

DEPARTMENT OF STATISTICS
Southern Methodist University

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Joseph E. Tepera

by

SCATTERING OF SOUND FROM LAYERED
HOLLOW ELASTIC CYLINDERS

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SYMBOLS AND DEFINITIONS

The following definitions are applicable to the symbolic notation used throughout this investigation. Localized variation to these definitions may exist within the discussion whenever clarity is enhanced.

Symbol	Definition
$\bar{A}$	vector potential field
a	mid-plane radius of a thin shell
$\bar{B}$	vector potential field
c	wave velocity in a non-viscous fluid
c_{bar}	wave velocity based on bar assumption
c_d	wave velocity of a distortion wave
c_λ	wave velocity of a dilatation wave
cm	centimeter
E	Young's Modulus
$\hat{e}_r, \hat{e}_\theta, \hat{e}_z$	unit directional vectors for cylindrical coordinates
g	grams
$H_n^{(1)}$	Hankel function of the first (1), or second (2)
$H_n^{(2)}$	kind of order v or n
h	shell half thickness
i	imaginary unit, $\sqrt{-1}$
J_n, J_v	Bessel function of the first kind of order v or n
k	ratio of wave frequency to wave velocity
KA, ka	general wave number range; specific wave number value

Definition

<u>Symbol</u>	<u>Definition</u>
n	integer index for series operations
p	pressure
P_o	amplitude of incident wave, pressure
Q_n	generalized force for n^{th} generalized coordinate
r	radius
r_a	outer radius of coating
r_b	inner radius of coating
r_c	outer radius of base cylinder
R/t	inner radius of base cylinder
	ratio of mid-plane radius of a shell and shell thickness
T	kinetic energy
t	time
$\underline{u}$	thickness of shell
	general displacement vector
u	component of displacement in radial direction
V	potential energy
v	component of displacement in tangential direction
x	wave number for a dilatation wave
	independent coordinate in a rectangular coordinate system
Y_v, Y_n	Bessel function of the second kind of order v or n
y	independent coordinate in a rectangular coordinate system
y	wave number for a shear wave
Z_n	mechanical impedance factor resulting from thin shell theory

GREEK SYMBOLS AND DEFINITIONS

Symbol	Definition
$\gamma_{r\theta}$	shear strain in r- θ plane
Δ	dilatation function
$\epsilon_{\theta\theta}, \epsilon_{rr}$	normal strain in the tangential or radial direction
θ	independent coordinate in cylindrical coordinates
κ	bulk modulus
λ	Lame's constant for an isotropic solid
μ	Lame's constant for an isotropic solid (identical to the shear modulus)
π	universal constant 3.1415926...
ρ	mass density
$\sigma_{\theta\theta}, \sigma_{rr}$	normal stress in the tangential or radial direction
$\sigma_{r\theta}$	shear stress in the r- θ plane
Φ	scalar potential function
χ_{θ}	change in curvature in thin shells
ψ	scalar potential function
ω	angular velocity, radians per second
2ω	rigid body rotation
∇	del operator = $\hat{e}_r \frac{\partial}{\partial r} + \frac{1}{r} \hat{e}_{\theta} \frac{\partial}{\partial \theta} + \hat{e}_z \frac{\partial}{\partial z}$
SUBSCRIPTS	
i	incident wave parameter
n	series index pertaining to the n th operation

Symbol

p, q	indices for matrix notation	p - row q - column
s	scattered wave parameter	
t	shear wave parameter	
1	external fluid parameter	
2	coating material parameter	
3	base cylinder parameter	
4	inner fluid parameter	

of the wave equation and the associated boundary conditions.

greater than unity. This solution was formulated through the application

scattering patterns for wave numbers from less than unity to numbers

restriction on the radius-wave length ratio. In doing so, Morse obtained

scattering of a plane wave by a rigid circular cylinder, deleting Rayleigh's

into the mathematical model. Later, Morse [14] discussed the solution to

rigid scatterers allowed Rayleigh to introduce simplifying assumptions

less than unity. The use of the assumptions of long wave lengths and

a rigid spherical obstacle having a radius-to-wave length ratio of much

analysis of the scattering problem. His study was limited to the case of

Lord Rayleigh [13] is credited with developing the first mathematical

although the obstacle response is easily obtained from the solution.

gation into scattering characteristics as opposed to obstacle response,

contributed mainly to one area or the other. This study is an investi-

made by Junger [1-5] and Payton [6,7] while other authors [8-12] have

scattered wave. Notable contributions to both of these areas have been

while others have concentrated on the characteristics of the resulting

studies have emphasized the response of the body to the incident wave

been the subject of investigation in numerous studies. Some of these

The interaction of acoustic waves and various geometric bodies has

INTRODUCTION

CHAPTER I

Another method of deriving the solution to a special class of scatterers has been developed around the various thin shell assumptions. Initially, Primakoff and Keller [8] applied basic thin shell theory to the problem of reflection and transmission of sound by a general thin curved shell. Then Keller and Keller [9] specialized Primakoff's general case to that of the thin elastic spherical shell. Junger [4] expanded this line of approach by studying the scattering of sound by the thin circular cylindrical shell. Other contributions which use this theoretical approach have been made by Carrier [15], Bordoni [16], and Huang [17].

Concurrent with the thin shell studies, the boundary value problem associated with the wave equation was extended to include the elastic scatterers. The use of the idealized rigid scatterer has been rejected by Faran [18], who demonstrated the necessity of including the effects of the induced waves within an elastic solid obstacle when computing the scattering pattern. Following this work, Doolittle and Uffner [19] outlined the general theory for scattering by thick walled hollow cylinders. The resulting matrix representation is shown to contain the special cases of the fluid filled interior, the void interior, as well as other special cases.

More sophisticated scattering solutions have been developed from both theoretical standpoints. Junger [5] provided a solution to the scattering of a plane wave by a thin cylindrical shell which contained a rigid core. Spencer and Granger [20] have produced a solution to the scattering of a prolate spheroidal obstacle. Multiple scattering by two arbitrary cylinders has been investigated by Zitron [21]. Burke [22, 23,

24] has discussed scattering by penetrable, soft and hard spheroids. The effects of a penetrable coating on a rigid spheroid has been investigated by Yeh [25]. His findings suggest that such a coating has significant effects on the resulting patterns.

In this study, two specific types of scattering problems will be discussed. The basic task will be to investigate the scattering of a plane wave by a layered hollow cylindrical body. A secondary investigation compares the scattering solution obtained from thin shell theory to that derived from the wave equation-boundary value approach. The purposes are to extend the scattering theory into areas in which thin shell analyses are invalid, to examine the effects of the coating on scattering patterns, and to define the limits of thin shell assumptions as they pertain to the scattering problem.

In the preceding commentary, it was noted that two fundamental approaches are available to scattering theory analysis: (1) the thin shell method and (2) the elasticity method. In this work, the solution to the primary problem is developed entirely through the elasticity theory. A two-dimensional simplification in the independent variables of the equations of elasticity is made by assuming a cylinder and a wavefront of infinite lengths. The fundamental wave equation for a stress wave in an elastic solid is assumed to be satisfied by the appropriate combination of a scalar and vector potential fields (the Helmholtz reduction). The solution to the second-order partial differential equations representing these fields are found to be infinite series of the product of circular or cylindrical functions and unknown "Fourier" coefficients. These unknown

known coefficients are evaluated through the appropriate restrictions on the pressures, stresses and displacements at the material boundaries. Far field scattered pressure patterns are then developed for various isotropic linearly elastic coatings on a basic elastic cylinder. The effects of material density and moduli are noted, and a method for predicting the effects of other coating materials is set forward.

In the secondary problem, the comparison of the thin shell and the elasticity solution, the unknown scattering coefficient for the thin shell is obtained from Junger [4]. Far field patterns as determined from Junger's coefficient are compared to patterns obtained from Doolittle and Uberall's [19] elasticity solution. The comparison is made at various radius-to-wall thickness ratios. Excellent pattern agreement is obtained up to the ratio $R/t = 10$; however, large magnitude differences exist at smaller ratios. Analysis of the terms of the thin shell scattering coefficient tends to lend support to the conclusions reached on the bases of the results of the elasticity solution.

In these studies, the theoretical development was supported by numerical analyses because of the mathematical complexity of the problem. These analyses were performed on a Univac 1108 digital computer.

derivation.

different from the external fluid has been assumed throughout the analytical

plane wave are reserved. The presence of an internal acoustic fluid

The concepts of the infinite acoustic fluid and the continuous incident which is composed of two layers of isotropic linearly elastic materials.

referenced studies has been replaced by a cylinder of infinite length

In this investigation, the obstacle considered in each of the above

cases, the material is assumed to be isotropic and linearly elastic.

that a hollow cylinder is substituted for the solid cylinder. In both

and Überall's [19] problem is identical to Faran's, with the exception

infinite in length, thereby producing a two-dimensional problem. Doolittle

plification of the analysis was achieved by assuming the cylinder to be

incident signal was assumed parallel to the axis of the cylinder. Sim-

coordinate system as shown in Figure 2.1. Thus, the wave front of the

of revolution coincident with the z-axis of an orthogonal cylindrical

propagates from the $\theta = -\pi$ direction toward a cylinder having its axis

freely suspended in an infinite acoustic fluid. The incident plane wave

plane waves. In his problem, a solid elastic obstacle was assumed to be

the analytical description of far field pressure patterns for scattered

As previously noted, Faran [18] derived from the wave equations

Introduction

THEORETICAL DEVELOPMENT

CHAPTER II

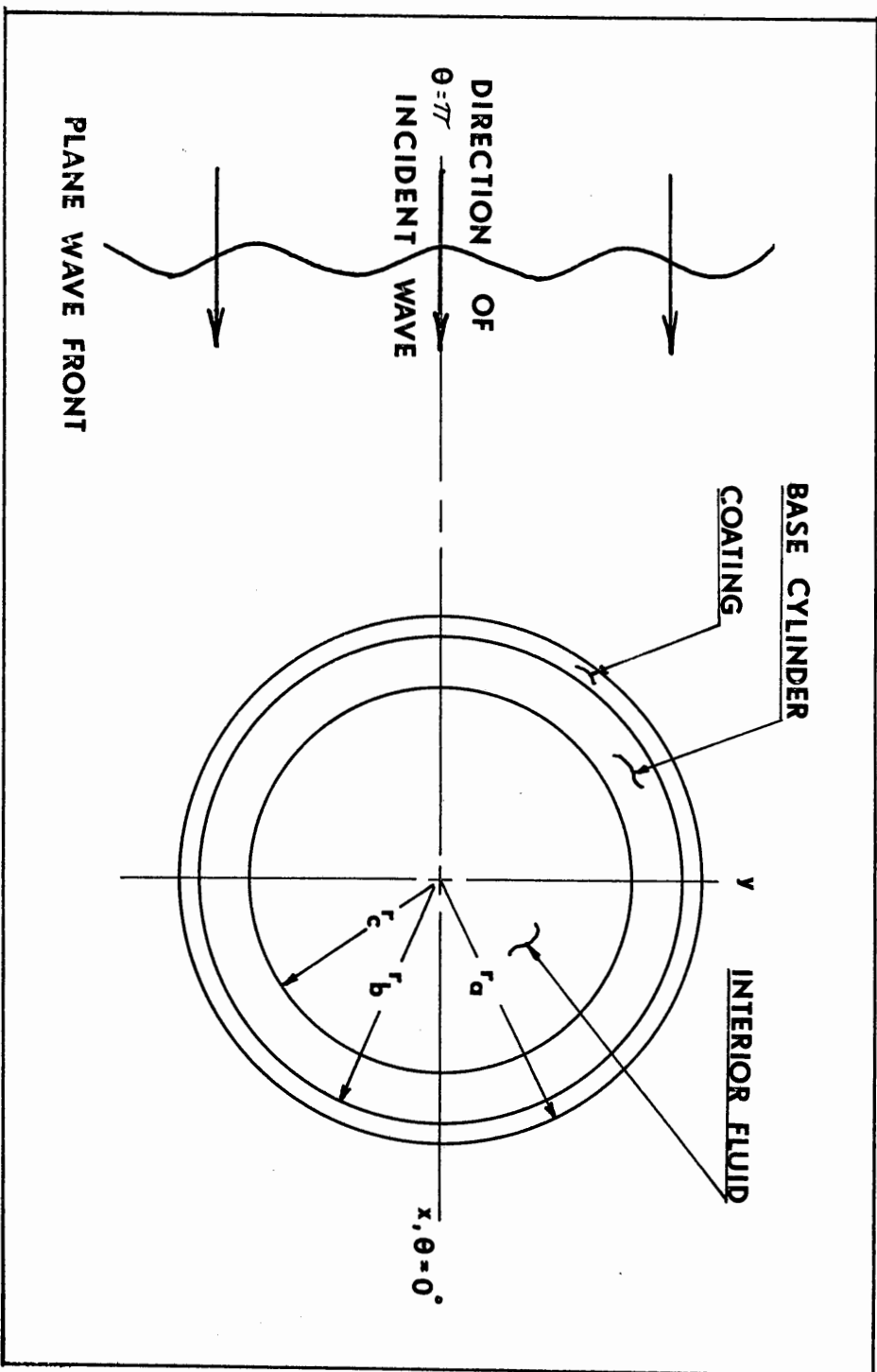


Figure 2.1. Schematic Representation of the Scattering Problem

Fundamental Equations

The fundamental partial differential equation that governs the propagation of elastic waves through a boundless elastic solid can be

written as

$$(2.1) \quad \rho \frac{\partial^2 \underline{u}}{\partial t^2} = (\lambda + 2\mu) \nabla \nabla - \mu \nabla \times (2\omega) \quad (2.1)$$

where ∇ and (2ω) are the dilatation and rotation of a differential

volume, respectively. These quantities are defined in terms of the displacement vector $\underline{u}$ as

$$(2.2) \quad \nabla = \nabla \cdot \underline{u}$$

and

$$(2.3) \quad (2\omega) = \nabla \times \underline{u},$$

where ∇ is the del operator.

Equation (2.1) can be separated into two distinct equations of

motion, each describing a different mode of wave propagation. In order

to obtain one of these equations, the divergence operator is allowed to

act on Equation (2.1). This operation yields the dilatation wave equation

tion

$$(2.4) \quad c_2^2 \nabla^2 \nabla = \frac{\partial^2}{\partial t^2} \nabla,$$

where

$$(2.5) \quad c_2 = [(\lambda + 2\mu)/\rho]^{1/2}$$

is the propagation velocity for the dilatation wave.

The remaining wave equation, which describes the motion of a dis-

tortion wave, is separated from Equation (2.1) by executing the curl operation.

This operation produces the differential equation

where

$$c_p^d = [u/p]^{1/2} \quad (2.7)$$

$$c_p^d \Delta^2 (2u) = \frac{\partial^2}{\partial t^2} (2u) \quad (2.6)$$

is the propagation velocity of the distortion wave.

It can be shown that the differential equation of motion for an

elastic solid can be reduced to two second-order partial differential equations, in which the dependent variables are a scalar or a vector

potential field. This reduction is accomplished through the Helmholtz

theorem which equates any vector to the sum of the gradient of a scalar

potential and the curl of a vector potential ([29], Page 47). That is,

the displacement vector $\underline{u}$ can be written as

$$\underline{u} = -\nabla\phi + \nabla \times \underline{A}, \quad (2.8)$$

where ϕ is a scalar potential and $\underline{A}$ is a vector potential.

Substituting Equation (2.8) into Equations (2.2) and (2.3) and

applying the results separately to Equation (2.1) yielded the two wave

equations

$$c_p^d \Delta^2 \phi = \frac{\partial^2}{\partial t^2} \phi \quad (2.9)$$

and

$$c_p^d \Delta^2 \underline{A} = \frac{\partial^2}{\partial t^2} \underline{A}. \quad (2.10)$$

Equations (2.4) and (2.6) are restricted to wave propagation in a

linear elastic material which is isotropic, homogeneous, and continuous.

However, it is possible to apply these equations of motion in media which

are nonlinear for large strains provided small perturbations from an am-

plient strain level are assumed. The equation of motion which governs the

propagation of a wave in a nonviscous fluid can be obtained in this man-

ner.

Wave Propagation in a Nonviscous Fluid

The shear modulus μ for a nonviscous fluid is defined to be zero and the bulk modulus is directly related to the Lamé constant λ by

$$\kappa = \lambda \quad (2.11)$$

under these conditions [26].

Furthermore, the bulk modulus κ is defined to be the ratio between the applied hydrostatic pressure and the dilatation; that is,

$$\kappa = p/\Delta \quad (2.12)$$

or

$$\Delta = -p/\kappa. \quad (2.13)$$

The substitution of Equations (2.12) and (2.13) into Equation (2.4) will

produce the results

$$c^2 \Delta^2 p = \frac{\partial^2}{\partial t^2} p, \quad (2.14)$$

where

$$c = [\kappa/\rho]^{1/2} \quad (2.15)$$

is the velocity of the pressure wave in the fluid. The counterpart to Equation (2.6), the distortion wave equation, is not formed since $\mu \equiv 0$ for the nonviscous fluid.

