Estimation of Residual Motion Errors in Airborne SAR Interferometry Based on Time-Domain Backprojection and Multisquint Techniques

Ning Cao, Hyongki Lee, Evan Zaugg, Ramesh Shrestha, William E. Carter, Craig Glennie, Zhong Lu, and Hanwen Yu

Abstract—For airborne repeat-pass synthetic aperture radar interferometry (InSAR), precise trajectory information is needed to compensate for deviations of the platform movement from a linear track. Using the trajectory information, motion compensation (MoCo) can be implemented within SAR data focusing. Due to the inaccuracy of current navigation systems, residual motion errors (RMEs) exist between the real and measured trajectory, causing phase undulations in the final interferograms. Up to now, MoCo and RME estimation have usually been combined in airborne InSAR to estimate ground deformation. Conventional MoCo methods generally involve azimuthal range resampling and phase correction. Then frequency-domain focusing techniques can be used to generate the SAR images. After focusing SAR images with MoCo, both multisquint and autofocus approaches can be used to estimate RME. In addition to the MoCo-based frequency-domain focusing, the time-domain backprojection (BP) technique can also focus the SAR data obtained from highly nonlinear platform trajectories. In this paper, we present, for the first time, the combination of BP and multisquint techniques for RME estimation. A detailed derivation of the implementation of the multisquint approach using the BP-focusing images is presented. Repeat-pass data from the SlimSAR system over Slumgullion landslide are used to demonstrate the feasibility of RME estimation for both stationary and nonstationary scenes. We conclude that the proposed method can effectively remove the RME.

Index Terms—Backprojection (BP), motion compensation (MoCo), residual motion error (RME), SAR interferometry (InSAR), synthetic aperture radar (SAR).

I. INTRODUCTION

Among the various remote-sensing techniques, synthetic aperture radar interferometry (InSAR) has proven to be a powerful tool for measuring ground deformation with millimeter- to centimeter-scale accuracy [1], [2]. In comparison with the spaceborne SAR, airborne SAR systems have the advantages of flexible scanning geometry and revisit times, easy to maintain and update, and no ionospheric distortion. Therefore, we hypothesize that the routine use of airborne InSAR could greatly expand access to observational data for researchers to better characterize, quantify, and understand the behavior of many natural hazards [3]. However, only a few airborne SAR systems have been successfully deployed for deformation mapping through repeat-pass interferometry, such as the E-SAR (and its successor F-SAR) system from the German Aerospace Center [4], UAVSAR from the Jet Propulsion Laboratory [5], and OrbiSAR from OrbiSat [6]. The major challenge of airborne InSAR for deformation mapping is correcting the observations for deviations in the trajectories estimated by a combination of Global Navigation Satellite System (GNSS) and Inertial Measurement Unit (IMU) observations. Using the trajectory information, motion compensation (MoCo) can be implemented in SAR data processing [7]. The basic procedure of MoCo is to: 1) generate a linear reference trajectory that best fits the actual trajectory; 2) calculate the offset between the reference trajectory and measured trajectory; and 3) apply motion corrections by interpolation and phase compensation. It is also necessary to use an external digital elevation model (DEM) in the MoCo procedure to make topography-dependent phase corrections [8]–[11].

Even after applying MoCo, errors in the trajectory estimated from the GNSS/IMU system, referred to as residual motion errors (RMEs), still exist. The RME can cause erroneous phase undulations in the final interferogram. Recent GNSS/IMU navigation systems can determine the aircraft trajectories with an estimated precision of 1–10 cm [15], [16]. In recent years, several advanced algorithms have attempted to estimate and remove the RME [12]–[17]. For example, in [12]–[16], multisquint techniques were used to estimate RME from the registration error. Through an aperture splitting procedure, SAR images with different squint angles can be obtained, and the corresponding interferograms can be generated. Then, the residual baseline errors can be estimated from the phase differences between the interferograms. In addition to the multisquint methods, advanced autofocus algorithms have also been proposed to compensate for RME [17].

In practice, RME estimation is combined with the MoCo procedure to obtain reliable interferograms. In addition to the MoCo methods mentioned previously, the time-domain backprojection (BP) technique can also focus the SAR data...
obtained from nonlinear platform trajectories [19]–[24]. Unlike the frequency-domain SAR image formation techniques, the time-domain BP method requires trajectory data as well as a reconstruction-grid (such as a DEM) to be known [23], [24]. Compared with frequency-domain focusing, the time-domain BP method has the advantages of: 1) being simple and straightforward; 2) focusing the data obtained from significantly nonlinear trajectories, which is common for airborne platforms (i.e., topography- and aperture-dependent MoCo is inherently taken into account); and 3) focusing the data directly on the desired reconstruction grid thus avoiding the need for geocoding [25]. The major drawback of the BP algorithm is its heavy computational burden, which can be mitigated by employing the fast-factorized BP method [22]. Until now, no study has investigated the use of the BP-based method to estimate RME for the purpose of deformation mapping. Therefore, in this paper, a time-domain RME estimation algorithm is proposed for airborne InSAR data by combining the BP focusing and multisquint RME estimation techniques.

