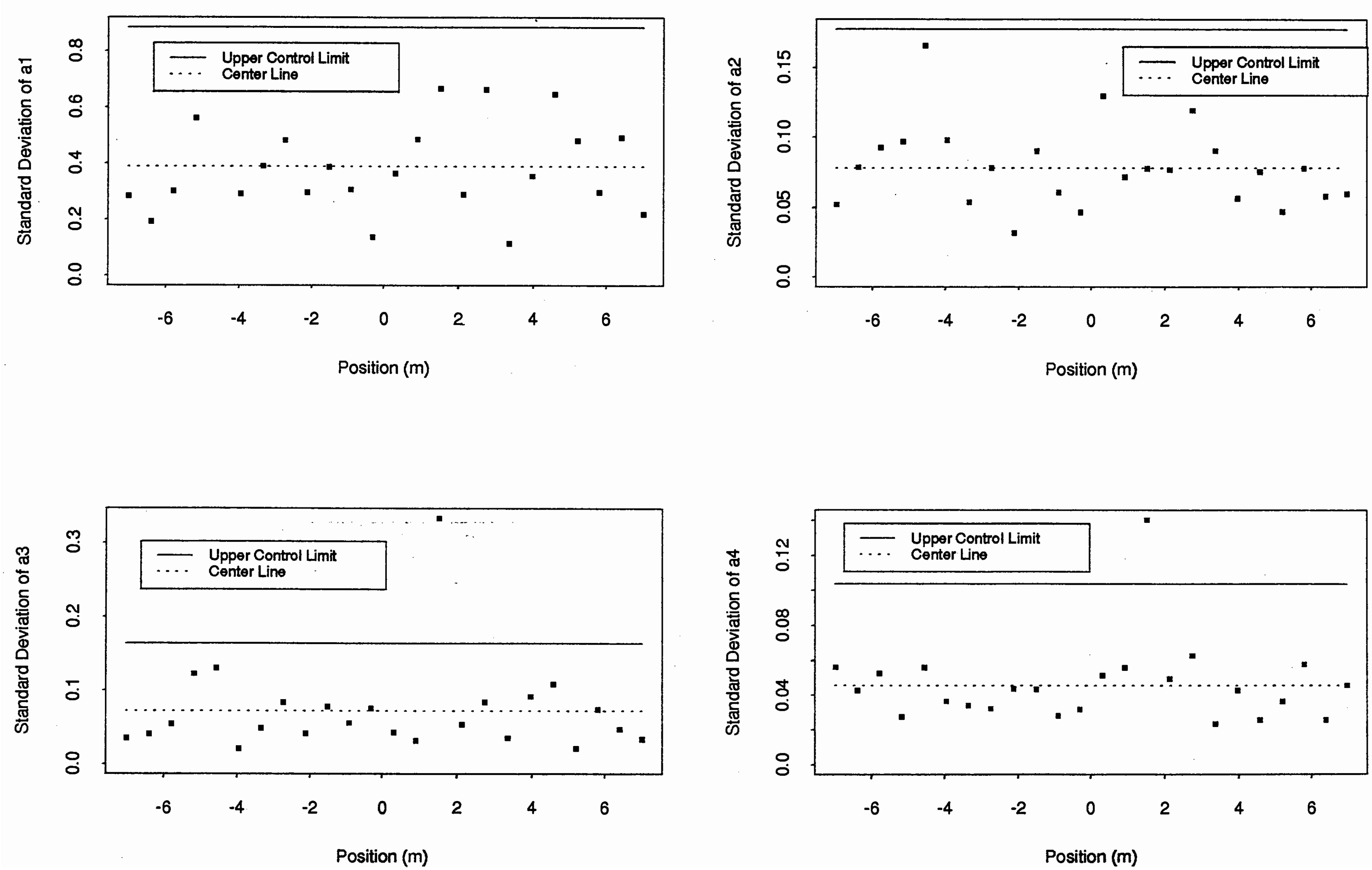


Figure A.4 Control Charts for Warm Multipole Standard Deviations, Magnet DCA318

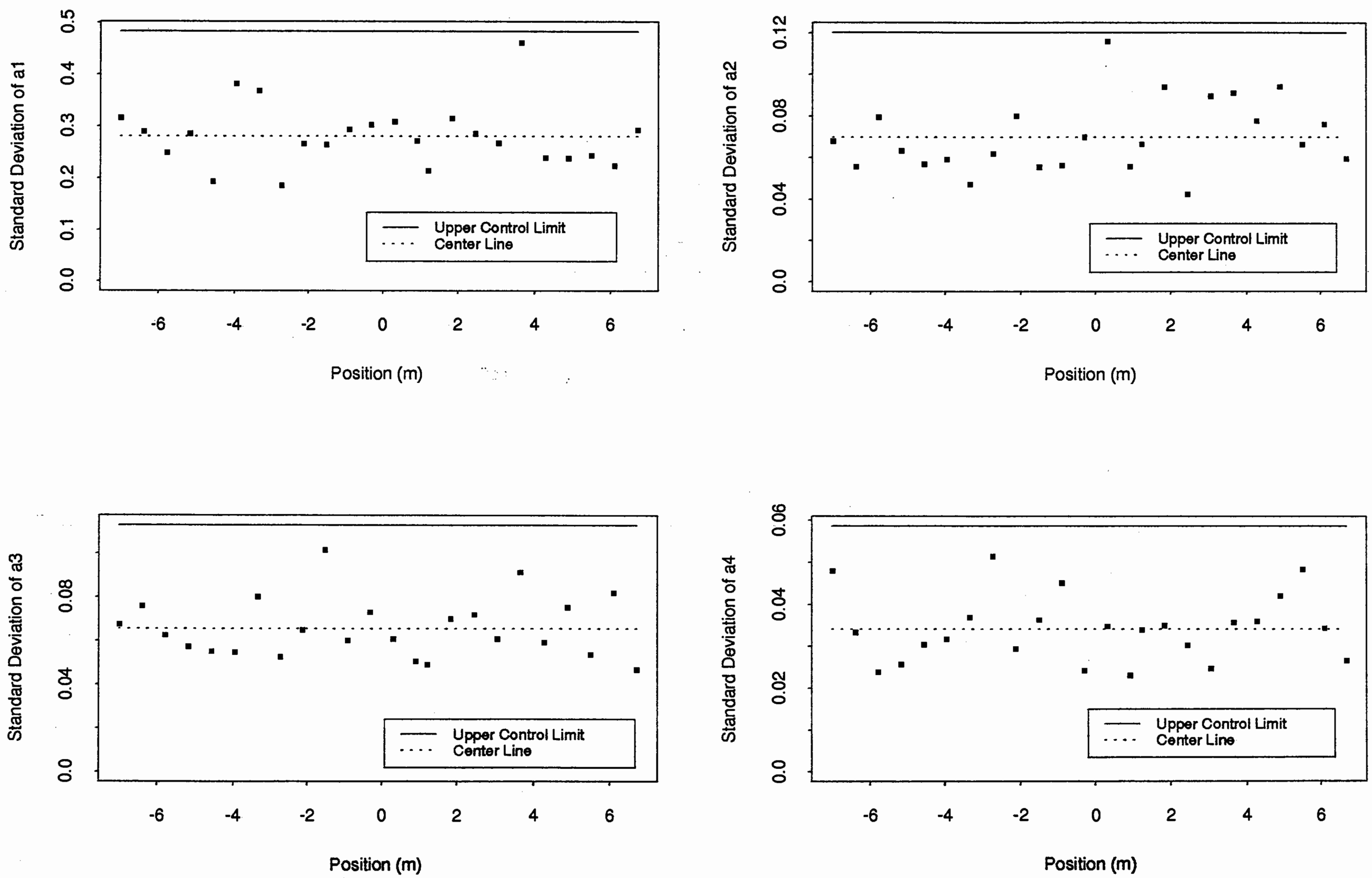
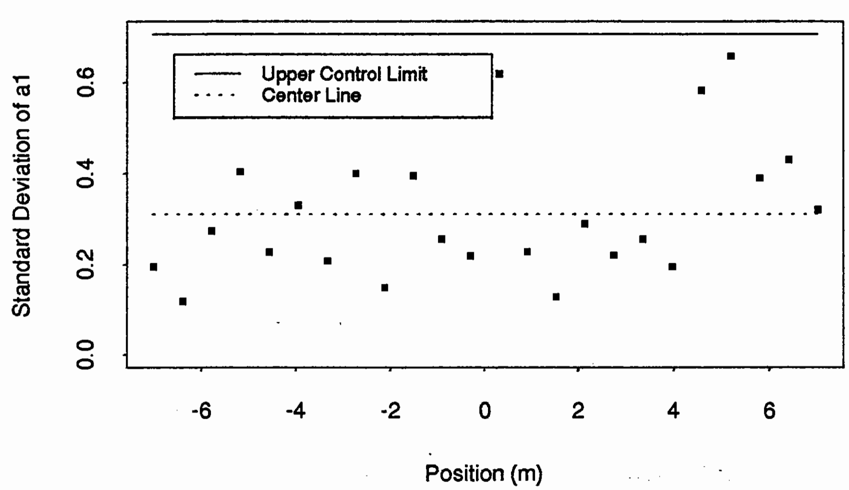
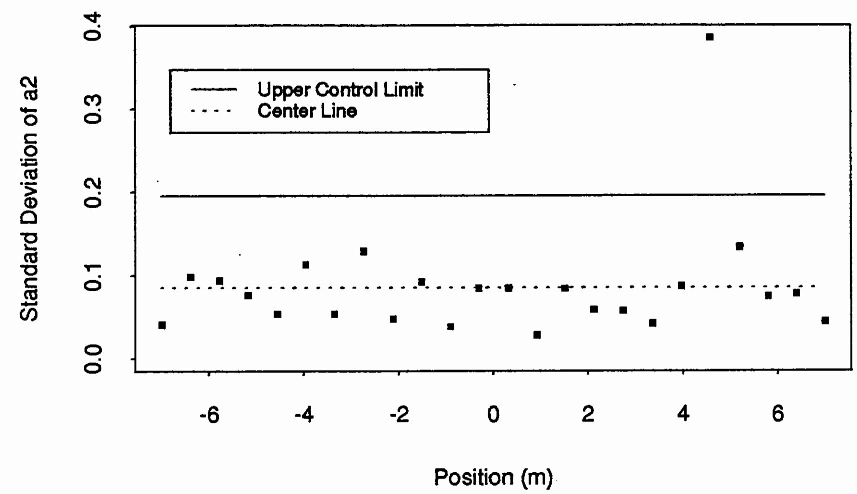
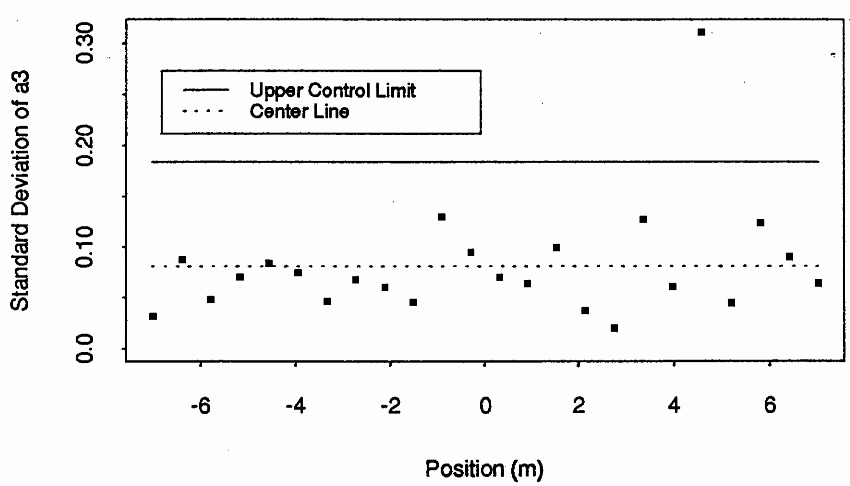


