Leverage and the Cross-Section of Equity Returns*

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Abstract

Building on the theoretical asset pricing literature, we examine the role of market risk and the size, book-to-market (BTM), and volatility anomalies in the cross-section of unlevered equity returns. Consistent with the theory, the unlevered market beta helps explain the cross-section of unlevered equity returns even when we control for the size and BTM factors. The value premium and the volatility puzzle disappear for unlevered returns, but the size discount remains consistent with Berk (1995). We revisit the relationship between leverage and levered stock returns, which is highly nonlinear. Accounting for heteroskedasticity in stock returns due to leverage confirms the empirical findings from unlevered returns.

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1 Introduction

How does financial leverage affect the cross-section of equity returns? Apart from its intrinsic interest, this question is also relevant because of its relation to well-documented stylized facts on the cross-section of stock returns: the inability of the unconditional CAPM to explain the cross-section of stock returns (Fama and French, 1992) and the robust presence of a wide variety of anomalies. In this paper we focus on unconditional market risk and three important anomalies: the size discount (Banz, 1981; Fama and French, 1992), the value premium (Rosenberg, Reid, and Lanstein, 1985; Fama and French, 1992),[1] and the negative cross-sectional relation between volatility and returns known as the volatility puzzle (Ang, Hodrick, Xing and Zhang, 2006). While there is widespread consensus on the existence of these anomalies, there is much less agreement on their economic interpretation.[2] This paper investigates if these empirical anomalies are (partly) due to financial leverage risk.

Our study differs from the existing literature on the role of leverage in asset pricing because it develops and empirically tests refutable hypotheses regarding the cross-section of *unlevered* equity returns. In developing these testable implications, we argue that for a large class of linear asset pricing models of *levered* (stock) returns that are considered in the literature, systematic (levered) equity risk can be decomposed into systematic unlevered (or asset) risk and (possibly non-linear) representations of leverage risk. This decomposition generates testable implications for the cross-section of *unlevered* returns. The most important testable implication is that the asset beta should be priced in the cross-section of unlevered equity returns.

Although the negative cross-sectional relation between volatility and returns is referred to as the volatility puzzle, we show that an important class of models predicts a negative relation between volatility and levered equity returns. In contrast, these models do not impose restrictions on the relation of volatility to unlevered equity returns. For book-to-market (BTM), some models predict a positive relation between unlevered returns and BTM, while others predict that BTM should not be priced in the cross-section of unlevered equity returns. Finally, various studies have investigated the role of size in an economy without leverage. Most of these studies incorporate the mechanism in Berk (1995), which predicts that the size effect remains in the cross-section of unlevered equity returns. Along a similar vein, Babenko, Boguth, and Tserlukevich (2016) argue

[1] The size and BTM anomalies have been documented for different sample periods, stock markets, and other security markets (Chan, Hamao, and Lakonishok, 1991; Fama and French, 1998, 2012; Asness, Moskowitz and Pedersen, 2013).

[2] The literature provides a variety of possible explanations for the value premium that range from interpreting it as a rational reward for risk (Zhang, 2005; Petkova and Zhang, 2005; Kumar, Sorescu, Boehme, and Danielson, 2008) to the view that they reflect some form of irrational investor behavior (La Porta, Lakonishok, Shleifer, and Vishny, 1997; Chan, Karceski, and Lakonishok, 2003).

that the size discount will emerge if the empirical test design does not correctly account for all sources of priced risk.

We test these theoretical implications using two different empirical approaches. First, we use the Merton (1974) model to infer unlevered equity returns from observed stock returns on levered firms. In our second empirical approach, we do not impose the restrictions of a particular parametric model, but we adjust for the heteroskedasticity induced by leverage and then conduct asset pricing tests using the adjusted returns. Our main findings are consistent across both approaches. Consistent with theory, unlevered market beta helps explain the cross-section of unlevered equity returns even when we control for size and BTM, and the estimates of the market risk premium are large and intuitively plausible. And, consistent with the theoretical predictions of structural credit risk models, our empirical findings confirm the volatility puzzle for levered returns but not for unlevered returns.

We find, however, that the value premium is not present in the cross-section of unlevered equity returns. This finding is inconsistent with theoretical restrictions imposed by a variety of real option models of unlevered firms with dynamic asset composition. Our results also suggest that BTM and volatility affect the cross-section of stock returns primarily through their influence on leverage risk, and suggest that leverage may help explain the value premium and the volatility puzzle. Finally, the size discount remains in the cross-section of unlevered equity returns, consistent with the intuition in Berk (1995) and Babenko et al. (2016). These empirical results are robust. They hold when using univariate and bivariate portfolio sorts as well as Fama-MacBeth regressions, and they are robust to using different structural credit risk models (e.g., Leland and Toft, 1996) and variations in empirical implementation.

But why has the importance of leverage for the BTM anomaly not been uncovered in existing work? Specifically, several studies have investigated the importance of leverage using regressions with (levered) stock returns. We argue that the existing regression approaches fail to fully uncover the importance of leverage for the cross-section of stock returns. The reason is that identifying the relation between leverage and stock returns is challenging because it is nonlinear, and highly so for low volatility stocks. Consequently, existing empirical approaches that use stock returns may not fully capture the impact of leverage because they do not account for the heteroskedasticity (in returns) that arises from the presence of leverage.

Our analysis is related to the extensive empirical literature that directly or indirectly suggests a role for financial leverage and distress in explaining the cross-sectional dispersion in expected stock returns.[3] Bhandari (1988) finds a positive relation between leverage and average stock

[3] In other related work, Engle and Siriwardane (2015) study the impact of leverage on volatility, and Penman, Richardson, and Tuna (2007) decompose book-to-price in enterprise book-to-price and a leverage component.

returns. Fama and French (1992) conclude that "the combination of size and book-to-market seem to absorb the role of leverage" (Fama and French, 1992, page 428). That is, size and BTM subsume not only systematic market risk but also leverage risk.[4]

To our knowledge, our paper is the first to analyze the role of leverage by mapping the universe of stock returns into unlevered equity returns in a convenient and methodologically appealing fashion by relying on structural credit risk models.[5] In contrast to the existing literature that analyzes asset returns, we find that the size and BTM anomalies are impacted by leverage risk in qualitatively different ways. We also document that there is no volatility puzzle for unlevered returns.

We are also the first to emphasize the importance of correcting for heteroskedasticity when analyzing leverage using the cross-section of stock returns. We show how to adjust stock returns to correct for the resulting econometric problems. We demonstrate that the results from unlevered equity returns are consistent with those obtained from cross-sectional regressions of stock returns on leverage, provided we take into account the heteroskedasticity in the data.

The remainder of the paper is structured as follows. Section 2 develops the implications of existing asset pricing models for the cross-section of unlevered returns. Section 3 discusses how to unlever equity returns and the role of heteroskedasticity in levered equity returns. Sections 4 and 5 analyze asset beta, size, and BTM in the cross-section of unlevered equity returns. Section 6 provides evidence on the cross-sectional relation between volatility and returns. Section 7 conducts an extensive robustness analysis and Section 8 concludes.

2 Theory and Testable Predictions

Building on the theoretical asset pricing literature, we distinguish between theoretical predictions for *unlevered equity* (asset) returns and *levered equity* (stock) returns. We differ from the existing literature by formulating testable hypotheses for unlevered returns. We first outline the general framework and fix the notation. Subsequently, we discuss testable hypotheses for several risk factors.

[4] Controlling for size, Fama and French (1992) find a weak negative relationship between book leverage and expected returns. Chan and Chen (1991) argue that the size effect appears to be driven by small firms in financial distress, while Fama and French (1995) highlight the depressed earnings of high BTM firms.

[5] Vassalou and Xing (2004) analyze stock returns for firms with different default probabilities implied by Merton's (1974) model and find that the size and BTM effects only exist in segments of the market with high default risk. They conclude that the size effect is a default effect, and that default risk is also intimately related to the BTM.

2.1 General Framework

Consider a firm with market values of levered equity and debt given by E and D, respectively. The *asset* (or unlevered equity) value of the firm is given by A. Leverage is denoted by L. In this section, leverage may refer to either $D/(D+E)$ or D/E, which will be indicated as needed.

We focus on the class of linear asset pricing models (see Hansen and Renault, 2009; Nagel and Singleton, 2011) in which the equilibrium expected stock return can be written as

$$\mu_L = r + \beta^L \lambda^L, \tag{2.1}$$

where r is the instantaneous riskfree rate, β^L is the systematic stock risk and λ^L is a risk-premium on the levered equity E. We focus on the implications of leverage for three risk factors that have attracted substantial attention in the empirical literature on the cross-section of stock returns: the book-equity-to-market-equity ratio ($BTM \equiv BE/E$) and size (E) (see, for example, Fama and French, 1992) and stock volatility (see Ang et al., 2006).[6] Like most of the literature, we focus on BTM as an indicator of value stocks. Basu (1983) instead uses the earnings-to-price ratio, which is another proxy of the value effect. While the empirical analysis of Ang et al. (2006) uses levered volatility σ^L, our conceptual framework below will also utilize unlevered volatility σ^A. We denote $\boldsymbol{\delta} = (BTM, E, \sigma)$ as the vector of risk factors of interest.

The empirical literature on financial leverage typically decomposes the systematic risk of levered equity β^L in terms of (1) the systematic risk of the underlying asset value A, denoted by β^A, and (2) shareholders' systematic risk due to bankruptcy exposure, denoted by the function $\Phi(L, \boldsymbol{\delta}, Z)$, where Z denotes a vector of covariates other than L and $\boldsymbol{\delta}$ that affect systematic bankruptcy risk. That is, β^L is represented through some functional $f(\beta^A, \Phi(L, \boldsymbol{\delta}, Z))$. Note that, because an unlevered firm does not face any systematic bankruptcy risk, we must have $\Phi(L = 0, \boldsymbol{\delta}, Z) = 0$.

While the functional form of f is theoretically unrestricted, for empirical tractability we focus on the cross-section of expected asset returns using the following linear specification:

$$\mu_A = r + \beta^A \lambda^A + \boldsymbol{\gamma}_1^A BTM + \boldsymbol{\gamma}_2^A E + \boldsymbol{\gamma}_3^A \sigma^A. \tag{2.2}$$

[6]Several other important cross-sectional anomalies may also be related to leverage. These include momentum (Jegadeesh and Titman, 1993; Asness, Moskowitz and Pedersen, 2013; Garlappi and Yan, 2011), betting against beta (Frazzini and Pedersen, 2013), investment (Zhang, 2005), and profitability (Novy-Marx, 2013; Fama and French, 2015; Hou, Chen, and Zhang, 2015). We limit ourselves to BTM, size, and volatility in order to maintain focus and because of space constraints.

We now develop testable hypotheses on equation (2.2).

2.2 Asset Market Beta

As noted above, the existing literature decomposes the systematic risk of stocks in terms of the asset (or unlevered) beta and systematic bankruptcy related risk. Below we discuss models of levered equity in the literature that utilize various parameterizations of Φ, as well as models of unlevered firms where $\Phi = 0$. However, all models we discuss share a common implication regarding the price of systematic asset risk in equation (2.2):

Implication I: Positive price of systematic asset risk, that is, $\beta^A > 0$.

2.3 Size

Empirical discussions of the size discount (Banz, 1981; Fama and French, 1992) refer to levered returns. But should a size discount also obtain in unlevered returns? Berk (1995) presents a stylized economy in which the size discount obtains due to a simple relation between risk and return. In a similar vein, Babenko et al. (2016) argue that a size discount will emerge if all sources of priced risk are not correctly accounted for. Notably, the arguments in Berk (1995) and Babenko et al. (2016) do not depend on leverage and therefore also apply to unlevered equity returns. That is, these arguments suggest that $\gamma_2^A < 0$ in equation (2.2).

A growing number of papers incorporates capital investment and growth options in equilibrium asset pricing models to provide economic intuition for the size effect. Importantly, many of these models consider *unlevered* firms and derive analytical representations of the firm's priced risk factors that include size. For instance, Carlson, Fisher, and Giammarino (2004) use the assumption of declining growth opportunities to argue that larger equity values (size) reflect a lower proportion of risky growth options, and hence imply a negative relation between size and expected returns for an unlevered firm. The mechanics of size in this model can be interpreted as providing further structure to the Berk (1995) economy. In Berk, Green and Naik (1999), controlling for BTM, the firm's equity value (size) is a state variable that captures the relative importance of the firm's assets-in-place (AIP) and growth options and is negatively related to the expected equity return on an unlevered firm. The role of the size as a risk factor is somewhat different here, because the size discount disappears in the absence of dynamic variation in the relative importance of growth options.

We conclude that the existing theoretical literature predicts that the size effect remains in the cross-section of asset (unlevered equity) returns in equation (2.2) (Berk, 1995; Babenko et al., 2016):

Implication II: Negative price of size in the cross-section of unlevered equity returns, that is, $\gamma_2^A < 0$.

2.4 Book-to-Market

Several strands of the theoretical literature make predictions regarding the book-to-market effects in unlevered and levered returns. We first consider the implications of real options models of unlevered firms. Subsequently we discuss models with financial leverage, including those with exogenous and strategic default.

In Carlson, Fisher, and Giammarino (2004), firms invest to exercise growth options and convert these options into AIP, which raises the firms' long run operating leverage. In economic downturns or periods of low demand, the risks of AIP increase. The systematic risk of high BTM firms is therefore especially amplified, because of their greater operating leverage. In a similar vein, Zhang (2005) models costly capital reversibility and a countercyclical price of risk to generate a greater systematic risk for high BTM firms, ceteris paribus. In Berk, Green and Naik (1999), BTM is a state variable that, at any point in time, is positively related to the future systematic risk of the firm's assets. And in Babenko et al. (2016), idiosyncratic cash flows are negatively related to BTM and the firm's systematic risk. These models therefore predict a positive cross-sectional relation between BTM and unlevered equity returns, that is, $\gamma_1^A > 0$.

On the other hand, several models of equilibrium *levered* returns imply the absence of BTM effects in the cross-section of unlevered equity returns. Consider, first, the widely used textbook treatment of asset and levered beta in a CAPM framework (Hamada, 1972), which posits $\beta^L = \beta^A \left[1 + (1-t)\frac{D}{E}\right]$. That is, the functional $f(\cdot,\cdot)$ is multiplicatively separable and $\Phi(L, \boldsymbol{\delta}, Z) = (1-t)\frac{D}{E}$, where t is the marginal corporate income tax. In this case leverage is the only risk factor besides asset beta that determines the levered beta and hence BTM is not priced in the cross-section of unlevered returns.