The major advantage of the proposed BP-based RME estimation method is that BP-based methods are more suitable to process airborne SAR data obtained with highly nonlinear flight tracks [23], [24]. The trajectories of such small aircrafts depend on factors, including rugged terrain, atmospheric turbulence, and curvilinear areas of interest, such as traffic routes, pipelines, and rivers. In such scenarios, acquiring SAR data along a nonlinear trajectory would be more cost-effective and time-saving compared with repeatedly flying linear tracks to cover the area of interest. Even though a patchwise frequency-domain focusing and mosaicking technique can be used to process SAR data obtained from highly nonlinear flight tracks, the patchwise processing method is limited and insufficient as discussed in [23]. Therefore, the study of BP-based RME estimation is needed to process such repeat-pass InSAR data obtained with highly nonlinear flight tracks, especially from small aircraft flying at low altitude.

This paper is organized as follows. A detailed description of the BP focusing algorithm and BP-based RME estimation method is given in Section II. Section III provides the experimental results to verify RME removal capabilities for both stationary and nonstationary scenes. Finally, we conclude with Section IV which summarizes our results.

## II. Algorithm

### A. Signal Analysis of the BP Focusing Technology

A brief derivation of the time-domain BP algorithm for a point target is given in this section. The transmitted signal of the radar is normally a “chirp” (linear frequency-modulated signal). For simplicity, the following analysis is based on stop–go approximation, i.e., the antenna remains at the same location during the transmitting and receiving of one pulse. Under the stop–go approximation, the SAR focusing can be broken into two separate steps (range and azimuth compressions). After demodulation and range-compression, the range-compressed signal can be given by

\[ s_{RC}(i) = A_i \exp(-j2\pi f_0 \tau_i) \]  

(1)

where \( i \) is the slow-time pulse index, \( A_i \) is the amplitude, \( f_0 \) is the center frequency, and \( \tau_i \) is the two-way propagation time from the radar antenna to the target. Assuming that \( R_i \) is the real range from the true radar antenna position to the phase center of a target, we have \( \tau_i = 2R_i/c \). An important step for the BP algorithm is to compensate the phase component in (1) using the estimated two-way propagation time \( \tilde{\tau}_i \). Here, \( \tilde{\tau}_i \) can be estimated from \( \tilde{\tau}_i = 2\tilde{R}_i/c \), where \( \tilde{R}_i \) is the estimated radar-target range from the radar position to the physical center of a scattering cell. In general, the target is assumed to be located at the center of the cell. Therefore, as shown in Fig. 1, \( \tilde{R}_i \) can be estimated from the SAR scanning geometry using the trajectory information and specified reconstruction grid. It should be noted that the physical center of the cell represents the estimated phase center of the target, since the exact phase center of the target is unknown. Then, we obtain the phase-compensated signal \( s_{RC}^{\text{comp}}(i) \) as

\[ s_{RC}^{\text{comp}}(i) = s_{RC}(i) \exp(j2\pi f_0 \tilde{\tau}_i) \]

\[ = A_i \exp(-j2\pi f_0(2R_i/c - 2\tilde{R}_i/c)) \]

\[ = A_i \exp(-jk\Delta R_i) \]  

(2)

where \( k = 4\pi/\lambda \) and \( \Delta R_i = R_i - \tilde{R}_i \). \( \lambda \) is the wavelength. It should be noted that a range migration correction procedure is not needed in the BP algorithm, because the range values can be directly estimated from the trajectory data and the reconstruction grid. A basic BP process is to coherently integrate the phase-compensated range-compressed radar echoes \( s_{RC}^{\text{comp}}(i) \) to form the image. For a point target, the backprojected signal is

\[ I_{BP} = \sum_{i=-N}^{M} s_{RC}^{\text{comp}}(i) \sum_{i=-N}^{M} A_i \exp(-jk\Delta R_i) \]  

(3)

Here, \( i \) is assumed to vary from \(-N\) to \( M \) during the integration along the azimuth. \(-N \) and \( M \) represent the range of the azimuth pulse index, which can be determined from the used aperture length. For a system with zero squint angle, we have \( N = M \). Therefore, we can see that the backprojected signal \( I_{BP} \) contains the “residual phase” caused by the range difference \( \Delta R_i \) [24]. The range difference \( \Delta R_i \) has two
components: the offset between the measured antenna position and the true antenna position; and the offset between the physical center and phase center of a scattering cell. In order to further study the properties of the BP algorithm, a detailed analysis of this “residual phase” is needed.

