

# A Model of Multi-Store Shoppers' Buying Decisions

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## **A Model of Multi-Store Shoppers' Buying Decisions**

### **Abstract**

We propose an analytical model of multi-store shoppers buying items from their shopping lists; specifically, “common items” that are available at competing stores. Multi-store shoppers buy some common items at the first store they visit, others are deferred to a competing store. These buying decisions depend on the prices observed at the first store and uncertainty about savings if purchases are deferred to a competing store. Analysis of our model shows that, if multi-store shoppers enjoy psychological benefits (in addition to rational economic benefits) from saving money, their purchase decisions depend on the ratio of expected savings to the variance in savings. If multi-store shoppers are motivated only by rational economic benefits, however, their purchase decisions depend on expected savings alone. To demonstrate empirically that some multi-store shoppers are motivated by psychological benefits, we develop a finite mixture model capable of capturing heterogeneity in shoppers' response to price savings. Using actual purchase data, we demonstrate that a substantial proportion of multi-store shoppers (38% in our sample) enjoy psychological benefits, in addition to rational economic benefits, from saving money. Our findings imply that retailers can affect where multi-store shoppers buy by pricing in a way that reduces or increases uncertainty about price savings. Thus, our analysis provides a new perspective on the connection between retail pricing and multi-store shopping behavior.

**Key Words:** Multi-Store Shopping, Cherry-Picking, Retailing, Behavioral Economics, Finite Mixture Model

## 1. Introduction

Consider a grocery shopper's purchase decisions. She goes shopping to buy certain items, which are usually recorded on a shopping list (Spiggle 1987, see Kahn and McAlister 1997, pp.118-9, for a discussion of shopping lists). Given that shopping list, store choice models assume that the shopper visits whichever store minimizes her total cost of shopping; i.e., the cost of travel and inconvenience plus the expected prices of items on the list (Bell, et al. 1998, Briesch, et al. 2009). But a multi-store shopper decides to buy the items on her shopping list at more than one store.<sup>1</sup> Some are *exclusive items* available at only one store, but most are *common items* available at multiple stores. At the first store a multi-store shopper visits, she must decide which common items to buy and which to defer buying until she visits another store. The implication is that the multi-store shopper compares common item prices *observed* at the first store with prices *anticipated* at another store and will presumably defer buying only if the prices at the second store are anticipated to be lower.

The presumption that multi-store shoppers will buy items at the store offering lower prices is consistent with rational economic behavior. However, evidence suggests that related factors affect buying decisions.

- First, multi-store shoppers often encounter substantial price variation between grocery stores. Fox and Hoch (2005) found an average price difference of 10% between competing retailers for common items (p. 46). Given the frequency of grocery shopping and multi-store shoppers' propensity to plan their trips and purchases (cf. Urbany, et al. 1991; Urbany et al. 2000; Fox and Hoch 2005; Talukdar, et al. 2010), we would expect multi-store shoppers to be aware of these price differences and to take advantage of them. However, the empirical data used in this paper (details about the dataset are in §4.1) show that shoppers who visit both of the leading grocery retailers in the Chicago market on the same day buy common items at the low price only 67.2% of the time. It is surprising that

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<sup>1</sup> The implication that multi-store shopping trips are pre-planned, rather than the decision to visit more than one store being made during the shopping trip, is supported empirically by Fox and Hoch (2005) and Talukdar, et al. (2010).

multi-store shoppers do not buy at the low price more often, given that most prior research assumes that the objective of multi-store shopping is to search for deals.

- Second, our empirical data (again, see §4.1 for details) show that if the first store that a shopper visits offers a price that is less than or equal to the second store's price, then the shopper buys at that low price 76.9% of the time. If, on the other hand, the first store that a shopper visits does not offer the low price (i.e., its price is higher than the price offered at the second store) then the shopper buys at the low price only 55.8% of the time.<sup>2</sup> This analysis suggests that the order in which multi-store shoppers visit stores affects the probability of buying at the low price.