General Solution in Cylindrical Coordinates

The solution to the wave equation in cylindrical coordinates is documented in numerous textbooks, e.g., McLachlan [27]. It is sufficient for this study to indicate that the solution in this coordinate system involves the Bessel and Hankel functions of the first and second kind. Herein, these functions are represented by symbols $J_n(z)$, $Y_n(z)$ for the

first and second kind of Bessel functions, respectively, and by $H_n^{(1)}(z)$ and $H_n^{(2)}(z)$ for the first and second kind of Hankel functions. Their explicit forms are given by the infinite series corresponding to Equations (4) on Page 24, (12) on Page 25, and (2) and (3) on Page 26 of McLachlan [27].

The second-order partial differential equation

$$\epsilon^2 \Delta^2 \xi(r, \theta, t) = \frac{\partial^2 \xi(r, \theta, t)}{\partial t^2} \quad (2.16)$$

can be shown through the technique of separation of variables to have a general solution of the form

$$\xi(r, \theta) = [A' J_\nu(kr) + B' Y_\nu(kr)] [C' \cos v\theta + D' \sin v\theta] \quad (2.17)$$

or

$$\xi(r, \theta) = [A'' H_\nu^{(1)}(kr) + B'' H_\nu^{(2)}(kr)] [C'' \cos v\theta + D'' \sin v\theta] \quad (2.18)$$

(McLachlan, Chapter II).

Here, the quantity v is a constant to be evaluated through the

boundary conditions, as can the arbitrary constants $A', B', \dots, C''$ and D'' . The constant k is a result of the assumption that the dependent variable $\xi(r, \theta, t)$ is a harmonic function of time, i.e.

$$\frac{\partial^2 \xi(r, \theta, t)}{\partial t^2} = -k^2 \xi(r, \theta, t), \quad (2.19)$$

and is defined as

$$k^2 = \omega^2 / \epsilon^2. \quad (2.20)$$

In this study, the solution to Equation (2.16) is periodic in θ , that is, single-valued. Consequently, the constant v is restricted to integer values [28]. As a result v will be replaced by n where n can take on the values $0, 1, 2, \dots$. Further, if the additional restriction of sym-

metry with respect to the $\theta = 0^\circ$, π line, is stipulated, then the

complete solution to Equation (2.16) can be reduced from Equations (2.17) and (2.18) to either

$$\xi(r, \theta) = [A_J^n(kr) + B_Y^n(kr)] \cos n\theta \quad (2.21)$$

or

$$\xi(r, \theta) = [C_H^n(kr) + D_H^n(kr)] \cos n\theta \quad (2.22)$$

The final form of the complete solution is an infinite series of

Bessel or Hankel functions since each integer value of n is an independent solution to Equation (2.15). In practice, the selection of either the

Bessel function or the Hankel function form of the solution is left en-

tirely to the discretion of the investigator. The numerical results to a particular problem will be the same (McLachlan, Page 27).

Scattering of Layered Cylinders

It has been shown in the preceding sections of this chapter that the

wave equation governs the propagation of a disturbance in an elastic solid as well as a nonviscous fluid. The solution to this form of partial dif-

ferential equation, as represented by Equation (2.16), is given by various Bessel and Hankel functions. These forms are given by Equations (2.21)

and (2.22) for those problems involving symmetry in θ . Equations (2.9),

(2.10) and (2.14) can be combined, formulating the boundary value problem

of plane wave scattering by a layered cylinder.

The propagation of a pressure wave in the acoustic fluid external

to the cylinder is governed by the equation

$$c_1^2 \nabla^2 p_1(r, \theta, t) = \frac{\partial}{\partial t} p_1(r, \theta, t), \quad r > r_a \quad (2.23)$$

where $c_2^2 = \kappa_1^2 / \rho_1^2$. Equation (2.23) can be separated into two parts by

use of the assumption that the external pressure, $p_1(r, \theta, t)$, is the superposition of the incident pressure, $p_1^i(r, \theta, t)$, and the scattered pressure, $p_1^s(r, \theta, t)$. Since the incident wave is a plane wave, by specification,

its description can be written as

$$p_1^i(x, t) = p_1^o e^{ik_1(x - ct)} \quad (2.24)$$

where $k_1 = \omega^2 / c_1^2$. Equation (2.24) is a "right-moving" wave and can be

shown to satisfy the wave equation. In cylindrical coordinates, the

relationship between the coordinates x and (r, θ) is given by

$$x = r \cos \theta, \quad (2.25)$$

which, when substituted into Equation (2.24), yields the equation

$$p_1^i(r, \theta, t) = p_1^o e^{ik_1(r \cos \theta - ct)} \quad (2.26)$$

It is convenient to represent Equation (2.26) as an infinite series of

cylindrical waves relative to the origin. This transformation can be

made by equating the right side of Equation (2.26) to the cosine series

(since symmetry is assumed)

$$p_1^o e^{ik_1(r \cos \theta - ct)} = p_1^o e^{-i\omega t} \sum_{n=0}^{\infty} \frac{e_n}{a^n} \cos n\theta \quad (2.27)$$

and solving for the coefficients. The result is

$$p_1^i(r, \theta, t) = e^{-i\omega t} p_1^o \sum_{n=0}^{\infty} e_n e^{ik_1 r} \cos n\theta, \quad (2.28)$$

where

$$e_n = \begin{cases} 1, & n = 0 \\ 2, & n \geq 1 \end{cases} \quad (2.29)$$

is the Neumann number.

*right moving: moving in the direction of increasing positive x .

Since the principle of superposition is assumed and the incident

wave has been specified by Equation (2.28), the external pressure

can be written as

$$p_1(r, \theta, t) = p_I(r, \theta, t) + p_S(r, \theta, t). \quad (2.30)$$

The governing Equation (2.23) can be written in terms of the scat-

tered pressure, $p_S(r, \theta, t)$, by the substitution of Equations (2.27) and

(2.28) therein and the performance of the indicated operations. Thus,

the equation

$$c_2^2 \Delta^2 p_S(r, \theta, t) = \frac{\partial^2 p_S(r, \theta, t)}{\partial t^2}, \quad r > r_a, \quad A(\theta, t) \quad (2.31)$$

governs the scattered wave in the external fluid.

Through the Helmholtz reduction, discussed on Page 8, the propaga-

tion of a disturbance through the elastic solids of the cylinders can be

written in terms of a scalar and vector potential field. Since the

velocities of propagation vary from material to material, a set of wave

equations similar to Equations (2.9) and (2.10) will be required for each

layer. Thus, in the outer layer, the governing equations

$$\frac{\partial^2 \chi_2}{\partial t^2} \Delta^2 \Phi(r, \theta, t) = \frac{\partial^2 \Phi(r, \theta, t)}{\partial t^2} \quad (2.32)$$

and

$$c_2^2 \Delta^2 \bar{A}(r, \theta, t) = \frac{\partial^2 \bar{A}(r, \theta, t)}{\partial t^2} \quad (2.33)$$

are applicable for all values of (θ, t) and r ranging from r_a to r_b .

The vector potential field can be expressed as a function of scalar

coefficients and unit vectors; that is,

$$\bar{A} = A_r \hat{e}_r + A_\theta \hat{e}_\theta + A_z \hat{e}_z. \quad (2.34)$$

However, in the specification of the problem, the displacement vector $\underline{u}$ must be symmetric with respect to $\theta = 0, \pi$. This displacement vector is given by Equation (2.8), repeated below:

$$\underline{u} = -\nabla\Phi + \nabla \times \underline{A}.$$

Then, the form of Φ must be even in θ , and $\underline{A}$ must be odd so that symmetry in $\underline{u}$ is respected. This is more clearly illustrated in Figure (2.2) and by the expanded form of Equation (2.8),

$$\underline{u} = \hat{e}_r \left(-\frac{\partial\Phi}{\partial r} + \frac{\partial\theta}{\partial r} \right) - \hat{e}_\theta \left(\frac{\partial\Phi}{\partial\theta} + \frac{\partial r}{\partial\theta} \right). \quad (2.35)$$

Furthermore, the scalar coefficients A_r and A_θ must be zero since the two-dimensional aspects of the overall problem are specified, i.e., there can be no z dependence. As a result of these restrictions, Equation (2.33)

can be reduced to the scalar equation

$$c_2^2 \nabla^2 A_z(r, \theta, t) = \frac{\partial^2}{\partial t^2} A_z(r, \theta, t). \quad (2.36)$$

The same conditions apply to the inner cylinder; and its governing equations can be written directly as

$$c_2^2 \nabla^2 \psi(r, \theta, t) = \frac{\partial^2}{\partial t^2} \psi(r, \theta, t) \quad (2.37)$$

and

$$c_2^2 \nabla^2 B_z(r, \theta, t) = \frac{\partial^2}{\partial t^2} B_z(r, \theta, t) \quad (2.38)$$

for all (θ, t) and r ranging from r_b to r_c . In Equations (2.32), (2.36), (2.37) and (2.38), the wave velocities are defined as

$$c_{\lambda_2}^2 = [(\lambda_2 + 2\mu_2)/\rho_2]^{1/2}, \quad c_{d_2}^2 = [\mu_2/\rho_2]^{1/2}, \\ c_{\lambda_3}^2 = [(\lambda_3 + 2\mu_3)/\rho_3]^{1/2} \quad \text{and} \quad c_{d_3}^2 = [\mu_3/\rho_3]^{1/2} \quad (2.39)$$

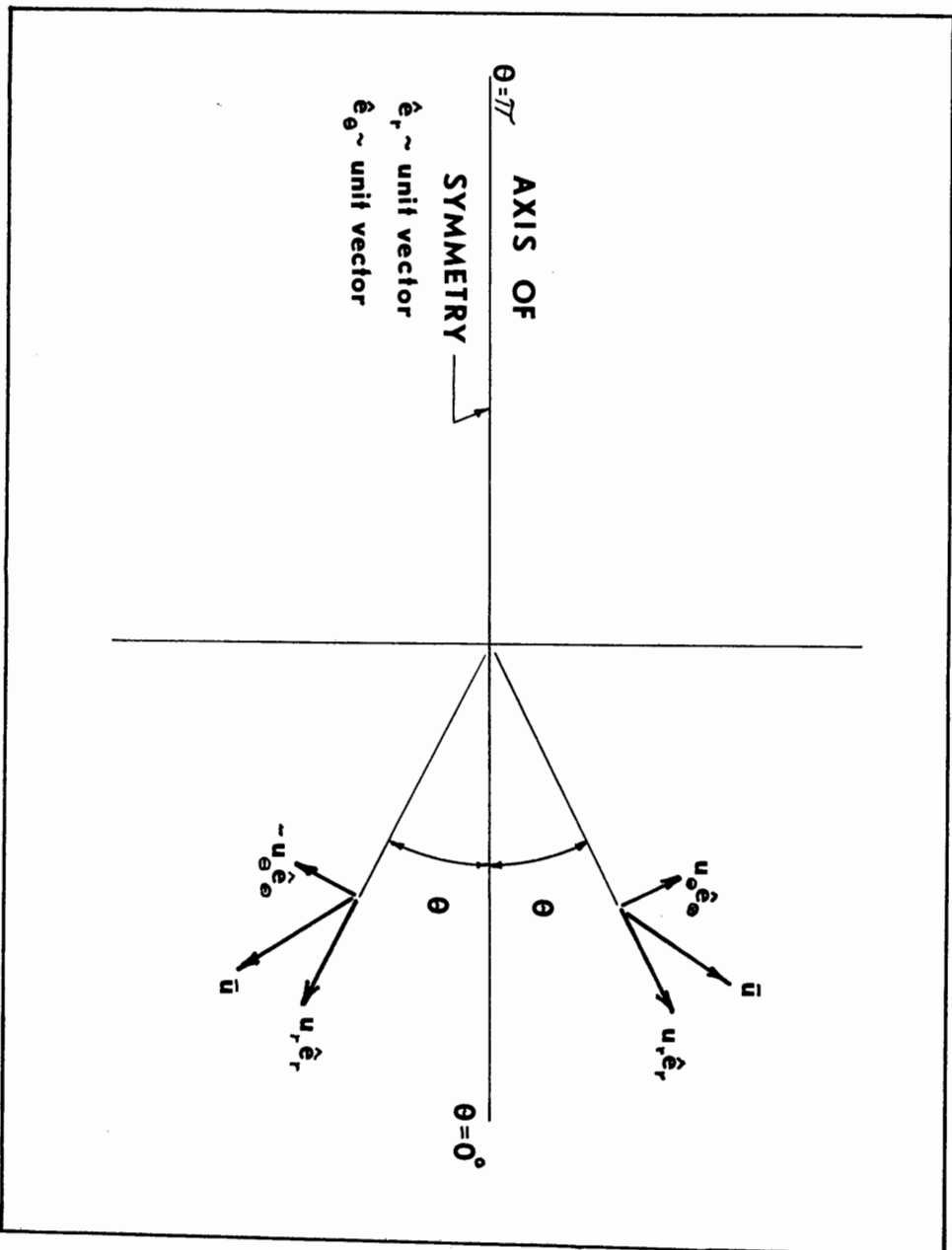


Figure 2.2. Schematic of Symmetry in Displacement Requirements

Finally, in the core fluid, the equation

$$\frac{\partial^2}{\partial t^2} \Delta^2 p_4(r, \theta, t) = \frac{\partial^2}{\partial t^2} p_4(r, \theta, t) \quad (2.40)$$

must hold for all (θ, t) and $r > r_c$. The wave velocity c_4 can be written

$$c_4 = [K_4/\rho_4]^{1/2} \quad (2.41)$$

In summary, Equations (2.27), (2.31), (2.32), (2.36), (2.37), (2.38)

and (2.40) govern all motion in their respective regions. The differential equations in ϕ and ψ have a solution of the form of either equation (2.21) or (2.22), that is, even in θ . Similarly, the solutions to the equations for A_z and B_z will be of the same form, except that the sine function replaces the cosine function in order that these quantities be odd in θ . Hence, the solutions to the Equations (2.32), (2.36), (2.37)

and (2.38) can be written directly from Equation (2.21) as

$$\Phi(r, \theta, t) = P_0^0 \sum_{n=0}^{\infty} i^n \epsilon_n^0 [c_n^0 J_n(k_2 r) + d_n^0 Y_n(k_2 r)] \cos n\theta, \quad (2.42)$$

$$A_z(r, \theta, t) = P_0^0 \sum_{n=0}^{\infty} i^n \epsilon_n^0 [e_n^0 J_n(k_{t_2} r) + f_n^0 Y_n(k_{t_2} r)] \sin n\theta, \quad (2.43)$$

$$\psi(r, \theta, t) = P_0^0 \sum_{n=0}^{\infty} i^n \epsilon_n^0 [g_n^0 J_n(k_3 r) + h_n^0 Y_n(k_3 r)] \cos n\theta \quad (2.44)$$

$$\text{and } B_z(r, \theta, t) = P_0^0 \sum_{n=0}^{\infty} i^n \epsilon_n^0 [j_n^0 J_n(k_{t_3} r) + \gamma_n^0 Y_n(k_{t_3} r)] \sin n\theta. \quad (2.45)$$

The solution form involving the Hankel functions can be used to write

the solution to the scattered pressure wave, Equation (2.27) as

$$p^s(r, \theta, t) = e^{-i\omega t} \sum_{n=0}^{\infty} \{C_n^H J_n(k_1 r) + D_n^H Y_n(k_1 r)\} \cos n\theta. \quad (2.46)$$

However, the scattered wave must be an outward travelling wave; hence, the

arbitrary coefficients, D_n , must be zero. (The Hankel function of the

first kind represents an outward travelling wave when the time function $e^{-i\omega t}$ is employed). Thus, the solution to the scattered wave equation

of motion can be written as

$$p^s(r, \theta, t) = p_o \sum_{n=0}^{\infty} e^{-i\omega t} \epsilon_n^b H_n^{(1)}(k_1 r) \cos n\theta \quad (2.47)$$

where the substitution

$$C_n = p_o \epsilon_n^b$$

has been introduced for later mathematical convenience [19].

In a similar manner, the solution to the wave equation for the

internal fluid can be written as

$$p_4(r, \theta, t) = p_o \sum_{n=0}^{\infty} e^{-i\omega t} \epsilon_n^b [A_n J_n(k_4 r) + B_n Y_n(k_4 r)] \cos n\theta.$$

However, the internal fluid contains the origin; consequently, any

solution to Equation (2.40) must be regular at $r = 0$. Thus, the coef-

ficients B_n must all be zero since the Bessel function, Y_n , is infinite

for an argument of zero. Hence, the internal pressure field can be

written as

$$p_4(r, \theta, t) = p_o \sum_{n=0}^{\infty} e^{-i\omega t} \epsilon_n^b J_n(k_4 r) \cos n\theta \quad (2.48)$$

where A_n is replaced by m_n and $B_n = 0$.

Boundary Conditions

There are ten separate boundary conditions that must be satisfied

by any solution to this problem. Three of these conditions relate the state of the external fluid to that of the outer layer at their common boundary. Four more conditions are prescribed at the interface between the outer layer and the inner cylinder. Finally, three conditions are established between the inner cylinder and the core fluid at the inner radius, r_c . At each of the two fluid-metal interfaces ($r = r_a, r_c$), the requirements are:

- (a) The radial stress in the metal equals the normal pressure in the fluid.