Figure A.5 Control Charts for Warm Multipole Standard Deviations, Magnet DCA320







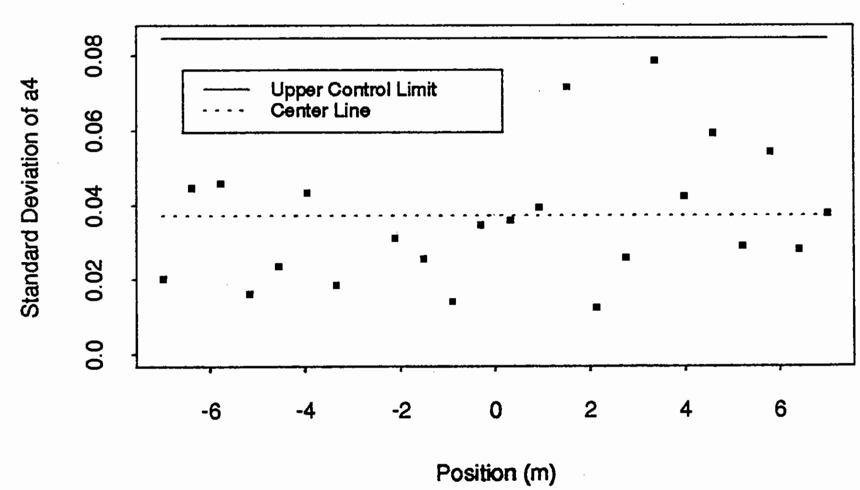


Figure A.6 Control Charts for Warm Multipole Standard Deviations
After Deleting Outliers, Magnet DCA311

