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ON THE ANALYSIS OF TRIANGULAR DESIGNS

by

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ON THE ANALYSIS OF TRIANGULAR DESIGNS

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ABSTRACT

This note gives an alternate formula for computing the adjusted treatment sum of squares for Triangular Partially Balanced Incomplete Block Designs (T-PBIB). The proposed formula is shown to be equal to one which is given in reference [1] and hence a check is provided in carrying out the computations.

INTRODUCTION

The definition of T-PBIB and various relationships which hold for such designs can be found in reference [1 - p.43]. The notations used here are exactly the same as those given there. The proposed formula to find the treatment (adjusted) sum of squares is

$$[8\lambda_1^2 + 2\lambda_1\lambda_2(3n-10) + \lambda_2^2(n-3)(n-4)]^{-1} \left\{ \frac{2k}{n-1}[2\lambda_1 + \lambda_2(n-3)] \sum_{i=1}^{v} Q_i^2 + \frac{4k(\lambda_1-\lambda_2)}{n(n-1)} \sum_{i=1}^{v} (2Q_1 + S_1(Q_i)) \right\}$$

which will be shown to be equal to one which is conventionally given by the formula

$$\sum_{i=1}^{v} \hat{t}_i Q_i \qquad [1],$$

where

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$$\hat{t}_i = \frac{k-c_2}{r(k-1)} Q_i + \frac{c_1-c_2}{r(k-1)} S_1(Q_i),$$

$$\text{(i)} \quad \frac{k-c_2}{r(k-1)} = \frac{n}{2} \frac{[2\lambda_1 + \lambda_2(n-3)] + 2(\lambda_1-\lambda_2)}{k\Delta},$$

$$\text{(ii)} \quad \frac{c_1-c_2}{r(k-1)} = \frac{\lambda_1-\lambda_2}{k\Delta},$$

and $\Delta = \frac{n(n-1)}{4k^2}\left[8\lambda_1^2 + 2\lambda_1\lambda_2(3n-10) + \lambda_2^2(n-3)(n-4)\right]$.

We shall now prove that

$$\sum_{i=1}^{\nu} \hat{t}_i Q_i = \left[8\lambda_1^2 + 2\lambda_1\lambda_2(3n-10) + \lambda_2^2(n-3)(n-4)\right]^{-1} \left\{\left(\frac{2k}{n-1}\right)\left[2\lambda_1 + \lambda_2(n-3)\right] \sum_{i=1}^{\nu} Q_i^2 + \frac{4k(\lambda_1-\lambda_2)}{n(n-1)} \sum_{i=1}^{\nu} (2Q_i + S(Q_i)Q_i\right\}.$$

Proof: $\sum_{i=1}^{\nu} \hat{t}_i Q_i = \frac{k-c_2}{r(k-1)} \sum_{i=1}^{\nu} Q_i^2 + \frac{c_1-c_2}{r(k-1)} \sum_{i=1}^{\nu} Q_i S_1(Q_i)$

$$= \frac{n}{2} \frac{[2\lambda_1+\lambda_2(n-3)] + 2(\lambda_1-\lambda_2)}{k\Delta} \sum_{i=1}^{\nu} Q_i^2 + (\frac{\lambda_1-\lambda_2}{k\Delta}) \sum_{i=1}^{\nu} Q_i S_1(Q_i)$$

$$= \frac{n}{2k\Delta}[2\lambda_1+\lambda_2(n-3)] \sum_{i=1}^{\nu} Q_i^2 + \frac{\lambda_1-\lambda_2}{k\Delta}\left[\sum_{i=1}^{\nu} (2Q_i + S_1(Q_i)Q_i\right]$$

$$= \left[8\lambda_1^2 + 2\lambda_1\lambda_2(3n-10) + \lambda_2^2(n-3)(n-4)\right]^{-1}.$$

$$\left\{\left(\frac{2k}{n-1}\right)[2\lambda_1 + \lambda_2(n-3)] \sum_{i=1}^{\nu} Q_i^2 + \frac{4k(\lambda_1-\lambda_2)}{n(n-1)} \sum_{i=1}^{\nu} (Q_i + S_1(Q_i)Q_i\right\}.$$

Hence the proposed formula gives the treatment (adjusted) sum of squares and a table showing calculation Q_i [1, Table 4.2 p. 47] is enough to find the adjusted treatment sum of squares since $2Q_i + S_1(Q_i)$ is the total of the row sum and the column sum in which the ith treatment is present.

It should be noted in the above formula that if $\lambda_1 = \lambda_2 = \lambda$, the second term in the braces is zero and we get the adjusted treatment sum of squares for the balanced incomplete block design which is

$$\frac{k}{\lambda\nu} \sum_{i=1}^{\nu} Q_i^2 .$$

It should also be noted that the estimated variance of the difference between two treatments which are 1st anociates (i.e. occur in the same row or column of the association scheme) is

$$\frac{2\lambda_1(n+1) + \lambda_2(n^2-3n-2)}{k\Delta} . s_e^2,$$

and the estimated variance of the difference between two treatments which are second anociates (i.e. do not occur in the same row or column of the association scheme) is

$$\frac{2\lambda_1(n+2) + \lambda_2(n-4)(n+1)}{k\Delta} . s_e^2 ,$$

where s_e^2 is error mean square which one gets from the analysis of variance table.

In order to illustrate the use of the formula, we shall give an example which is the same as given on pages 43-49, [1] and from table 42[1, p. 47] we get the following results:

Treat No.	Qi	Table $2Q_i + S_1(Q_i)$
1	- .2700	-.4875
2	- .2500	-.8075
3	- .1075	.6650
4	.2225	-.5850
5	- .5725	-.4850
6	.3350	.9875
7	.4250	-.2625
8	1.0450	.6675
9	- .6250	-.5825
10	- .2050	.8900

Since $\sum_{i=1}^{\nu} Q_i^2 = 2.3407$ and $\sum_{i=1}^{\nu} Q_i[2Q_i + S_1(Q_i)] = 1.5101$,

we have treatment adjusted

$$\text{sum of squares} = (36)^{-1}[12(2.3407 - \tfrac{4}{5}(1.5101)$$
$$= .7467.$$

The above example shows that a table showing calculation of Q_i is enough to find the adjusted treatment sum of squares and hence a check is provided in carrying out the computations.

REFERENCE

[1] Base, R.C., Clatworthy, W.H. and Shrihande, S.S., Tables of Partially Balanced Designs with Two Associate Classes. Raleigh, North Carolina: North Carolina Agricultural Experiment Station Technical Bulletin, No. 107. 1954.

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