- (b) The radial displacement of the particles of the fluid and metal are continuous.

- (c) The shearing stresses in the cylinders are zero at the surfaces which are common with the fluids.

At the single interface between the two layers of the cylinder, boundary conditions similar to the three above are specified. If the bond between these two materials is assumed to be a perfect bond, then the boundary condition requirements are:

- (d) The radial stresses of each material are equal.

- (e) The radial displacements are continuous.

- (f) The tangential stresses are equal.

- (g) The tangential displacements are continuous.

These boundary conditions are expressed as follows:

At $r = r_a$ (external fluid - outer cylinder);

$$(2.49) \left\{ \begin{array}{l} 1. p_1(r_a, \theta, t) = p_1(r_a, \theta, t) + p_s(r_a, \theta, t) = -\sigma_{rr}(r_a, \theta, t), \\ 2. u_1(r_a, \theta, t) = u_1(r_a, \theta, t) + u_s(r_a, \theta, t) = u_2(r_a, \theta, t), \\ 3. \sigma_{r\theta_2}(r_a, \theta, t) = \sigma_{r\theta_1}(r_a, \theta, t) = 0. \end{array} \right.$$

and

At $r = r_p$ (outer cylinder-inner cylinder);

$$\left. \begin{aligned} 4. \sigma_{rr_2}(r_p, \theta, t) &= \sigma_{rr_3}(r_p, \theta, t), \\ 5. u_2(r_p, \theta, t) &= u_3(r_p, \theta, t), \\ 6. \sigma_{r\theta_2}(r_p, \theta, t) &= \sigma_{r\theta_3}(r_p, \theta, t), \\ 7. v_2(r_p, \theta, t) &= v_3(r_p, \theta, t). \end{aligned} \right\} \quad (2.50)$$

At $r = r_c$ (inner cylinder-internal fluid);

$$\left. \begin{aligned} 8. -\sigma_{rr_3}(r_c, \theta, t) &= p_4(r_c, \theta, t), \\ 9. u_3(r_c, \theta, t) &= u_4(r_c, \theta, t), \\ 10. \sigma_{r\theta_3}(r_c, \theta, t) &= \sigma_{rz_3}(r_c, \theta, t) = 0. \end{aligned} \right\} \quad (2.51)$$

The boundary conditions expressed by Equation (2.49) are the Farn

[18] conditions for the solid elastic cylinder. Similarly, Equation (2.49)

and Equation (2.51) are the conditions for the hollow elastic cylinder

with an internal fluid as given by Doolittle and Uberall [19]. These

equations are applicable to the case of no internal fluid when modified

so that the right side of Item (8) of Equation (2.51) will be zero and

Item (9) deleted completely from the field of equations.

The total boundary value problem which is under consideration here

is summarized in Table 2.1. The ten unknown coefficients, $b_n, c_n, \dots, m_n$

found in Equations (2.42) through (2.45), (2.47) and (2.48) can be

evaluated through these boundary conditions. Identification of the coef-

ficient b_n leads directly to the scattered pressure through Equation (2.47).

The remaining coefficients will control various functions from which the

stresses and displacements can be obtained. In order to identify these

TABLE 2.1

ILLUSTRATION OF THE TOTAL BOUNDARY VALUE PROBLEM

REGION APPLICABLE	GOVERNING PARTIAL DIFFERENTIAL EQUATIONS	BOUNDARY CONDITIONS	EVALUATED AT RADIUS
Outer Fluid $r > r_a$	$c_1 \Delta^2 p_1 = \ddot{p}_1$		
Outer Coating $r_b < r < r_a$	$c_2 \Delta^2 \phi = \ddot{\phi}$ $c_2 \Delta^2 A_z = \ddot{A}_z$	$\begin{cases} p_1 = -\sigma r r_2 \\ u_1 = u_2 \\ r \theta_2 = 0 \end{cases}$	$@ r = r_a$
Base Cylinder $0 \leq r \leq r_b$	$c_2 \Delta^2 \psi = \ddot{\psi}$ $c_2 \Delta^2 B_z = \ddot{B}_z$	$\begin{cases} \sigma r r_2 = \sigma r r_3 \\ u_2 = u_3 \\ r \theta_2 = \sigma r \theta_3 \\ v_2 = v_3 \end{cases}$	$@ r = r_b$
Inner Fluid	$c_2 \Delta^2 p_4 = \ddot{p}_4$	$\begin{cases} -\sigma r r_3 = p_4 \\ u_3 = u_4 \\ r \theta_3 = 0 \end{cases}$	$@ r = r_c$

The second partial derivative of the dependent variable with respect to time is represented by the double dot notation.

terms, it is necessary to express the boundary equations of pressures, displacements and stresses in Equations (2.49), (2.50) and (2.51) as functions of the solutions given by Equations (2.42) through (2.45), (2.47) and (2.48).

The Boundary Equations

One can write the stresses in an elastic solid as a function of the

displacements

$$\sigma_{rr}(r, \theta) = \lambda \Delta + 2\mu \epsilon_{rr}, \quad (2.52)$$

$$\sigma_{\theta\theta}(r, \theta) = \lambda \Delta + 2\mu \epsilon_{\theta\theta}, \quad (2.53)$$

and

$$\sigma_{r\theta}(r, \theta) = \mu \gamma_{r\theta}, \quad (2.54)$$

where the strains are given by

$$\epsilon_{rr}(r, \theta) = \left(\frac{\partial u}{\partial r} \right), \quad (2.55)$$

$$\epsilon_{\theta\theta}(r, \theta) = \left(\frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{v}{r} \right), \quad (2.56)$$

and

$$\gamma_{r\theta}(r, \theta) = \left(\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \right) \quad (2.57)$$

([29], Page 288). The radial displacement u and the tangential displacement v can be written as functions of ϕ and A_z from Equation (2.35).

These expressions are

$$u = \left(-\frac{\partial \phi}{\partial r} + \frac{\partial v}{\partial z} \right) \quad (2.58)$$

and

$$v = \left(-\frac{\partial \phi}{\partial \theta} + \frac{\partial u}{\partial z} \right) \quad (2.59)$$

(Since the expressions for the stresses and displacements as functions of the scalar and vector potentials for each layer of the cylinder differ only in symbolic representation, this discussion will be made in terms of

the outer layer. Inner layer notation will be introduced as necessary

to avoid ambiguity).

By taking the appropriate derivatives of u and v in Equations (2.58)

and (2.59) and substituting these results into Equations (2.52), (2.53),

and (2.54), the following stress equations can be written:

$$\sigma_{rr_2}(r, \theta) = -\lambda_2 \left[\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} \right] + 2\mu_2 \left[-\frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} \right], \quad (2.60)$$

$$\sigma_{\theta\theta_2}(r, \theta) = -\lambda_2 \left[\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} \right] + 2\mu_2 \left[\frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} \right] \quad (2.61)$$

and

$$\sigma_{r\theta_2}(r, \theta) = \mu_2 \left[\frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial \Phi}{\partial r} - \frac{1}{r} \frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} - \frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} \right] \quad (2.62)$$

The final expressions for the stresses as functions of the independent

variables (r, θ) can be written by substituting Equations (2.42) and

(2.43) into the above Equations (2.60), (2.61) and (2.62). The results

of this operation yield

$$\sigma_{rr_2}(r, \theta) = p_0 \sum_{n=0}^{\infty} \epsilon_n \{ [\lambda_2 k_2^2 J_n''(k_2 r) - 2\mu_2 k_2^2 J_n''(k_2 r)] c_n$$

$$+ [\lambda_2 k_2^2 Y_n''(k_2 r) - 2\mu_2 k_2^2 Y_n''(k_2 r)] d_n$$

$$+ 2\mu_2 \frac{r}{z} [k_{t_2 J_n''}(k_{t_2} r) - J_n(k_{t_2} r)] e_n$$

$$+ 2\mu_2 \frac{r}{z} [k_{t_2 Y_n''}(k_{t_2} r) - Y_n(k_{t_2} r)] f_n \cos n\theta, \quad (2.63)$$

$$\sigma_{\theta\theta_2}(r, \theta) = P \sum_{n=0}^{\infty} \epsilon_n \{ [\lambda_2 k_2^2 J_n(k_2 r) + 2\mu_2 \frac{r}{n^2} J_n(k_2 r) - \frac{r}{k_2^2} J_n'(k_2 r)] c_n$$

$$+ [\lambda_2 k_2^2 Y_n(k_2 r) + 2\mu_2 \frac{r}{n^2} Y_n(k_2 r) - \frac{r}{k_2^2} Y_n''(k_2 r)] d_n$$

$$+ 2\mu_2 [\frac{r}{n^2} J_n(k_2 r) - \frac{r}{n} k_{t_2} J_n'(k_{t_2} r)] e_n$$

$$+ 2\mu_2 [\frac{r}{n} Y_n(k_{t_2} r) - \frac{r}{n} k_{t_2} Y_n'(k_{t_2} r)] f_n \} \cos n\theta, \quad (2.64)$$

and

$$\sigma_{r\theta_2}(r, \theta) = P \sum_{n=0}^{\infty} \epsilon_n \{ 2n [\frac{r}{k_2^2} J_n'(k_2 r) - \frac{1}{r} J_n(k_2 r)] c_n$$

$$+ 2n [\frac{r}{k_2^2} Y_n'(k_2 r) - \frac{1}{r} Y_n(k_2 r)] d_n$$

$$- [k_2^2 J_n''(k_{t_2} r) - \frac{r}{k_{t_2}^2} J_n'(k_{t_2} r) + \frac{r}{n^2} J_n(k_{t_2} r)] e_n$$

$$- [k_2^2 Y_n''(k_{t_2} r) - \frac{r}{k_{t_2}^2} Y_n'(k_{t_2} r) + \frac{r}{n^2} Y_n(k_{t_2} r)] f_n \} \sin n\theta. \quad (2.65)$$

In a similar manner, the expressions for the displacements u and v

of Equations (2.58) and (2.59) can be written in terms of the solution

equations, Equations (2.42) and (2.43). Hence, the equations

$$u_2(r, \theta) = P \sum_{n=0}^{\infty} \epsilon_n \{ -k_2 [c_n J_n'(k_2 r) + d_n Y_n'(k_2 r)]$$

$$+ \frac{r}{n} [e_n J_n(k_{t_2} r) + f_n Y_n(k_{t_2} r)] \} \cos n\theta \quad (2.66)$$

and

$$v_2(r, \theta) = P \sum_{n=0}^{\infty} \epsilon_n \{ \frac{r}{n} [c_n J_n(k_2 r) + d_n Y_n(k_2 r)]$$

$$- k_{t_2} [e_n J_n'(k_{t_2} r) + f_n Y_n'(k_{t_2} r)] \} \sin n\theta \quad (2.67)$$

are developed.

The expressions for u_3 , v_3 , σ_{rr_3} , $\sigma_{\theta\theta_3}$ and $\sigma_{r\theta_3}$ in the inner cylinder can be obtained from Equations (2.63) through (2.67) by making the transformation of symbols indicated in Table 2.2.

TABLE 2.2

TRANSFORMATION IDENTITIES

OUTER CYLINDERS			INNER CYLINDERS		
Subscript 2			Subscript 3		
ϕ	A	z	ψ	B	z
c_n	d_n		e_n	h_n	
e_n	f_n		j_n	k_n	
f_n			l_n		

In a fluid, the displacement of any particle can be written in terms of the pressure gradient from a force balance of a differential volume. In the radial direction, the relationship

$$-\frac{\partial p}{\partial r} = \rho \frac{\partial^2 u}{\partial t^2} = -\rho \omega^2 u \quad (2.68)$$

can be established if displacement is harmonic in time, $\ddot{u} = -\omega^2 u$. Solving this equation directly for the displacement, u can be expressed as

$$u = \frac{1}{\rho \omega^2} \frac{\partial p}{\partial r} \quad (2.69)$$

The substitution of Equations (2.28), (2.47) and (2.48) separately into Equation (2.69) yields the displacement functions

$$\begin{aligned}
 u_I(r, \theta) &= \frac{\rho_1 \omega^2}{P} \sum_{n=0}^{\infty} e^{-n} k_{1J}'(k_1 r) \cos n\theta, & (2.70) \\
 u_S(r, \theta) &= \frac{\rho_1 \omega^2}{P} \sum_{n=0}^{\infty} e^{-n} k_{1H}'(1)'(k_1 r) \cos n\theta, & (2.71) \\
 u_4(r, \theta) &= \frac{\rho_4 \omega^2}{P} \sum_{n=0}^{\infty} e^{-n} k_{4J}'(k_4 r) \cos n\theta & (2.72)
 \end{aligned}$$

and

for the incident wave, the scattered wave, and the internal fluid, respectively.

The boundary conditions expressed in Equations (2.49), (2.50) and (2.51) can be written as functions of (r, θ) by using Equations (2.63)

through (2.67), their respective transforms, and Equations (2.70), (2.71) and (2.72).

The result of this substitution is a set of ten equations in terms of the ten unknown coefficients. These equations are written out in detail in Table 2.3. Although these equations involve infinite series, a solution for the unknown quantities can be made at each particular value of the index n , independent of the other values. This solution is possible since each $n = k$ value represents an independent solution to the governing wave equations. Hence, the boundary conditions must be satisfied at each index, $n = 0, 1, 2, \dots$, separately, independent of the remaining indices. The final solution to the scattered pressure, stresses, etc. will be the sum of all the independent solutions. The sum total of Table 2.3 can be written in matrix notation as

$$[A]_n \{Z\}_n = \{F\}_n, \quad (2.73)$$

for $n = 0, 1, 2, \dots$.

1.
$$P \sum_{n=0}^{\infty} e^{-n} \{ H^{(1)}(x) b_n + x^2 \}$$
2.
$$P \sum_{n=0}^{\infty} e^{-n} \{ x H^{(1)}(x) b_n' \} \cos n\theta$$
3.
$$P \sum_{n=0}^{\infty} e^{-n} \{ 2 [x J'(x)]^2 - y^2 J'(y) + n^2 J^2(y) \} \sin n\theta = 0$$
4.
$$P \sum_{n=0}^{\infty} e^{-n} \{ x^2 [\lambda J^2(x) - x^2 \lambda J^3(x) - 2 \mu J''(x^4)] q_n - x^2 [\lambda J^3(x^4) -$$
5.
$$P \sum_{n=0}^{\infty} e^{-n} \{ -x J'(x) c_n - 0$$
6.
$$P \sum_{n=0}^{\infty} e^{-n} \{ 2 n \mu [x J'(x) - J'(y) + n^2 J^2(y)] f_n - 2 n \mu [x J^4(x) - (y^4) + n^2 J^2(y^4)] \}$$
7.
$$P \sum_{n=0}^{\infty} e^{-n} \{ n J(x) c_n + n = 0$$
8.
$$P \sum_{n=0}^{\infty} e^{-n} \{ 2 n \mu [x J'(x) - (y^5) + n^2 J^2(y^5)] \sin n\theta = 0$$
9.
$$P \sum_{n=0}^{\infty} e^{-n} \{ -x J'(x) g_n -$$
10.
$$P \sum_{n=0}^{\infty} e^{-n} \{ -x^2 [\lambda J^3(x) - c^2 J^2(x^6)] \cos n\theta = 0$$

$$[A]_n \{Z\}_n = \{F\}_n \quad (3.0)$$

used, the equation

not available in the double precision mode for the particular computer maximum numerical accuracy was desired, and because complex arithmetic was many are void in either the imaginary component or totally void. Since Each of the elements in Equation (2.73) is a complex number, though

Numerical Solution

been obtained through numerical analysis. methods. Consequently, solutions to the layered cylinder problem have tions, a closed form solution cannot be easily obtained through algebraic Table 2.3, Page 26). As a result of this increase in the number of equations to ten in the layered elastic cylinder problem of this investigation (see in obtaining a similar solution [19]. The number of equations increases case of the hollow elastic cylinder, six boundary equations are involved in the scattering coefficient, has been developed by Faran [18]. In the equations. A closed form solution, i.e., an analytical expression explicit elastic cylinder can be derived by the simultaneous solution of three The solution to the problem of plane wave scattering by a solid

Introduction

NUMERICAL ANALYSIS AND RESULTS

CHAPTER III

was expanded into the real part

$$(3.1) \quad [a]_{pq}^n \{x\}^n - [b]_{pq}^n \{y\}^n = \{f\}^n \{x\}^n,$$

and the imaginary part

$$(3.2) \quad [a]_{pq}^n \{y\}^n + [b]_{pq}^n \{x\}^n = \{f\}^n \{y\}^n,$$

where

$$(3.3) \quad [A]_n = [a]_{pq}^n + i[b]_{pq}^n$$

$$(3.4) \quad \{Z\}_n = \{x\}_n + i\{y\}_n$$

and

$$(3.5) \quad \{F\}_n = \{f\}_n + i\{f\}_n \{y\}_n$$

for $p, q = 1, 2, \dots, 10$ and $n = 0, 1, 2, \dots$

Equation (3.2) can be solved for the imaginary component of vector

Z , since

$$(3.6) \quad \{f\}_n \{y\}_n = \{0\}_n$$

for all values of n . Hence, the solution for the real part of Z is

$$(3.7) \quad \{x\}_n = \langle [a]_{pq}^n + [b]_{pq}^n [a]_{pq}^n - [b]_{pq}^n \rangle^{-1} \{f\}_n$$

and for the imaginary part

$$(3.8) \quad \{y\}_n = -[a]_{pq}^n - [b]_{pq}^n \{x\}_n.$$

The square matrix of the expanded coefficient matrix A of Equation (3.3),

$$(3.9) \quad [b]_{pq}^n,$$

can be shown to be void in all elements except b_{11} and b_{21} . As a result,

the modified matrix

$$(3.10) \quad [a]_{pq}^n \text{MOD}$$

can be formed where

$$a_{12MOD}^n = a_{12}^n + b_{11}^n [b_{11}a_{11} + b_{21}a_{12}]^n \quad (3.11)$$

and

$$a_{21MOD}^n = a_{21}^n + b_{21}^n [b_{11}a_{11} + b_{21}a_{12}]^n. \quad (3.12)$$

All remaining elements of Matrix (3.10) are identical to the corresponding elements of the matrix

$$[a_{pq}^n] \quad (3.13)$$

The terms a_{11} and a_{12} in Equations (3.11) and (3.12) are the elements of

the inverse of Matrix (3.13). Consequently, Equation (3.7) can be written

as

$$\{x_p\}^n = [a_{pq}^n]_{MOD}^{-1} \{f_{x_p}\}^n \quad (3.14)$$

and the vector Z can be expressed as

$$\{Z\}^n = \left\langle [I] - I[a_{pq}^n]_{MOD}^{-1} [b_{pq}^n] \right\rangle^n \left\langle [a_{pq}^n]_{MOD}^{-1} \{f_{x_p}\}^n \right\rangle^n, \quad (3.15)$$

where I is the identity matrix.