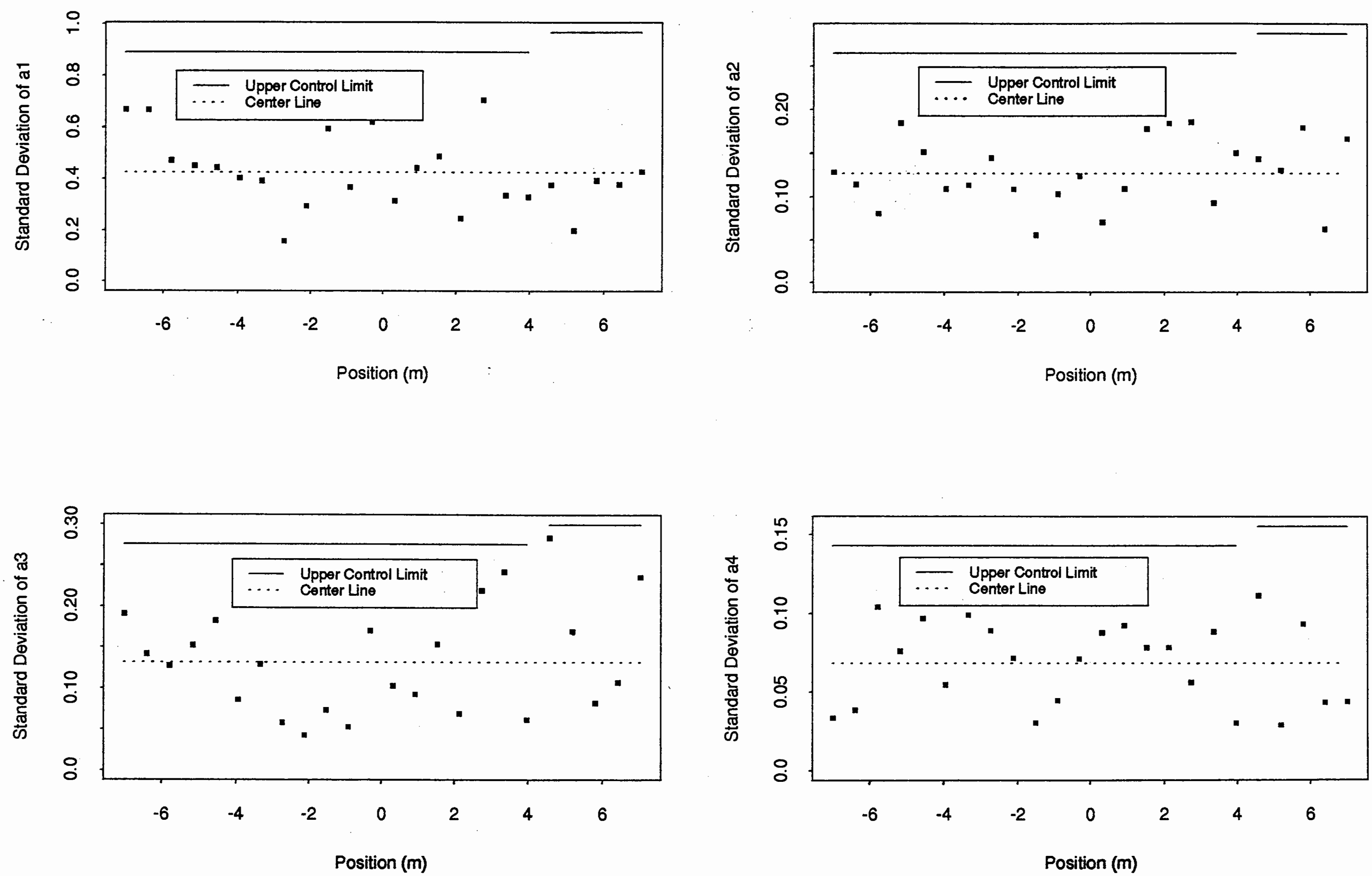
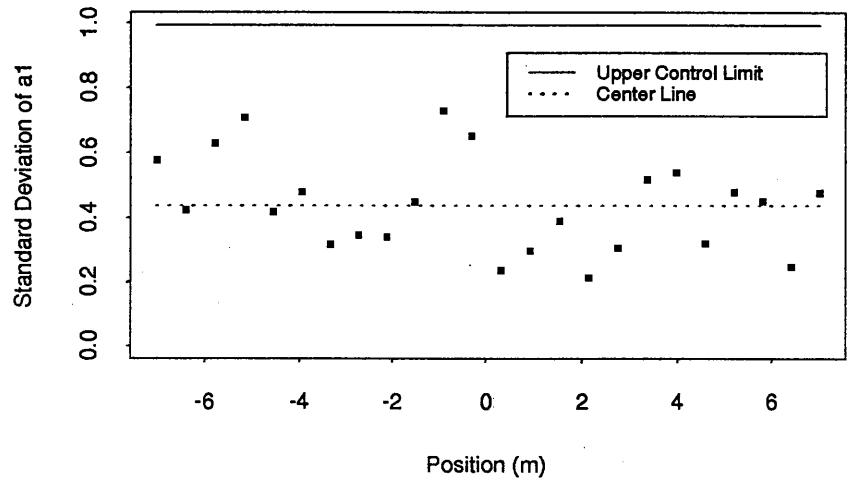
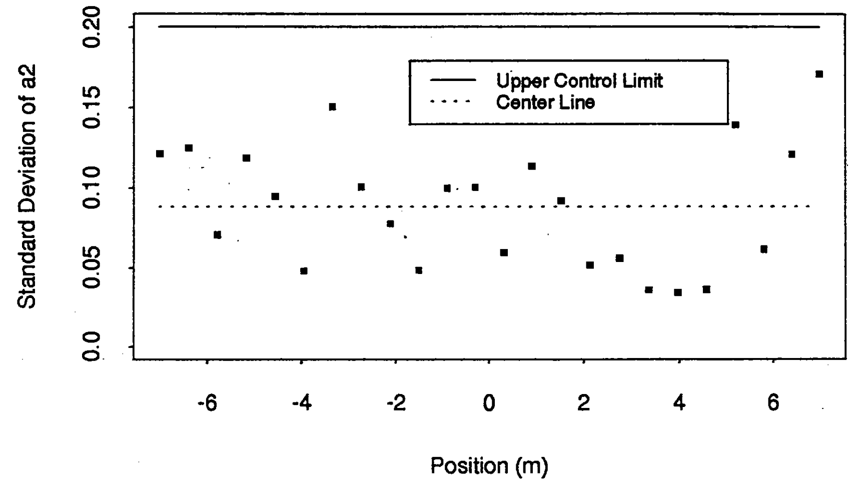
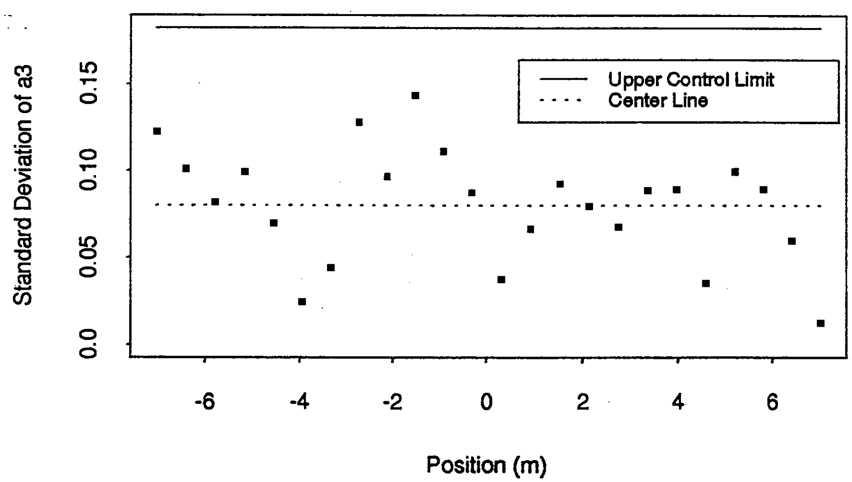


Figure A.7 Control Charts for Warm Multipole Standard Deviations After Deleting Outliers, Magnet DCA314







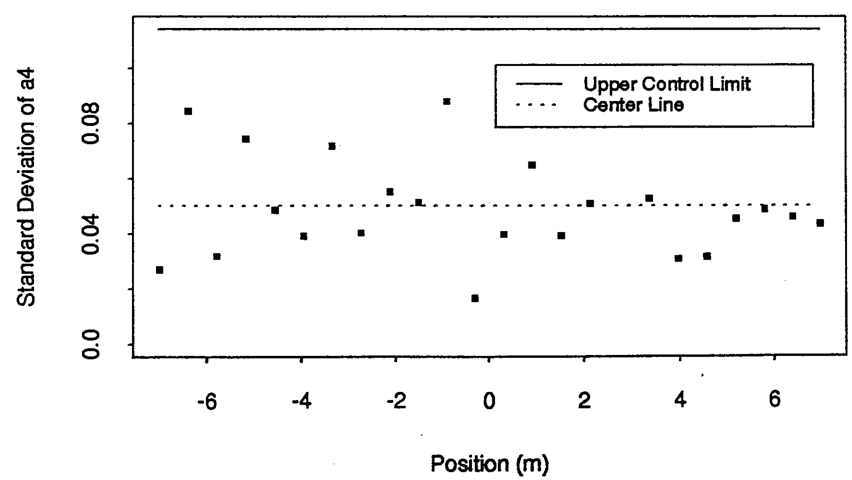


Figure A.8 Control Charts for Warm Multipole Standard Deviations After Deleting Outliers, Magnet DCA315