While the textbook approach takes leverage as fixed and does not consider financial distress costs, a more recent literature develops real options models for levered firms; these models make leverage endogenous and/or allow financial distress costs. In several of these models (Gomes and Schmid, 2010; Garlappi and Yan, 2011), BTM is related to leverage and financial distress costs and the levered beta is decomposed into the asset beta and the bankruptcy related systematic risk in a multiplicative fashion, i.e. $\beta^L = \beta^A \left[1 + \Phi(L, \boldsymbol{\delta}, Z)\right]$. For our purpose the most critical observation is that these predictions regarding the role of BTM are about the cross-section of levered returns. For unlevered returns, BTM is not priced.

In sum, based on the predictions from real option models of unlevered firms with dynamic asset composition as well as models of levered firms with financial distress costs, we conclude:

Implication III: Non-negative price of BTM in the cross-section of unlevered equity returns, that is, $\gamma_1^A \geq 0$.

2.5 Volatility

The existing empirical literature has examined the relation between levered equity (stock) returns and levered equity volatility σ^L, following Ang et al. (2006) who document a negative cross-sectional relation between a stock's volatility and its subsequent return. The empirical literature has subsequently investigated and largely confirmed this cross-sectional relation (Chen et al., 2014), which is often thought of as an anomaly and referred to as the "volatility puzzle."

From a theoretical perspective, the relation between expected stock returns (μ_E) and levered volatility (σ_E) is ambiguous. The observed empirical relation seems inconsistent with the classical asset pricing view that only systematic risk should be priced in equilibrium. Theoretical deviations from this classical paradigm typically predict a positive relation between IVOL and expected returns (Merton, 1987; Barberis and Huang, 2001; Malkiel and Xu, 2002). Meanwhile, a growing literature provides various possible explanations for the negative IVOL-return relation (e.g., Chen et al., 2014; Stambaugh, Yu, and Yuan, 2015; Babenko et. al., 2016; Herskovic et al., 2016). It is important to note that these theoretical models do not explicitly consider leverage, and therefore the predictions also implicitly hold for unlevered returns.

When explicitly considering the relation between volatility, leverage, and returns, the class of structural credit risk models provides some interesting predictions. In particular, the literature largely appears to have ignored that in these models, the role of volatility in the cross-section of stock returns emerges naturally due to their option-theoretic features, and that the effects of volatility are closely tied to leverage. We now briefly discuss these implications. Appendix A presents results for the Merton (1974) model, which is relatively simple, but these results also obtain for more complex models. We show that the model-implied derivative of expected levered equity returns with respect to levered equity volatility is negative:

$$\frac{\partial(\mu_E - r)}{\partial\sigma_E} = \frac{\frac{\partial(\mu_E - r)}{\partial\sigma_A}}{\frac{\partial\sigma_E}{\partial\sigma_A}} < 0. \tag{2.3}$$

For our purpose, it is critical that these effects hold only in case of non-zero debt. For $L = 0$ we have $\frac{\partial\mu_E}{\partial\sigma_E} = \frac{\partial\mu_E}{\partial\sigma_A} = \frac{\partial\mu_A}{\partial\sigma_A} = 0$. However, note that this model predicts no relationship between unlevered equity returns and unlevered (or levered) volatility *by design*, in the sense that it models a simple asset dynamic which can best be thought of as exogenous. For our purpose, the model's most important implication is that it endogenously implies a negative relation between levered equity returns and equity volatility.

We conclude that existing models have very different implications for the price of volatility in the cross-section of unlevered equity returns:

Implication IV: Volatility in the cross-section of unlevered equity returns may not be priced, or may be positively or negatively priced; hence, the sign of γ_3^A in Equation (2.2) is unrestricted (i.e., $\gamma_3^A \lesseqqgtr 0$).

In sum, we argue that empirically testable implications from existing models regarding the cross-section of equity returns can usefully be expressed in terms of unlevered (asset) returns. It is critical that these implications for unlevered returns (*Implications I-IV*) are radically different from the stylized facts regarding the cross-section of levered equity returns, which we verify below for our sample. In the cross-section of levered returns, the stylized facts are that β^L is not priced, while BTM is positively priced, and size and volatility are negatively priced. In contrast, for the cross-section of unlevered returns, theory predicts that the asset beta β^A is positively priced and size is negatively priced. Some theories predict a positive relation between BTM and unlevered returns, while others predict no relation. With respect to volatility, different theories predict either a positive, negative, or no relationship with unlevered equity returns.

We now describe our empirical test design in Section 3 and then present the empirical analysis. In light of the prominence of the size and BTM anomalies in the literature, which have typically been considered together and separately from the volatility anomaly, we first present results for asset beta β^A, BTM, and size (E) in Sections 4 and 5. Subsequently, we present results for volatility in Section 6. Segmenting the presentation of our analysis in this way facilitates comparison of results with the existing literature.

3 Leverage and Heteroskedasticity in Stock Returns

How can we test the implications *I-IV* given that we do not directly observe a representative sample of unlevered equity returns? We adopt a two-step approach whereby we first unlever stock returns based on an explicit parametric model for (the unknown) Φ. (Below we discuss how to unlever stock returns with a parametric model.) In the second step, we test the implications using the unlevered returns specification given by equation (2.2). This approach is relatively straightforward and convenient, but it has two drawbacks. First, because we proceed in two steps, the question arises how to address the sampling error resulting from the first step. Second, we may introduce biases if the model used for unlevering is incorrectly specified.

For robustness, we therefore also utilize a very different approach that is data-driven. Consider Panel A of Figure 1, which shows monthly stock returns plotted against leverage in our sample. We use below-median asset volatility values because for higher levels of asset volatility there is a high incidence of very high and low return observations, which makes the figure hard to interpret. The figure clearly indicates that leverage induces multiplicative heteroskedasticity (Harvey, 1976) in the data. It is well known that such heteroskedasticity patterns greatly complicate statistical inference. Hence, we proceed by correcting for heteroskedasticity in stock returns using a standard econometric argument when investigating the risk factors. We show that while the first approach is model-based and the second one is data-driven, they are related and lead to the same basic conclusions.

Before we outline these two distinct approaches to account for leverage in more detail, we investigate the size and BTM anomalies in our sample of (levered) stock returns, to ensure that the stylized facts of our sample are consistent with the existing literature on the cross-section of stock returns. We also discuss the cross-section of leverage in our sample.

3.1 Data

We obtain stock returns and the number of shares outstanding from the Center for Research in Security Prices. We refer to the stock return for firm i at time t as $R_{E,i}(t)$, to emphasize that these are levered returns. We limit ourselves to non-financial firms, consistent with Fama and French (1992). The risk-free rate is obtained from Kenneth French's website.

Panel A of Table 1 reports stock returns for twenty-five size- and book-to-market portfolios, computed according to Fama and French (1993). Consistent with the available literature, small firms and high BTM firms have higher stock returns in our sample. The average risk-free rate in the sample period is 0.43% per month. Hence, the excess returns for the twenty-five size- and book-to-market portfolios are similar to the excess returns in Fama and French (1993), who report on a very different sample period. Panel A also shows the differences between the fifth and first size quintiles, conditional on book-to-market, and the differences between the fifth and the first book-to-market quintiles, conditional on size. We also report the t-statistics for these differences. The differences between the fifth and the first book-to-market quintiles are positive and economically large in all five cases. They are statistically significant in three of the five cases.

Panel B of Table 1 complements the results in Panel A by reporting on univariate sorts on size and BTM. The signs of the long-short portfolios are consistent with the existing literature, and as in Panel A the statistical significance of the BTM anomaly is stronger than that of the size anomaly.

We next use Fama and French's (1992) regression approach and consider how size and book-to-market affect the cross-section of expected stock returns when they are considered as firm characteristics (see Daniel and Titman, 1997). The results are shown in Panel C of Table 1. We use as regressors the firm's levered market beta, the logarithm of the firm's market capitalization $ln(E)$, the logarithm of book value over market value $ln(BE/E)$, the logarithm of book value of assets over market value of equity $ln(BA/E)$, and the logarithm of book value of assets over book value of equity $ln(BA/BE)$. The results confirm the existence of two important anomalies for our sample: First, regardless of the other regressors, the coefficient on $ln(E)$ is significantly negative, confirming the size effect, whereas the coefficient on $ln(BE/E)$ is significantly positive, confirming the BTM effect; indeed, the size and BTM anomalies are statistically more significant compared to the sorting results in Panels A and B. Second, the levered market beta is not a priced risk factor when we control for size and BTM. It is not statistically significant and its loading has the wrong sign.

In sum, despite using a very different sample from Fama and French (1992), we confirm the statistical and economic significance of the size discount and the value premium in the cross-section of stock returns. We also confirm that the levered market beta is not significant when controlling for size and BTM.

If cross-sectional differences in leverage help explain the value premium, but not the size discount, then we should find confirming evidence in leverage patterns. Panel A of Table 2 presents average leverage, defined as the ratio of total liabilities to the sum of total liabilities and the market value of equity, for each of the 25 size and BTM portfolios. Panel B presents average leverage for univariate size and BTM quintile portfolios.

Both the univariate and bivariate sorts indicate a strong positive relation between BTM and leverage. Indeed, it is striking in Panel A that leverage increases with book-to-market in every size quintile, and that this increase is monotonic within each size quintile. Moreover, the difference between the average leverage of BTM 5 and BTM 1 quintiles is highly statistically significant for each size group. Because stock returns and financial leverage are positively related (Bhandari, 1988), this provides support for the hypothesis that the positive BTM effect on stock returns is due to an underlying positive relation between BTM and leverage. In contrast, the relation between size and leverage is much less pronounced.

3.2 Unlevering Stock Returns Using a Parametric Model

We proceed by inferring unlevered returns from levered (or stock) returns for all available firms and subsequently conduct empirical tests using the unlevered returns. In principle, we can unlever stock returns using any model discussed in Section 2 that explicitly describes the mapping from unlevered to levered returns, such as Hamada (1972), Garlappi and Yan (2011), and Gomes and Schmid (2010). We utilize Merton's (1974) structural credit risk model, which provides several advantages. Within the class of models that allow for a nonlinear relation of stock returns to leverage, the Merton model is the simplest one, which provides expositional convenience and facilitates intuition regarding our empirical results. It is also the most parsimonious model in this class, which avoids ad hoc assumptions regarding ancillary parameters. Finally, the Merton model has been widely studied in the credit risk literature, and as a result the existing literature is quite instructive regarding robust implementation of this model.

In the Merton (1974) model, equity holders have the option to pay back the face value of the debt at maturity. They will exercise the option if the value of the firm's assets exceeds the value of the debt. This insight makes it possible to value the firm's equity, and by extension its risky debt, using standard option pricing techniques. To understand the nature of the leverage adjustment, consider the relation between the instantaneous expected excess return on levered equity $\mu_L - r$ and the instantaneous expected excess return on unlevered equity $\mu_A - r$ in the Merton model:[7]

$$\mu_L - r = (\mu_A - r)\left[\frac{\partial E}{\partial A}\frac{A}{E}\right] \tag{3.1}$$

where $\Delta_E = \partial E/\partial A > 0$ is the call option delta. This is equation (20) in Merton (1974). Merton (1974) focuses almost exclusively on the valuation of risky debt, and does not provide much evidence on expected levered equity returns; however, Figure 9 in Merton studies expected levered equity returns as a function of D/E.[8]

Panel A in Figure 2 illustrates this relation for three different values of the volatility of unlevered firm equity σ_A. On the horizontal axis, we have the ratio between the face value of the debt D and the market value of the firm's equity E. We assume that the yearly expected return on the unlevered firm equity μ_A is 6%. The risk-free rate is assumed to be 3%, and the initial value of the unlevered firm is assumed to be 100. Given the 6% expected return on unlevered equity, the expected value of the unlevered firm after one month is equal to 100.5. We compute the initial levered equity value and the levered equity value after one month for different face

[7]Studies that use the Merton model usually use the term "equity" to refer to the equity on the levered firm and refer to the unlevered equity as "firm assets" or "the value of the firm". We refer to levered and unlevered equity returns instead to clarify that we use the model to filter out the effects of leverage.

[8]The purpose of the Merton (1974) paper is the valuation of risky debt, but the model can also be used to study various other security returns in levered firms. Campello, Chen, and Zhang (2008) use the model to infer the expected returns on the firm's equity from the prices, yields, and expected returns on risky corporate bonds. Friewald, Wagner, and Zechner (2014) use the model to study the relationship between firms' credit risk premia and equity returns.

values of debt and different values of unlevered equity volatility. On the vertical axis, we have the expected excess return on the levered firm's equity $\mu_E - r$, which is concave in the D/E ratio.

For our purposes, it is useful to consider Panel B of Figure 2, which graphs expected excess equity returns as a function of leverage, defined as $D/(D+E)$, which by definition is bounded between zero and one. The difference between levered and unlevered equity returns increases with leverage, as expected. Moreover, at high levels of leverage, the relationship between leverage and expected stock returns becomes highly convex for realistic values of unlevered equity volatility σ_A. When adjusting firms' equity returns for leverage, it is important to take these nonlinearities into account. This may be especially relevant to understanding the returns of firms with high BTM, because we will show below that these firms typically are also more highly levered.

Note that when the firm's unlevered equity return is negative, leverage once again amplifies this effect; however, this now means that levered returns become more negative and thus *decline*. Note that expected returns are positive in equilibrium, but in empirical tests ex-post returns are used as proxies for expected returns, and these are frequently negative. We conclude that the sign of the unlevered equity return determines the implications of leverage for levered returns. This is illustrated in Panel C of Figure 2. We discuss this implication of the Merton (1974) model in more detail below.

Finally, the intuition from Equation (3.1) as captured by Panel A of Figure 1 can also be demonstrated using the weighted cost of capital. See Appendix B for details. Choi (2013) uses this approach and studies the resulting asset returns. His analysis focuses on the value premium and does not address the size discount, the volatility puzzle, and the cross-sectional performance of market beta. He documents the cross-sectional relation between asset beta and leverage, but his analysis largely focuses on conditional betas. During downturns, the asset beta and leverage of value firms increase, raising the equity beta, while the equity beta of growth firms remains relatively stable over the cycle. In the cross-section, the levered beta is therefore positively related to BTM.

3.3 Leverage-Induced Heteroskedasticity in Stock Returns

Unlevering returns using the Merton model may bias our results if the model is incorrectly specified. We therefore also use a very different approach, which is data-driven. We directly addresses the leverage-induced heteroskedasticity in stock returns using standard econometric techniques.

To see the role leverage plays in generating heteroskedasticity, consider again Panels B and

C of Figure 2. If the firm's unlevered return is positive, leverage will increase the levered return, but if the unlevered equity return is negative, higher leverage will decrease the levered return. We analyze the role of leverage while explicitly accounting for the multiplicative heteroskedasticity in the levered return data. We adjust for multiplicative heteroskedasticity in a straightforward way, by simply rescaling the regressand to take into account the heteroskedasticity.