The solution to Equation (2.73) was obtained in the following manner:

• The value of each of the elements of the coefficient matrix, A,

was computed from information about the incident signal and the

physical parameters of the specified problem. Matrix A was

then separated into its real and imaginary components to form

Matrices (3.13) and (3.9), respectively.

• Matrix (3.13) was subsequently inverted through the application

of the Gauss-Jordan method; thus, elements a_{11} and a_{12} were

identified. The modified matrix, Matrix (3.10), was formed by

replacing a_{11} and a_{12} of Matrix (3.13) with the results of

Equations (3.11) and (3.12). Equation (3.14) was solved for the vector $\{x\}_n^d$ through the application of a method of solution of simultaneous equations. Since the vector $\{y\}_n^d$, as well as the total problem, is dependent upon the accuracy of the solution $\{x\}_n^d$, an error reduction procedure was utilized to improve $\{x\}_n^d$ (see Appendix A). All computations required to identify the vector $\{x\}_n^d$ and $\{y\}_n^d$ have been carried out in double precision arithmetic.

The elements of $\{x\}_n^d$ and $\{y\}_n^d$ were reduced to single precision numbers and combined to form the complex vector $\{Z\}_n$. An incremental far field pressure value was computed from the equation

$$(3.16) \quad \frac{p_o}{p_s} \left[\frac{\pi k r}{2} \right]^{1/2} = i^n e^{i b_n} \cos n\theta$$

which is a single term of Equation (2.47) with the Hankel function replaced with its asymptotic equivalent for large argument,

$$(3.17) \quad H_n^{(1)} \sim \left[\frac{\pi k r}{2} \right]^{1/2} e^{i[kr - \frac{1}{2}n\pi - \frac{1}{4}\pi]}$$

(The exponential function does not appear in Equation (3.16)

since the absolute magnitude of the scattered pressure is computed, i.e. $|e^{i\theta}| = 1$).

This procedure was executed initially with the index n set to zero, then repeated with the index n increasing monotonically. Each solution to Equation (3.16) was added with the $n = 0$ value until the series

$$(3.18) \quad \left| \frac{p_o}{p_s} \right| \left[\frac{\pi k r}{2} \right]^{1/2} = \sum_{n=0}^{\infty} e^{i b_n} \cos n\theta$$

The study of effects on the far field scattering patterns of a hollow elastic cylinder due to the addition of an external coating is the primary objective of this investigation. Insight into the mechanics of scattering has resulted from the comparative analysis of the patterns produced by various coating materials. Consequently, material properties can be ranked as to degree of influence and can be related to other reflection phenomena. Similarly, the range in which a thin shell approach to single layer hollow cylinder scattering is valid has been identified as a function of radius-to-thickness ratio.

The comparative analysis procedure was arranged to show the relation between the patterns produced by a steel cylinder and that same cylinder with a coating. The steel cylinder, hereafter referred to as the "base cylinder," was assumed to have an outer radius (r_o) of 25 cm. Its inner radius (r_i) varied from 23.5 cm to 24.5 cm depending upon the case to be considered and the effect to be illustrated. In addition, this procedural approach permitted comparisons to be made between patterns produced by different coating materials. Each problem was considered for the condition of a fluid-filled interior and the condition of a void interior.

Results

The number of terms required to satisfy the convergence criteria was established by the $\theta = 0^\circ$ position. Justification of the use of a single pressure ray on which to base convergence was established through the large number of problems run in which the boundary conditions were checked and were found to be verified.

The void interior condition has yielded patterns, both in magnitude and

shape, not unlike an air-filled interior.

The different isotropic materials that were used in this investiga-

tion are given in Table (3.1). Listed with each material are the proper-

ties that were used. The values of Lamé constants, λ and μ , and Young's

Moduli, E , were computed from acoustic velocity data [30] using Equations

(2.5), (2.14) and the bar velocity equation

$$C_{\text{bar}} = [E/\rho]^{1/2} \quad (3.19)$$

Similarly, Poisson's ratio was calculated from the Equation

$$\nu = [E/2\mu] - 1 \quad (3.20)$$

Table (3.2) is a tabulation of the various configurations considered

throughout this study. Coating thicknesses were varied from a maximum

thickness of 0.5 cm to a minimum thickness of 0.125 cm. In general, a far

field scattered pressure pattern was computed for each of these configura-

tions at wave numbers varying from 0.25 to 5.0.

The resulting patterns are presented in polar form. In each plot,

the center of the cylinder is positioned at the origin of the polar grid

($r = 0$). Each plot will include the wave number, ka , coating material,

base cylinder inner radius and coating outer radius (r_a). (The inner

radius of the coating and the outer radius of the base are equal). The

incident plane wave is assumed to advance from the 180° position toward

the cylinder. In general, all discussion will be in terms of the back-

scattering conditions unless otherwise specified. All of the patterns

presented are for an external fluid with the acoustic properties of fresh

water. Similarly, the internal fluid, whenever present, is also considered

TABLE 3.1

MATERIAL PROPERTIES

MATERIAL	DENSITY	LAME CONSTANTS			YOUNG'S MODULUS	POISSON'S RATIO
	ρ / cm^3	λ	μ	E	ν	
Lead	11.34	.0197(13)	.049(12)	.0138(13)	.408	
Copper	8.93	.1321(13)	.460(12)	.1256(13)	.365	
Nickel	8.9	.1645(13)	.801(12)	.2137(13)	.334	
Titanium	4.5	.0779(13)	.439(12)	.1161(13)	.322	
Beryllium	1.87	.0158(13)	.1475(13)	.3097(13)	.05	
Magnesium	1.74	.0256(13)	.1619(12)	.0425(13)	.312	
Steel	7.84	.0779(13)	.7779(12)	.2015(13)	.293	
Fresh Water	.998	--	--	.00218(13)*	--	

These properties are from the data given in C.R.C. (30). The units for λ , μ , and E are dynes per cm^2 .

*Bulk modulus

TABLE 3.2

COMPUTATION CONFIGURATIONS

BASE CYLINDER THICKNESS (cm)	COATING THICKNESS (cm)	C	O	A	T	I	N	G	S
2.0	.500 .250 .125	Lead	Copper	Nickel	Titanium	Beryllium	Magnesium		
1.0	.500 .250 .125	X	X	X	X	X	X	X	X
		X	X	X	X	X	X	X	X
		X	X	X	X	X	X	X	X
0.5	.500 .250 .125	X	X	X	X	X	X	X	X
		X	X	X	X	X	X	X	X
		X	X	X	X	X	X	X	X

The outer radius (r_p) of the base cylinder was maintained at 25 cm.

to be fresh water. Figure (3.0) is provided as a reference pattern and represents the far field scattered pressure pattern for a cylindrical void at a wave number of $ka = 1.0$. This is a polar plot of the parameter

$$\frac{|p_s|}{p_o} \left(\frac{\pi k r}{2} \right)^{1/2} \text{ versus } \theta$$

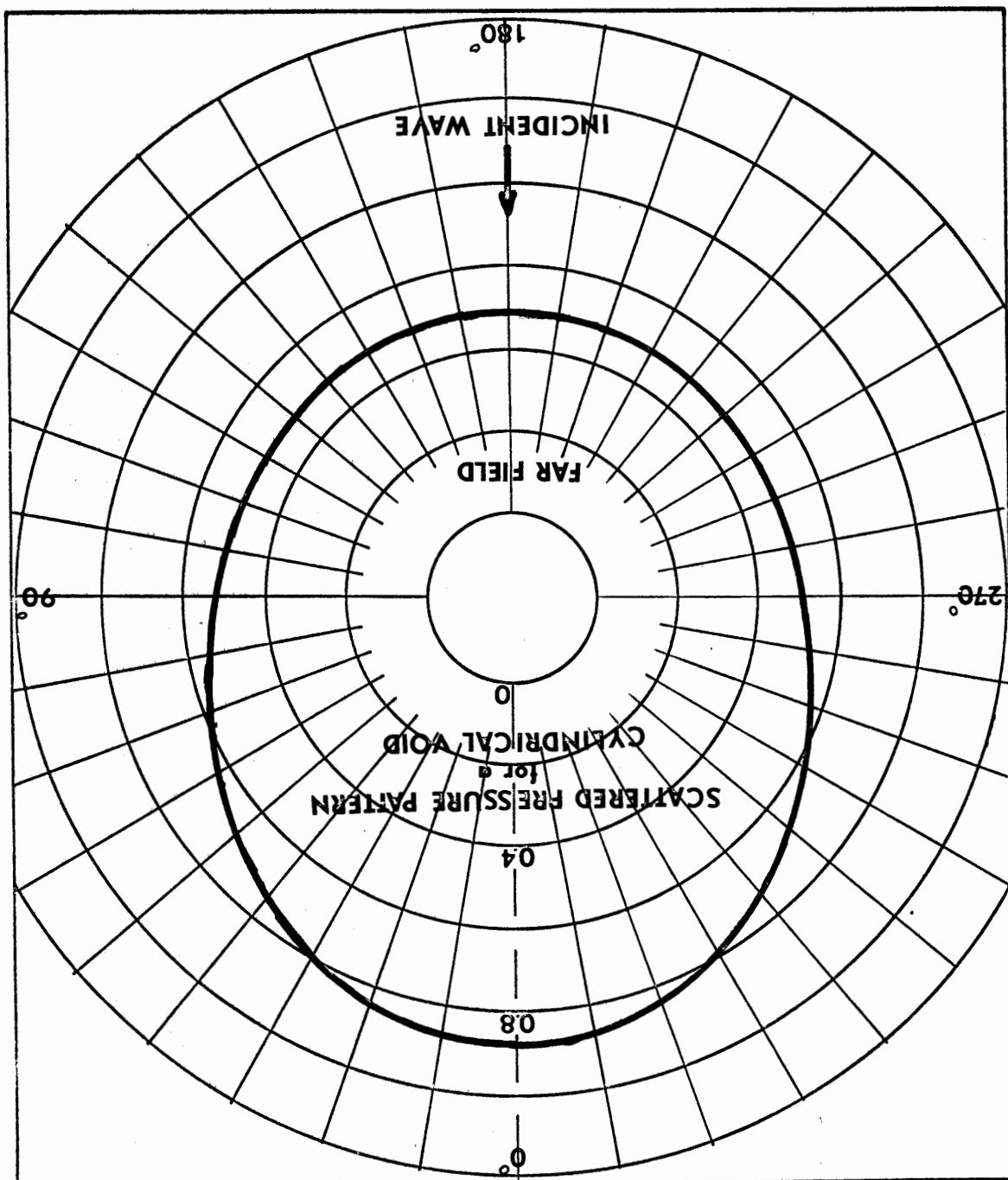
as computed from Equation (3.18).

In the discussion of the results, it is convenient to divide the presentation into five separate parameter studies. These parameters are; (1) wave number, (2) coating thickness, (3) base cylinder wall thickness, (4) coating moduli, and (5) coating mass.

Wave Number Effects

Figures (3.1) through (3.4) are the far field scattered pressure patterns for wave numbers ranging from 0.25 to 5.0. The base cylinder for these four conditions has a wall thickness of 2 cm. ($r_c = 23$ cm.) and is fluid-filled. In each case, the coating is held constant in configuration with a thickness of 0.5 cm. ($r_a = 25.5$ cm.) and has the material properties of nickel. Each plot is divided into two parts using the $\theta = 0^\circ - 180^\circ$ line as the demarcation between the left half-plane and the right half-plane. The patterns on the left half represent the result of water filled cylinders; whereas, the right half represent dry interior conditions. A dashed curve indicates the uncoated base cylinder's pattern and the pattern for the coated case is defined by the solid line. The effect of increasing wave number is seen to be to increase the magnitude of the scattered signal as well as to shift the pattern from a

Figure 3.0. Scattering Pattern for a Void Inclusion. The far field scattered pressure pattern for a cylindrical void of infinite length in an unbounded fluid, water. $ka = 1.0$



some degree of destructive interference is experienced. can enter this region. Under certain conditions, it may be possible that

interior is filled with an acoustic fluid, then the intercepted energy tered outward with a net effect of a pattern shift. Conversely, when the the interior region. Consequently, the intercepted energy must be scat-

In the void configuration, none of the intercepted energy can enter impedance between the base cylinder and the internal and external fluids. into a lens. That is, the coating appears to enhance the matching of the

caused by a quarter wave plate often used to enhance light transmission possible that this effect is a parallel to the impedance matching phenomena (c)). The reasons for this reversal are not immediately obvious. It is

the left half-plane of these same plots (comparing curves (a) to curves (d)). The opposite is true for the water filled configurations shown on than those of their coated counterparts (comparing curves (b) to curves pressures of the uncoated void configuration are less in backscattering

cylinders. Note, that in Figures (3.1) through (3.4), the scattered in the relation between the scattering patterns of the coated and uncoated Paradoxically, this inclusion of water is seen to cause a reversal

of the dry cylinder. cylinder is seen to increase the magnitude of back-scattering over that

scattering sector. The addition of water to the interior of the base

creases, these two curves begin to merge, especially in the forward cylinder exceeds that of the uncoated cylinder. As the wave number in- case of the void interior cylinder, the back scattering of the coated predominate back scatterer to a predominate forward scatterer. In the

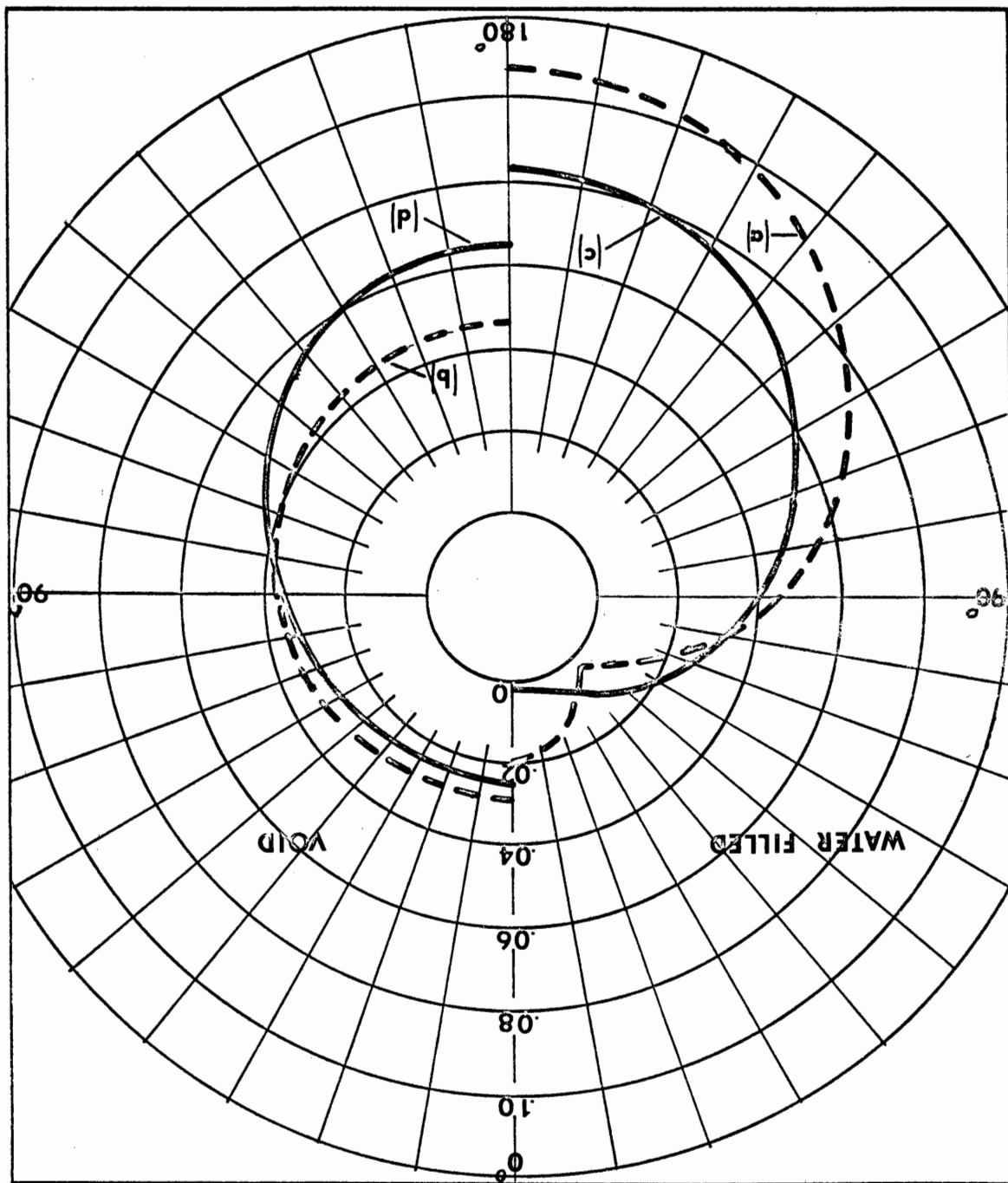


Figure 3.1. Scattering Patterns for a Nickel Coated Cylinder $k_a = .25$. Curves (a) and (b) represent the pressure patterns for the uncoated base cylinder with a wall thickness of 2 cm., Curves (c) and (d) are the shifts in the basic patterns caused by the addition of a Nickel coating of 0.5 cm. $k_a = .257$

Figure 3.2. Scattering Patterns for a Nickel Coated Cylinder $KA = 1$. Curves (a) and (b) represent the pressure patterns for the uncoated base cylinder, wall thickness of 2 cm., $r_0 = 25$ cm., $ka = 1.008$. Curves (c) and (d) are the shifts in the basic patterns caused by the addition of a Nickel coating of .5 cm., $ka = 1.028$.