For further analysis, the coordinates of Points $A_i$, $A'_i$, $B$, and $C$ in Fig. 1 are set to $(x_{i,A}, y_{i,A}, z_{i,A})$, $(x'_{i,A}, y'_{i,A}, z'_{i,A})$, $(x_B, y_B, z_B)$, and $(x_C, y_C, z_C)$, respectively. Assuming $\varepsilon(x, y, z) = C - B$ and $\sigma_i(x_{i,x}, y_{i,y}, z_{i,z}) = A'_i - A_i$, the off range $R_i$ and $\tilde{R}_i$ can be expressed as [24], [32]

$$\tilde{R}_i = \|A_i - B\| = \sqrt{(x_{i,A} - x_B)^2 + (y_{i,A} - y_B)^2 + (z_{i,A} - z_B)^2}$$

$$R_i = \|A'_i - C\| = \|A_i - B - (\varepsilon - \sigma_i)\|$$

$$\approx \tilde{R}_i - \frac{(A_i - B) \cdot (\varepsilon - \sigma_i)}{\|A_i - B\|}. \quad (4)$$

Therefore, the range difference $\Delta R_i$ can be estimated from (4) as

$$\Delta R_i = R_i - \tilde{R}_i \approx \frac{(B - A_i) \cdot (\varepsilon - \sigma_i)}{\|B - A_i\|} = \mu_i \cdot (\varepsilon - \sigma_i). \quad (5)$$

where $\mu_i = (B - A_i)/\|B - A_i\|$ is the unitary vector from Point $A_i$ to Point $B$. Therefore, we can see that the range difference $\Delta R_i$ can be estimated by the length of the component of vector $\varepsilon - \sigma_i$ in the instantaneous $A_iB$ direction during scanning. The approximation in (5) is valid, since vector $A_iB$ and vector $A'_iC$ can be assumed to be parallel. Since vector $\varepsilon - \sigma_i$ is not in the slant-range and along-track plane like vector $A_iB$, only the partial component of vector $\varepsilon - \sigma_i$ in this plane can make a contribution to the range difference in (5). Therefore, we assume the components of vectors $\varepsilon$ and $\sigma_i$ in the slant-range and along-track planes are $\varepsilon'$ and $\sigma'_i$, respectively. Therefore, we have $\mu_i \cdot (\varepsilon - \sigma_i) = \mu_i \cdot (\varepsilon' - \sigma'_i)$. Due to the aircraft motion, the slant-range and along-track planes here mean the averaged slant-range and along-track plane during the scanning, respectively (when $i$ changes from $-N$ to $M$).

We further decompose the offset vectors $\varepsilon'$ and $\sigma'_i$ into the slant-range and along-track directions as $\varepsilon' = \varepsilon_r + \varepsilon_a$ and $\sigma'_i = \sigma_{i,r} + \sigma_{i,a}$, respectively. As shown in Fig. 2, (5) can be rewritten as

$$\Delta R_i = \frac{\mu_i \cdot (\varepsilon - \sigma_i) \cdot (\varepsilon' - \sigma'_i)}{\|B - A_i\|} = \mu_i \cdot (\varepsilon_r - \sigma_{i,r}) + (\varepsilon_a - \sigma_{i,a}) \sin \alpha_i \quad (6)$$

where $\alpha_i$ is the angle between vector $A_iB$ and the slant-range direction. It should be noted that the slant-range direction here is the slant range direction at zero Doppler time. During scanning, vector $A_iB$ can be regarded as the instantaneous line of sight (LOS) direction. Consequently, $\alpha_i$ can be regarded as the instantaneous squint angle. Substituting (6) into (2), we can obtain

$$s_{RC}^{\text{comp}}(i) = A_i \exp[-j(k\varepsilon_r - \sigma_{i,r}) \cos \alpha_i - j(k\varepsilon_a - \sigma_{i,a}) \sin \alpha_i] \approx A_i \exp[-j(k\varepsilon_r - \sigma_{i,r}) - j(k\varepsilon_a - \sigma_{i,a}) \alpha_i]. \quad (7)$$

Since $\alpha_i$ is generally very small (i.e., $2^\circ \sim 5^\circ$ for airborne SAR and $<3 \times 10^{-6}$ for spaceborne SAR), we have $\cos \alpha_i \approx 1$ and $\sin \alpha_i \approx \alpha_i$. Consequently, $\alpha_i$ can be seen as a linear function of $i$. Therefore, (3) becomes

$$I_{BP} \approx \sum_{i=-N}^{M} A_i \exp\{-jk(\varepsilon_r - \sigma_{i,r}) - jk(\varepsilon_a - \sigma_{i,a})\alpha_i\}. \quad (8)$$

In order to analyze the effect of the RME on the backprojected signal $I_{BP}$, four scenarios can be considered as in the following. For simplification, we assume that the system squint angle is zero ($M = N$) and the response of the target is symmetric with respect to $i = 0$ (i.e., $A_i \approx A_{-i}$ and $\alpha_i = -\alpha_{-i}$). Moreover, because phase component $k\sigma_{i,a}\alpha_i$ in (8) tends to be much smaller than the other terms, we can safely assume that RME mainly comes from the range error instead of the azimuth error (i.e., $\sigma_{i,a} \approx 0$).