The two approaches we use to account for leverage may seem very different but they are in fact related and represent a classical trade-off between bias and efficiency. The first approach uses a model to unlever stock returns and introduces biases if the model is incorrectly specified. Because the second approach does not use a model, it circumvents this problem. However, while it makes fewer assumptions, it may be less efficient than unlevering using a parametric model if the model is correctly specified.

Similarly, while it may seem that the first approach does not explicitly address the heteroskedasticity in the data, Panel B of Figure 1 depicts a large amount of monthly stock returns simulated from the Merton model for various values of leverage. The figure shows that the Merton (1974) model performs quite well in reproducing the heteroskedasticity in the data in Panel A. Therefore, unlevering the data with the Merton model or another reasonable parametric model implicitly addresses the heteroskedasticity problem.

4 Size, BTM, and Asset Beta in the Cross-Section of Unlevered Equity Returns

This section contains empirical results and tests on asset beta, size and BTM in the cross-section of equity returns unlevered using the Merton (1974) model, as discussed in Section 3.2. We first discuss the implementation of the model. We then present average unlevered portfolio returns based on univariate and double sorts on size and BTM. Subsequently, we use Fama-MacBeth regressions to assess the role of β^A, BTM, and E in explaining the cross-sectional variation in unlevered equity returns.

4.1 Implementation

To implement the Merton model, we compute monthly unlevered equity returns by solving for the value of the unlevered firm A and unlevered equity volatility σ_A at the end of every month t. Using Ito's lemma, levered equity volatility depends on unlevered equity volatility and the value of the unlevered firm as follows:

$$\sigma_L = \sigma_A \frac{\partial E}{\partial A} \frac{A}{E} \tag{4.1}$$

We can then infer the value of the unlevered firm and unlevered equity volatility at time t by using two equations: (4.1) and the option valuation formula that values levered equity E in terms of unlevered assets A. These two equations depend on the levered firm's equity value, levered equity volatility, the face value of debt, debt maturity, the risk-free rate, the value of the unlevered firm, and unlevered equity volatility. All of these quantities, except the value and volatility of unlevered equity, are observable. We observe the face value of the debt, as well as the equity value of the levered firm E (its market capitalization). Note that in our implementation, we adjust E for any distributions. We measure the equity volatility of the levered firm using the annualized standard deviation of the past year's daily returns. We therefore have two equations in two unknowns, and we can solve these two equations to infer A at the end of every month t. We then compute the monthly return on unlevered equity using A_t and A_{t+1}.

While our computation of the value of the unlevered firm using the two-equations-in-two-unknowns approach is the most direct one, we have to perform this computation at each time t for all the firms in the sample, which is time-consuming. We therefore compute the value of the unlevered firm once a month, and subsequently compute monthly returns on unlevered equity. This implementation also makes comparison with the available literature on cross-sectional stock returns easier, because most of these studies use monthly returns.

The estimation of the value of the unlevered firm using the firm's levered equity and levered equity volatility requires information on the face value of debt. We obtain quarterly data on the firm's debt and liabilities from Compustat. Quarterly Compustat debt data are available from 1971, so our sample period is from 1971 to 2012. In our benchmark implementation, we follow Eom, Helwege, and Huang (2004), who measure the face value of the firm's debt as total liabilities. For the cross-section of firms in our sample, it is not possible to compute the exact debt maturity. Thus, we specify the maturity of the debt equal to 3.38 years, which is the average maturity of debt obtained in Stohs and Mauer (1996) using a much smaller sample. In robustness tests in Section 7, we provide results for alternative definitions of firm debt and debt maturity.

4.2 Univariate and Bivariate Sorts

Panels A and B of Table 3 present average returns on unlevered equity using univariate and double sorts on size and book-to-market. We present the average value-weighted returns for each of the size and book-to-market portfolios, where the weights are determined by the market value of the levered equity. The structure of Table 3 is the same as that of Table 1. In Table 3, however, we use the unlevered equity returns computed using the Merton (1974) model instead of the (levered) stock returns. The face value of debt in the Merton model is assumed to be equal to the total liabilities and the maturity of debt is assumed to be 3.38 years. Panel A of Table 3 presents the results for the twenty-five size and BTM portfolios. The firms in each of the twenty-five cells are exactly the same ones as in Panel A of Table 1.

The results are striking. The pattern of returns on unlevered equity as a function of BTM is markedly different from the pattern for levered stock returns shown in Panel A of Table 1. In contrast to the monotone positive relation between stock returns and BTM, which constitutes the value premium, the relation between unlevered equity returns and BTM is complex and non-monotone. In particular, for Size 2 through Size 4 quintiles, the relation of unlevered equity returns to BTM is best described as a cubic function, where returns first fall, then rise, and fall again for the highest BTM (BTM 5) quintile. Indeed, for this middle 20%–80% firm size groups, the *highest* BTM quintile has the *lowest* average return on unlevered equity. Meanwhile, for the smallest size quintile (Size 1), the relation is also described by a cubic function, but here the returns first rise, then fall, and rise for the highest BTM group. Finally, for the largest size group (Size 5), the relation is quadratic, with the returns first rising and then declining in BTM.

Given the complex and polynomial relation between unlevered equity returns and BTM across all size groups, it is not surprising that the difference between the return on the highest and lowest BTM quintiles is not statistically significant for any size group. This is in striking contrast to stock returns, where the difference between the return on the highest and lowest BTM quintiles is positive and statistically and economically significant for Size 1 through Size 3. Thus, we conclude that de-levering returns through a widely-used structural credit risk model (Merton, 1974) eliminates the positive BTM effect.

The size effect observed for stock returns appears to survive adjustments for leverage. We find that the difference between the return on the highest and lowest size quintiles is negative in all BTM groups, and highly statistically significant for the largest BTM quintile, as is the case for stock returns in Panel A of Table 1.

The univariate sorts in Panel B of Table 3 confirm the results from the bivariate sorts in Panel A. The long-short portfolio return associated with BTM is much smaller compared to Panel B of Table 1, and no longer statistically significant. The online appendix shows that similar results obtain when using two other measures of value: earnings-to price and sales-to-price. The long-short portfolio return associated with size is almost identical to the one in Panel B of Table 1, and statistically more significant.

4.3 Fama-MacBeth Regressions on Unlevered Returns

Panel C of Table 3 presents the results of an analysis that repeats the regressions in Panel C of Table 1 for unlevered equity returns. In sharp contrast to the results in Panel C of Table 1, the coefficient on asset beta in the first row is positive and significant. Thus, correcting for leverage effectively resurrects the effect of systematic asset risk.

The positive and statistically significant coefficient on the asset beta is consistent with theory. The estimate for the price of market risk is 0.68% per month, somewhat large compared to the historical average of the market excess return of approximately 6% per year. The estimates of the price of market risk in rows 3 and 6 are more in line with the historical average. All three estimates are statistically significant. This result is meaningful for the large literature (Fama and French, 1992, and onwards) on the role of market beta in the cross-section of stock returns.

A number of papers argue that time-varying (or conditional) market betas and risk premia perform well in explaining cross-sectional variation of stock returns (see, e.g., Jagannathan and Wang, 1996). However, Nagel and Singleton (2011) question if the proposed conditional models are a good fit for the data. Our analysis shows that the *unconditional* unlevered (or asset) beta has statistically and economically significant explanatory power in explaining the cross-section of unlevered returns. As discussed in Section 2, the positive relation of expected unlevered equity returns and systematic asset risk is a common feature of a large number of models, including real option models.

Why does the market model perform so much better when using unlevered returns? The online appendix documents that unlevered returns are less variable over time, and Figure 3 suggests that this may be helpful when estimating market betas. Recall that we follow the implementation of Fama and French (1992), who group the stocks in one hundred portfolios and compute market betas for these portfolios. Figure 3 studies the relation between market beta and average return for these portfolios, because the corresponding figure for stocks is very noisy. Panel A in Figure 3 reports on the levered returns, and Panel B on the unlevered returns. The relation between market beta and returns in Panel A seems to be noisier than the one in Panel B, especially for the high betas. This may complicate reliable estimation of the security market line.

The remaining rows of Panel C of Table 3 augment asset beta with various permutations of size and BTM, following Panel C of Table 1. In sharp contrast to the results in Table 1, the coefficient on asset beta remains positive and significant. The coefficient on BTM is negative and not statistically significant and the size effect remains negative and statistically significant. Note that in Tables 1 and 3, the regressions capture the size effect better than the portfolio sorts. Besides resurrecting the effect of systematic asset risk, correcting for leverage effectively eliminates the BTM effect. These findings with respect to BTM and size are consistent with the results in Panels A and B of Table 3.

The estimates of the price of market risk in rows 3 and 6 are smaller than in the first row but intuitively plausible. The unconditional unlevered beta has statistically and economically significant explanatory power in explaining the cross-section of unlevered returns even when controlling for size and BTM.

In summary, the principal results of this section are as follows. Consistent with theory reviewed in Section 2 (implication *I*), the *unlevered* market beta is priced in the cross-section of unlevered equity returns even when we control for size and BTM, and the estimates of the market risk premium are large and intuitively plausible. Furthermore, there is a negative effect of size on unlevered returns, consistent with the general argument of Berk (1995) and real option models of unlevered firms (implication *II*). Finally, the zero price for BTM in the cross-section of unlevered equity returns is consistent with theoretical models that interpret book-to-market effects as arising from leverage and costs of financial distress (implication *III*).[9]

5 Size, BTM, and Asset Beta in the Cross-Section of Heteroskedasticity-Adjusted Equity Returns

Our empirical findings on unlevered returns and the relation between leverage and BTM raise two important questions. First, we use the Merton model for unlevering returns, and a natural question is if the results are robust to the use of other models. We address this in Section 7.3 below. Second, why has the importance of leverage for the BTM anomaly not been uncovered in existing work? Specifically, several studies include leverage as a covariate when analyzing stock returns and explicitly consider if leverage can capture the book-to-market anomaly.[10] We now address this question.

Our analysis suggests that existing regression approaches fail to fully uncover the importance of leverage for the cross-section of stock returns because of econometric complications, specifically

[9] The existing literature finds weak evidence for asset beta and *both* the size and BTM anomalies using shorter sample periods and/or more restrictive sample sizes (Hecht, 2000; Charoenrook, 2004; Ozdagli, 2012; Choi and Richardson, 2015). For example, Choi and Richardson (2015) consider the shorter 1981-2012 period when the size effect is known to be relatively weak and exclude zero leverage firms from their sample. Alternatively, these differences may also be due to the method used to unlever returns. Some existing studies use market returns for debt, which has certain advantages but reduces the size of the cross-section.

[10] See for instance Bhandari (1988), Fama and French (1992), George and Hwang (2010), and Trigeorgis and Lambertides (2014).

leverage-induced heteroskedasticity in the stock return data. Once heteroskedasticity is taken into account, the empirical results are consistent with our findings in Table 3.

5.1 Leverage in Stock Return Regressions

We now investigate how estimates of the implications of leverage for the cross-section of stock returns are affected if heteroskedasticity is not taken into account. First, rather than analyze this issue using the return data in our sample, for which we do not know the data generating process, we use a controlled experiment. Panel A of Table 4 is based on return data simulated using the Merton model. The values of the model parameters used in the simulations are $r = 3\%$, and $T = 3.38$ years. The volatility σ_A is chosen to be uniformly distributed between 10% and 150%. The drift μ_A is chosen to be uniformly distributed between -6% and 6%. Leverage, defined as the ratio of debt to asset value, is chosen to be uniformly distributed between 0 and 1. We simulate 10,000 monthly equity returns using these parameters and regress the simulated returns on leverage and higher order terms of leverage.

An additional advantage of simulated data is that we can keep the analysis simple because we do not have to consider additional explanatory variables, such as size and BTM. The data are generated using the Merton model, and we therefore know that leverage should be significant in these regressions. Panel A of Table 4 reports on regressions using the simulated data and linear as well as higher order terms in leverage. Leverage is not significant, regardless of whether higher order terms in leverage are included. The point estimates of the leverage terms are not statistically significant and the R-squares are very small.

Note that it is possible to extend this simulation exercise to incorporate market risk, which would allow us to investigate if correcting for heteroskedasticity allows us to retrieve this factor structure. This exercise requires additional assumptions and we leave it for future work.

Panel B of Table 4 documents the implications of multiplicative heteroskedasticity using the sample data. We include polynomials of leverage in Fama-MacBeth regressions with market betas, size, and BTM. The first row repeats the results from the last row of Panel C of Table 1, which illustrates the size and BTM anomalies. Rows 2 through 4 of Panel A include linear, quadratic, and cubic leverage terms into the regression. Leverage is often insignificantly estimated. Consistent with the existing literature, including leverage has little impact on the point estimates and statistical significance of market beta and the size and BTM characteristics.

5.2 A Simple Correction for Heteroskedasticity

Table 5 analyzes the role of leverage while explicitly accounting for the multiplicative heteroskedasticity in the levered return data. We adjust for multiplicative heteroskedasticity in the simplest possible way, by simply rescaling the regressand to take into account the heteroskedasticity. Because the scaling variable in Figures 1 and 2 is leverage itself, we scale excess levered equity returns $R_{E,i}(t)$ by $1 - L_i(t)$, where L_i is proxied by the ratio of total liabilities to the sum of total liabilities and the market value of equity. Panels A and B of Table 5 present the rescaled returns for univariate and bivariate sorts. In Panel C of Table 5, we run a Fama-MacBeth regression of the rescaled $R_{E,i}(t)(1 - L_i(t))$ on various permutations of market beta, size and BTM. Note that once leverage is incorporated this way, there is no need to include it as a regressor, and that the market betas are recomputed using a market portfolio that aggregates the rescaled returns.

The results in Panel C of Table 5 strongly differ from the results in Panel B of Table 4. The size anomaly persists and is strongly statistically significant, consistent with the results in Table 3. Point estimates are similar to Panel C of Table 1. We conclude for Tables 3 and 5 that the size effect is stronger in unlevered returns. One possible explanation for this finding is that there is a negative relation between the market value of debt and size. Market beta is estimated with a positive sign and it is statistically significant. Panel C of Figure 3 provides additional insight into the performance of the market model by graphing the market betas for the portfolios used in the regressions and the corresponding security market line. Comparing Panel C with Panel A suggests that the relation between market betas and average returns in Panel A is noisier. This confirms our conclusion from the comparison of Panels A and B. If anything, the relation between market betas and returns in Panel C is less noisy than the one in Panel B.

The BTM effect is no longer statistically significant in Panel C of Table 5, and when size is included, it has a negative sign. The bivariate and univariate sorts in Panels A and B are consistent with the regression results in Panel C.