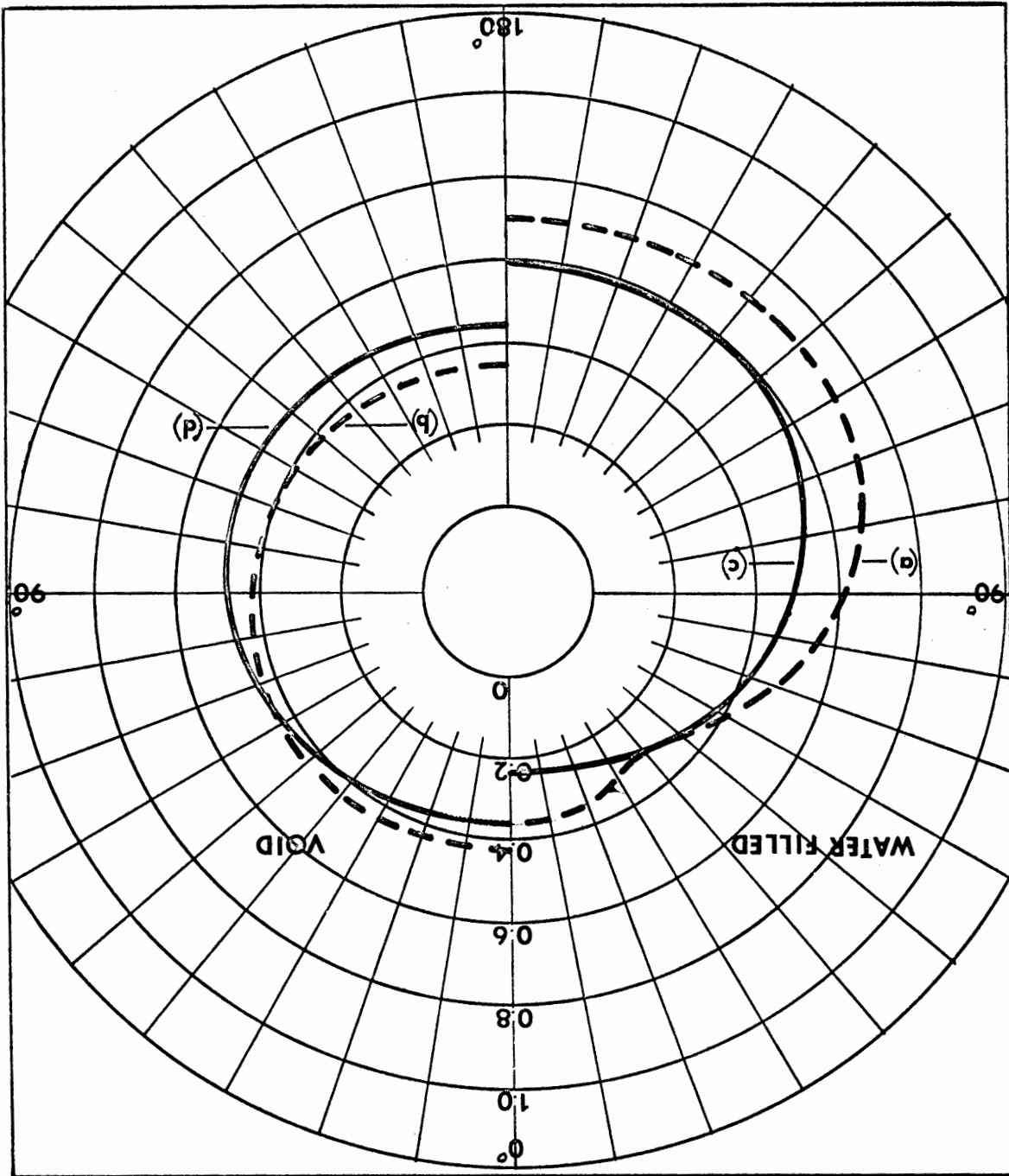


Figure 3.3. Scattering Patterns for a Nickel Coated Cylinder $KA = 3$. Curves (a) and (b) represent the pressure patterns for the uncoated base cylinder with a wall thickness of 2 cm., Curves (c) and (d) are the shifts in the basic patterns caused by the addition of a Nickel coating of .5 cm., $ka = 3.023$.

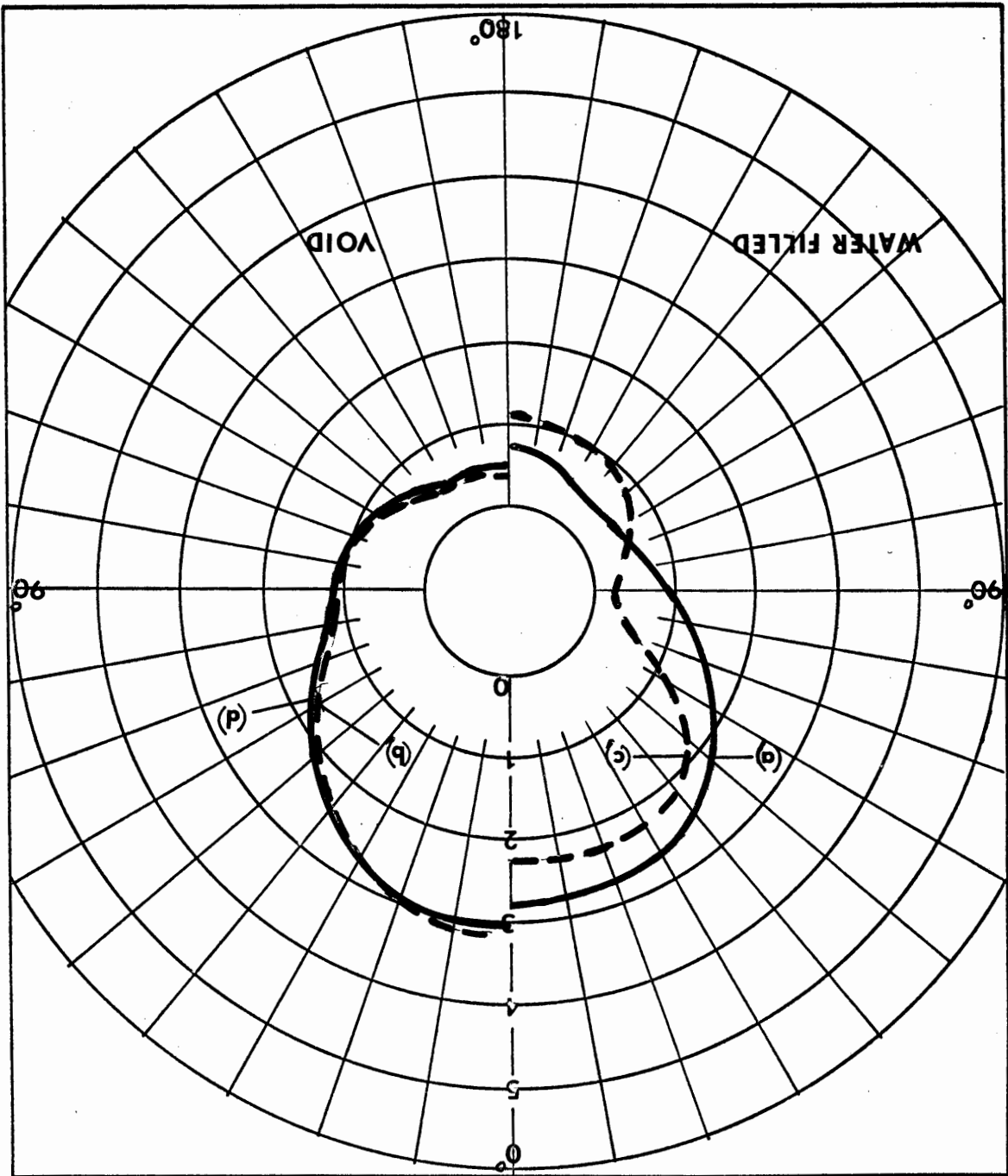
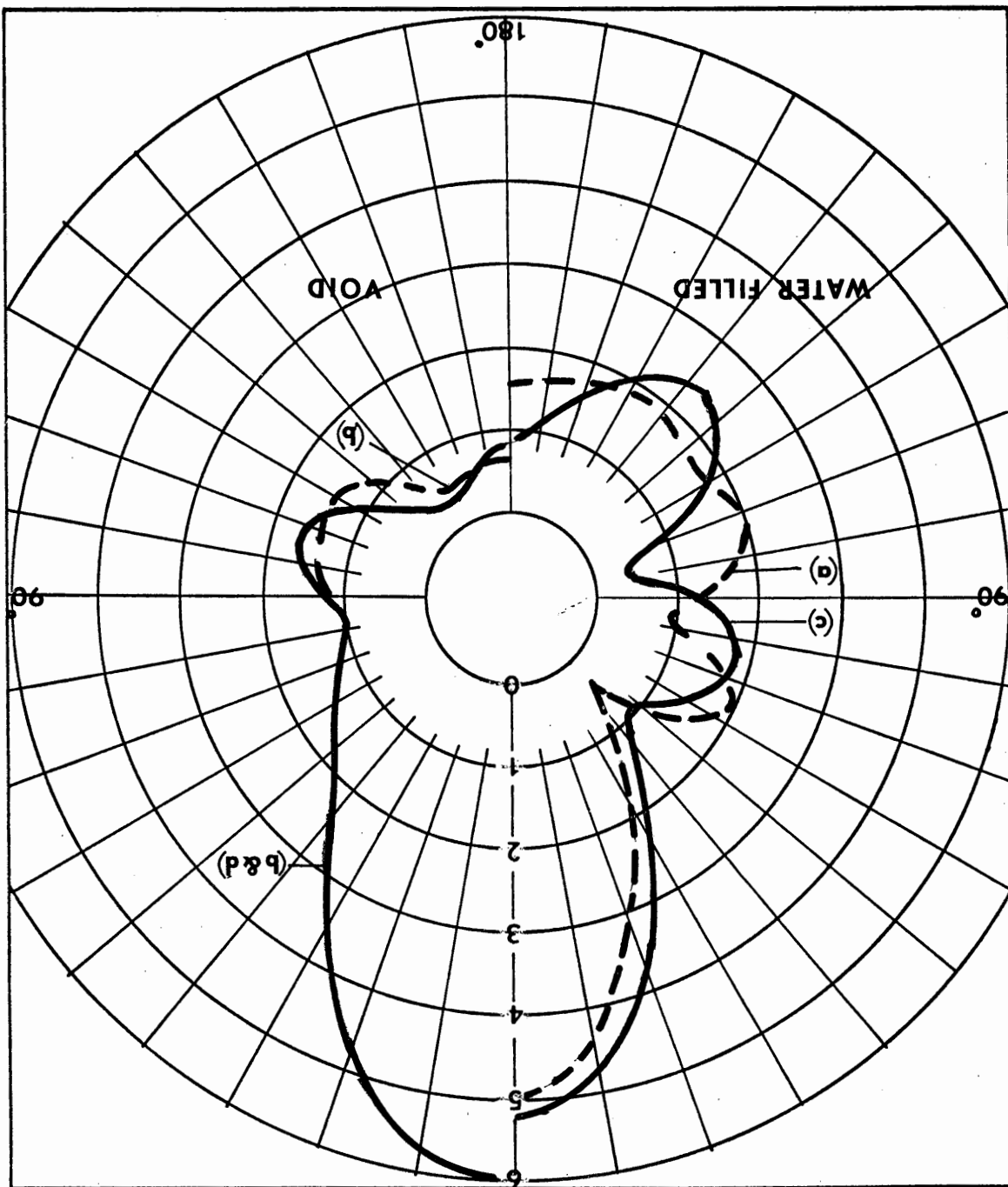


Figure 3.4. Scattering Patterns for a Nickel Coated Cylinder $KA = 5$.
 Curves (a) and (b) represent the pressure patterns for the
 uncoated base cylinder with a wall thickness of 2 cm.,
 $r_b = 25$ cm., $ka = 5.038$. Curves (c) and (d) are the shifts
 in the basic patterns caused by the addition of a Nickel
 coating of .5 cm., $ka = 5.139$.



For the purpose of this discussion, it is sufficient to note that variations of wave number in the range specified (0.25 to 5.0) do not cause a change in the trends for either the water filled or the void configurations. Furthermore, the trends at a wave number of 1.0 are compatible with those observed both at lower values and at higher values of wave numbers. Hence, for the sake of brevity, the remaining results will generally be presented for a wave number of unity.

Variations in Coating Thickness

The patterns of several lead coated cylinders and their respective

uncoated base cylinders are shown in Figure (3.5). Again, the left half-plane represents the water-filled configuration and the void interior condition is shown in the right half-plane. In the case of no interior fluid, the effect of adding the lead coating of 0.5 cm. to the 1 cm. thick base is seen to be to increase the backscattering and decrease the forward scattering, curve (b) compared to curve (d). The reduction of the coating to a thickness of 0.125 cm., causes the pattern (curve (f)) to shift towards the uncoated case. In each of these cases the distortion of the base cylinder's patterns appear to be minimal.