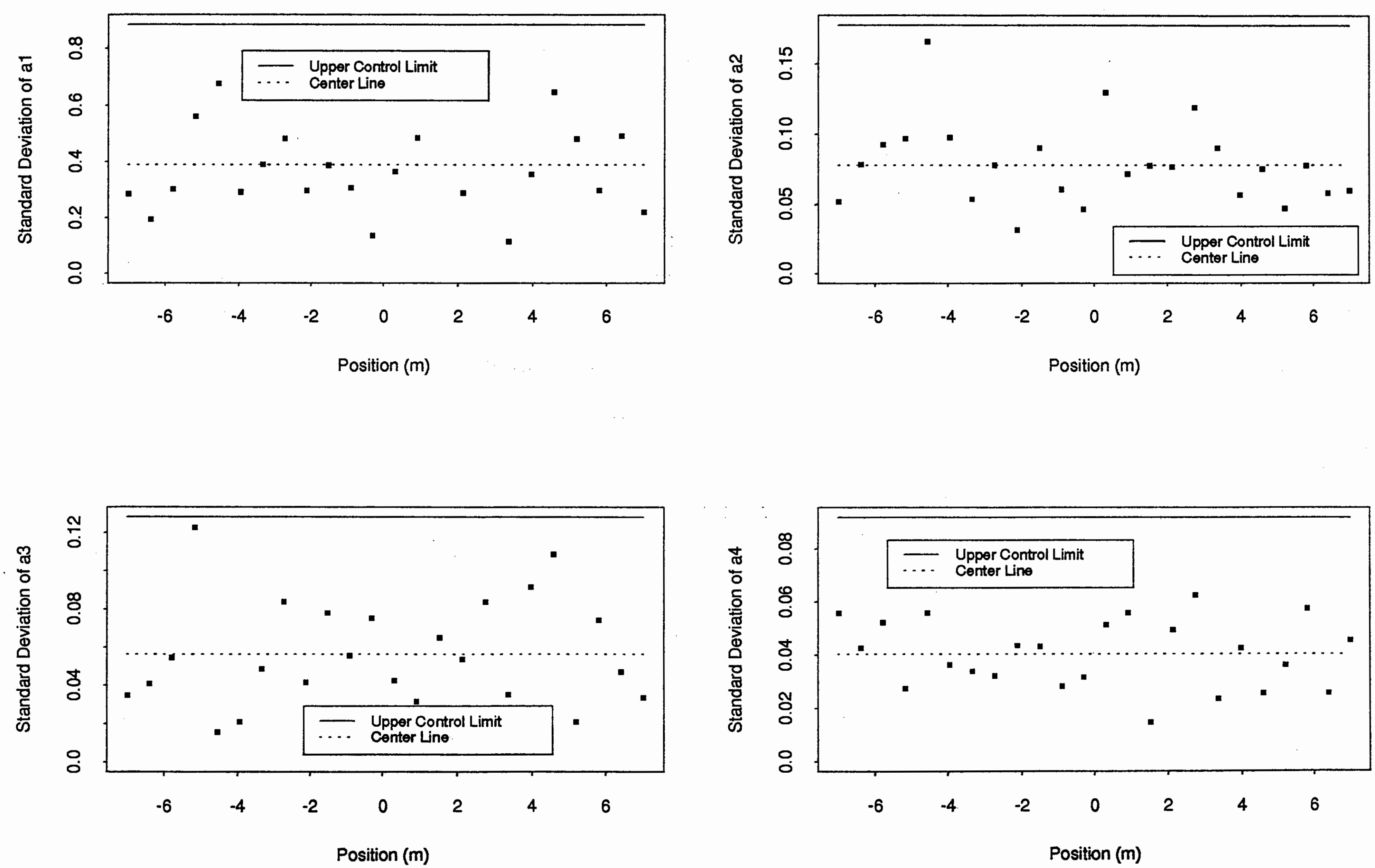


Figure A.9 Control Charts for Warm Multipole Standard Deviations
After Deleting Outliers, Magnet DCA320