**Case 1 (No RME, i.e., $\sigma_{i,a} = 0$):** If no RME exists, (8) can be rewritten as

$$I_{BP} = A_0 \exp\{-jk\varepsilon_r\} + \sum_{i=1}^{N} (A_i \exp\{-jk\varepsilon_r\} \cdot 2 \cos(k\varepsilon_a\alpha_i))$$

$$= A_0 \exp\{-jk\varepsilon_r\} + \sum_{i=1}^{N} (A'_i \exp\{-jk\varepsilon_r\}) \quad (9)$$

where $A'_i = 2A_i \cos(k\varepsilon_a\alpha_i)$. From (9), it can be seen that the essence of the BP algorithm is a coherent summation procedure. The BP algorithm also only contains the information for the range offset $\varepsilon_r$. Therefore, the backprojected signal $I_{BP}$ for a scattering cell has the form

$$I_{BP} \approx |I_{BP}| \exp\{-jk\varepsilon_r\}. \quad (10)$$

It is known that the azimuth-compressed signal for one target in the frequency domain has the following form:

$$I_{Freq} \approx |I_{Freq}| \exp\{-jkR_0\} \quad (11)$$

where $R_0$ is the shortest range from the antenna to the target at time $i = 0$, in which case $\Delta R_0 = R_0 - \tilde{R}_0 = \varepsilon_r$. The range
offset $\varepsilon_r$ is caused by the uncompensated delay. It should be noted that the system squint angle is assumed to be zero and the slant-range direction here is the slant range direction at zero Doppler time. For the cases of nonzero Doppler centroid, a similar result can be obtained with $R_0$ corresponding to the sensor location where the target is illuminated with the peak antenna pattern. Since $R_0$ is known from the geometry, we can add back the phase component of $R_0$ into $I_{BP}$ as

$$I_{BP} \exp(-jk\bar{R}_0) = |I_{BP}| \exp(-jk(\varepsilon_r + \bar{R}_0)) = |I_{BP}| \exp(-jkR_0). \quad (12)$$

Comparing (12) with (11), it can be seen that the BP algorithm obtains the same phase component as frequency-domain-focusing methods. Therefore, we can perform the SAR interferometry using the backprojected signal $I_{BP}$. In practice, it is not necessary to add back the phase component of $\bar{R}_0$ into $I_{BP}$. The interferogram can be directly obtained by calculating the complex conjugate multiplication of the backprojected signals $I_{BP}$ of the Master and Slave data. In other words, unlike frequency-domain interferometry, time-domain interferometry can be implemented without flattening and removal of topographic phase components. For clarification, in this paper, all the obtained single look complex (SLC) images were generated from signals $I_{BP}$ without adding back the phase component of $\bar{R}_0$. Therefore, the obtained interferograms do not contain topographic phase components. It should also be noted that the BP-focused images are inherently georegistered and projected onto the reconstruction grid [23], [25]. Instead of focusing the image onto the natural range/azimuth geometry of conventional SAR-focusing techniques, BP can directly focus the image onto a desired 3-D reconstruction-grid, which can be generated using an accurate DEM. Therefore, no geocoding is needed in the interferometry procedure.

In the BP method, the motion of the platform is compensated for by calculating the range $\bar{R}_i$ and removing the corresponding phase component. In other words, the topography- and aperture-dependent MoCo is inherently applied to every individual target. Therefore, the BP technique can fully compensate for the aberrations caused by trajectory deviations, given precise antenna positions.

Case 2 (Constant RME, i.e., $\sigma_{i,r} = \sigma_r$): For constant RME, similar to (9) and (10), (8) can be rewritten as

$$I_{BP} \approx |I_{BP}| \exp(-jk(\varepsilon_r - \sigma_r)) \quad (13)$$

which means that a constant RME induces a phase offset directly related to the RME in range.

Case 3 (Linear RME, i.e., $\sigma_{i,r} = \sigma_r \alpha_i$):

$$I_{BP} \approx \sum_{i=-N}^{N} A_i \exp(-jk(\varepsilon_r - \sigma_r) - jk\delta_a \alpha_i) = \sum_{i=-N}^{N} A_i \exp(-jk\varepsilon_r - jk\delta_a \alpha_i) = |I_{BP}| \exp(-jk\varepsilon_r). \quad (14)$$

We can see from (14) that a linear RME does not induce a phase offset. Comparing (8) and (14), a linear RME $\sigma_r \alpha_i$ can shift the target impulse response by $\sigma_r$ in the azimuth direction.