However, for the fluid-filled interior, the addition of the 0.5 cm.

lead coating reduces backscattering and distorts the pattern slightly,

curve (a) to curve (c) in Figure (3.5). This is a reappearance of the

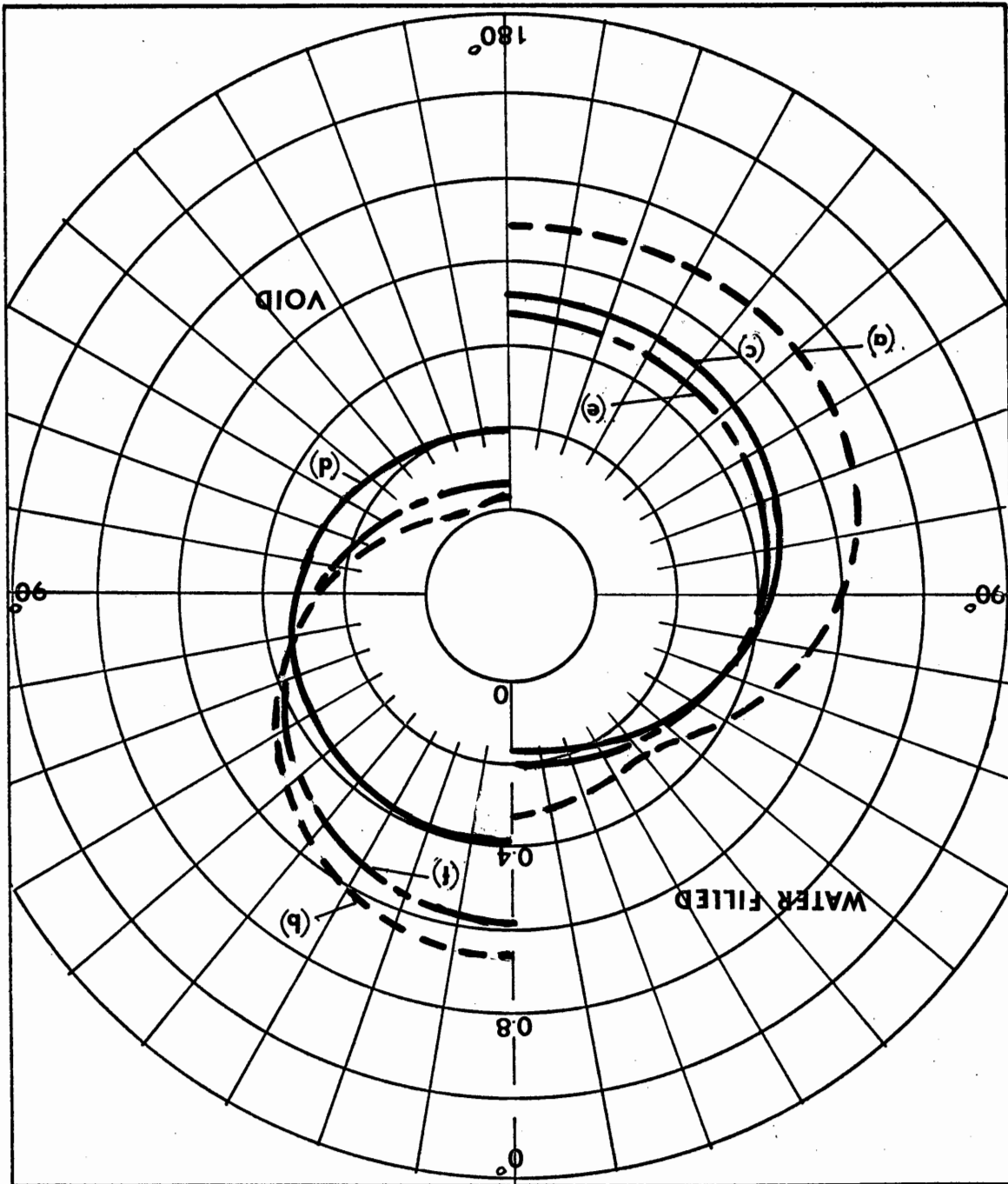
phenomenon mentioned previously, and observed throughout this study. In

each case, the coating seems to perform as an agent of acoustic impedance

matching. Note that reduction of the coating from 0.5 cm., curve (c), to

0.125 cm. thick, curve (e), further reduces the backscattering, moving the

Figure 3.5. Scattering Patterns for a Lead Coated Cylinder. Curves (a) and (b) are for the uncoated base cylinder with a wall thickness of 1 cm., $k_a = 1.008$. Curves (c) and (d) are the shifts caused by a lead coating of .5 cm. and $k_a = 1.028$, whereas, curves (e) and (f) are for a lead coating of .125 cm. and $k_a = 1.013$.



pattern away from the uncoated case. This behavior suggests the possibility of the existence of a particular thickness at which the backscattering is a minimum.

Variations in Base Cylinder Thickness

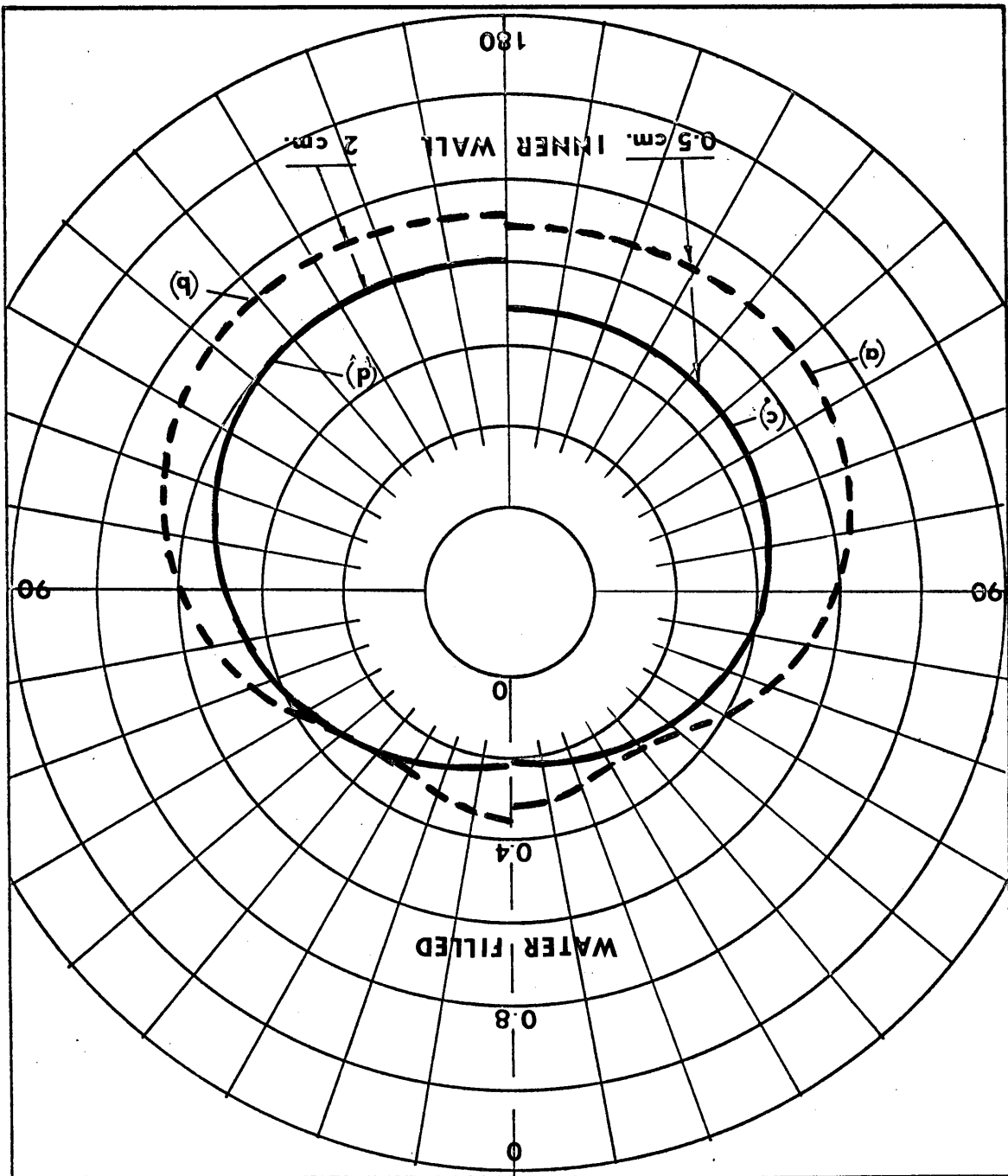
The effects of base cylinder thickness on the scattered pressure

patterns are shown in Figures (3.6) and (3.7). A different presentation of the results is being used. Note that both the left and right half-plane of Figure (3.6) are for the water filled configuration; whereas, both planes are for the void condition in Figure (3.7). However, in both plots, the wall thicknesses of the base cylinder are 0.5 cm. (left half-plane) and 2 cm. (right half-plane). The coating is a 0.5 cm. layer of nickel in all four conditions presented in these two plots. The dashed curves are the reference patterns as established by the uncoated cylinders.

The effects of reducing the base cylinder thickness are similar to the effects produced by reducing the coating thickness. In the water-filled configuration, Figure (3.6), reducing the inner wall thickness reduces the scattering pattern, curve (d) compared to curve (c). The shift is away from the base cylinder patterns, curves (a) and (b), identical to the effect seen in the left half-plane of Figure (3.5). In a similar manner, the reduction of wall thickness in the void configuration, Figure (3.7), moves the pattern to a condition of reduced backscattering and increased forward scattering, curves (d) to (c). The unusual pattern shown on the left half-plane of Figure (3.7) is what is usually expected for vibration near a natural mode for the uncoated cylinder ([4], page 372).

Addition of the coating causes a significant shift away from the natural

Scattering Patterns for a Nickel Coated Cylinder - Water Filled. Curve (a) is the pattern produced by an uncoated cylinder with a wall thickness of .5 cm., $k_a = 1.008$. Curve (b) is the pattern produced by an uncoated cylinder with a wall thickness of 2 cm., $k_a = 1.008$. Curves (c) and (d) are shifts in curve (a) and (b), respectively, due to the addition of a .5 cm. Nickel coating.



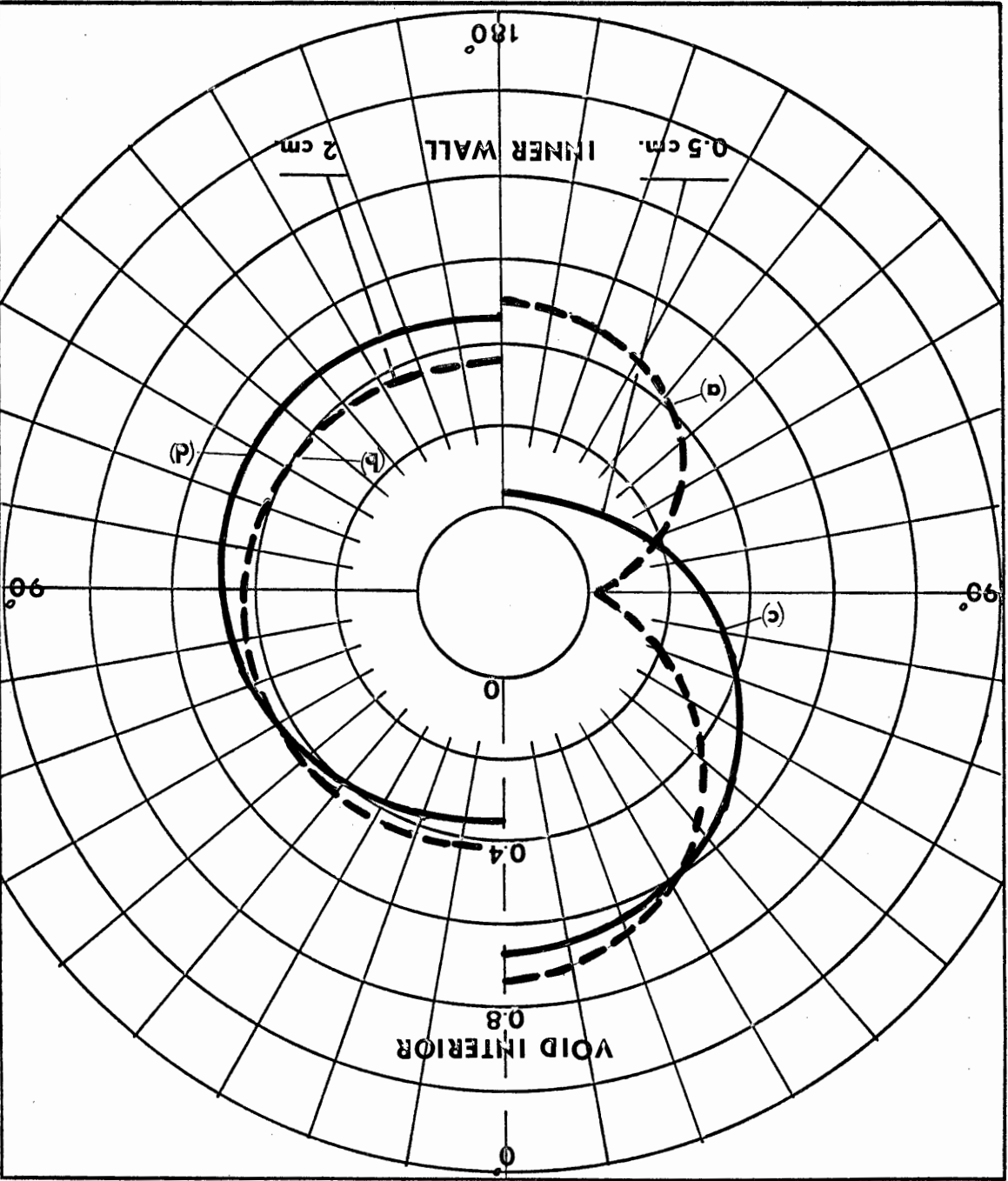


Figure 3.7. Scattering Patterns for a Nickel Coated Cylinder - Void Interior. Curve (a) is the pattern produced by an uncoated cylinder with a wall thickness of .5 cm., $k_a = 1.008$. Curve (b) is the pattern produced by an uncoated cylinder with a wall thickness of 2 cm., $k_a = 1.008$. Curves (c) and (d) are the shifts in curves (a) and (b) respectively, due to the addition of a .5 cm. Nickel coating.

frequency of the uncoated cylinder, as can be noted by the change in the scattering pattern.

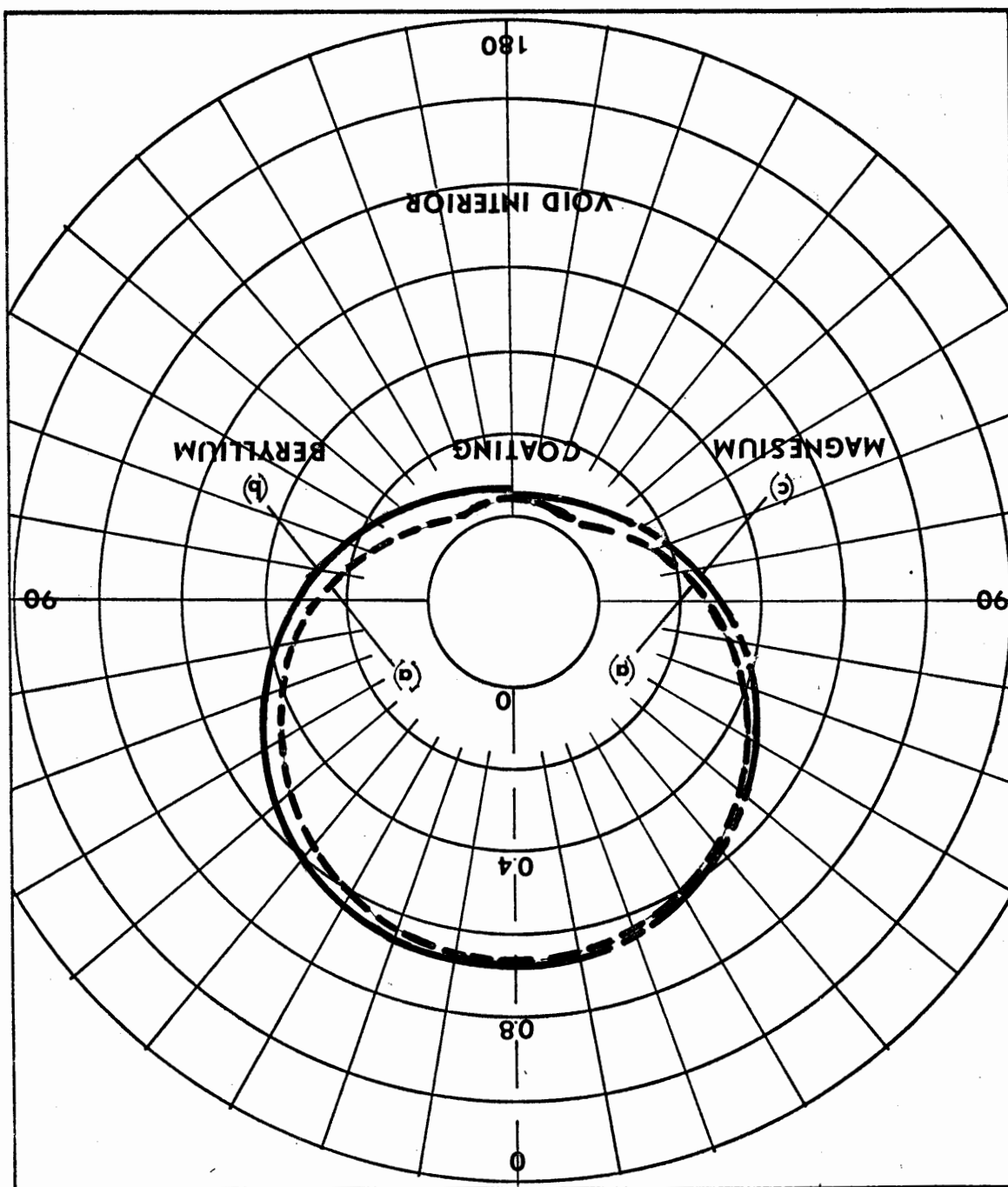
Variation in Coating Moduli

A graphic illustration of the influence of the elastic modulus of the coating is given in Figure (3.8). Again, the dashed line, curve (a), represents the scattered pressure pattern for the base cylinder (outside radius of 25 cm. and wall thickness of 1 cm.). The solid line, curve (b), is the pattern produced by this base cylinder when a 0.5 cm. coating of beryllium is present. A broken line, curve (c), defines the pattern when the base cylinder has a 0.5 cm. layer of magnesium. Note in Table (3.2) that the densities of beryllium and magnesium are very similar, i.e., 1.87 grams per cm.³ for beryllium as compared to 1.74 grams per cm.³ for magnesium. The value of Young's modulus for beryllium is listed as 31×10^{10} dynes per cm.² versus 4.3×10^{10} dynes per cm.² for magnesium. More significantly, the dilatational wave velocity for beryllium is approximately 2-1/4 times greater than that of magnesium, which is indicative of a much stiffer material for the same mass. Thus, it is seen that a large change in material modulus is required to produce even a small change in the scattered pressures. Hence, despite the large difference in elastic moduli, the scattered pressure patterns are very nearly identical.