### 1.1. Research Overview and Contribution

To account for these putative anomalies, we propose an analytical model for the *buying decisions of shoppers who visit two grocery stores* in order to predict whether multi-store shoppers buy common items at the first store they visit or defer purchase to the second store. Our model incorporates the uncertainty inherent in deferral decisions having observed prices only at the first store. Our model also accommodates systematic deviations from rational economic behavior by recognizing that shoppers may enjoy psychological benefits from saving money. This is consistent with several theories of the psychology of shopping, including transaction utility (Thaler 1985), smart-shopper feelings (Schindler 1989, 1998) and market mavenism (Feick and Price 1987). While we do not advocate for any particular theory *per se*, we observe that they all imply that shoppers may derive utility from saving money beyond the economic value of that savings. Our analysis results in a surprisingly simple and parsimonious specification for the utility of multi-store shoppers. If these shoppers enjoy psychological (in addition to economic) benefits from saving money, we prove that the decision to defer purchase of common items to the second store depends on a simple ratio of expected savings to the variance of savings.<sup>3</sup> The

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<sup>2</sup> These proportions are different ( $z$ -value=7.04,  $p$ -value $\leq$ 0.0001). Note that multi-store shoppers make only 57.0% of their purchases at the first store visited.

<sup>3</sup> More precisely, it increases according to an approximately piecewise-linear sigmoidal function of this ratio.

implication is that uncertainty about prices at the second store affects multi-store shoppers who enjoy the psychological benefits of saving money differently from those who do not.

Observe that we assume multi-store shoppers visit two stores. While this assumption admittedly simplifies our analytical arguments, it is also consistent with time-constrained shopping (Morgan and Manning 1985) and retail duopoly markets; moreover, it has very strong empirical support.<sup>4</sup> It is important to note that, for our purposes, we are not interested in why the shopper visits both stores--she may be motivated to find bargains (Fox and Hoch 2005) but she may also be buying each store's exclusive products and/or taking advantage of each store's category-specific offerings (Briesch, et al. 2013).

We provide an empirical demonstration of the proposed model by developing a finite mixture specification capable of capturing heterogeneity in shoppers' responses to price savings. This specification is estimated using actual purchase data from a panel dataset of shopping trips and item purchases in the Chicago market.<sup>5</sup> We demonstrate that a substantial proportion of multi-store shoppers (38% in our sample) are driven by the psychological benefits, in addition to the economic benefits, of saving money. Taken together, our analytical and empirical results represent a new perspective on the connection between retail price competition and the purchase decisions of multi-store shoppers. They suggest that retailers could increase sales among multi-store shoppers by matching the timing and depth of competitor discounts for items that the retailer generally prices below competition (thereby reducing the variance of price savings) while

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<sup>4</sup> The vast majority of multi-store shopping trips appear to involve two stores. Fox and Hoch (2005) found that, in 99% of cases in which shoppers visited multiple grocery stores on a single day, they visited exactly two stores. Talukdar, et al. (2010) found that 98% of shoppers' total grocery expenditures are made at their two most frequently visited stores.

<sup>5</sup> Our empirical application requires us to identify multi-store shopping visits (see Carlson and Gieseke 1983, Dreze 1999, Fox and Hoch 2005 and Talukdar, et al. 2010 for different approaches to identifying multi-store shopping in panel data) and we must be careful to distinguish multi-store shopping from store switching (e.g., Rhee and Bell 2002).

discounting independent of competitors for items that the retailer generally prices above competition.

## **1.2. Organization**

The remainder of the paper is organized as follows. We begin by discussing the relevant literature in §2. In §3, we introduce and develop a mathematical framework for analyzing multi-store purchasing behavior, resulting in two testable propositions. With a view toward testing these propositions, we formulate an empirical model of multi-store purchasing in §4. There we also describe the dataset used to demonstrate the analytically-derived propositions, develop a constrained latent mixture model, and report the empirical results. In §5, we discuss our findings and conclude with limitations and suggested topics for future research.