Some of the results in Panel C of Table 5 quantitatively differ from the results in Panel C of Table 3. For instance, the estimates of the market risk premium are smaller. But qualitatively, the results are remarkably consistent. First, the size effect remains. Second, the BTM anomaly disappears. Third, market beta is estimated with the intuitively plausible positive sign and statistically significant. The market risk premium is much larger than in the existing literature based on stock returns, as evidenced by the estimates in Panel C of Table 1.

The results in Table 5 rescale excess levered equity returns $R_{E,i}(t)$ by $1 - L_i(t)$ to address the presence of heteroskedasticity, but alternatively this approach can also be thought of as the application of an entirely different model to unlever returns. In particular, recall from Section 2 that $\beta^L = \beta^A [1 + \Phi(L, \boldsymbol{\delta}, Z)]$, where in the textbook treatment of asset and levered beta in a CAPM framework (Hamada, 1972), we have $\Phi(L, \boldsymbol{\delta}, Z) = (1-t)\frac{D}{E}$. Therefore, ignoring taxes, our rescaling of the levered equity return, which is motivated by econometric concerns, is consistent with unlevering according to the textbook treatment of leverage, because $1+\Phi(L, \boldsymbol{\delta}, Z) = 1+\frac{D}{E} = \frac{D+E}{E}$. Table 5 therefore also provides a robustness test with respect to the use of the structural credit risk model in the benchmark results.

The latter argument also shows that stock returns exhibit heteroskedasticity in a variety of models. Figure 2 uses the Merton model to illustrate the implications of leverage, but similar results will obtain for most reasonable models of leverage. The exact nonlinear relation of course depends on the structure and parameterization of the Merton model and will differ somewhat from other models, but the existence of the heteroskedasticity pattern is a general result.

5.3 Alternative Corrections for Heteroskedasticity

Table 5 presents results using the simplest possible approach to deal with multiplicative heteroskedasticity. Table 6 presents results for more sophisticated approaches motivated by equation (3.1). The first row repeats the results from the last row of Panel C of Table 5, obtained by regressing the rescaled return $R_{E,i}(t)(1 - L_i(t))$ on market beta, size and BTM. In rows 2 through 4, we address the presence of heteroskedasticity by scaling the levered equity return using increasingly sophisticated factors suggested by structural credit risk models. Note that the increased sophistication of these approaches comes at the cost that unlike in row 1, the rescaling in rows 2 through 4 involves ancillary parameters that need to be estimated in a first step.

In row 2, we first estimate α_1 from the cross-sectional regression

$$\log(R_{E,i}^2) = \alpha_1 \log\left(\frac{A_i}{E_i}\right)^2 + \epsilon_i, \qquad i = 1, ..., N \tag{5.1}$$

where N is the number of stocks in the cross-section, A_i is a proxy for the market value of assets measured by the sum of total liabilities and market value of equity for firm i, E_i is the market value equity and $R_{E,i}$ is the excess levered equity return for firm i. We then rescale the levered equity return by the square root of the regression function, i.e., we regress the rescaled return $R_{E,i}/\text{sqrt}(\exp(\alpha_1 \log\left(\frac{A_i}{E_i}\right)^2))$ on market beta, size and BTM. Note that if $\alpha_1 = 1$, this adjustment is equivalent to the textbook adjustment in row 1. In row 3, we first estimate α_1

from the cross-sectional regression

$$\log(R_{E,i}^2) = \alpha_1 \log\left(\Delta_{E,i}\frac{A_i}{E_i}\right)^2 + \epsilon_i, \qquad i = 1, ..., N \tag{5.2}$$

where $\Delta_{E,i}$ denotes the option delta, which is computed using the Merton model with value of assets assumed to be equal to A_i, the face value of debt equal to the total liabilities, the debt maturity equal to 3.38 years, and volatility equal to the levered equity volatility times $(1-L_i(t))$. Subsequently we regress the rescaled return $R_{E,i}/\text{sqrt}(\exp(\alpha_1 \log\left(\frac{\Delta_{E,i}A_i}{E_i}\right)^2))$ on market beta, size and BTM. In row 4, we estimate α_1 and α_2 from the cross-sectional regression.

$$\log(R_{E,i}^2) = \alpha_1 \log\left(\frac{A_i}{E_i}\right)^2 + \alpha_2 \log\left(\Delta_{E,i}^2\right) + \epsilon_i, \qquad i = 1, ..., N \tag{5.3}$$

and we then regress the rescaled return $R_{E,i}/\text{sqrt}(\exp(\alpha_1 \log\left(\frac{A_i}{E_i}\right)^2 + \alpha_2 \log\left(\Delta_{E,i}^2\right)))$ on market beta, size and BTM. For each specification, we compute the market betas for the rescaled returns using a market portfolio that aggregates the rescaled returns, and we subsequently implement Fama-MacBeth regressions.

Table 6 indicates that the results are very robust across the different implementations. We conclude that multiplicative heteroskedasticity is a serious problem when incorporating leverage into cross-sectional regressions of stock returns. When heteroskedasticity is taken into account, results on the effects of leverage on the cross-section of stock returns are similar to the findings for the cross-section of unlevered equity returns. The size effect remains, the BTM anomaly disappears, and market beta plays a more prominent role in explaining the cross-section of returns.

6 Volatility in the Cross-Section of Equity Returns

As noted in Section 2, existing asset pricing models impose different restrictions on the effect of unlevered volatility on *unlevered* equity returns. Here, we investigate empirically the role of volatility in the cross-section of equity returns. First we highlight differences between asset volatility and stock volatility. Then we document the cross-sectional relation between volatility and levered as well as unlevered returns. We use asset volatility computed as levered equity volatility times $(1 - L_i(t))$, based on our approach in Section 5.3.

6.1 Asset Volatility: Stylized Facts

Panel A of Table 7 presents descriptive statistics for stock volatility and unlevered equity volatility. There are clearly important differences. For instance, unlevered equity volatility is on average 17.3% lower than stock volatility. Panel B reports average leverage for portfolios sorted on volatility. The leverage spread is much higher in the case of unlevered equity volatility. These findings confirm existing results in the literature. For instance, Choi and Richardson (2015) emphasize the differences between stock volatility and asset volatility, and the critical role leverage plays in explaining these differences.

6.2 Volatility and the Cross-Section of Levered Equity Returns

As in Section 4, we first analyze the relation between volatility and stock returns. Panel C of Table 7 replicates the results of Ang et al. (2006) for our sample, sorting on *levered* equity volatility. We confirm the negative relation documented by Ang et al. (2006) using a univariate sort.

Panel D of Table 7 reports on Fama-MacBeth regressions that include volatility as well as beta, size, and BTM. Levered volatility is estimated with a negative sign but it is not statistically significant. Note that most existing empirical evidence points to a negative sign, consistent with Ang et al. (2006), but the evidence is not unequivocal. For example, Bali and Cakici (2008) find no robust relation between volatility and returns, while Boehme et al. (2009) document a positive volatility-return relation for small firms with non-binding shorting constraints.[11] Compared to Panel C of Table 1, the size effect is smaller and less statistically significant. The BTM effect remains very strong both economically and statistically.

6.3 Asset Volatility and the Cross-Section of Unlevered Equity Returns

Panel E of Table 7 reports on a univariate volatility sort and Panel F reports on Fama-MacBeth regressions. In Panel E, the unlevered equity return increases monotonically with unlevered equity volatility for the first four quintiles but declines from the fourth to the fifth quintile. The long-short portfolio return spread is positive but not statistically significant. Thus, consistent with the predictions of the asset pricing models discussed in Section 2, we find no significant

[11] Furthermore, Bartram, Brown, and Stulz (2012) and Segal, Shaliastovich, and Yaron (2015) highlight the existence of "good" volatility or uncertainty at the macro or country level, which predicts better aggregate economic outcomes. Kumar and Li (2016) find a positive cross-sectional relation between IVOL and subsequent returns for large innovative firms.

relation of unlevered equity returns and unlevered volatility in the univariate sorts. The Fama-MacBeth regressions in Panel F report a positive and statistically significant loading on volatility. While the theory reviewed in Section 2 does not suggest any relation between unlevered equity volatility and unlevered equity returns, we conclude that there is some evidence of a positive relation in our sample. We can reject with a high degree of confidence the null hypothesis of a significant *negative* relation between unlevered equity returns and unlevered volatility.

In sum, we conclude that the negative cross-sectional relation between levered volatility and stock returns reported in the literature may be due to leverage, consistent with the predictions of structural credit risk models.

7 Robustness

In this section we report on several robustness tests. First, we discuss results for different definitions of the firm's debt and different debt maturities. Second, we present results for a different computation of unlevered equity returns. Third, we present results for the Leland and Toft (1996) model, which has a richer structure, to investigate if our results are due to the use of the Merton (1974) model. Fourth, we discuss the importance of the weights used to compute portfolio returns. Fifth, we report on the robustness of the impact of leverage over time. Finally, we present evidence on zero leverage firms. In the online appendix, we also discuss robustness with respect to the computation of the leverage variable, and we highlight the implications of our findings for factor models. Table 8 presents results for sorts using BTM, size, and volatility. Table 9 presents the results of Fama-MacBeth regressions using market beta, size, and BTM.

7.1 Measuring Debt and Debt Maturity

Structural credit risk models can be implemented with relatively few assumptions, and we use the classical Merton (1974) model because it uses as few assumptions as possible. One assumption is on the definition and the maturity structure of the debt. Two approaches are used in the existing literature. Most implementations use a maturity of one or five years, and they measure the debt as the sum of the short-term debt and one-half of the long-term debt.[12] This approach is appropriate for these studies because their main focus is the computation of expected one-year or five-year default probabilities.

Our focus is different and for our analysis presented above in Tables 2 through 7, we therefore

[12] See Bharath and Shumway (2008), Campbell, Hilscher, and Szilagyi (2008), Crosbie and Bohn (2003), Duffie, Saita, and Wang (2007), Ericsson, Reneby, and Wang (2015), and Vassalou and Xing (2004) for examples.

follow Eom, Helwege and Huang (2004), who measure the firm's debt as total liabilities. For our cross-section of firms, we do not have sufficiently detailed information to compute the exact maturity of the debt for each firm. We therefore use the average maturity from Stohs and Mauer (1996), who use a much smaller sample, which allows them to compute the exact maturity structure of the firms' debt. The average maturity in their study is 3.38 years.

We now investigate the robustness of our results to alternative assumptions. Panels A and B of Tables 8 and 9 report results for the Merton (1974) model using a debt maturity of one and five years respectively. In Panel A, we measure the debt as the sum of the short-term debt and one-half of the long-term debt, while in Panel B we define the debt as equal to the total liabilities. In Panel C, we present results obtained using firm-specific debt maturities. The Compustat data do not provide us the exact maturity of the debt or liabilities, but they provide information on the debt maturing in one, two, three, four and five years. For debt maturing in two to five years, we use Compustat items DD2 to DD5. Current liabilities are our measure of debt maturing in one year. We treat the remaining liabilities as the long-term debt maturing in 10 years. Using this information, we compute the average maturity of the debt as follows:

$$T = \sum_{t=1}^{5} w_t \times t + \left(1 - \sum_{t=1}^{5} w_t\right) \times 10,$$

where w_t is the proportion of total debt maturing in t years. The average of the firm-specific debt maturities used in this table is $T = 4.75$, higher than the maturity used for all firms in the benchmark results. Note that the detailed debt information is available annually, therefore we hold the debt maturity to be same for a given fiscal year even though we use quarterly debt data as our measure of default boundary F. The debt is defined as equal to the total liabilities in Panel C.

Finally, In Panel D, we report the results for the Merton model, defining debt as the sum of long-term and short-term debt, with maturity $T = 3.38$ years.

The results are clear. In Table 8, the calibration of the maturity and the definition of the debt somewhat affects the level of the unlevered equity returns, but not the cross-sectional patterns as a function of BTM, size, and volatility. These assumptions do not impact the cross-sectional differences among firms, and our results are robust in this dimension. In Table 9, market beta is not always statistically significant and the estimates for the price of market risk are somewhat smaller than the estimates in Table 3. However, they are much larger and more statistically significant than the estimates obtained using levered returns in Table 1.

7.2 Computing Unlevered Equity Returns

In our benchmark implementation we infer the value of the unlevered equity at times t and $t+1$ using two equations in two unknowns, and then compute the monthly unlevered equity return using A_t and A_{t+1}. We now report on a different implementation that directly uses equation (3.1). At time t, we solve the model for the value of unlevered equity A_t and its volatility σ_{A_t}. We compute ex-post (levered) monthly stock returns $R_{E,t}$ as an estimate of the expected levered equity return μ_E. Using the estimates A_t, σ_{A_t}, and $R_{E,t}$ and equation (3.1), we then compute the ex-post unlevered equity return $R_{A,t}$.

This implementation has both advantages and disadvantages compared to the benchmark implementation. It is intuitively appealing because it uses the theoretical relationship (3.1) and it only requires information at one point in time. By using the ex-post stock return to estimate μ_E, dividends are also taken into account in a straightforward way, while in the benchmark approach they have to be added into returns after computing A_t and A_{t+1}. A drawback of this implementation is that our sampling frequency is monthly, thus, we are effectively using monthly stock returns as an estimate of instantaneous expected returns μ_E. Hence, implicitly we are assuming that important model features, including leverage, remain constant over a one-month period.

Panel E of Tables 8 and 9 show the results of this alternative implementation. The size effect seems to be economically and statistically stronger in Panel E of Tables 8 and 9 than in Table 3. The result that higher BTM is not associated with higher returns is robust for all but the smallest size quintile. The estimates for the market price of risk are smaller and less statistically significant compared to Table 3. The results for volatility are very similar. We conclude that our results are mostly confirmed when using this alternative procedure for computing unlevered equity returns.

7.3 The Leland-Toft Model

Panel F of Table 8 presents the results obtained using the Leland-Toft (1996) model instead of the Merton (1974) model. Other aspects of the implementation, such as the debt maturity and the definition of debt, are the same as in Table 2. The Leland-Toft model is a more richly parameterized and more complex model than the Merton (1974) model, allowing for an endogenous default boundary.[13] The model implementation is identical to the method used to estimate the

[13] See for instance Black and Cox (1976), Collin-Dufresne and Goldstein (2001), Cremers, Driessen, and Maenhout (2008), Geske (1977), Kim, Ramaswamy and Sundaresan (1993), Leland (1994), and Longstaff and Schwartz (1995) for examples of other extensions of the Merton (1974) model.

Merton model, i.e. at each point in time, we solve two equations to obtain two unknowns. However, this model requires additional inputs besides the information used to estimate the Merton model. The Leland-Toft model also requires information about the tax rate, the payout ratio, and the costs of financial distress. We fix the tax rate at 20%, which is consistent with the previous literature (Leland, 1998). We assume that the firm loses 15% of its assets in financial distress, which is within the range estimated in Andrade and Kaplan (1998). We compute the payout rate each quarter using the Compustat and CRSP data. The payout rate is defined as follows.

$$\delta = \frac{IE}{TL} \times \frac{TL}{TL\ +\ E} + DY \times \left(1 - \frac{TL}{TL\ +\ E}\right)$$

where IE is the interest expense obtained from Compustat, TL is the total liabilities, E is the market value of equity, and DY is the dividend yield.