Variation in Coating Mass

The effect of mass has been isolated in a similar manner by comparing the patterns produced by a copper coating to that produced by a titanium

Figure 3.8. Scattering Patterns for Beryllium and Magnesium. Curve (a) is the pattern produced by the uncoated base cylinder with a 1 cm. wall thickness, $ka = 1.008$. Curve (b) is the shift caused by the addition of .5 cm. of Beryllium and curve (c) is the shift caused by .5 cm. of magnesium.

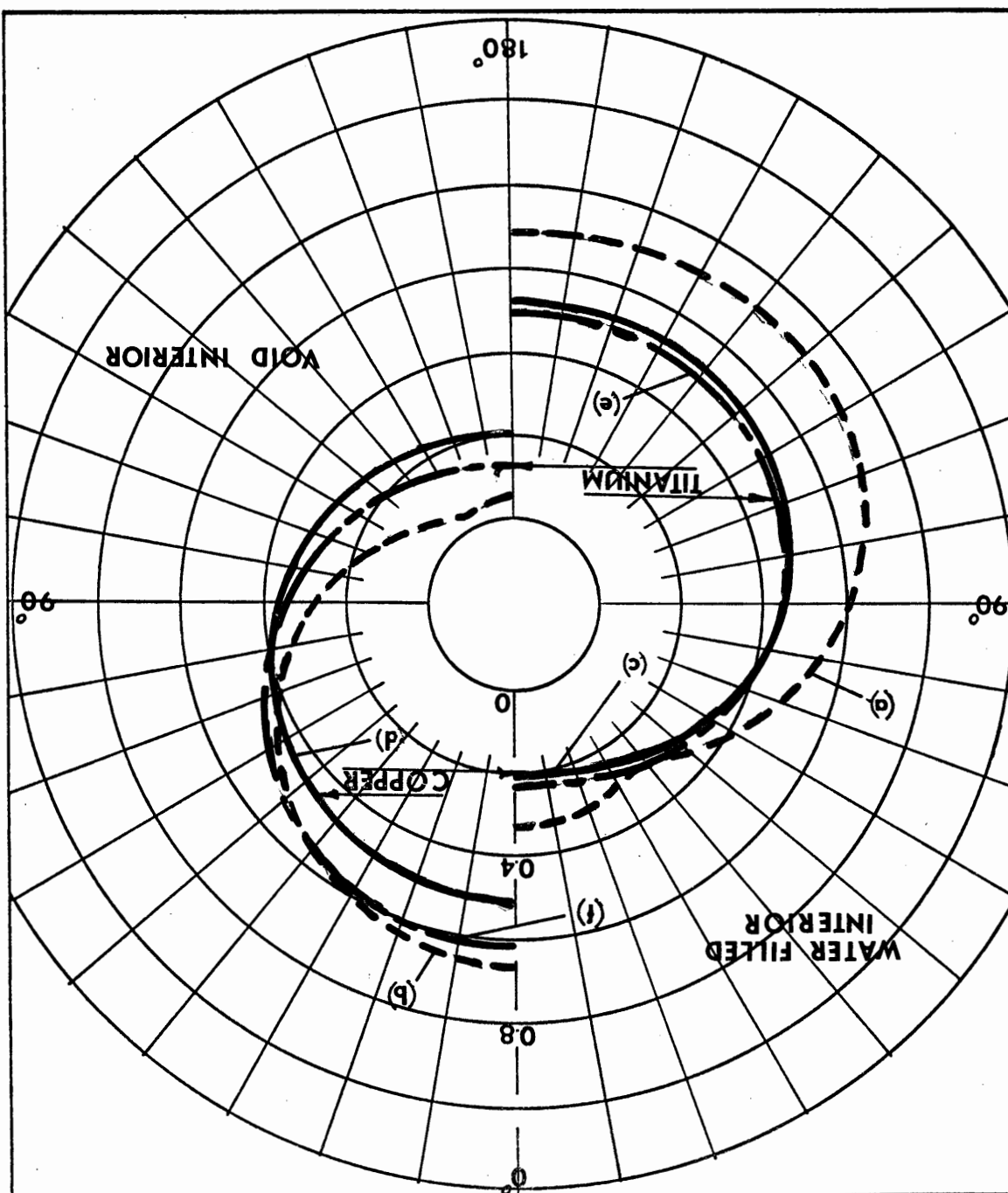


coating. Titanium is fifty-plus percent less dense than copper, but possesses approximately the same value of Young's modulus, i.e., 12.6×10^{10} dynes per cm^2 for copper versus 11.6×10^{10} dynes per cm^2 for titanium. Figure (3.9) is an illustration of the patterns produced by coatings of these two materials on the base cylinder. (Here the base cylinder has a wall thickness of 1 cm. and an outside radius of 25 cm.). Curves (a) and (b) are the reference patterns of the base cylinder. The patterns for the copper coating (0.5 cm. thick) are designated as curves (c) and (d). The titanium coating, also 0.5 cm. thick, produces the scattered pressure patterns given by curves (e) and (f). Again, the left half-plane is for the water-filled configuration and the void interior conditions are on the right half-plane of Figure (3.8). Hence, it is seen that the effect of reduction of coating mass is to reduce backscattering and increase forward scattering, i.e., a shift in the scattered pressure pattern away from the incident signal.

Conclusions

The effect of wave number has been shown to control the overall magnitude level of the scattered pressure pattern as well as its direction, i.e., predominate backscatterer or a forward scattered signal. Consequently, within the scope of this investigation, the presence of a coating does not influence the directivity of the pattern or cause large changes in the overall magnitude. However, in the parametric study, it was seen that the effects produced by a coating are controlled mainly by the density and the elastic moduli of the material. The density is seen to exert a greater influence on the scattered pressure pattern than that caused by

Figure 3.9. Scattering Patterns for Copper and Titanium. Curves (a) and (b) are the patterns produced by the uncoated base cylinder with a wall thickness of 1 cm., $k_a = 1.008$. Curves (c) and (d) are the shifts caused by the addition of .5 cm. of copper. Curves (e) and (f) are the shifts caused by the addition of .5 cm. of Titanium.



the elastic modul. Although there appears to be an effect due to the

thickness of the scattering object, it is, in reality, an influence

which can be related to changes in mass, i.e., density effect, caused by

the removal or addition of material.

The role of elastic modul in the scattering phenomenon appears to

be better explained in terms of the associated wave propagation velocities

than in terms of stiffness. The suggestion is that the scattering charac-

teristics of various isotropic materials might be related through their

material properties, such as density and modulus. Figure (3.10) is a

method that could be used to relate the scattering characteristics of

these types of materials. This graph was developed directly from the re-

sults of this investigation. The magnitude of the backscattered signal

at the 180° position for the various materials tested was normalized with

respect to the largest value (which was for nickel). These ratios for

the water-filled conditions (flagged symbols) and the void interior (open

symbols) were plotted versus material density. It can be seen that two

curves are thereby suggested (dashed lines), both peaking at the symbol

for nickel. Hence, those materials are subordinate to nickel in backscat-

tering capabilities. However, to develop a plot of this type documenting

all isotropic linearly elastic materials would require an investigation of

considerable magnitude and labor.

The scattered pressure pattern of a cylinder without any internal

substance converges to the pattern obtained by scattering from a cylindrical

void as the wall thickness decreases to zero. At any specific wall thick-

ness condition, the addition of a coating tends to stiffen the base cylin-

der and to therefore produce a shift in the pattern with very little dis-

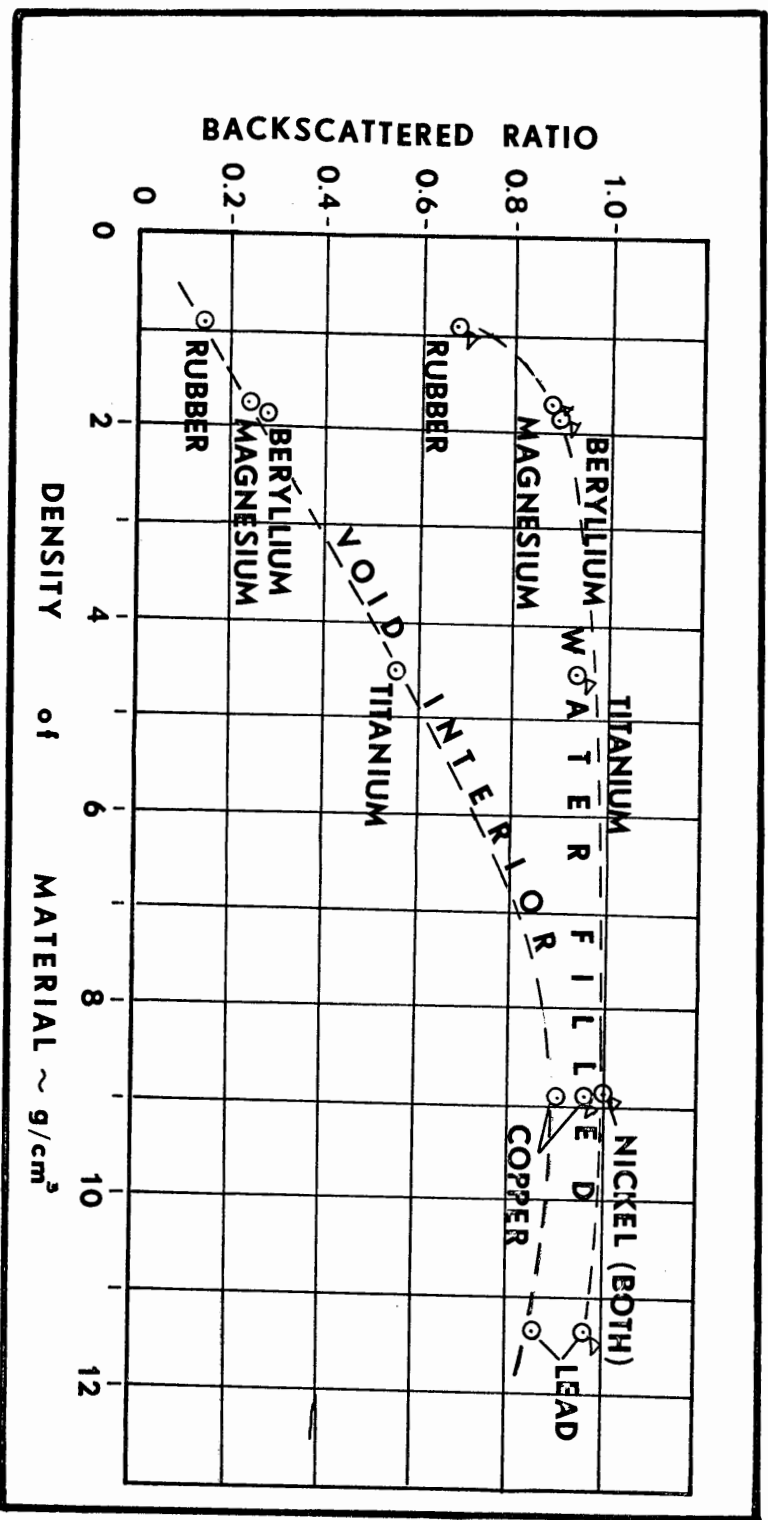


Figure 3.10. Normalized Backscattered Pressure Magnitude versus Material Density

As a result, there is an increase in backscattering along with a decrease in forward scattering; that is, no overall reduction in scattered signal. However, note in Figure (3.2) for example, that the effects of a coating produces an overall reduction in scattering; that is, a partial collapsing of the uncoated pattern. This suggests that the coating serves to change the acoustic impedance, causing a phase shift in the energy transmitted into the interior fluid. Any shift in phase, less than one radian in phase angle, when reflected from the opposite wall of the cylinder, would tend to reduce, or cancel the reflected wave. Since the void interior cylinder cannot admit energy into its interior, preventing the exposure of the signal to possible cancellation effects, the total signal must be reflected.

Junger's formulation of the equations of motion was developed through

Thin Shell Formulation

comparison was made.

ter III for the same cylinder dimensions were computed as well, and a
10 to 500. Patterns based on the elasticity solution formulated in Chap-
for a steel cylinder at various radius-to-thickness ratios ranging from
 b_n , (Equation 16 of Reference 4) was used to compute scattering patterns
In order to accomplish this, Junger's solution for the coefficient
based on radius-to-thickness ratio, for which thin shell theory is valid.
cribed. One of the objectives of this study was to establish the region,
solutions of thin shell theory with the elasticity method previously des-
type of approach, e.g., simpler mathematics, it was desired to compare the
quantities being negligible. Since there are certain advantages to this
are the major contributors to the potential energy expression, all other
ficient, b_n , Junger assumed that changes in curvature and in hoop strains
thin shell theory. In obtaining the equation for the scattering coef-
mulated the differential equation of motion through the application of
hollow cylinder (single layer) has been presented by Junger [4]. He for-
Another method for obtaining the solution of the scattering of a

Introduction

THIN SHELL COMPARISON

CHAPTER IV

the energy method. Initially, the radial and tangential displacements

are defined in terms of two Fourier series

$$(4.1) \quad u = \sum_{n=0}^{\infty} r_n \cos n\theta$$

and

$$(4.2) \quad v = \sum_{n=0}^{\infty} t_n \sin n\theta,$$

where r_n and t_n are generalized coordinates. The Lagrangian, L , can be

expressed as a function of these coordinates where the kinetic energy is

given by

$$(4.3) \quad T = ah\rho_3 \int_{2\pi}^0 (u_z^2 + v_z^2) d\theta,$$

the potential energy is given by

$$(4.4) \quad V = \frac{Eha}{(1-\nu^2)} \int_{2\pi}^0 \epsilon_z^2 \theta d\theta + \frac{Eh^3}{3a} \int_{2\pi}^0 \chi_z^2 \theta d\theta$$

and the generalized forces are given by

$$(4.5) \quad Q_n = -a \int_{2\pi}^0 p_1 \cos n\theta d\theta,$$

where p_1 is the external pressure field at the surface of the cylinder,

ref. Equations (2.28), (2.30) and (2.47). The scattering coefficient b_n

is introduced into the equation of motion through Equation (4.5) and the

definition of p_1 as given by Equation (2.30). Two equations of motion are

obtained in terms of r_n , t_n and b_n which can be solved for the scattering

coefficient. This expression is given as

$$(4.6) \quad b_n = \frac{I_n(Z/\rho_1 c_1) k_1 J_n'(x_1) + J_n(x_1)}{I_n(Z/\rho_1 c_1) k_1 J_n'(x_1) - I_n(x_1) [H_n^{(1)}(x_1)] k_1} \left[\frac{I_n(Z/\rho_1 c_1) k_1 J_n'(x_1) + J_n(x_1)}{I_n(Z/\rho_1 c_1) k_1 J_n'(x_1) - I_n(x_1) [H_n^{(1)}(x_1)] k_1} \right]$$

where

$$Z_n = -2i \frac{h_3}{a_3} \left\{ c_{10} x_1 - \frac{E_3}{E_3} \frac{c_1 (1 - \nu_3^2) x_1}{1 + \frac{n^2}{4} \left(\frac{h_3}{a_3} \right)^2} \right. \\ \left. + \frac{E_3 n^2 \left[1 + \frac{n^2}{4} \left(\frac{h_3}{a_3} \right)^2 \right]}{\left[\omega^2 a_3^2 \rho_3 (1 - \nu_3^2) \right] - E_3 n^2 \left[1 + \frac{n^2}{4} \left(\frac{h_3}{a_3} \right)^2 \right]} \right\} \quad (4.7)$$

is the mechanical impedance.

Results

The results of this comparative investigation are shown in Figures

(4.1), (4.2) and (4.3) for radius-to-thickness ratios of 10, 20 and 50.

Although these figures are for a single wave number, i.e., $k_{1a} = 1.61$,

they are representative of the range investigated, 0.4 to 2.4. Good

agreement in pattern shapes can be observed over the entire ratio range.

However, the maximum error in magnitude can be shown to vary from approximately eighteen percent at the lower end of $R/t = 10$, to a minimum of

three percent at $R/t = 50$. Consequently, in this low frequency range,

thin shell assumptions, as made by Junger, can produce a significant error for R/t ratios less than 50, especially in the forward scattered signal.