## **2. Literature Review**

Shoppers who visit multiple stores have long been of interest to both researchers and practitioners. There is ample evidence that multi-store shopping is pervasive and that it materially affects retailer sales and profitability. For example, Fox and Hoch (2005) found that 13.2% of all grocery store visits are made by shoppers who also visit a competing grocery store on the same day. On these visits, multi-store shoppers pay higher prices at their “primary” stores compared to the other, or “secondary” stores, that they visit. Talukdar, et al. (2010) found that the “opportunists” shopper segment--35.5% of shoppers in their sample--“regularly shop and split their purchases between primary and secondary stores” (p. 347). Moreover, Talukdar and colleagues found that “opportunists” are generally less profitable for retailers than the average shopper. Food Marketing Institute’s *U.S. Grocery Shopper Trends 2010* (2010) reported that nearly a third of shoppers compare prices across stores and nearly half visit multiple stores to find bargains. Food Marketing Institute also reported that shoppers are spending less at their

primary grocery store, increasingly buying food and packaged goods at other stores and retail formats.

Three existing literature streams are particularly relevant to multi-store shopping: marketing researchers have studied cherry-picking, or shopping for bargains across stores; economists have considered models of sequential search for grocery products; and social psychologists have discussed the psychological (i.e., non-economic) benefits of saving money.

### **2.1. Cherry-Picking**

A number of recent studies have investigated cherry-picking--shopping for bargains across stores--focusing on who cherry-picks, how much they cherry-pick and how retailers' pricing and store location decisions affect cherry-picking behavior. Note that the term "cherry-picking" implies that the motivation for multi-store shopping is to buy at a lower price; as we will explain in §3, our model of multi-store shopping allows for other motivations as well.

Cherry-picking has been used in game theoretic models of the retailer/shopper interaction. For example, Lal and Rao (1997) developed a model that segments shoppers into those who are time constrained and those who cherry-pick. Dreze (1999) analyzed a segment of shoppers who are price sensitive with low travel costs and so can be induced to cherry-pick by retailer price deals. Both studies argue that cherry-pickers will travel to multiple stores to take advantage of price deals because of their low opportunity cost of time. Moreover, cherry-picking shoppers are generally assumed to be less profitable than other shoppers for retailers.

Other cherry-picking studies have taken a more empirical approach. Fox and Hoch (2005) found that cherry-picking is materially important for retailers, with an average of 13.2% of a store's visitors cherry-picking on any given day (p. 50). The authors found that at the shopper's *primary* store, she i) makes the large majority of her purchases, ii) pays higher prices

on average, and iii) is less likely to secure bargains. As a result, the primary store is not adversely affected by cherry-picking as much as the *secondary* store. Fox and Hoch also found evidence that cherry-picking trips are planned, with more than twice as much spent on such trips compared to single-store shopping trips. In terms of demographics, cherry-picking behavior was found to be positively associated with household size, home ownership and senior citizenship, but negatively associated with working adult females and income. The authors also found that, depending upon the shopper's wage rate, cherry-picking is economically rational behavior for a substantial proportion of households.

Talukdar, et al. (2010) investigated the profit impact of extreme cherry-picking on retailers, where extreme cherry-pickers were defined as those who generated a negative profit contribution at their secondary store--2.1% of shoppers were found to generate such a negative contribution. While cherry-picking behavior was not measured directly, the authors divided households into i) regular, ii) occasional and iii) rare cherry-pickers. They found that the first two segments are more motivated to search and believe they are better at price search than shoppers who rarely cherry-pick. Interestingly, the authors found that a household's opportunity cost of shopping is not monotonically related to its profit contribution at the secondary retailer.

Gauri, et al. (2008) decomposed shoppers' price search into two strategic dimensions: i) across stores and ii) across time within a store (search strategies were self-reported). The cross-store dimension is consistent with cherry-picking. The authors found that shoppers' opportunity costs are negatively related to both dimensions of price search. They also found that shoppers who principally engage in cross-store price search (cherry-picking) realize roughly the same savings as those who engage in temporal price search but are substantially less profitable for retailers. Interestingly, the authors measured shoppers' self-perceptions of market mavenism and



search skills, finding that both are positively related to the two dimensions of price search (including cherry-picking) and to shoppers' savings from price search. These mavenism and perceived search skill findings suggest that psychological factors may affect search behavior and consequently support the inclusion of psychological benefits in our model of multi-store shopping behavior.