The results in Panel F of Tables 8 and 9 indicate that the choice of model does not drive our results. Just as in Table 3, high BTM firms in Panel F of Table 8 do not have higher unlevered equity returns than low BTM firms. The results for the size effect and the market beta are similar to those in Table 3, and the results for volatility are similar to those in Table 7.

7.4 Levered and Unlevered Equity Weights

Panel G of Tables 8 and 9 report alternative results where the unlevered equity returns are computed using different weights. In Table 3, portfolio unlevered equity returns are computed using the weights used in Panel A of Table 1, which are based on (levered) stock market value. It may be preferable to use weights based on the value of unlevered equity, as some of the stock-based weights may be affected by leverage. The differences between the resulting returns should be more pronounced for those portfolios that contain firms with high leverage. Compared to the results in Table 3, the resulting portfolio returns in Table 8 are significantly smaller for high book-to-market firms, especially for small ones. More importantly for our purposes, the implications are that our result is strengthened. High book-to-market firms do not offer higher returns after correcting for leverage. In fact, for three out of the five size quintiles, they offer statistically significant lower returns. The results for market beta and volatility are similar to those in Tables 3 and 7 respectively.

We conclude that our results are somewhat sensitive to the choice of unlevered versus levered weights, but that using unlevered weights actually strengthens some of the results. We, therefore, use the levered weights in our benchmark analysis, because they yield more conservative results.

7.5 Zero Leverage Firms

Rather than unlevering returns, an alternative approach is to restrict the sample to zero leverage or low leverage firms. Unfortunately, zero leverage firms represent a relatively small subsample of the universe of public firms. Using zero leverage may also lead to selection bias because firms choose to remain unlevered, i.e. the sample of zero leverage firms is not random (Strebulaev and Yang, 2013).

Figure 4 addresses this issue by depicting long-short book-to-market returns for various subsamples defined by leverage. As the leverage percentile increases, we include firms with higher and higher leverage ratios. Leverage is defined as the total liabilities scaled by the sum of total liabilities and market value of equity. Figure 4 indicates that as we include firms with more and more leverage, the long-short return increases, consistent with our other results. As leverage decreases, the sample becomes smaller and the confidence interval widens. Restricting the sample to firms with zero or low leverage results in very large standard errors. We obtain a similar conclusion for the size effect (not reported): it is not significant when using zero leverage or low leverage firms, but this is due to the fact that the estimates are extremely noisy.

7.6 Cumulative Returns

One question that comes to mind is if the effect of leverage on stock returns changes through time. In Figure 5 we answer this question by graphing the cumulative long-short return for levered and unlevered equity over the sample, starting with a $1 investment in 1971. Panel A graphs the cumulative long-short return based on book-to-market for levered equity. It is well known that the return changes through time and that this time variation is related to the business cycle.

Panel B of Figure 5 graphs the corresponding cumulative long-short unlevered return. The figure indicates that the absence of a book-to-market effect in the unlevered equity returns is not due to a small part of the sample period. But there is substantial variation in the long-short return on unlevered equity. Somewhat surprisingly, the cumulative long-short investment breaks even only because of the large return on unlevered equity during the recent recession.

8 Summary and Conclusions

How financial leverage affects the cross-section of stock returns is an important topic that remains unresolved in the literature. We build on the existing theoretical asset pricing literature to develop and empirically test refutable hypotheses regarding the cross-section of unlevered equity returns. We adjust observed stock returns for leverage to compute unlevered returns and investigate the role of unlevered market betas and three important anomalies: size, book-to-market (BTM), and volatility. This approach is particularly useful because many asset pricing theories formulate testable implications in terms of unlevered returns.

Our sample period is different from the one used by Fama and French (1992, 1993) but we confirm the existence of the size effect and the value premium in levered returns and also find that levered beta is not priced. Our main results are that asset beta is priced in the cross-section of unlevered equity returns; the value premium and volatility puzzle disappear; and the size discount remains. We establish the robustness of our results to a wide variety of implementations.

Theory suggests that leverage induces multiplicative heteroskedasticity in levered returns, and we empirically confirm this. We show that it is critical to adjust levered returns for this heteroskedasticity when investigating the role of market beta and the presence of anomalies. After accounting for heteroskedasticity, the empirical results confirm our benchmark results. Some of the adjustments for heteroskedasticity can be interpreted as unlevering using an alternative model, which further confirms the robustness of our results.

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Appendix A: Proof of Equation (2.3)

In this appendix, we show that the derivative of the expected excess levered equity returns $(\overline{\mu_E} = \mu_E - r)$ with respect to levered equity volatility (σ_E) is negative $\left(\frac{\partial \overline{\mu_E}}{\partial \sigma_E} < 0\right)$. Note that

$$\frac{\partial \overline{\mu_E}}{\partial \sigma_E} = \frac{\frac{\partial \overline{\mu_E}}{\partial \sigma_A}}{\frac{\partial \sigma_E}{\partial \sigma_A}} \tag{A.1}$$

and

$$\overline{\mu_E} = \overline{\mu_A}\left[\frac{\partial E}{\partial A}\frac{A}{E}\right]$$

where E is the levered equity value, A is the unlevered equity value, $\overline{\mu_E}$ is the excess levered equity returns and $\overline{\mu_A}$ is the excess unlevered equity returns.

Therefore, in order show that the derivative in equation (A.1) is negative, we show that the numerator is negative and the denominator is positive. Specifically, we first show that the derivative of the levered equity returns with respect to unlevered equity volatility is negative i.e.,

$$\frac{\partial \overline{\mu_E}}{\partial \sigma_A} = \frac{A}{E}\overline{\mu_A}\frac{1}{E}\left[E\frac{\partial \Delta_E}{\partial \sigma_A} - \Delta_E\frac{\partial E}{\partial \sigma_A}\right] < 0.$$

Since, $\frac{A}{E^2}$ and $\overline{\mu_A}$ are positive, in order to show the relationship, we need to show that $\left[E\frac{\partial \Delta_E}{\partial \sigma_A} - \Delta_E\frac{\partial E}{\partial \sigma_A}\right] < 0$. The proof is as follows.

$$E\frac{\partial \Delta_E}{\partial \sigma_A} - \Delta_E\frac{\partial E}{\partial \sigma_A} = En(d_1)\frac{\partial d_1}{\partial \sigma_A} - AN(d_1)n(d_1)\sqrt{T},$$

where $N(x)$ indicates the cumulative distribution function (cdf) of a standard normal distribution and $n(x)$ indicates the probability density function (pdf) of a standard normal distribution. Substituting the derivative $\frac{\partial d_1}{\partial \sigma_A}$ where

$$d_1 = \frac{\ln\frac{A}{F} + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}},$$

we obtain the following.

$$\begin{aligned} E\frac{\partial \Delta_E}{\partial \sigma_A} - \Delta_E\frac{\partial E}{\partial \sigma_A} &= E \times n(d_1)\left(\sqrt{T} - \frac{d_1}{\sigma_A}\right) \\ &\quad -AN(d_1)n(d_1)\sqrt{T}. \end{aligned}$$

Simplifying further we get:

$$
\begin{aligned}
E\frac{\partial \Delta_E}{\partial \sigma_A} - \Delta_E \frac{\partial E}{\partial \sigma_A} &= n(d_1)\left(-\sqrt{T}Fe^{-rT}N(d_2) - E\frac{d_1}{\sigma_A}\right) \\
&< 0.
\end{aligned}
$$

Next, we show that the denominator in equation (A.1) is positive. Using Ito's lemma, levered equity volatility depends on unlevered equity volatility and the unlevered equity value as follows:

$$
\sigma_E = \sigma_A \frac{\partial E}{\partial A}\frac{A}{E}. \tag{A2}
$$

Therefore, the derivative of levered equity volatility with respect to unlevered equity volatility is

$$
\frac{\partial \sigma_E}{\partial \sigma_A} = \frac{A}{E}\left[\sigma_A \frac{\partial \Delta_E}{\partial \sigma_A} + \Delta_E\left[1 - \frac{\sigma_A}{E}\frac{\partial E}{\partial \sigma_A}\right]\right] > 0.
$$

Rewriting the bracket term after substituting $\frac{\partial \Delta_E}{\partial \sigma_A}$, Δ_E and $\frac{\partial E}{\partial \sigma_A}$

$$
\begin{aligned}
\sigma_A \frac{\partial \Delta_E}{\partial \sigma_A} + \Delta_E\left[1 - \frac{\sigma_A}{E}\frac{\partial E}{\partial \sigma_A}\right] &= \sigma_A n(d_1)\left(\frac{\partial d_1}{\partial \sigma_A} - N(d_1)\frac{A}{E}\sqrt{T}\right) \\
&\quad + N(d_1).
\end{aligned}
$$

Simplifying further after substituting $\frac{\partial d_1}{\partial \sigma_A} = \sqrt{T} - \frac{d_1}{\sigma_A}$

$$
\sigma_A \frac{\partial \Delta_E}{\partial \sigma_A} + \Delta_E\left[1 - \frac{\sigma_A}{E}\frac{\partial E}{\partial \sigma_A}\right] = N(d_1) - n(d_1)\left(d_1 + N(d_2)\frac{F}{E}e^{-rT}\sigma_A\sqrt{T}\right).
$$

The above equation is always positive except in some extreme parameter range where $F > A$.

Appendix B

Note that the intuition from Equation (3.1) captured by Panel A of Figure 1 is identical to the intuition we obtain using the formula for the weighted cost of capital. Defining the value of the firm's assets A as $A = E + D$, we have

$$
r_A = r_E \frac{E}{A} + r_D \frac{D}{A} \tag{B1}
$$

where r_V is the required return on the firm's assets, r_E is the required return on equity, and r_D is the required return on debt. Rearranging, we get an expression for the return on levered equity as a function of $\frac{A}{E}$:

$$r_E = (r_A - r_D \frac{D}{A}) \frac{A}{E} \tag{B2}$$

Taking expectations and substituting the required return on debt in the Merton model

$$\mu_D - r = (\mu_A - r) \left[\frac{\partial D}{\partial A} \frac{A}{D} \right] \tag{B3}$$

into Equation (B2), it can be shown that Equation (B1) is equivalent to Equation (3.1) in the Merton (1974) model.

Table 1: Average Stock Returns and Regressions on Firm Characteristics

Panel A: Average Stock Returns for 25 Size and Book-to-Market Portfolios

	BTM 1	BTM 2	BTM 3	BTM 4	BTM 5	5 - 1	t-stat
Size 1	0.66%	1.14%	1.24%	1.37%	1.47%	0.81%	4.20
Size 2	0.90%	1.13%	1.26%	1.30%	1.32%	0.42%	2.18
Size 3	0.91%	1.15%	1.15%	1.27%	1.42%	0.51%	2.71
Size 4	1.00%	1.00%	1.12%	1.17%	1.29%	0.28%	1.53
Size 5	0.84%	0.98%	0.93%	1.00%	1.02%	0.18%	0.93
5 - 1	0.19%	-0.16%	-0.31%	-0.37%	-0.44%		
t-stat	0.70	-0.70	-1.55	-1.82	-2.15		

Panel B: Average Stock Returns for 5 Size and Book-to-Market Portfolios

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t - stat
BTM	0.85%	0.99%	1.00%	1.10%	1.23%	0.38%	2.31
Size	1.21%	1.19%	1.13%	1.07%	0.88%	-0.33%	-1.63

Panel C: Fama-MacBeth Regressions of Stock Returns on Firm Characteristics

Market Beta	ln(E)	ln(BE/E)	ln(BA/E)	ln(BA/BE)
0.27				
(0.90)				
	-0.18			
	(-4.27)			
-0.15	-0.19			
(-0.51)	(-4.73)			
		0.29		
		(5.15)		
			0.29	-0.31
			(4.94)	(-5.61)
-0.06	-0.16	0.16		
(-0.22)	(-3.86)	(3.17)		

Notes to Table: Panel A presents average value-weighted stock returns for 25 double-sorted size and book-to-market portfolios. Size 1 and BTM 1 indicate the lowest size and book-to-market portfolios respectively. Size 5 and BTM 5 indicate the highest size and book-to-market portfolios. Panel B presents the average value-weighted stock (levered equity) returns for five book-to-market (BTM) and size portfolios based on single sorts. Panel C presents the results of Fama-MacBeth regressions of stock returns on firm characteristics. We run a cross-sectional regression of stock returns on market beta and firm specific characteristics in each month. We report the time-series mean of the estimated coefficients and the corresponding t-statistics (in parentheses). Market beta is computed as in Fama and French (1992). E indicates market value of equity, BE indicates book value of equity, and BA indicates book value of assets. The sample period is from 1971 to 2012.

Table 2: Ex-Ante Leverage

Panel A: Average Leverage for 25 Size and Book-to-Market Portfolios

	BTM 1	BTM 2	BTM 3	BTM 4	BTM 5	5 - 1	t-stat
Size 1	26.9%	36.7%	45.7%	51.9%	63.3%	36.4%	33.8
Size 2	22.6%	36.8%	49.0%	57.4%	70.0%	47.4%	47.2
Size 3	23.0%	38.5%	51.9%	61.3%	70.0%	47.0%	53.6
Size 4	23.0%	40.8%	53.5%	61.8%	72.2%	49.1%	64.1
Size 5	24.4%	44.5%	57.2%	64.1%	72.4%	48.0%	47.1
5 - 1	-2.5%	7.8%	11.5%	12.2%	9.1%		
t-stat	-2.4	9.6	15.2	12.1	7.5		

Panel B: Average Leverage for 5 Size and Book-to-Market Portfolios

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t - stat
BTM	24.3%	38.3%	49.1%	55.7%	64.9%	40.6%	53.4
Size	48.6%	45.2%	44.7%	44.5%	44.4%	-4.1%	-3.5

Notes to Table: Panel A presents average leverage defined as the ratio of total liabilities to the sum of total liabilities and market value of equity for 25 double-sorted size and book-to-market portfolios. Size 1 and BTM 1 indicate the lowest size and book-to-market portfolios respectively. Size 5 and BTM 5 indicate the highest size and book-to-market portfolios. Panel B presents average leverage for five BTM and size portfolios based on single sorts.