Case 4 (Quadratic RME, i.e., $\sigma_{i,r} = \sigma_r \alpha_i^2$):

$$I_{BP} \approx \sum_{i=-N}^{N} A_i \exp\{ -jk(\varepsilon_r - \sigma_r \alpha_i^2) - jk\delta_a \alpha_i \} = \sum_{i=-N}^{N} A_i \exp\{ -jk\varepsilon_r - jk\delta_a \alpha_i + jk\sigma_r \alpha_i^2 \}. \quad (15)$$

For quadratic and higher order RME, both defocusing and phase offset will occur due to the nonlinear phase component. From the above-mentioned analysis, we can see the effect of RME on BP focusing, and a constant RME causes a phase offset; a linear RME induces an azimuth shift of the target impulse response; and a quadratic (and higher order) RME can cause both defocusing and phase offsets. Similar conclusions have also been stated in the literature in the case of frequency-domain focusing [13], [26], which is expected, since time-domain BP focusing is theoretically equivalent to frequency-domain focusing techniques.

B. BP-Based RME Estimation

For simplification, the derivation in Section II-A has been based on analyzing the signal model of a point target. In order to implement the interferometry, it is necessary to consider a distributed target (DT). Fig. 3 shows a diagram of a DT. Corresponding to (7), the $s_{RC}^{\text{comp}}(i)$ signal for this DT can be expressed as

$$s_{RC}^{\text{comp}}(i) = \int \int A(x, y)W(x, y) \times \exp\left\{ -jk\varepsilon_r(x, y) - jk\varepsilon_a(x, y)\alpha_i \right\} dxdy. \quad (16)$$

where $A(x, y)$ is the complex backscatter at each point on the ground, $W(x, y)$ is the system impulse response, and $\varepsilon_r(x, y)$ and $\varepsilon_a(x, y)$ are the offset functions (corresponding to $\varepsilon_r$ and $\varepsilon_a$, respectively) in the slant-range and along-track direction. Fig. 3. Diagram of a distributed target centered at (0, 0) with size of 2X and 2Y in along-track and ground-range directions, respectively. The red rectangle indicates a small scatterer with a complex response $A(x, y)$. 


obtained by CAO BP can be obtained by splitting the integration procedure in frequency-domain focusing, aperture splitting in time-domain. In order to estimate the RME using a multisquint method of flights, respectively, and \( \delta \) is the range resolution. Consequently, (23) becomes

\[
\langle I_M \cdot I_S^* \rangle = C \cdot \exp\{ jk \Delta \epsilon_r + jk \Delta \epsilon_a \alpha_{(m+n)/2} \} \cdot \exp\{ -jk \cos \theta \delta \theta \} \cdot \delta x \cdot \delta y.
\]

The integration over y can be regarded as a Fourier kernel, which leads to the following expression after normalization:

\[
\langle I_M \cdot I_S^* \rangle = \left( 1 - \frac{2 \cos \theta \delta \theta |R_s|}{\lambda} \right) \cdot \exp\{ jk \Delta \epsilon_r + jk \Delta \epsilon_a \alpha_{(m+n)/2} \} \cdot \exp\{ jk \sigma_{M,S} \}.
\]

Equation (26) gives the interferometry of Master and Slave scanning for subaperture \( m \leq i \leq n \) with effective squint angle \( \alpha_{(m+n)/2} \). It is easy to see that the interferometric phase contains components induced by range direction motion \( \Delta \epsilon_r \), along-track motion \( \Delta \epsilon_a \) weighted by effective squint angle, and RME difference \( \sigma_{M,S} \) between Master and Slave scans. From (26), we can also obtain the spatial baseline decorrelation function as

\[
\delta_{\text{spatial,BP}} = \frac{2 \cos \theta |\delta \theta| |R_s|}{\lambda}.
\]