## **2.2. Grocery Store Price Search**

In economics, there have been several studies of the search for information about frequently purchased goods across grocery stores (e.g., Stigler 1961). These studies assumed sequential search for a predetermined list of goods to be purchased. Burdett and Malueg (1981) proposed a model of sequential search across stores to purchase multiple goods. Their analysis linked the search for multiple goods (where the price of each good at each store is randomly drawn from a known distribution that is common to all stores) to the search for a single good. The authors found that, as with a single good, multiple goods have threshold prices. If the price of any item at the current store is below that threshold, then it should be purchased; otherwise, the shopper visits another store. Threshold prices depend on the known distribution of prices for each good. The authors focused on the search for two goods and considered two specific cases--with and without recall. In the latter case, the consumer cannot return to stores visited previously without incurring a cost (this approximates the problem that grocery shoppers face). Carlson and McAfee (1984) extended Burdett and Malueg's analysis from 2 to  $n$  different goods. Compared to these models, we: 1) assume that multi-store shoppers visit a fixed sample size of two stores, consistent with time-constrained shopping; 2) allow prices at the two stores to have different distributions; and 3) generalize the cost minimization objective function commonly specified in optimal price search (Maier 1990) to include psychological benefits from saving money.

Carlson and Gieseke (1983) tested hypotheses from these sequential search models using panel data. They found that shoppers with lower search costs tend to visit more stores and pay lower prices; however, they also found decreasing benefits to search. Carlson and Gieseke's study was the first to provide empirical support for grocery store search theory using actual purchase data. Subsequent studies found that the extent of search across grocery stores is negatively related to shoppers' opportunity cost of time, using variables such as wage rate and income (Marmorstein, et al. 1992; Ratchford and Srinivasan 1993). Petrevu and Ratchford (1997) found that the economic benefits of search, which depend on price dispersion and per capita income, are positively related to the amount of search. They also found that the costs of search, including opportunity cost of time and perceived costs such as time pressure, difficulty in comparing stores, and lack of physical energy are negatively related to the amount of search.

Urbany, et al. (1991) and Urbany, et al. (2000) used survey data to investigate price search and multi-store shopping for groceries, developing a detailed behavioral profile. Among other findings, they reported that:

- I. 22%-24% of consumers regularly shop at multiple stores,
- II. 25% shop other than their principal store to get advertised specials,
- III. 42% of shoppers compare prices across stores at least once per month,
- IV. 19% regularly shop specials at multiple stores, and
- V. 57%-80% of shoppers read fliers to compare prices across stores.

### **2.3. Psychological Benefits of Saving Money**

We now briefly review theories predicting that people may enjoy psychological benefits from saving money. Common to all of these theories is the implication that saving money offers shoppers psychological benefits, in addition to the economic value of the item(s) purchased.

- Thaler (1985) argued that shoppers derive utility in proportion to the amount they pay below their reservation price, which he termed *acquisition utility*, while deriving

additional utility from paying less than they expected to pay, which he termed *transaction utility*. Transaction utility thus represents the psychological benefit from getting a bargain, independent of the economic benefit inherent in the purchase.

- Schindler (1989, 1998) proposed that “smart-shopper feelings” are an emotional consequence of finding bargains. Smart-shopper feelings are self-perceptions of one’s shopping prowess. They “involve a sense of efficacy and competence” (Schindler 1989, p. 449) as well as “pleasure in beating the system” (Schindler 1989, p. 450). Because smart-shopper feelings are pleasurable emotions, this theory explains extreme bargain hunting behavior as an emotional dependency. Schindler (1998) determined that, if shoppers attribute bargain purchases to themselves (i.e., their own effort or acumen), then they are more likely to share information about the bargain purchase with others and to buy again. Thus, he found a relationship between smart-shopper feelings and post-purchase behaviors. Prelec and Bodner (2003) suggested that smart-shopper feelings might act as a type of diagnostic utility, a signal that the individual knows more than other shoppers.
- Feick and Price (1987) introduced the related idea of market mavens, consumers who purposefully gather information about markets, stores, products and prices. The authors argued that market mavens benefit by sharing that information with other consumers. Feick and Price found that, across product types, mavenism is correlated with coupon usage and, more importantly, with shopping enjoyment. The former finding suggests that market mavens may seek bargains more than other shoppers do. Together, their findings suggest that market mavens derive psychological benefit (i.e., enjoyment) from shopping and saving.
- Rick, et al. (2007) proposed a theory that reflects individual differences in the pain of paying for goods. “Tightwads” feel more pain and consequently spend less than their own rational analysis would dictate, while “spendthrifts” feel less pain and so spend more than they would like. Across several large samples, the authors found that about 24% of people are tightwads, for whom the pain of paying for goods leads to less spending than they would rationally prefer. In related work, Lastovicka, et al. (1999) developed and measured the construct of “frugality.” While tightwads are driven by pain induced by