Table 3: Leverage and Portfolio Returns: Univariate, Bivariate Sorts and Fama-MacBeth Regressions

Panel A: Average Unlevered Equity Returns for 25 Size and Book-to-Market Portfolios

	BTM 1	BTM 2	BTM 3	BTM 4	BTM 5	5 - 1	t-stat
Size 1	1.39%	1.43%	1.35%	1.35%	1.50%	0.11%	0.45
Size 2	1.38%	1.23%	1.36%	1.21%	1.03%	-0.35%	-1.57
Size 3	1.35%	1.34%	1.20%	1.23%	1.07%	-0.28%	-1.09
Size 4	1.20%	1.11%	1.14%	1.14%	1.06%	-0.14%	-0.89
Size 5	1.01%	1.14%	1.29%	1.23%	1.09%	0.08%	0.20
5 - 1	-0.38%	-0.28%	-0.06%	-0.13%	-0.41%		
t-stat	-1.56	-1.53	-0.26	-0.55	-3.21		

Panel B: Average Unlevered Equity Returns for 5 Size and Book-to-Market Portfolios

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t - stat
BTM	1.02%	1.13%	1.28%	1.25%	1.11%	0.09%	0.54
Size	1.42%	1.25%	1.26%	1.13%	1.08%	-0.34%	-2.38

Panel C: Fama-MacBeth Regressions of Unlevered Equity Returns on Firm Characteristics

Market Beta	ln(E)	ln(BE/E)	ln(BA/E)	ln(BA/BE)
0.68				
(2.83)				
	-0.14			
	(-4.36)			
0.49	-0.10			
(2.17)	(-3.53)			
		-0.04		
		(-0.57)		
			-0.04	-0.26
			(-0.41)	(-2.13)
0.42	-0.11	-0.12		
(2.37)	(-3.37)	(-1.70)		

Notes to Table: Panel A presents average value-weighted unlevered equity returns for 25 double-sorted size and book-to-market portfolios. Size 1 and BTM 1 indicate the lowest size and book-to-market portfolios respectively. Size 5 and BTM 5 indicate the highest size and book-to-market portfolios. Panel B presents average value-weighted unlevered equity returns for five BTM and size portfolios based on single sorts. In Panels A and B, the weights for the value-weighted portfolios are determined by the market value of equity. Panel C presents the results of Fama-MacBeth regressions of unlevered equity returns on firm characteristics. We run the cross-sectional regression of unlevered equity returns on market beta and firm-specific characteristics in each month. We report the time-series mean of the estimated coefficients and the corresponding t-statistics (in parentheses). Market beta is computed using the unlevered equity market returns as the independent variable. E indicates market value of equity, BE indicates book value of equity, and BA indicates book value of assets. The unlevered equity returns are computed using the unlevered firm value inferred from the Merton structural credit risk model. The face value of debt in the Merton model is assumed to be equal to the total liabilities and the maturity of debt is assumed to be 3.38 years.

Table 4: Regressions of Simulated Equity Returns and Stock Returns on Leverage and Other Firm Characteristics

Panel A: Regressions of Simulated Equity Returns on Leverage

Lev	Lev^2	Lev^3	Adj. R^2
-0.03			0.01%
(-1.26)			
-0.03	0.01		0.00%
(-0.41)	(0.08)		
-0.13	0.29	-0.21	-0.01%
(-0.70)	(0.59)	(-0.58)	

Panel B: Fama-MacBeth Regressions of Stock Returns on Firm Characteristics Including Leverage

Beta	ln(E)	ln(BE/E)	Lev	Lev^2	Lev^3
-0.06	-0.16	0.16			
(-0.22)	(-3.86)	(3.17)			
-0.06	-0.10	0.20	-0.10		
(-0.21)	(-2.48)	(4.71)	(-0.58)		
-0.02	-0.10	0.21	1.59	-1.89	
(-0.06)	(-2.50)	(4.91)	(3.31)	(-4.19)	
-0.02	-0.10	0.22	0.90	0.00	-1.38
(-0.06)	(-2.50)	(5.03)	(1.09)	(0.00)	(-1.16)

Notes to Table: Panel A presents the estimated coefficients and the t-statistics (in brackets) for regressions of simulated levered equity returns on leverage. The equity returns are simulated using the Merton Model. The values of the model parameters used in the simulations are $r =$ 3%, and $T = 3.38$ years. The volatility of the asset value (σ_A) is chosen to be uniformly distributed between 10% and 150%. The drift of the asset value (μ_A) is chosen to be uniformly distributed between -6% and 6%. The leverage defined as the ratio of debt to asset value is chosen to be uniformly distributed between 0 and 1. We simulate 10,000 monthly equity returns using these parameters and regress the simulated returns on leverage and higher order terms of leverage. Panel B presents the results of Fama-MacBeth regressions of stock returns on firm characteristics. We run a cross-sectional regression of stock returns on market beta and firm specific characteristics in each month. We report the time-series mean of the estimated coefficients and the corresponding t-statistics (in parentheses). Market beta is computed as in Fama and French (1992). E indicates market value of equity, BE indicates book value of equity, BA indicates book value of assets and *Lev* indicates firm leverage. The sample period is from 1971 to 2012.

Table 5: Portfolio Sorts and Fama-MacBeth Regressions of Stock Returns Adjusted for Leverage on Firm Characteristics

Panel A: Average Stock Returns Adjusted for Leverage for 25 Size and Book-to-Market Portfolios

	BTM 1	BTM 2	BTM 3	BTM 4	BTM 5	5 - 1	t-stat
Size 1	1.05%	1.29%	1.30%	1.36%	1.32%	0.27%	1.39
Size 2	1.09%	1.15%	1.15%	1.11%	1.08%	-0.01%	-0.04
Size 3	1.01%	1.16%	0.97%	1.11%	1.00%	0.00%	-0.04
Size 4	1.11%	0.99%	1.00%	1.03%	0.89%	-0.21%	-1.07
Size 5	0.83%	0.86%	0.85%	0.80%	0.81%	-0.02%	-0.88
5 - 1	-0.22%	-0.43%	-0.46%	-0.56%	-0.51%		
t-stat	-1.01	-2.38	-3.23	-4.22	-5.51		

Panel B: Average Stock Returns Adjusted for Leverage for 5 Size and Book-to-Market Portfolios

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t - stat
BTM	0.85%	0.90%	0.88%	0.90%	0.91%	0.05%	0.40
Size	1.26%	1.11%	1.04%	1.00%	0.80%	-0.46%	-3.23

Panel C: Fama-MacBeth Regression of Stock Returns Adjusted for Leverage

Market Beta	ln(E)	ln(BE/E)	ln(BA/E)	ln(BA/BE)
0.25				
(1.85)				
	-0.16			
	(-6.53)			
0.25	-0.15			
(1.81)	(-6.19)			
		0.06		
		(0.98)		
			0.04	-0.21
			(0.67)	(-4.68)
0.24	-0.15	-0.02		
(1.89)	(-5.91)	(-0.49)		

Notes to Table: Panel A presents average value-weighted stock returns adjusted for leverage for 25 double-sorted size and book-to-market portfolios. Size 1 and BTM 1 indicate the lowest size and book-to-market portfolios respectively. Size 5 and BTM 5 indicate the highest size and book-to-market portfolios. Panel B presents average value-weighted stock returns adjusted for leverage for five BTM and size portfolios based on single sorts. In Panels A and B, the weights for the value-weighted portfolios are determined by the market value of equity. Panel C present the results of Fama-MacBeth regressions of returns on different characteristics using the stock returns adjusted for leverage. The adjustment for leverage follows the textbook implementation, $R_{E,i}(t)(1 - L_i(t))$, where $L_i(t)$ indicates the market leverage of firm i at time t and $R_{E,i}(t)$ indicates excess stock returns for firm i at time t. Market leverage is computed as the ratio of total liabilities to the sum of total liabilities and the market value of equity. We recompute market betas for adjusted returns using an adjusted market portfolio. We run the cross-sectional regressions of returns on beta and firm-specific characteristics in each month. We report the time-series mean of the estimated coefficients and the corresponding t-statistics (in parentheses). Beta is computed as in Fama and French (1992). E indicates the market value of equity, and BE indicates the book value of equity.

Table 6: Portfolio Sorts and Fama-MacBeth Regressions of Stock Returns Adjusted for Leverage on Firm Characteristics: Alternative Adjustments

Panel A: Average Stock Returns Adjusted for Leverage for 5 Book-to-Market Portfolios

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t-stat
$R_{E,i}(1-L_i)$	0.85%	0.90%	0.88%	0.90%	0.91%	0.05%	0.40
$R_{E,i}/sqrt(exp(\alpha_1 log((A_i/E_i)^2)))$	0.86%	0.89%	0.86%	0.86%	0.86%	0.00%	-0.01
$R_{E,i}/sqrt(exp(\alpha_1 log((\Delta_{E,i}A_i/E_i)^2)))$	0.85%	0.86%	0.82%	0.80%	0.78%	-0.07%	-0.55
$R_{E,i}/sqrt(exp(\alpha_1 log((A_i/E_i)^2)+\alpha_2 log(\Delta_{E,i}^2)))$	0.88%	0.89%	0.85%	0.84%	0.80%	-0.08%	-0.58

Panel B: Average Stock Returns Adjusted for Leverage for 5 Size Portfolios

$R_{E,i}(1-L_i)$	1.26%	1.11%	1.04%	1.00%	0.80%	-0.46%	-3.23
$R_{E,i}/sqrt(exp(\alpha_1 log((A_i/E_i)^2)))$	1.24%	1.11%	1.03%	0.99%	0.79%	-0.46%	-3.40
$R_{E,i}/sqrt(exp(\alpha_1 log((\Delta_{E,i}A_i/E_i)^2)))$	1.22%	1.08%	1.00%	0.96%	0.76%	-0.46%	-3.65
$R_{E,i}/sqrt(exp(\alpha_1 log((A_i/E_i)^2)+\alpha_2 log(\Delta_{E,i}^2)))$	1.22%	1.11%	1.04%	0.99%	0.79%	-0.43%	-3.57

Panel C: Fama-MacBeth Regressions of Stock Returns Adjusted for Leverage

Returns	Beta	ln(E)	ln(BE/E)
$R_{E,i}(1-L_i)$	0.24	-0.15	-0.02
	(1.89)	(-5.91)	(-0.49)
$R_{E,i}/sqrt(exp(\alpha_1 log((A_i/E_i)^2)))$	0.24	-0.14	-0.04
	(1.88)	(-6.26)	(-0.91)
$R_{E,i}/sqrt(exp(\alpha_1 log((\Delta_{E,i}A_i/E_i)^2)))$	0.28	-0.14	-0.03
	(2.39)	(-6.55)	(-0.69)
$R_{E,i}/sqrt(exp(\alpha_1 log((A_i/E_i)^2)+\alpha_2 log(\Delta_{E,i}^2)))$	0.34	-0.11	-0.06
	(2.63)	(-5.77)	(-1.22)

Notes to Table: Panels A and B present average value-weighted stock returns adjusted for leverage for five book-to-market and size portfolios respectively based on single sorts. In Panels A and B, the weights for the value-weighted portfolios are determined by the market value of equity. Panel C presents the results of Fama-MacBeth regressions of returns on market beta and firm characteristics. The dependent variable is the stock return adjusted for leverage. In row 1 of each panel, the adjustment for leverage follows the textbook implementation, $R_{E,i}(t)(1-L_i(t))$, where $L_i(t)$ indicates the market leverage of firm i at time t and $R_{E,i}(t)$ indicates excess stock returns for firm i at time t. Market leverage is computed as the ratio of total liabilities to the sum of total liabilities and the market value of equity. In rows 2 to 4 of each panel, the coefficients α_1 and α_2 are estimated to adjust for leverage induced heteroskedasticity. In these specifications, A_i is defined as the sum of total liabilities and market value of equity, E_i is the market value equity. $\Delta_{E,i}$ is computed using the Merton model with the value of assets assumed equal to A_i, the face value of debt equal to the total liabilities, the debt maturity equal to 3.38 years, and the volatility equal to the stock volatility times $(1-L_i(t))$. For each specification, we recompute market betas for adjusted returns using an adjusted market portfolio. We run the cross-sectional regressions of returns on market beta and firm-specific characteristics in each month. We report the time-series mean of the estimated coefficients and the corresponding t-statistics (in parentheses). Beta is computed as in Fama and French (1992). E indicates the market value of equity, and BE indicates the book value of equity.

Table 7: Volatility and the Cross-Section of Returns

Panel A: Summary Statistics of Levered and Unlevered Volatility

	Mean	Median	Std. Dev.	p25	p75
Levered	54.4%	44.7%	35.8%	30.7%	67.1%
Unlevered	33.3%	25.9%	26.8%	15.3%	43.5%

Panel B: Average Ex-Ante Leverage for Portfolios Sorted on Volatility

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t - stat
Levered	53.4%	47.5%	43.8%	42.7%	48.0%	-5.4%	-2.53
Unlevered	74.4%	50.2%	39.9%	31.8%	25.8%	-48.6%	-46.70

Panel C: Average Stock Returns for Portfolios Sorted on Volatility

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t - stat
Levered	0.96%	0.99%	1.01%	0.78%	0.14%	-0.82%	-2.04

Panel D: Fama-MacBeth Regressions for Stock Returns

Beta	ln(E)	ln(BE/E)	Vol.
0.00	-0.07	0.28	-0.39
(0.00)	(-2.07)	(5.36)	(-1.20)

Panel E: Average Asset Returns Sorted on Volatility

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t - stat
Vol.	0.74%	0.83%	0.97%	1.15%	0.90%	0.17%	0.51

Panel F: Fama-MacBeth Regressions for Asset Returns

Beta	ln(E)	ln(BE/E)	Vol.
0.14	-0.10	0.03	0.94
(1.95)	(-5.95)	(0.96)	(2.44)

Notes to Table: Panel A presents summary statistics for levered and unlevered volatility, where p25 and p75 indicate the 25^{th} and 75^{th} percentile respectively. Panel B presents average ex-ante leverage for five levered and unlevered volatility portfolios based on single sorts. Panel C presents the average value-weighted stock (levered equity) returns in five levered volatility portfolios based on single sorts. Panel D presents the results of Fama-MacBeth regressions of stock returns on firm characteristics including levered volatility. The coefficients and the t-statistics are computed as in Panel C of Table 1. Panel E presents the average value-weighted asset returns for five unlevered volatility portfolios based on single sorts. Panel F presents the results of Fama-MacBeth regressions of asset returns on firm characteristics including unlevered volatility. The asset returns in Panels E and F are computed by adjusting for leverage induced heteroskedasticity using the regression specification in equation (5.2). Coefficients and t-statistics are computed as in Panel C of Table 3. The unlevered volatility is computed as the levered volatility times $(1 - L_i(t))$ where $L_i(t)$ indicates market leverage. E indicates market value of equity and BE indicates book value of equity.