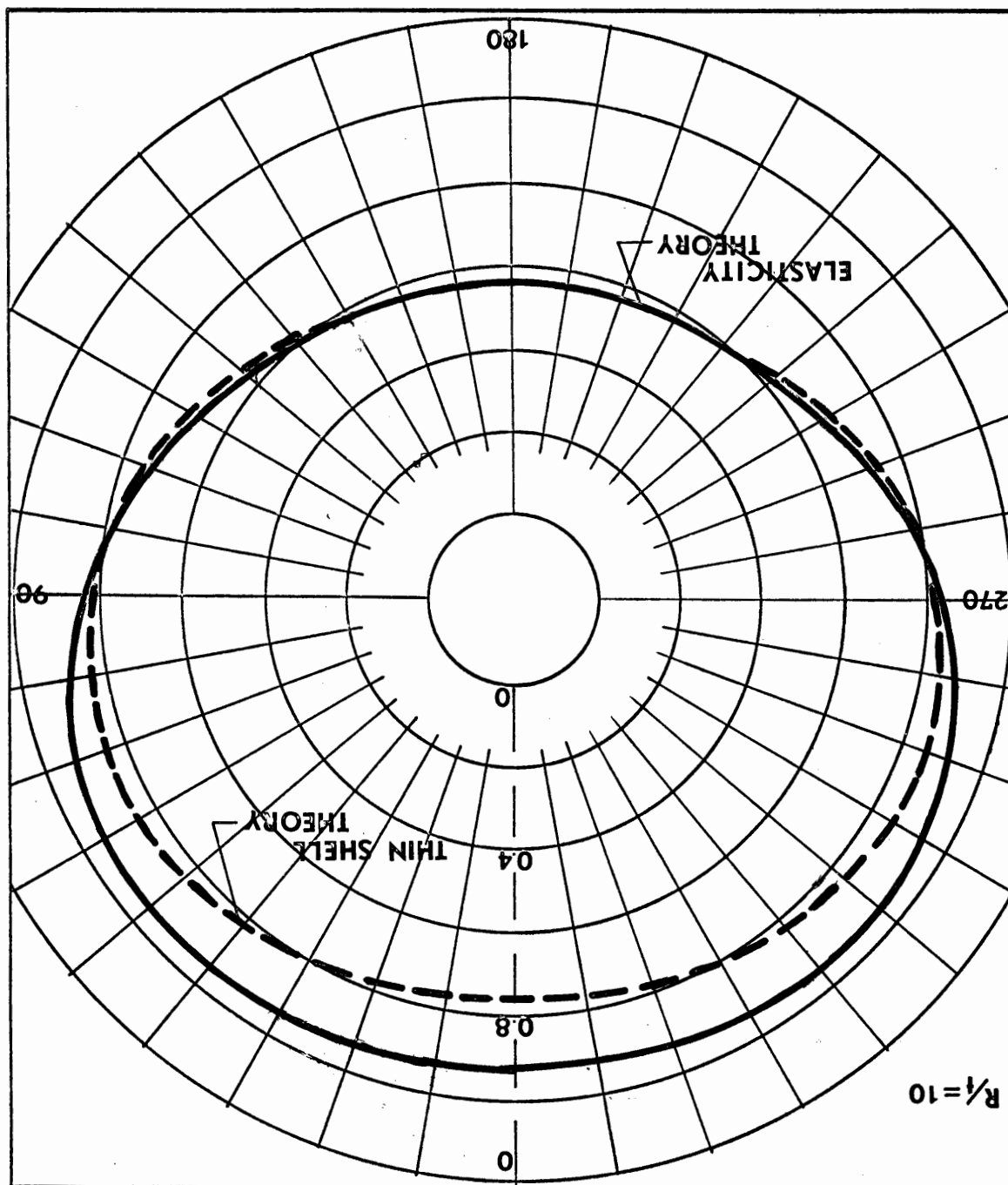


Figure 4.1. Scattered Pressure Patterns for R/t Ratio of 10

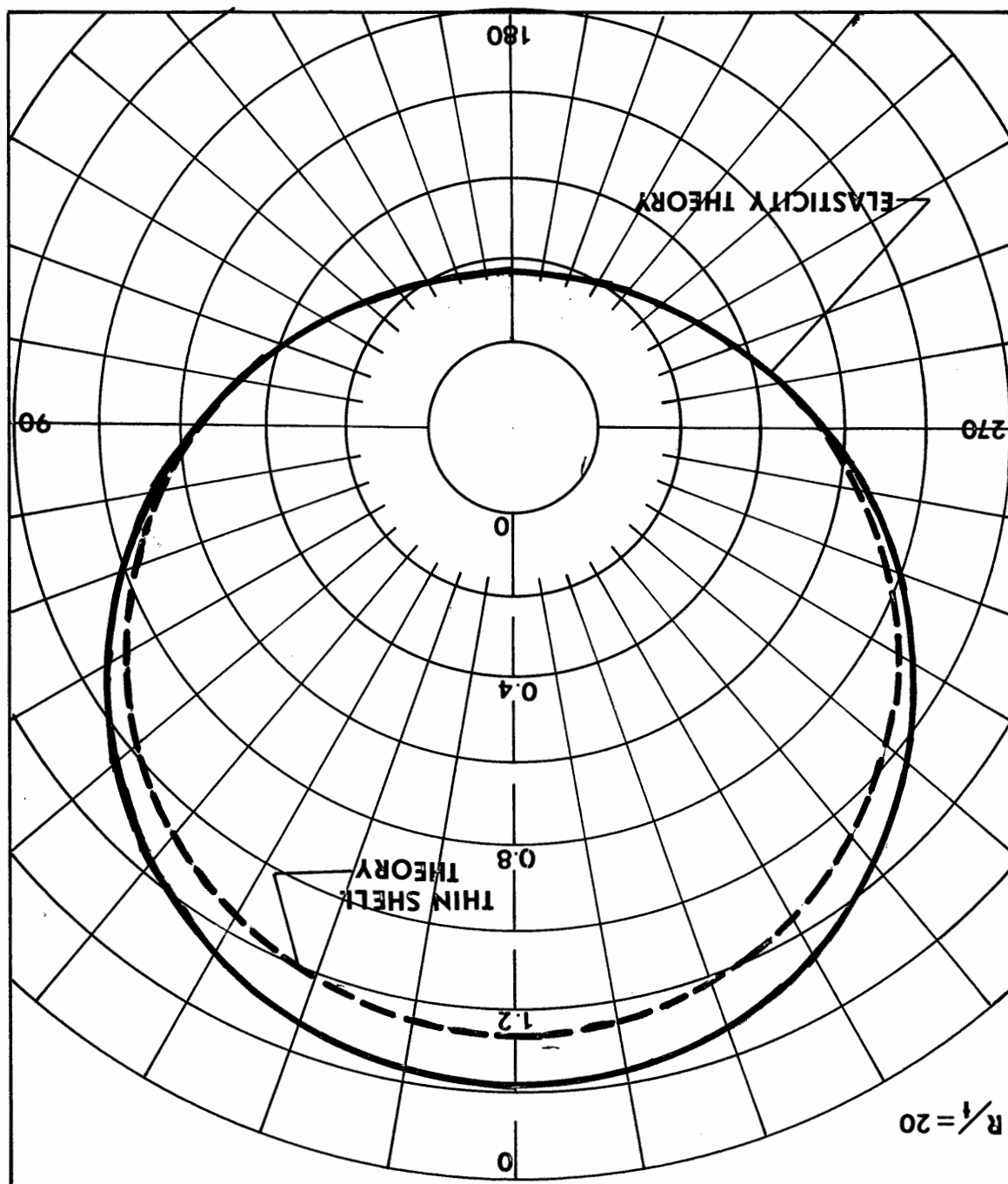
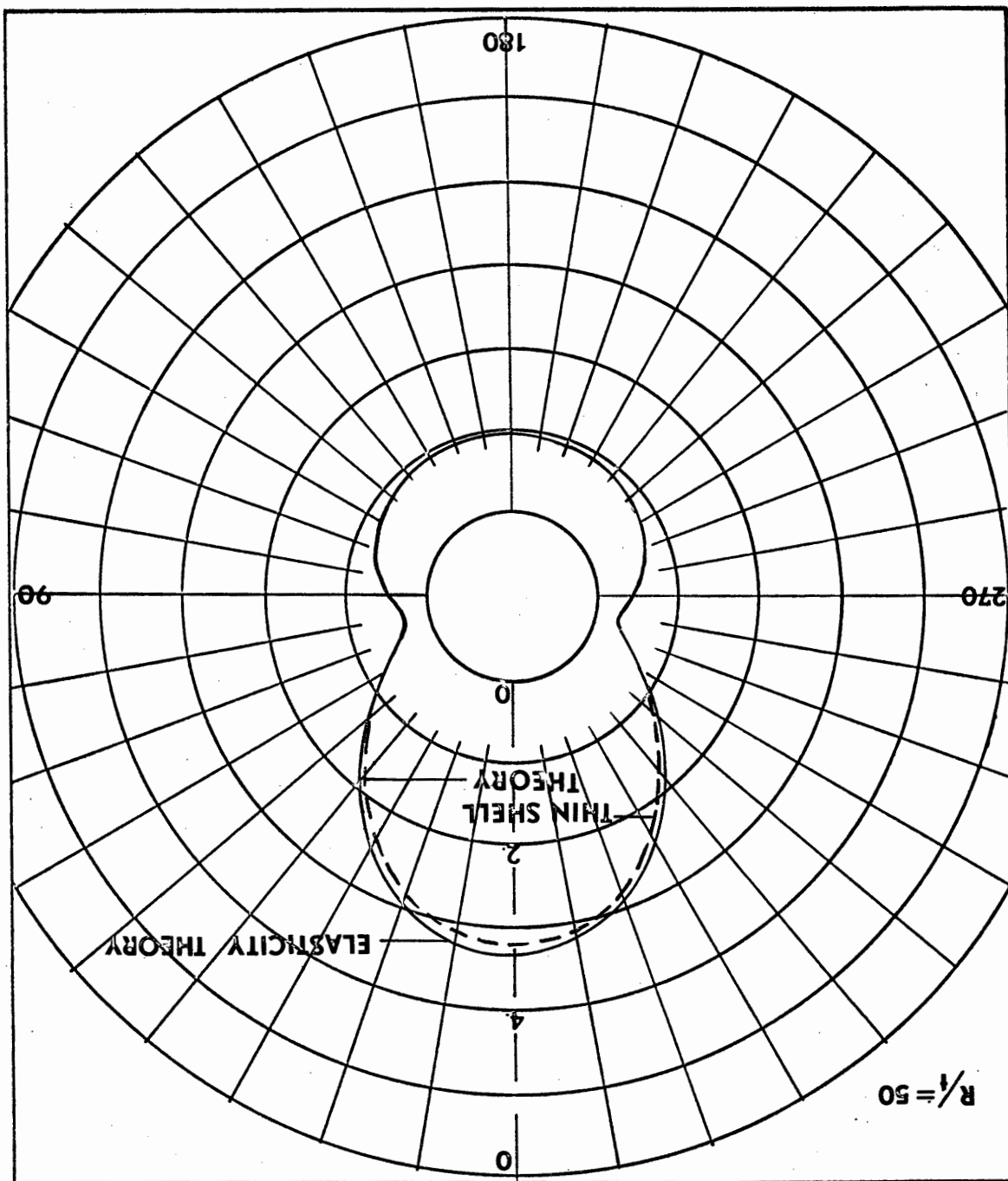


Figure 4.2. Scattered Pressure Patterns for a R/t Ratio of 20

Figure 4.3. Scattered Pressure Patterns for a R/t Ratio of 50



In conclusion, it was found that the scattering of a continuous plane wave by the presence of a layered elastic cylinder can be controlled by the

Conclusions

ing properties, was proposed.

homogeneous isotropic coating materials, based upon their common engineering properties of the

Finally, a qualitative measure of relative scattering properties of the ratios for which the shell theory begins to be in significant error.

theories. This study has also determined the range of radius-to-thickness pattern shapes and magnitudes for thin wall cylinders from two different factor in the solution technique was developed by obtaining agreement in affect the scattering are identifiable. Furthermore, a high confidence analysis of the computed results that the major material properties which terization of the scattered field. It has been shown, from a comparative double precision computations of the complex numbers necessary for character which utilized the peculiar properties of the coefficient matrix to obtain up of elastic materials. A technique was provided to solve this model,

computation of the scattering patterns for a layered hollow cylinder made

This investigation has produced a mathematical model which allows the

Summary

SUMMARY, CONCLUSION AND CLOSURE

CHAPTER V

Several interesting problems have been revealed by this study. In future investigations, the question of some positive measure of coating material scattering properties should be studied. Similarly, the optimum coating thickness should be determined for the case of a fluid-filled cylinder. Note, in Figure (3.5), for a dry interior, the reduction of coating thickness moves the pattern toward the uncoated base cylinder pattern. However, the same reduction in coating thickness for the wet

Closure

mass and elastic moduli of a coating material. Furthermore, the back-scattered pressure level of a dry interior cylinder cannot be reduced below the backscattered pressure level of the base cylinder through the use of coatings. However, the wet interior cylinder, because of a probable "transmission" effect, does allow a reduction in the backscattering when a coating is applied. The shift in pattern has been found to be due largely to the mass of the coating rather than the moduli of the coating. The effects of these two properties are cumulative. Over the range of wave numbers, 0.25 to 5., these trends remain unaffected. Because of the influence of mass and modulus, a relative ranking of the scattering properties of metals can be qualitatively established. In addition, it has been found that the solution to the scattering coefficients developed through thin shell theory become effective in magnitude for radius-to-thickness ratios of 50 or greater. Induced error at low frequencies for a ratio of 50 has been shown to be approximately 3 percent at the zero degree position (forward scattering). Scatterings hold good shape agreement at a ratio of 10 or greater.

work will stimulate additional interest in this field.

It is certain that the questions uncovered herein are more numerous than those that were answered. It is hoped that the presentation of this

reopened.

exigencies. If further support for this work is available, it shall be the research program was postponed to a later date because of scheduling ing this study was not optimized for eigenvalue searches. This part of was found to be excessive; mainly because the computer program support-study to determine the fundamental frequency. The computer time required a characteristic value for the frequency. An attempt was made in this then the determinant of $[A]$ must be zero. Hence, this would indicate of the matrix $[a_{pq}]$ for the homogeneous form of Equation (3.2) is zero, of Equation (3.0) vanishes. It can be shown that, if the determinant is to determine the frequencies at which the determinant of matrix $[A]$ able. One technique, which was given some consideration in this study, ever, explicit equations for the thick-walled conditions are not available. How-natural frequencies of thin shell structures is straightforward. How- ders is of interest as well. In a relative sense, the determination of The question of natural frequencies of thick-walled layered cylinder would be of significant interest.

to various combinations of base cylinder material and coating material hold under these conditions? The effects on the scattered pressure due proaches the coating thickness, is of interest. Do these same trends Similarly, the case of short wave length, the size of which ap-the wet cylinder to achieve minimum backscattering.

The implication is that there may be an optimum coating thickness for cylinder, actually drives the pattern away from the uncoated pattern.

APPENDIX A

The solution to a numerical equation obtained through a digital computer will be in error from the true solution due to round-off, truncation or ill-conditioning. Improvement in the solution can be accomplished through the use of error reduction techniques. One such technique is provided by McCalla [32].

A digital solution to the set of simultaneous equation

$$(A.1) \quad [A]\{X\} = \{B\}$$

can be written as $\{X_1\}$ where the actual solution will be $\{X_1^A\}$. Premultiplying the computer solution $\{X_1\}$ yields a new value for the right side of

Equation (A.1)

$$(A.2) \quad [A]\{X_1\} = \{B_1\}.$$

The error will be the difference between $\{B\}$ and $\{B_1\}$. Hence, the new

matrix equation

$$(A.3) \quad [A]\{\delta X_1\} = \{E_1\}$$

where

$$(A.4) \quad \{B\} - \{B_1\} = \{E_1\}$$

The addition of the solution vector of Equation (A.3), $\{\delta X_1\}$, to the previous solution $\{X_1\}$ will result in a new solution, $\{X_2\}$, which will agree more closely to the actual solution vector. That is,

$$(A.5) \quad \{B\} = \{B_1\} + \{E_1\}$$

or

$$(A.6) \quad [A]\{X\} = [A]\left[\{X_1\} + \{\delta X_1\}\right]$$

Further purification of the digital computer solution can be made by repeating this process, forming a $\{B_2\}$ by

$$(A.7) \quad [A] \begin{bmatrix} \{X_1\} + \{\delta X_1\} \\ \{B_2\} \end{bmatrix} =$$

and a new error vector

$$(A.8) \quad \{B\} - \{B_2\} = \{E_2\} \cdot$$

Hence, the solution to the equation

$$[A] \{\delta X_2\} = \{E_2\}$$

will produce a more refined solution vector, where

$$(A.9) \quad \{X\} = \{X_1\} + \{\delta X_1\} + \{\delta X_2\} \cdot$$

This procedure can be continued until the desired accuracy is obtained or the maximum significant figure of the computer is approached.

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13. ABSTRACT

The scattering of a continuous plane wave by a layered cylinder was investigated through the solution of hyperbolic wave equations and the associated boundary conditions. A basic steel cylinder was assumed to be coated with a single layer of an isotropic linearly elastic material, e.g., copper, nickel, etc., and the resulting far field scattered pressure patterns were computed considering fluid-filled interiors as well as void interiors. Comparative analyses of the resulting patterns over a wave number range of 0.25 to 5.0 have shown that the mass and elastic moduli of the coating materials are of significant influence, and that there is the presence of an apparent transmission phenomenon. In addition, the thin shell solution to the scattering of a single layer cylinder resulted in magnitude differences in the forward scattered portion for radius-to-thickness (R/t) ratios up to 50 when compared to the 2-dimensional elasticity solution. However overall pattern shape and backscattered signal magnitudes remain in good agreement for R/t ratios as low as ten. The solution to the layered cylinder problem was obtained from a matrix of ten boundary conditions and ten unknown "Fourier" coefficients. Equations were developed from which cylinder response and internal stress conditions were acquired. A digital computer was used to solve the equations for the unknown quantities and to compute the patterns of scattered pressure.