spending, frugal shoppers are driven, in part, by good feelings induced by saving. Both studies, however, imply a psychological benefit due to saving (or not spending) money.

- Additional evidence that consumers derive psychological benefits from saving money comes from work on the hedonic and utilitarian benefits of promotions (Chandon, et al. 1999) and from surveys that show that highly deal-prone consumers get a sense of achievement from buying items on special (Garretson and Burton 2003).

### 3. Analytical Model

We now develop a formal analytical model of the multi-store shopper's buying decisions for common items. Before embarking on trip  $t$ , the shopper compiles a *shopping list* of common items  $i = 1, 2, \dots, n$ , along with purchase quantities  $q_{1t}, q_{2t}, \dots, q_{nt}$  of each item. Purchase quantities are treated as exogenous and fixed. There are two retailers from which the items can be purchased. The order in which the retailers are visited is important, as discussed in §1. On trip  $t$ , the first retailer visited, denoted Retailer 1, offers a vector of stochastic prices  $p_t^{(1)} = (p_{1t}^{(1)}, p_{2t}^{(1)}, \dots, p_{nt}^{(1)})$ . The second retailer visited, denoted Retailer 2, offers a possibly different vector of stochastic prices  $p_t^{(2)} = (p_{1t}^{(2)}, p_{2t}^{(2)}, \dots, p_{nt}^{(2)})$ . Prices at the two retailers may be correlated. For clarity of exposition, we say that both store visits take place during the same shopping trip, although the store visits could just as easily take place on separate trips.

The decision regarding where to buy the common items on the list is based on the buyer's knowledge of price differences between the two retailers for those items, reflecting the possible savings. In choosing to visit both retailers, the shopper incurs a fixed cost  $k$  ( $k > 0$ ) associated with the purchase of common items at the second retailer. This fixed cost may include the direct cost of travel and indirect cost of time spent shopping. However, the customer may have independent motives for visiting the second store that offset or nearly eliminate any fixed cost associated with purchasing common items. For example, if Retailer 2 carries exclusive items or

offers attractive assortments in categories of interest, then the customer might choose to visit the second store regardless of potential savings on common items. In this case, all travel costs would be treated as sunk costs, so the fixed cost associated with purchasing common items would be nearly 0. In other cases, the shopper may be motivated to visit the second store solely save money on common items, in which case  $k$  would include all costs associated with the store visit.

To describe the shopper's situation in more detail, we introduce the *decision vector*  $\lambda_t = (\lambda_{1t}, \lambda_{2t}, \dots, \lambda_{nt})$ , where component  $\lambda_{it}$  is the proportion of item  $i$  purchased at Retailer 2 on trip  $t$ . Observe that we do not require  $\lambda_{it}$  to be binary, a relaxation that facilitates our analytic arguments. The elements of the decision vector and their interpretation will be discussed in greater detail once our model is formally proposed. The shopper may consider two criteria when making deferment decisions: the expected savings from purchasing deferred items at Retailer 2 and the probability of realizing a savings by deferring purchases. The first criterion reflects the magnitude and sign, positive or negative, of expected savings from deferring purchases to Retailer 2. The second criterion allows the shopper to enjoy psychological benefits from saving money that are not proportional to the magnitude of the savings but instead depend on whether a savings is realized. We will show that placing a *positive* weight versus *zero* weight on the probability of realizing a savings leads to a fundamentally different utility function.