Table 8: Average Unlevered Equity Returns. Alternative Model Specifications

Panel A: Merton with T = 1 year

	BTM 1	BTM 2	BTM 3	BTM 4	BTM 5	5 - 1	t-stat
Size 1	1.32%	1.39%	1.35%	1.34%	1.31%	0.00%	0.02
Size 2	1.32%	1.22%	1.30%	1.18%	1.09%	-0.23%	-1.21
Size 3	1.22%	1.18%	1.07%	1.12%	1.03%	-0.19%	-1.03
Size 4	1.12%	0.96%	0.98%	1.01%	0.84%	-0.28%	-1.44
Size 5	0.82%	0.84%	0.76%	0.78%	0.85%	0.03%	0.18
5 - 1	-0.50%	-0.55%	-0.59%	-0.56%	-0.46%		
t-stat	-2.19	-2.98	-3.83	-3.71	-3.02		

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t-stat
UEV	0.80%	0.81%	1.01%	1.28%	0.97%	0.16%	0.53

Panel B: Merton with T = 5 years

	BTM 1	BTM 2	BTM 3	BTM 4	BTM 5	5 - 1	t-stat
Size 1	1.36%	1.45%	1.34%	1.37%	1.29%	-0.07%	-0.29
Size 2	1.38%	1.22%	1.37%	1.22%	1.08%	-0.30%	-1.30
Size 3	1.33%	1.35%	1.21%	1.23%	1.18%	-0.15%	-0.48
Size 4	1.20%	1.13%	1.18%	1.19%	1.07%	-0.13%	-0.82
Size 5	1.01%	1.17%	1.04%	1.00%	1.23%	0.23%	0.48
5 - 1	-0.35%	-0.28%	-0.30%	-0.37%	-0.06%		
t-stat	-1.37	-1.45	-1.81	-2.41	-1.02		

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t-stat
UEV	1.01%	0.99%	1.15%	1.38%	1.48%	0.47%	1.15

Panel C: Merton with Variable Debt Maturities

Size 1	1.39%	1.44%	1.25%	1.30%	1.31%	-0.08%	-0.34
Size 2	1.27%	1.12%	1.32%	1.18%	1.03%	-0.24%	-0.96
Size 3	1.26%	1.23%	1.11%	1.21%	1.30%	0.04%	0.33
Size 4	1.12%	1.05%	1.14%	1.16%	1.13%	0.01%	0.16
Size 5	0.96%	1.09%	1.00%	0.98%	1.08%	0.12%	0.36
5 - 1	-0.43%	-0.34%	-0.25%	-0.32%	-0.23%		
t-stat	-1.69	-1.67	-1.49	-1.98	-2.26		

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t-stat
UEV	1.11%	1.01%	1.15%	1.41%	1.61%	0.51%	1.03

Panel D: Merton with T = 3.38 years and F = Total Debt

Size 1	1.30%	1.50%	1.47%	1.45%	1.36%	0.06%	0.28
Size 2	1.39%	1.34%	1.40%	1.33%	1.20%	-0.20%	-0.98
Size 3	1.23%	1.35%	1.25%	1.29%	1.13%	-0.10%	-0.53
Size 4	1.25%	1.11%	1.16%	1.20%	0.98%	-0.27%	-1.46
Size 5	0.94%	1.04%	0.98%	0.95%	1.04%	0.11%	0.60
5 - 1	-0.36%	-0.46%	-0.49%	-0.50%	-0.31%		
t-stat	-1.47	-2.29	-2.87	-3.05	-1.94		

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t-stat
UEV	0.97%	1.01%	1.17%	1.33%	1.19%	0.22%	0.60

Panel E: Merton with T = 3.38 years and Returns Computed using Equation (2.2)

Size 1	1.04%	1.30%	1.29%	1.40%	1.47%	0.44%	2.07
Size 2	1.07%	1.11%	1.19%	1.10%	1.10%	0.02%	0.15
Size 3	1.09%	1.18%	1.00%	1.07%	1.07%	-0.02%	-0.11
Size 4	1.05%	1.02%	1.00%	0.98%	0.94%	-0.11%	-0.77
Size 5	0.84%	0.93%	0.81%	0.83%	0.79%	-0.05%	-1.27
5 - 1	-0.20%	-0.37%	-0.48%	-0.57%	-0.68%		
t-stat	-0.77	-1.96	-3.17	-4.16	-6.31		

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t-stat
UEV	0.72%	0.93%	0.96%	1.08%	0.80%	0.08%	0.21

Panel F: Leland-Toft with T = 3.38 years

Size 1	1.24%	1.38%	1.26%	1.18%	1.11%	-0.13%	-0.61
Size 2	1.34%	1.19%	1.20%	1.08%	0.83%	-0.51%	-2.49
Size 3	1.26%	1.26%	1.00%	0.93%	0.77%	-0.49%	-2.47
Size 4	1.09%	0.97%	0.89%	0.94%	0.64%	-0.44%	-2.40
Size 5	0.77%	0.91%	0.75%	0.67%	0.77%	0.00%	0.01
5 - 1	-0.47%	-0.47%	-0.51%	-0.51%	-0.34%		
t-stat	-2.07	-2.41	-3.22	-3.77	-2.13		

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t-stat
UEV	0.70%	0.78%	0.92%	1.12%	1.11%	0.42%	1.26

Panel G: Merton with T = 3.38 years and F = Total Liabilities. Alternative Portfolio Weights.

Size 1	1.13%	1.22%	1.02%	0.96%	0.56%	-0.57%	-2.10
Size 2	1.30%	1.09%	1.17%	0.99%	0.58%	-0.71%	-3.04
Size 3	1.27%	1.22%	1.11%	1.06%	0.77%	-0.51%	-2.03
Size 4	1.19%	1.03%	1.03%	0.93%	0.89%	-0.30%	-1.51
Size 5	1.07%	1.17%	0.97%	0.88%	0.87%	-0.20%	-1.34
5 - 1	-0.06%	-0.05%	-0.05%	-0.08%	0.31%		
t-stat	-0.18	-0.40	-0.29	-0.58	1.30		

	Ptf 1	Ptf 2	Ptf 3	Ptf 4	Ptf 5	5 - 1	t-stat
UEV	0.95%	0.93%	1.09%	1.30%	1.20%	0.25%	0.65

Notes to Table: We present average value-weighted unlevered equity returns for each of the 25 double-sorted size and book-to-market (BTM) portfolios as well as the five single-sorted unlevered equity volatility (UEV) portfolios, using alternative specifications and implementations of the structural credit risk models. In each panel, the weights are determined by the market value of equity. Panel A presents the average unlevered equity returns using the Merton model with the debt defined as the sum of half the long term debt and the short term debt and all debt assumed to mature in 1 year. Panel B uses the Merton model with debt equal to total liabilities and all debt assumed to mature in 5 years. Panel C uses the Merton model with the debt equal to total liabilities and maturity computed every fiscal year using firm-specific debt maturity data. Panel D uses the Merton model with debt equal to the sum of long-term and short-term debt and maturity T=3.38 years. Panel E uses the Merton model with debt equal to total liabilities and maturity T=3.38 years, and unlevered returns computed using equation (3.1). Panel F uses the Leland-Toft model with debt equal to total liabilities and maturity T=3.38 years. Panel G uses the Merton model with debt equal to the total liabilities and maturity T=3.38 years. In this panel, the weights for computing the portfolio unlevered equity returns are determined using the market value of unlevered equity.

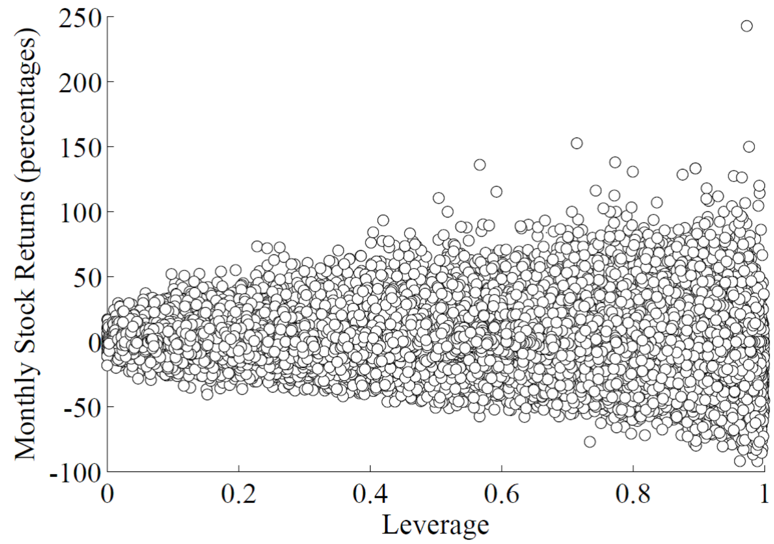
Table 9: Fama-MacBeth Regressions of Unlevered Equity Returns on Firm Characteristics. Alternative Model Specifications

Beta	ln(E)	ln(BE/E)	Beta	ln(E)	ln(BE/E)
Panel A: Merton with T = 1 year			**Panel B: Merton with T = 5 years**		
0.41			0.62		
(2.31)			(2.65)		
0.23	-0.16	-0.07	0.34	-0.12	-0.10
(1.58)	(-5.30)	(-1.05)	(2.02)	(-3.56)	(-1.36)
Panel C: Merton with Variable Debt Maturities			**Panel D: Merton with T = 3.38 years and F = Total Debt**		
0.53			0.60		
(2.27)			(2.39)		
0.31	-0.14	-0.11	0.31	-0.14	-0.08
(1.69)	(-4.22)	(-1.56)	(1.62)	(-3.91)	(-1.24)
Panel E: Merton with T = 3.38 years and Returns Computed using Equation (2.2)			**Panel F: Leland-Toft with T = 3.38 years**		
0.40			0.43		
(2.10)			(2.26)		
0.15	-0.19	0.14	0.37	-0.11	-0.14
(0.97)	(-5.58)	(2.14)	(2.36)	(-4.08)	(-2.09)
Panel G: Merton with T = 3.38 years and F = Total Liabilities. Alternative Portfolio Weights.					
0.59					
(2.46)					
0.36	-0.11	-0.10			
(1.98)	(-3.52)	(-1.61)			

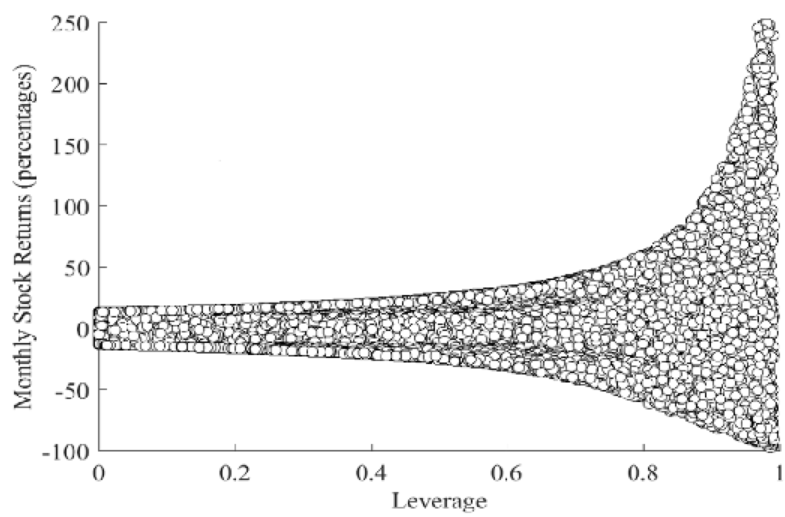
Notes to Table: We present the results of Fama-MacBeth regressions of unlevered equity returns on firm characteristics. We run the cross-sectional regression of unlevered equity returns on market beta and firm-specific characteristics in each month. We report the time-series mean of the estimated coefficients and the corresponding t-statistics (in parentheses). Market beta is computed using unlevered equity market returns as the independent variable. E indicates the market value of equity and BE indicates the book value of equity. Panel A presents the results for unlevered equity returns computed using the Merton model with the debt defined as the sum of half the long term debt and the short term debt and all debt assumed to mature in 1 year. Panel B uses the Merton model with debt equal to total liabilities and all debt assumed to mature in 5 years. Panel C uses the Merton model with the debt equal to total liabilities and maturity computed every fiscal year using firm-specific debt maturity data. Panel D uses the Merton model with debt equal to the sum of long-term and short-term debt and maturity T=3.38 years. Panel E uses the Merton model with debt equal to total liabilities and maturity T=3.38 years, and unlevered returns computed using equation (3.1). Panel F uses the Leland-Toft model with debt equal to total liabilities and maturity T=3.38 years. In Panels A to F, the value-weighted market portfolio is computed using the weights based on market value of equity. In Panel G, the value-weighted market portfolio is computed using the weights based on the market value of unlevered equity.