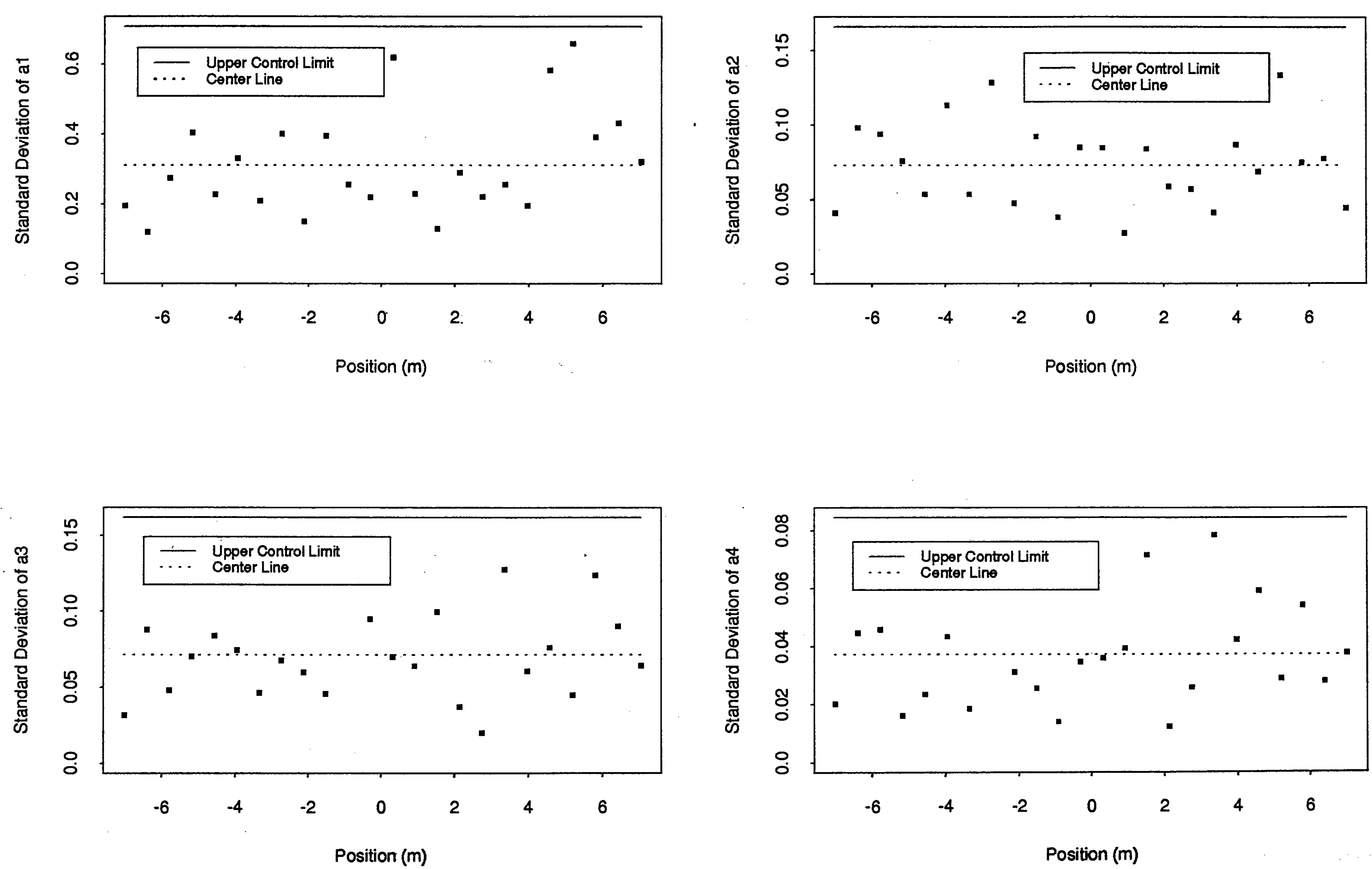


Figure B.1 Control Charts for Warm Multipole Standard Deviations, Magnet DCA311

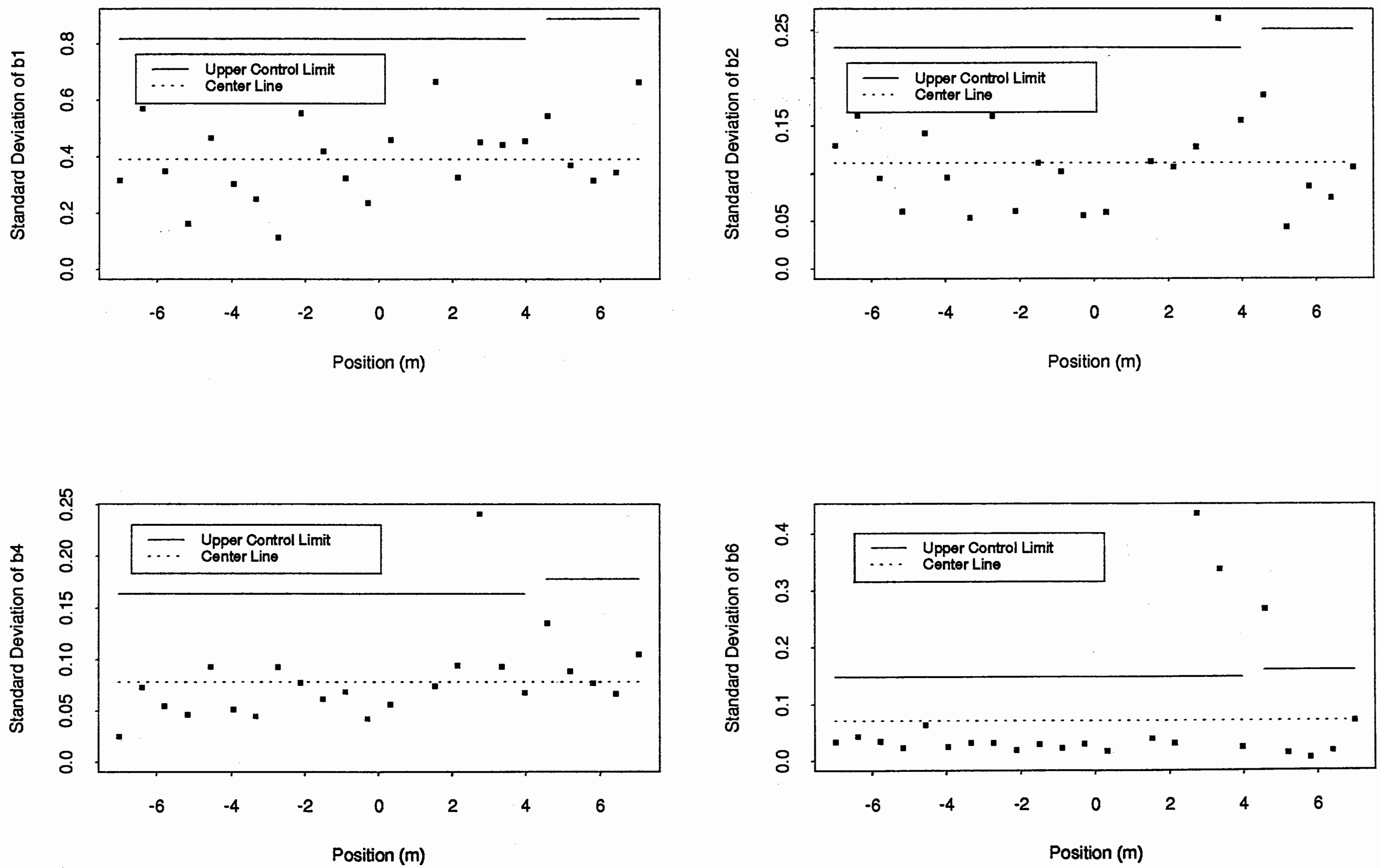


Figure B.2 Control Charts for Warm Multipole Standard Deviations, Magnet DCA314

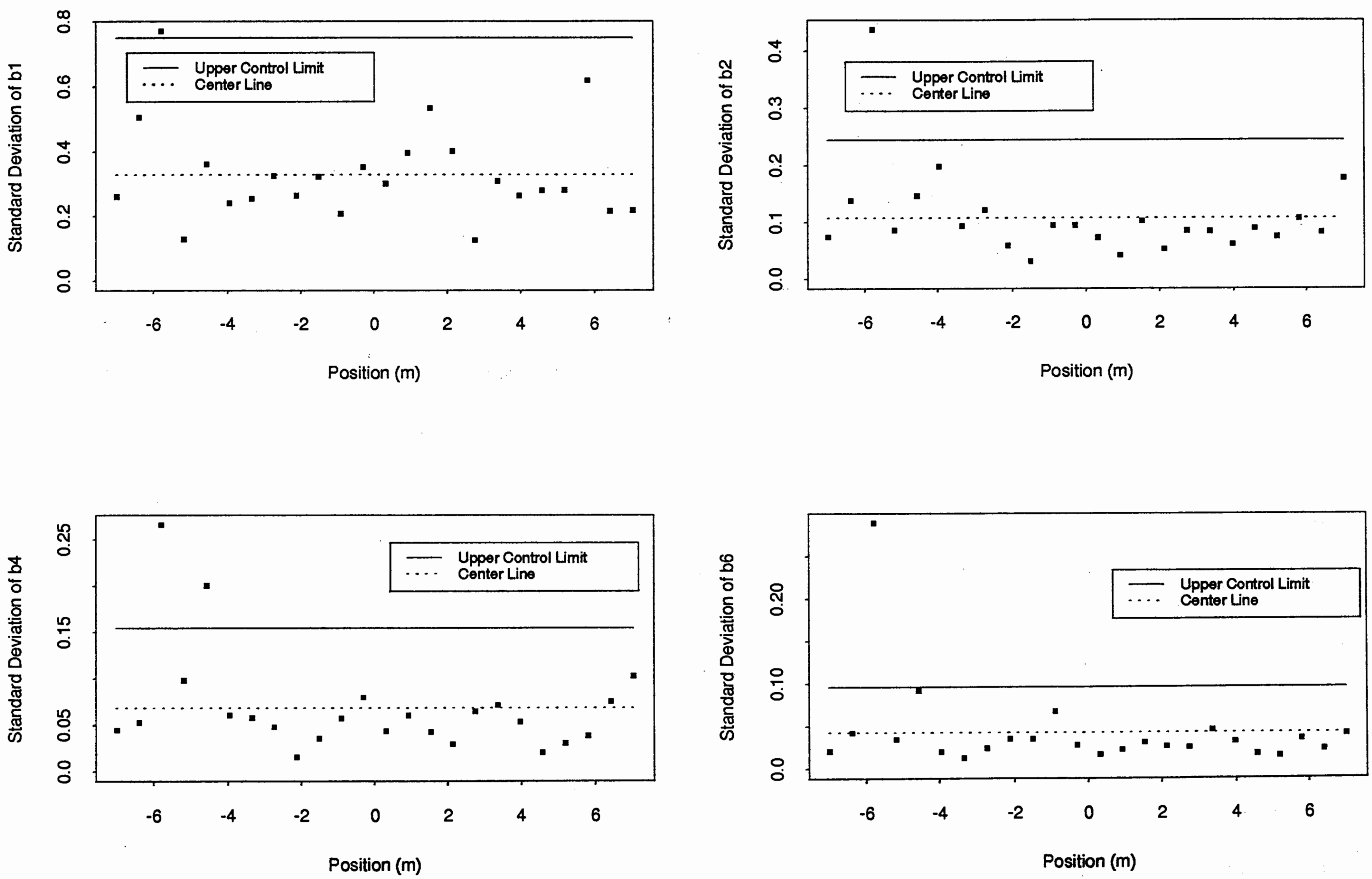