Equation (27) is identical to the decorrelation function given in [27]. Based on (26), in order to estimate the RME \( \sigma_{M,S} \),
the differential phases for the two interferograms generated from the two subapertures can be obtained as

\[ \Phi^{s1,s2} = \Phi^{s1} - \Phi^{s2} = \arg \left\{ \left( I_M^{s1} \cdot I_S^{s1} \right) \cdot \left( I_M^{s2} \cdot I_S^{s2} \right)^* \right\} \]
\[ = k(\Delta \varepsilon_a(\alpha_{s1} - \alpha_{s2}) + \sigma_{M,S}(s1) - \sigma_{M,S}(s2)) \]  \hspace{1cm} (28)

where \( s1 \) and \( s2 \) denote two subapertures with effective squint angles \( \alpha_{s1} \) and \( \alpha_{s2} \), respectively, and \( \Phi^{s1} \) and \( \Phi^{s2} \) represent the interferograms generated from subapertures \( s1 \) and \( s2 \), respectively. It should be noted that the differential phases are less sensitive to along-track motion \( \Delta \varepsilon_a \) than the RME \( \sigma_{M,S} \) due to the scale factor \( \alpha_{s1} - \alpha_{s2} \). In other words, a short separation of subapertures can reduce the effect of along-track motion on the RME estimation. From (28), the RME estimation can be classified into two cases: 1) stationary and 2) nonstationary scenes.

For stationary scenes without along-track deformation (i.e., \( \Delta \varepsilon_a = 0 \)), (28) becomes

\[ \Phi^{s1,s2} = k(\sigma_{M,S}(s1) - \sigma_{M,S}(s2)). \]  \hspace{1cm} (29)

We can see that the differential phase \( \Phi \) is proportional to the derivative of the RME. The distance between the two looks is given as

\[ \Delta x(R_0) = R_0(\tan \alpha_{s1} - \tan \alpha_{s2}). \]  \hspace{1cm} (30)

From (29) and (30), the RME can be obtained by integrating the differential phase \( \Phi(x, R_0) \) as

\[ \sigma_{M,S}(x, R_0) = \int_0^x \left( \frac{\Phi(x', R_0)}{k\Delta x(R_0)} \right) dx' + \text{Constant} \]  \hspace{1cm} (31)

where \( x \) indicates the along-track position. Equation (31) is identical to the result given in [13]. In other words, the multisquint technology can also be implemented using the BP algorithm to estimate RME.

For nonstationary scenes with along-track deformation (i.e., \( \Delta \varepsilon_a \neq 0 \)), RME cannot be directly estimated from the integration of differential phase \( \Phi(x, R_0) \) because of the phase component caused by along-track deformation \( \Delta \varepsilon_a \). One way to estimate RME for nonstationary scenes is to calculate the difference between two adjacent differential phases \( \Phi^{s1,s2} \) and \( \Phi^{s2,s3} \) as [28]

\[ \Phi = \Phi^{s1,s2} - \Phi^{s2,s3} \]
\[ = k(\sigma_{M,S}(s1) - \sigma_{M,S}(s2)) - k(\sigma_{M,S}(s2) - \sigma_{M,S}(s3)) \]  \hspace{1cm} (32)

where \( s3 \) denotes a third subaperture with effective squint angle \( \alpha_{s3} = 2\alpha_{s2} - \alpha_{s1} \). The phase component caused by \( \Delta \varepsilon_a \) can be canceled out by the double differential phase \( \Phi \), which is the second derivative of the RME \( \sigma_{M,S} \). Therefore, the RME for nonstationary scenes can be obtained by a two-step integration of \( \Phi \) as

\[ \sigma_{M,S}(x, R_0) = \int_0^x \int_0^{x'} \left( \frac{\Phi(x, R_0)}{k\Delta x^2(R_0)} \right) dx'' dx' + \text{Constant}. \]  \hspace{1cm} (33)

Because of the two-step integration, the RME becomes more sensitive to phase noise than the one-step integration given in (31).

A flowchart of the BP-based multisquint technique with three looks is shown in Fig. 4. First, using the BP algorithm, the aperture can be split into \( N_{\text{Look}} \) adjacent looks (e.g., \( N_{\text{Look}} = 3 \) in Fig. 4) and \( N_{\text{Look}} \) interferograms can be generated from corresponding looks of the Master and Slave data sets. Then \( N_{\text{Look}} - 1 \) differential interferograms can be obtained from the \( N_{\text{Look}} \) adjacent interferograms. For stationary scenes without along-track motion, RME can be estimated by integrating the combined \( N_{\text{Look}} - 1 \) differential interferograms. The combination of differential interferograms from different looks can reduce the noise, especially for low-coherence areas. For nonstationary scenes with along-track deformation, \( N_{\text{Look}} - 2 \) double differential interferogram can be generated from the phase difference of the adjacent differential interferograms. Finally, the RME can be estimated from a two-step integration.