Exhibit 1 summarizes the notation that we use in our analytic model.

Place Exhibit 1 about here

### 3.1. The Rational Economic Benefits (*REB*) Shopper

For the first case, we assume that the shopper derives utility only from the rational economic benefits (*REB*) of saving money. In other words, the shopper's disutility is increasing in price, so the deferment decision depends on expected savings. The *REB* shopper does not derive

psychological benefits from saving money; thus, the deferment decision is not influenced by the probability of realizing a savings. The **REB** shopper maximizes expected savings (across all items on the list), taking into account the fixed cost of a visit to Retailer 2

$$\text{Max}_{\lambda_t} E \{ \lambda_t^T Q_t d_t \} - k, \quad (1)$$

where  $d_t$  is a vector of price differences,  $d_t = p_t^{(1)} - p_t^{(2)}$ ,  $Q_t$  is a diagonal matrix of required purchase quantities,  $Q_t = \text{diag}(q_{1t}, q_{2t}, \dots, q_{nt})$ , and  $E$  is the expectation operator. Observe that positive values in the price difference vector imply a positive contribution to savings by deferring purchase to Retailer 2. The **REB** shopper's maximization of expected savings in (1) implicitly assigns  $\lambda_{it} = 1$  for each product where  $E(d_t) > 0$  and  $\lambda_{it} = 0$  otherwise. Because multi-store shoppers visit both retailers by definition, the optimality of this decision requires that the **REB** shopper's expected savings (including the fixed cost  $k$ ) is positive.

**Proposition 1** – The **REB** shopper's decision of whether to defer purchase of an item to Retailer 2 is an increasing function of the contribution to expected savings at Retailer 2.

### 3.2. The Psychological Benefits and Rational Economic Benefits (**PB+REB**) Shopper

For the second case, we consider the shopper who derives utility from the psychological benefits, in addition to the rational economic benefits, of saving money. Because psychological benefits (unlike rational economic benefits) are not proportional to the amount saved, we model psychological benefits as depending on *whether or not the shopper realizes a savings*. The **PB+REB** shopper therefore places a positive weight not only on the expected savings from deferring purchase of the item but also on the probability of saving money by deferring purchase. The two components are combined into a single objective function with the use of decision weight functions  $g$  and  $h$ . The particular choice of  $g$  and  $h$  reflects the **PB+REB** shopper's utility associated with economic and psychological benefits. We make only the mild assumptions that

these decision weight functions are nonnegative and strictly monotone increasing. These assumptions ensure that, all things being equal, i) the **PB+REB** shopper is more likely to defer item purchases if the expected savings of doing so is greater, holding the probability of realizing a savings fixed; and ii) the shopper is more likely to defer item purchases if the probability of realizing a savings (by deferring) is greater, holding the expected savings fixed. The mathematical program for this decision model in period  $t$  is

$$\text{Max}_{\lambda_t} g\left(E\{\lambda_t^T Q_t d_t\} - k\right) + h\left(\text{Pr}\{\lambda_t^T Q_t d_t > k\}\right), \quad (2)$$

where  $E\{\lambda_t^T Q_t d_t\} - k$  is the *expected savings* from deferring a proportion of purchases  $\lambda_t$  to Retailer 2 and  $\text{Pr}\{\lambda_t^T Q_t d_t > k\}$  is the *probability of realizing a savings* (more specifically, a positive savings  $> \$0$ ) by deferring a proportion of purchases  $\lambda_t$  to Retailer 2. We now consider this probability term in more detail.

Recall that there are many theoretical explanations for why shoppers may derive psychological benefits from saving money. The **PB+REB** shopper realizes a savings only if the purchases deferred to Retailer 2 more than offset the fixed cost of buying common items at the second retailer. Moreover, our assumptions that the weighting function  $h$  is nonnegative and strictly monotone increasing imply *risk aversion*; that is, the shopper's utility is decreasing in the uncertainty of realizing a savings. Incorporating risk aversion in this manner will yield a different specification for shopper utility than what is currently found in the literature.