Figure B.3 Control Charts for Warm Multipole Standard Deviations, Magnet DCA315

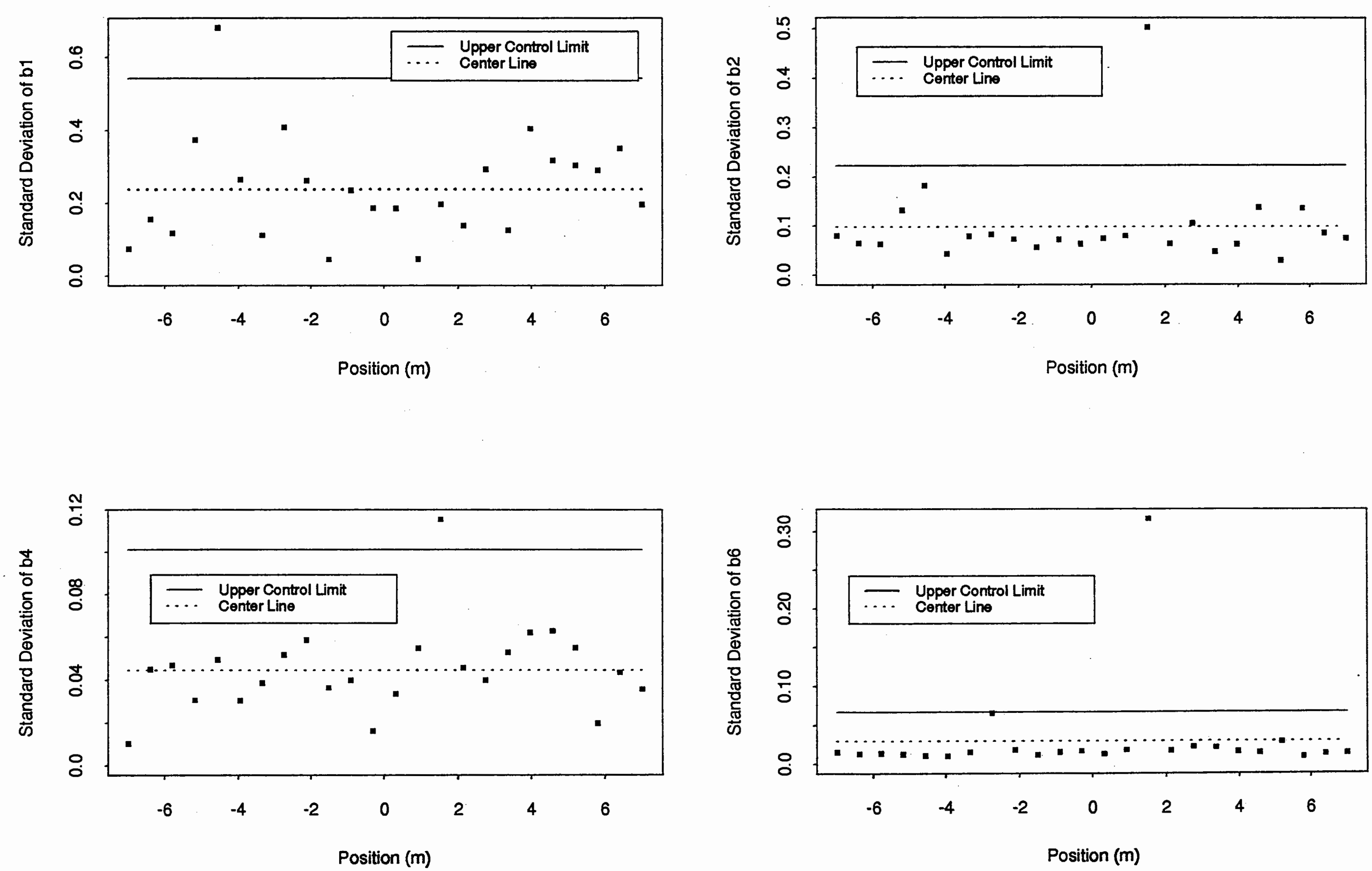


Figure B.4 Control Charts for Warm Multipole Standard Deviations, Magnet DCA318

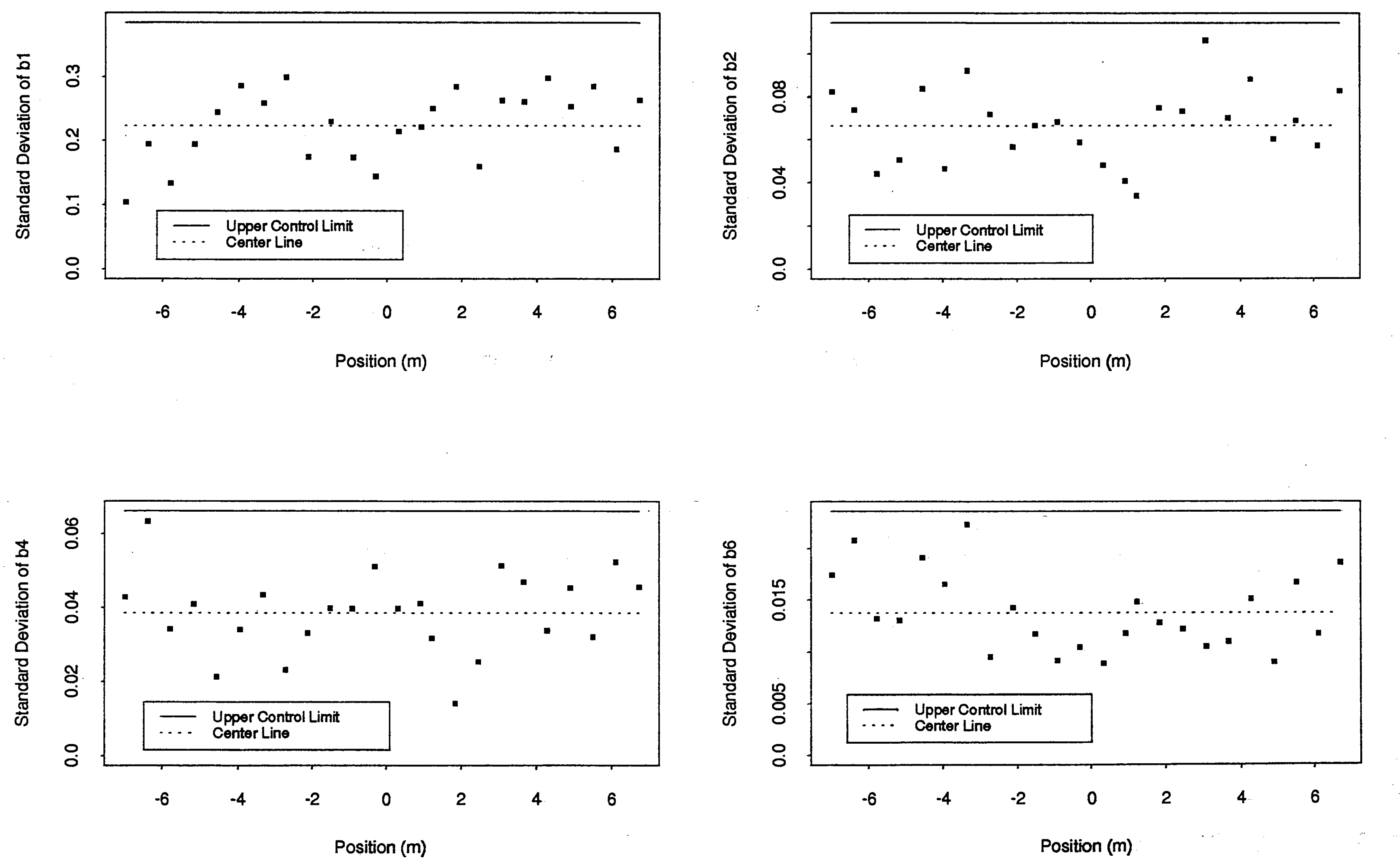
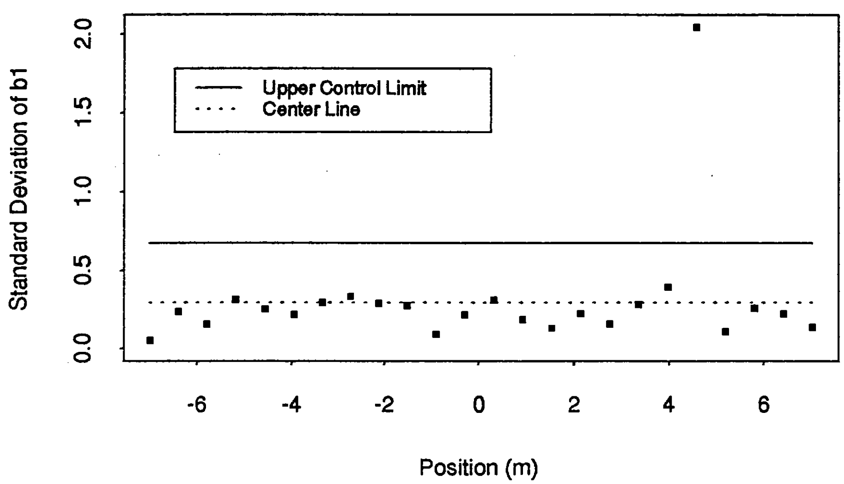
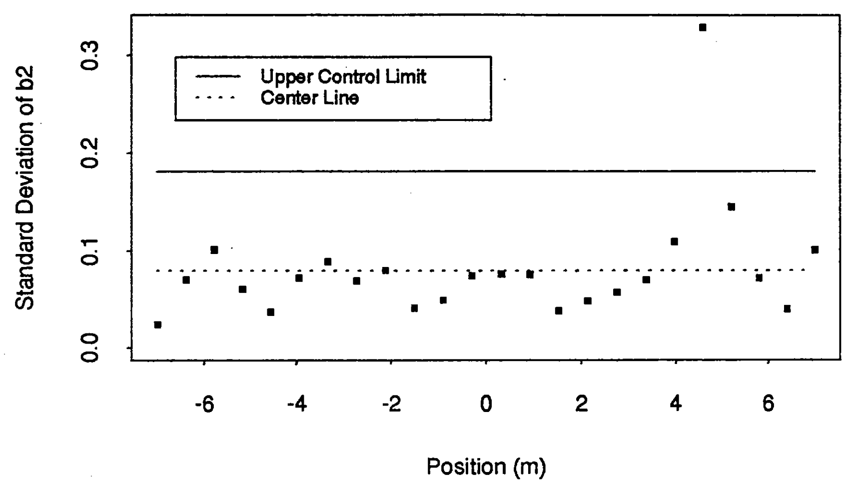
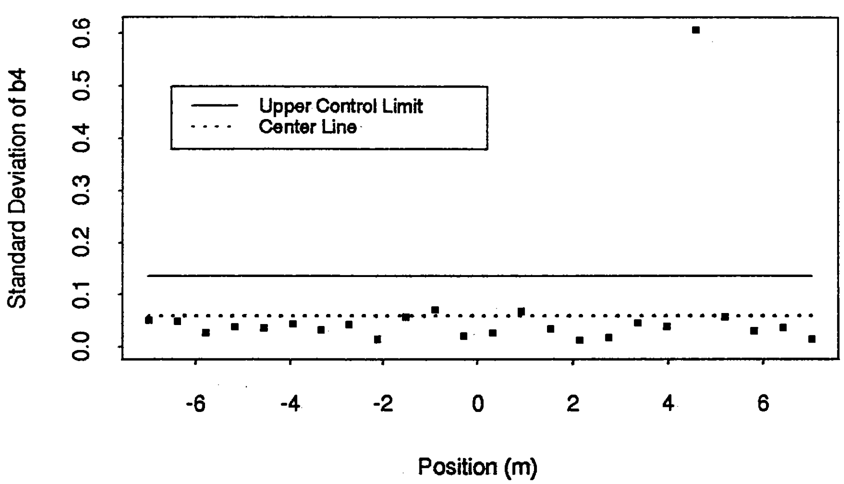