It should be noted that the multisquint technique can only be used to estimate low frequency nonlinear RME. Globally constant and linear components of RME cannot be directly estimated in the integration procedure of the multisquint technique. One way to estimate the constant and linear RME is described in [11]: fitting the interferogram with a model and estimating the unknown parameters using weighted least squares estimation. For nonstationary scenes, the quadratic component of the RME should also be estimated from the fitted model. One key component of the multisquint technique is the bandwidth of the subaperture. A long subaperture cancels out RME with frequency higher than the subaperture’s bandwidth [12]. In other words, multisquint techniques can only estimate the average of the true RME. The larger the subaperture, the less accurate the estimation of the high-frequency RME. However, if the subaperture is too short, the differential phases become noisy. It should be noted that three subapertures (\( N_{\text{Look}} = 3 \)) are used in Fig. 4 for simplification. In practice, multiple looks can be used to estimate RME, such as \( N_{\text{Look}} = 5 \) in [14], \( N_{\text{Look}} = 8 \) in [17], and \( N_{\text{Look}} = 9 \) in [28]. As discussed in [14], the optimal
Fig. 5. Optical image of the Slumgullion landslide area from Google Earth with the red outline indicating the boundary of the active portion of the landslide. Two dotted arrows indicate two flight paths, one flown across the earthflow and one flown along the earthflow. The blue and green rectangles denote the corresponding coverage areas, respectively.

### Table I

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Values</th>
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</thead>
<tbody>
<tr>
<td>Center Frequency</td>
<td>1.3075 GHz</td>
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<tr>
<td>Bandwidth</td>
<td>185 MHz</td>
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<tr>
<td>Chirp Time</td>
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<td>Polarization</td>
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</tbody>
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The number of looks and bandwidth depend on the coherence and magnitude of RME and should be selected on a case-by-case basis. In this paper, $N_{\text{Look}} = 6$ was used to estimate RME.

### III. Experimental Results

#### A. Study Area and Data

The Slumgullion landslide located in the San Juan Mountains near Lake City, CO, USA, is a long-term slow moving landslide that moves downhill continuously [29]–[31]. The “neck” of the landslide has the fastest speed of ~2 cm/day. An optical image of the Slumgullion landslide is shown in Fig. 5. In 2015, we conducted a pilot study over the Slumgullion landslide using an L-band airborne SAR system known as ARTEMIS SlimSAR, which is a compact, modular, and multifrequency radar system [3], [32]. The specifications of the SlimSAR system are summarized in Table I.

Two pairs of InSAR data are used to evaluate the performance of the proposed method for both stationary and nonstationary cases. As shown in Fig. 5, the blue dotted arrow indicates the flight path across the earthflow and the green dotted arrow indicates the flight path along the earthflow. The repeat-pass baselines for the across and parallel flights are shown in Fig. 6. The acquisition dates for the across and parallel flights are 20150703-20150710 and 20150703-20150707, respectively. Since the across path were nearly perpendicular to the landslide motion direction, the along-track motion was assumed to be zero and RME could be estimated for a stationary scene case. On the other hand, the InSAR pair for the parallel flight lines should be evaluated as a nonstationary scene due to the landslide motion. Therefore, the across and parallel flights are used to verify RME removal for stationary and nonstationary scenes, respectively.

#### B. RME Removal for a Stationary Scene

For the across path, the initial interferogram and coherence generated using the full aperture are shown in Fig. 7(a) and (b), respectively. The movement of the landslide is clearly visible when compared with the nonmoving areas surrounding the slide. We observe that the surface displacement due to the landslide gradually increases from the bottom to the top. It is known that RME can result in a phase undulation in azimuth

![Fig. 6. Repeat-pass baselines for (a) across and (b) parallel flights, respectively. The deviations of the Master and Slave tracks are shown, within a 10-m tube (black circle) and 20-m tube (green circle). The baselines for approximately 75% of the flight lines are contained within the 10-m tube.](image)

![Fig. 7. Interferograms and coherence maps for the across path collections. (a) Initial interferogram. (b) Initial coherence. (c) Final interferogram after RME removal. (d) Final coherence after RME removal. All interferograms are wrapped and geocoded.](image)
direction [12], [13], which can be seen in Fig. 7(a). Such phase undulation can cause bias in the deformation estimate.

In order to remove the RME, six looks \((N_{\text{Look}} = 6)\) with a bandwidth of 35 Hz each and with center equal to ±8.75, ±26.25, and ±43.75 Hz were generated. In this case, the total used bandwidth is 122.5 Hz and split into six segments with overlap of 17.5 Hz. Then, five differential interferograms were obtained as shown in Fig. 8(a). The phase undulations in the azimuth direction caused by RME are immediately apparent. The RME can be estimated by integrating the combined differential interferometric phases along the azimuth direction. We estimated the RME by iteratively implementing the BP-based multisquint algorithm, as shown in Fig. 9. Since the RME is range-dependent, the estimated residual baseline error shown in Fig. 9 is for the midswath. We can see that the estimated RME was reduced from the maximum of 2.1 cm to less than 0.6 mm after four iterations. The averaged STD of the five differential interferogram was reduced from 0.85 to 0.5 rad after RME removal.