To analyze the probability of realizing a savings, we now assume that *price differences* between the two stores on trip  $t$ ,  $d_t = p_t^{(1)} - p_t^{(2)}$ , can be modeled by a vector equation of the general form

$$d_t = \delta_t(\psi_t) + \eta_t,^6 \quad (3)$$

where  $\psi_t$  is a trip-specific matrix of relevant price information, including known price histories of items on the shopping list as well as observed prices of those items at Retailer 1 on the current trip  $t$  and any advertised specials;  $\eta_t$  is a vector of random errors such that  $\eta_t \sim N(0, \Sigma_t)$ . If we further assume that the components of the error vector  $\eta_t$  are serially uncorrelated and uncorrelated across items, then we have a simpler form for the variance-covariance of random error, namely,  $\Sigma_t = \text{diag}(\sigma_{1t}^2, \sigma_{2t}^2, \dots, \sigma_{nt}^2)$ . Because these errors apply to price differences, the model does not preclude the possibility of item price correlation between retailers, as one might expect to arise through normal price competition.

It is important to emphasize that we do not assume that the marginal distributions of individual prices are normal, an assumption that would be overly optimistic in many settings. Rather, we assume that the shopper has learned about price differences through shopping and can therefore approximate the expectation and variance of the price differences between stores for common items. Only residual uncertainty (which cannot be learned) is assumed to be normally distributed. The normality assumption on  $\eta_t$  implies that  $E(d_t) = \delta_t(\psi_t)$ ; note that we suppress the conditioning information  $\psi_t$  in the treatment below to simplify the remaining exposition.

In order to arrive at a distribution for the probability term from (2) we take its normalized version

$$\Pr\left(\frac{\lambda_t^T Q_t d_t - [\lambda_t^T Q_t \delta_t]}{\sqrt{\lambda_t^T Q_t \Sigma_t Q_t \lambda_t}} > \frac{k - [\lambda_t^T Q_t \delta_t]}{\sqrt{\lambda_t^T Q_t \Sigma_t Q_t \lambda_t}}\right), \quad (4)$$

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<sup>6</sup> If we assume that the prices at the first retailer are observed, then this can be rearranged to represent the conditional distribution of prices at Retailer 2 given prices at Retailer 1.



where  $\delta_t$  is the vector of expected price differences. The probability of realizing a savings in (4) reduces to

$$\Pr\left(z > \frac{k - [\lambda_t^T Q_t \delta_t]}{\sqrt{\lambda_t^T Q_t \Sigma_t Q_t \lambda_t}}\right), \quad (5)$$

where  $z \sim N(0,1)$ . The probability of realizing savings by purchasing proportion  $\lambda_t$  at Retailer 2 is therefore

$$1 - \Phi\left(\frac{k - [\lambda_t^T Q_t \delta_t]}{\sqrt{\lambda_t^T Q_t \Sigma_t Q_t \lambda_t}}\right), \quad (6)$$

where  $\Phi(\bullet)$  is the c.d.f. of the standard normal.

By substituting terms we can now formally rewrite the maximization posed in (2) as

$$\text{Max}_{\lambda_t} g(\lambda_t^T Q_t \delta_t - k) + h \left[ 1 - \Phi\left(\frac{k - [\lambda_t^T Q_t \delta_t]}{\sqrt{\lambda_t^T Q_t \Sigma_t Q_t \lambda_t}}\right) \right]. \quad (7)$$

The maximization (7) has no closed-form solution, yet it is still possible to determine important properties of its structure. To do so, we must first define the index set  $I_t^+ = \{i : \delta_{it} > 0\}$  where  $\delta_{it}$  is the  $i^{\text{th}}$  component of  $\delta_t$ . The set  $I_t^+$  is comprised of the items that the **PB+REB** shopper expects to find at lower prices at Retailer 2 on trip  $t$ . Consequently,  $\delta_{it}$  represents the item's contribution to expected savings for  $i \in I_t^+$ . Assuming that **PB+REB** shoppers visit both stores, the positive elements of the optimal decision vector  $\lambda_t^*$  are limited to the items in  $I_t^+$ . This and other properties of the optimal solution are addressed in the following theorem.