Figure B.5 Control Charts for Warm Multipole Standard Deviations, Magnet DCA320







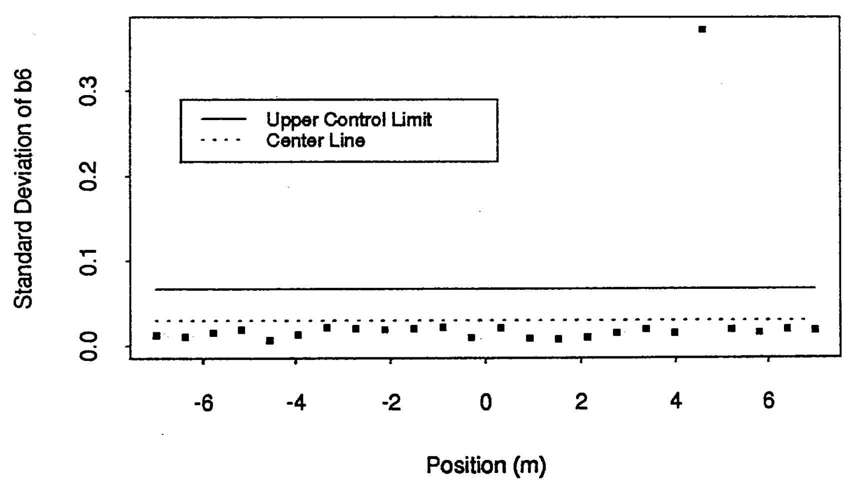


Figure B.6 Control Charts for Warm Multipole Standard Deviations After Deleting Outliers, Magnet DCA311

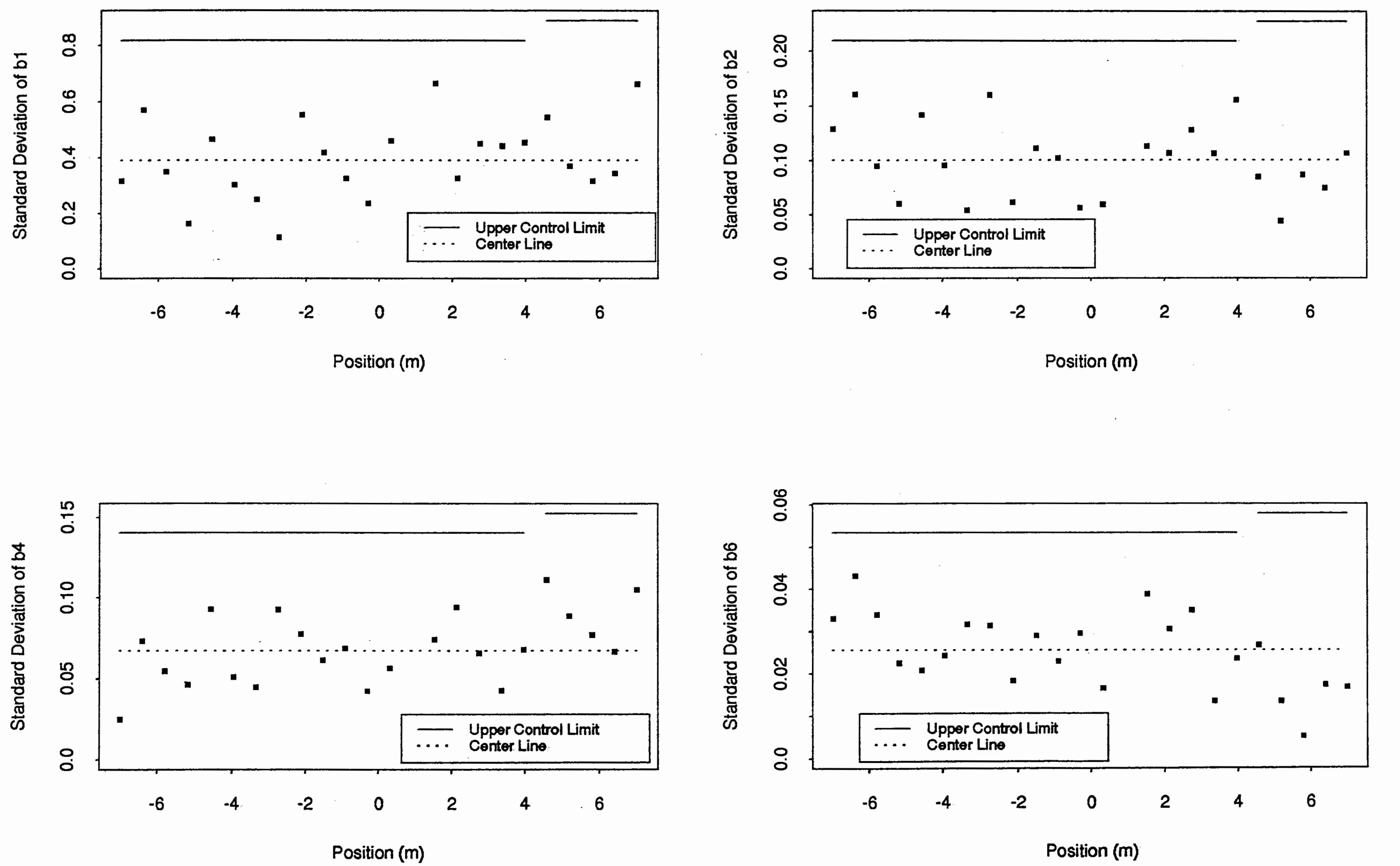


Figure B.7 Control Charts for Warm Multipole Standard Deviations After Deleting Outliers, Magnet DCA314