Figure 1: Leverage and Levered Equity Returns

Panel A: Historical Data

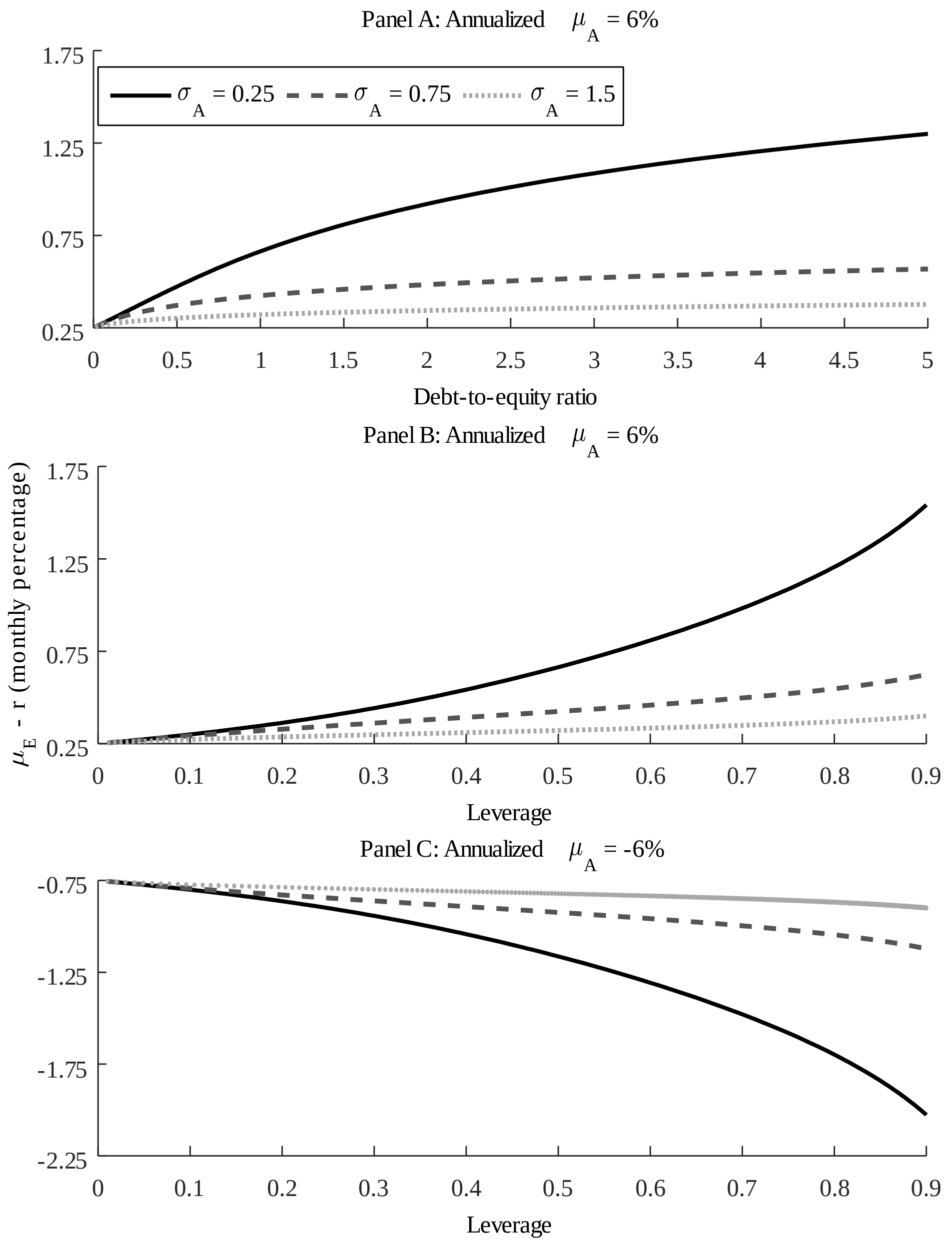


Panel B: Simulated Data



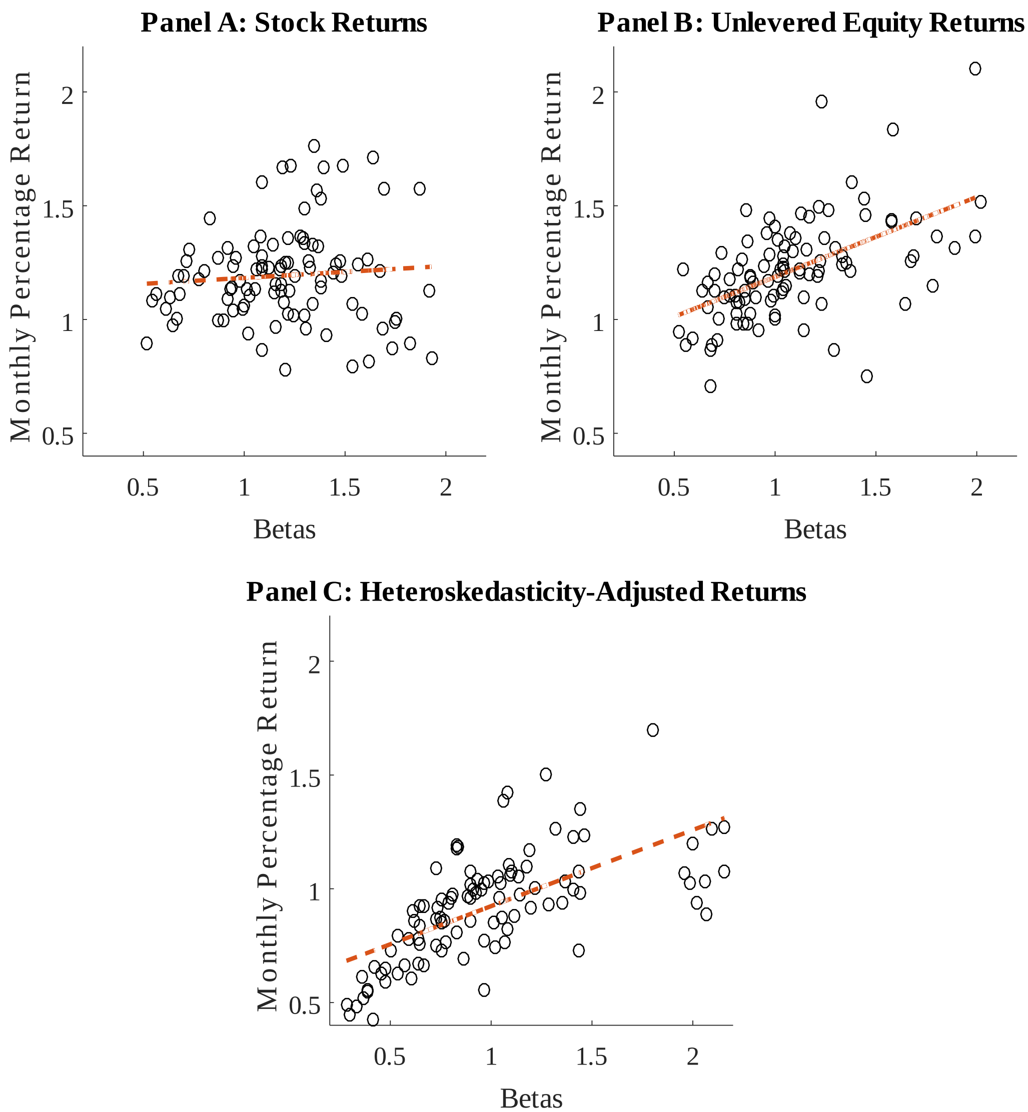
Notes to Figure: We present the relation between leverage and levered equity (stock) returns using simulated data from the Merton model and historical data. Panel A presents the relation between historical monthly stock returns and leverage. The graph depicts the monthly stock returns for firms with asset volatility below the sample median (27%). The circles indicate the individual firm-month stock returns. Panel B presents the relation between simulated monthly stock returns and leverage. We simulate individual firm-month returns for various levels of firm leverage, unlevered equity volatility and unlevered equity return. The annualized risk-free rate is assumed to be $r = 3\%$, the debt maturity $T = 3.38$ years, the firm leverage is drawn from uniform distribution between 0 and 1, the unlevered equity volatility is randomly drawn from a uniform distribution between 5% and 150%, and the monthly unlevered equity return is drawn from a uniform distribution between -13% and 13%.

Figure 2: Leverage and Levered Equity Returns



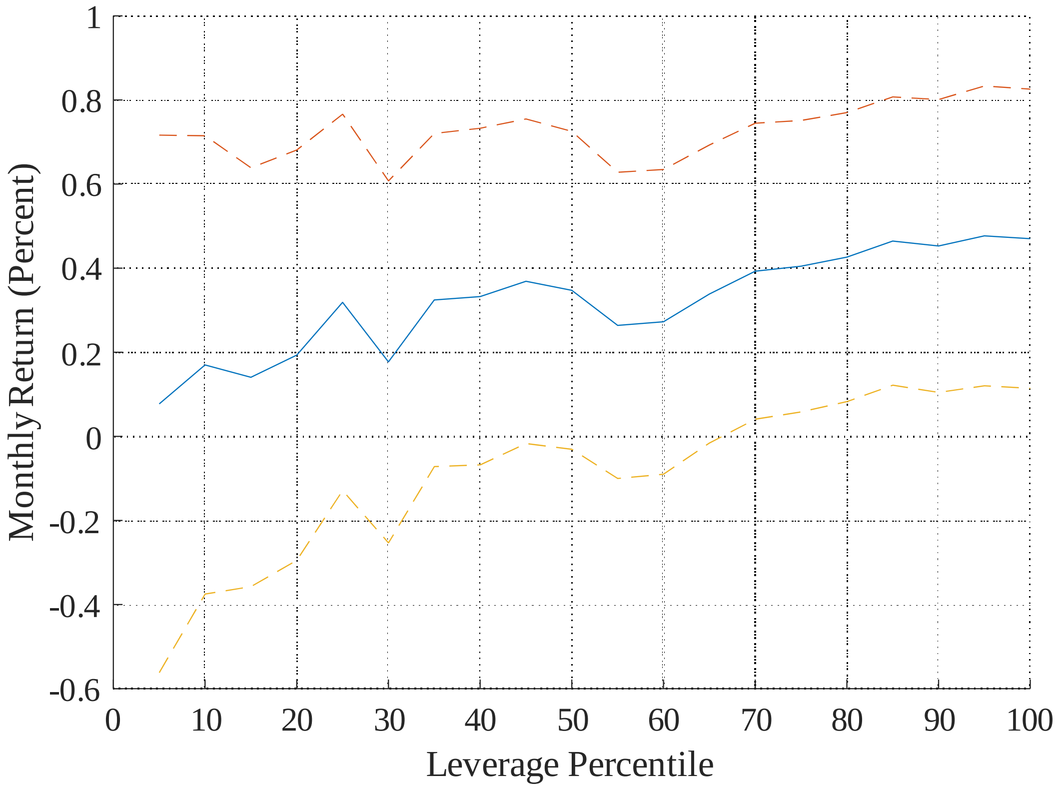
Notes to Figure: We present the relation between leverage and excess equity returns generated using the Merton model. Panel A presents stock returns generated from the model for different values of the debt-to-equity ratio and unlevered equity volatilities. Panel B presents the same returns but as a function of leverage defined as the ratio of debt to the sum of debt and equity. Panel C presents stock returns as a function of leverage defined as the ratio of debt to the sum of debt and equity, but with $\mu_A = -6\%$ annually. Panels A and B are generated using $\mu_A = 6\%$ annually. The values of the other model parameters are $r = 3\%$, and $T = 3.38$ years.

Figure 3: Scatter Plot of Monthly Returns Against Portfolio Market Betas



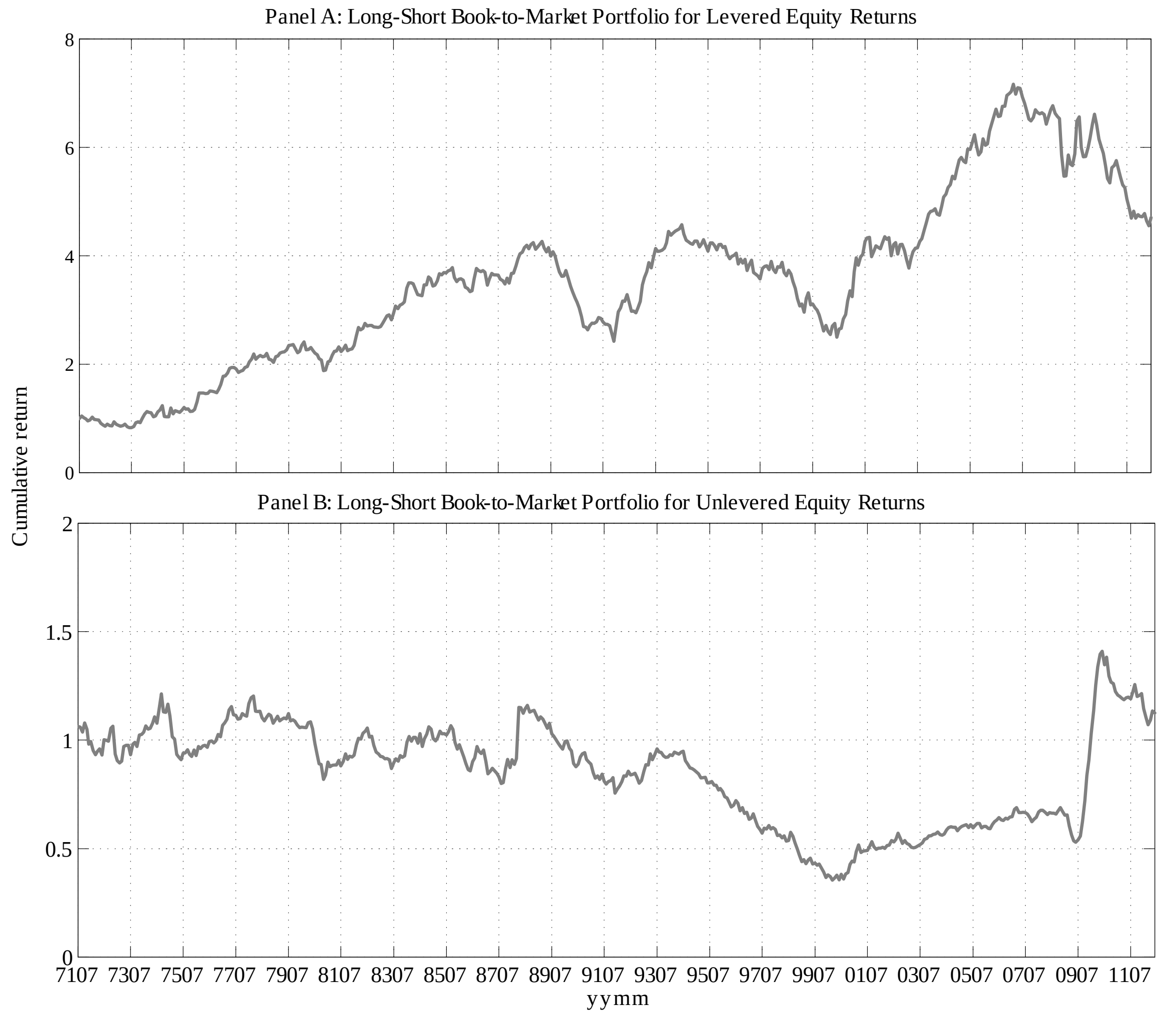
Notes to Figure: We scatter plot the average returns of the 100 portfolios used in the Fama-MacBeth regressions in Tables 1, 3 and 5 against market beta. Panel A presents the results for stock returns. The slope of the regression line in Panel A is 0.05 with a t-statistic of 0.80. Panel B presents the results for unlevered equity returns. The unlevered equity returns are computed using the Merton model with debt equal to total liabilities and all debt assumed to mature in 3.38 years, as in Table 3. The slope of the regression line in Panel B is 0.35 with a t-statistic of 6.37. Panel C presents the results for heteroskedasticity-adjusted returns using the textbook adjustment for leverage, as in Table 5. The slope of the regression line in Panel C is 0.33 with a t-statistic of 8.36.

Figure 4: Leverage and Long-Short Book-to-Market Portfolio Returns



Notes to Figure: We present the return for long-short book-to-market portfolios (solid line) in various subsamples based on leverage. The dashed line presents the 95% confidence interval. In each month, we drop all firms that are above a given leverage percentile (x-axis) and form five book-to-market portfolios using this sample. Leverage is defined as the total liabilities scaled by the sum of total liabilities and market value of equity.

Figure 5: Cumulative Return of Long-Short Book-to-Market Portfolios



Notes to Figure: We present the cumulative return for long-short book-to-market portfolios. Panel A presents the growth of a one dollar initial investment in levered equity (stock) made in June 1971. Panel B presents the growth of a one dollar initial investment in unlevered equity. In both cases, we construct five book-to-market portfolios. The long-short returns correspond to buying the highest book-to-market portfolio and selling the lowest book-to-market portfolio.