The final interferogram and coherence after removing RME are shown in Fig. 7(c) and (d), respectively. It can be seen from Fig. 7(c) that the phase undulation in the azimuth direction has been correctly removed. For nonlandslide area without motion, the STD of the interferogram has been reduced from 1.13 to 0.75 rad. Comparing Fig. 7(b) and (d), we can see that the coherence has been improved after removing RME, especially for the upper part of the image. The average coherence has improved from 0.38 to 0.45 after the RME removal. Therefore, we can see that the BP-based multisquint technique can successfully estimate the RME for stationary scenes.

### C. RME Removal for Nonstationary Scenes

For the parallel path, the initial interferogram and coherence generated using the full aperture are shown in Fig. 10(a) and (b), respectively. We can see from Fig. 10(a) that the azimuth phase undulation is more severe than the across pair in Fig. 7(a), which indicates stronger RME.

Similar to the stationary case, six looks with a bandwidth of 35 Hz each and with center frequencies equal to ±8.75, ±26.25, and ±43.75 Hz were generated. Five differential interferograms can also be obtained and are shown in Fig. 11(a). The phase stripes in the range direction caused by RME are clearly visible. Besides the phase component caused by the RME, the along-track landslide motion also induced phase components in the differential interferograms. Due to the small separation between two adjacent looks \((\sim 2.5°)\), the effect of along-track motion is not clear in Fig. 11(a). Then, four double differential interferograms were generated as shown in Fig. 12(a). As expected, the double differential interferograms are noisier than the differential interferograms in Fig. 11(a). The RME can be estimated by a two-step integration of the combined double differential interferometric phases along the azimuth direction, as shown in Fig. 13. We can see that the estimated RME was reduced from a maximum of 3 cm to less than 1.2 mm after four iterations. The five differential interferograms and four double differential interferograms after removing RME are shown in Figs. 11(b) and 12(b), respectively. We can see that the stripes have been effectively reduced after RME correction. For nonlandslide area, the averaged STD of the five differential interferogram was reduced from 0.77 to 0.47 rad after RME removal. The averaged STD of the four double differential interferogram was reduced from 0.77 to 0.39 rad after RME removal.
The final interferogram and coherence after RME removal are shown in Fig. 10(c) and (d), respectively. It can be seen from Fig. 10(c) that the phase undulation in the azimuth direction has been correctly removed. For nonlandslide, the STD of the interferogram has been reduced from 1.39 to 0.76 rad after RME removal. The phase noise shown in the lower-right part of Fig. 10(c) is caused by volume decorrelation due to vegetation. The average coherence has been improved from 0.44 to 0.51 after RME removal, as shown in Fig. 10(d). Therefore, we can see that the BP-based multisquint technique can also successfully estimate RME for nonstationary scenes.

The major limitation of the proposed method is the heavy computational burden of the BP algorithm. However, thanks to progress in computing technology, it can be easily mitigated by using alternate computing devices, such as a graphics processing unit (GPU). For example, in this paper, a desktop with a 1.8-GHz Quad Core CPU processor and two GeForce GTX TITAN X GPU devices were used to perform the BP focusing. For an SAR SLC image containing 4228 x 4404 pixels, the average processing time for BP focusing was 73 s. Therefore, the computational burden of the BP method was not a significant concern in this paper.

It should be noted that a conclusion about whether the proposed method outperforms the existing MoCo-based RME estimation methods is difficult to make due to the fact that a “well-designed” MoCo method can obtain similar results to BP [7], [12]. The motivation of this paper is to propose a method that is suitable for an SAR system mounted on a small aircraft toward promoting airborne InSAR applications. Because of its low altitude, it is expected that nonlinear flight tracks are more prevalent for the SlimSAR campaigns. Therefore, BP-based methods are more suitable for the SlimSAR system and can provide more flexibility in the future mission design. The study of BP-based RME estimation is needed for the successful application of small SAR systems, such as the SlimSAR system. Therefore, the comparison of the proposed and existing techniques is beyond the scope of this paper.

IV. CONCLUSION

For the first time, this paper demonstrates the capability of using the BP-based multisquint technique to estimate RME. First, the signal properties of the BP algorithm are discussed to show how the BP algorithm focuses the SAR image with a nonlinear trajectory. It has been shown that both pixel-dependent MoCo and aperture-dependent MoCo are inherently taken into account in the BP algorithm. Then, the effect
of RME on the focused image is discussed and details for implementing the multisquint technique based on BP is given. Experimental results acquired over the Slumgullion landslide area by the SlimSAR system demonstrated that the proposed BP-based multisquint technique has the potential to reduce the RME for both stationary and nonstationary cases. In the long run, the proposed method can be a complement to existing methods to estimate RME and acquire reliable surface deformation from airborne InSAR data.

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REFERENCES


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