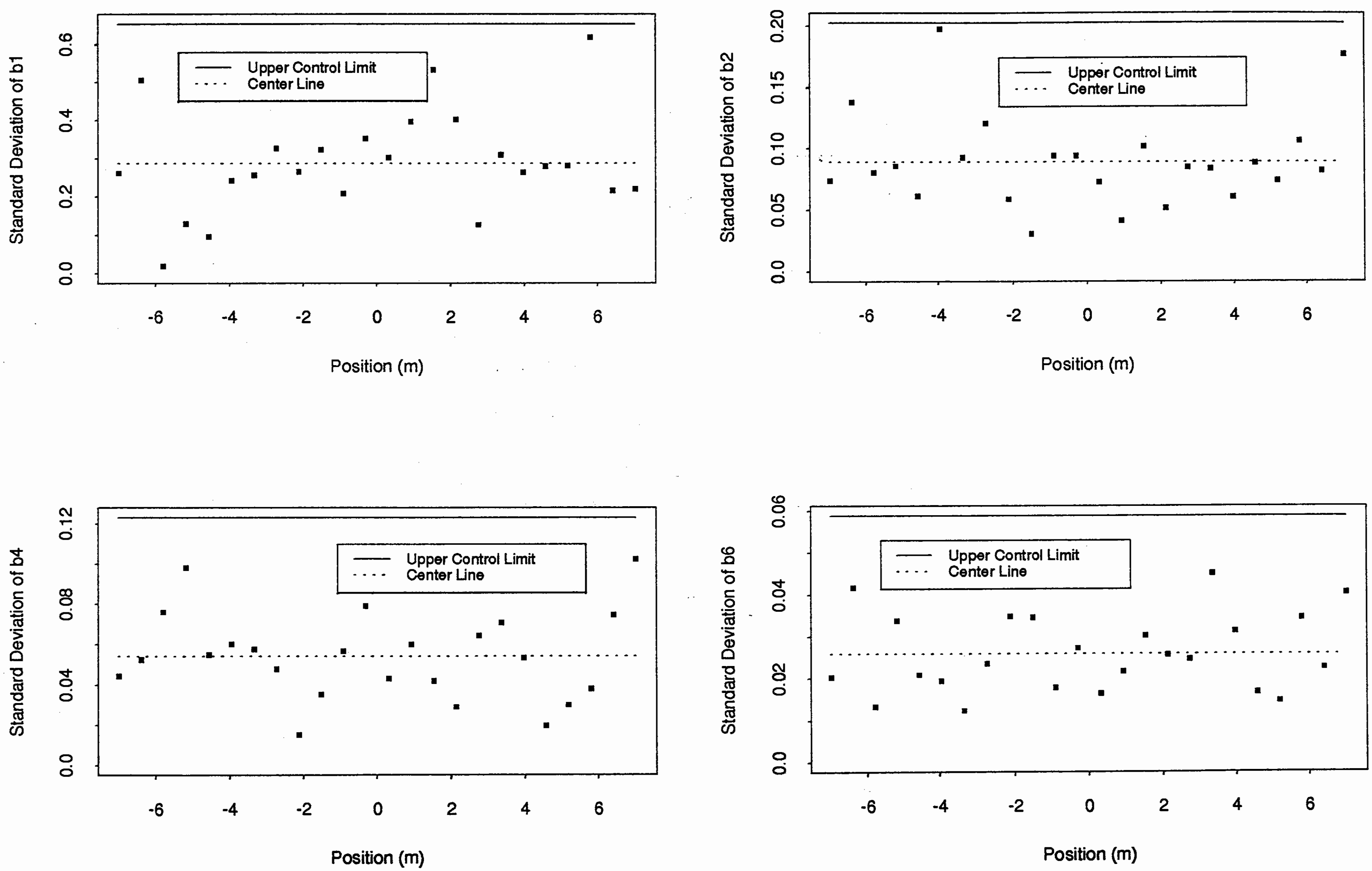


Figure B.8 Control Charts for Warm Multipole Standard Deviations
After Deleting Outliers, Magnet DCA315

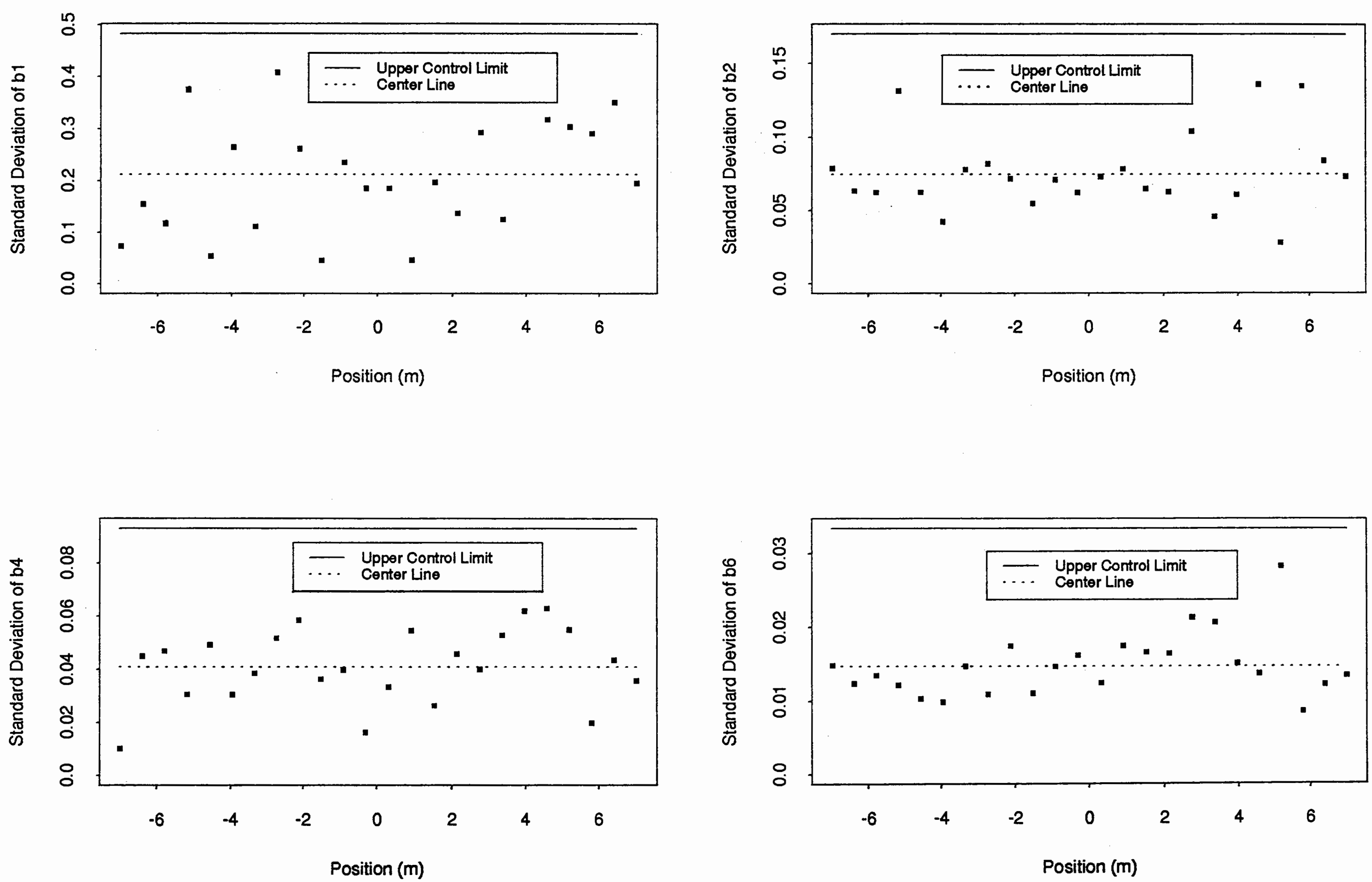


Figure B.9 Control Charts for Warm Multipole Standard Deviations After Deleting Outliers, Magnet DCA320