**Theorem:** Suppose  $\sum_{i \in I_t^+} q_{it} \delta_{it} > k$ . Then the optimal solution to (7) satisfies

1.  $\lambda_{it}^* = 0$  for  $i \notin I_t^+$

2.  $\lambda_{it}^* > 0$  for  $i \in I_t^+$
3. For any  $i, j \in I_t^+$  with  $\lambda_{it}^* < 1, \lambda_{jt}^* < 1$ ,  $\frac{\lambda_{it}^*}{\lambda_{jt}^*} = \frac{\delta_{it}}{\delta_{jt}} \cdot \frac{\sigma_{jt}^2}{\sigma_{it}^2} \cdot \frac{q_{jt}}{q_{it}} = \frac{\delta_{it} / q_{it} \sigma_{it}^2}{\delta_{jt} / q_{jt} \sigma_{jt}^2}$ .
4. If  $\lambda_{it}^* > \lambda_{jt}^*$ , then  $\frac{\delta_{it}}{q_{it} \sigma_{it}^2} > \frac{\delta_{jt}}{q_{jt} \sigma_{jt}^2}$ .

We provide a proof of these conditions in Appendix A. Observe that these conditions do not depend on the fixed cost  $k$ , a construct of the model which is neither measured nor calculated.

Parts 1 and 2 of the theorem tell us that every item whose expected price is lower at Retailer 2,  $i \in I_t^+$ , should be purchased in some proportion at Retailer 2, while every item whose expected price is *not* lower at Retailer 2,  $i \notin I_t^+$ , should not be purchased at Retailer 2. Part 3 of the theorem defines the relationship between any two products that should be bought in some proportion at both retailers. Part 4 of the theorem specifies the relationship that must hold if the **PB+REB** shopper prefers purchasing item  $i$  in greater proportion than item  $j$  at Retailer 2. The necessary condition is that the ratio  $\frac{\delta_{it}}{q_{it} \sigma_{it}^2}$  for item  $i$  must exceed the same ratio for item  $j$ . This is an intuitively appealing condition since it incorporates both expected savings and price uncertainty in a simple and parsimonious way. Moreover, it confirms the simple intuition that the **PB+REB** shopper should defer those purchases to the second retailer that have the greatest certainty of contributing to savings. To our knowledge, no existing models of decision-making under price uncertainty use this ratio. We note that it is similar to the Sharpe ratio used in financial portfolio theory, except that the denominator in our expression uses the variance, instead of the standard deviation, and includes a quantity scale factor. The quantity scale factor arises because buying larger quantities increases the expected contribution to savings but also, to a greater extent, the uncertainty of the contribution to savings by purchasing at Retailer 2.

**Proposition 2** – For **PB+REB** shoppers, the decision to defer purchase of an item to the second retailer visited is an increasing function of the contribution to expected savings, divided by the product of quantity and contribution to variance of savings.

An important consequence of the theorem is the structure of the *optimal proportion function*, which is the proportion of an item that should be purchased at the second retailer. Per Part 1 of the Theorem, the **PB+REB** shopper's optimal purchase proportion for an item is 0 if  $\delta_{it} \leq 0$ . By Part 4 of the theorem, if the **PB+REB** shopper chooses to buy all of item  $j$  at Retailer 2, then any item  $i$  whose ratio satisfies

$$\frac{\delta_{it}}{q_{it}\sigma_{it}^2} > \frac{\delta_{jt}}{q_{jt}\sigma_{jt}^2}$$

should also be purchased in its entirety at Retailer 2. Thus, there exists a critical ratio such that all items with higher ratios are purchased in their entirety at Retailer 2. By Part 3 and Part 4 of the theorem, the optimal purchase proportion at the second retailer *increases* in the ratio  $\frac{\delta_{it}}{q_{it}\sigma_{it}^2}$  prior to reaching its maximum of 1 at this critical ratio. In summary, all other things being equal, the **PB+REB** shopper's optimal proportion function is a monotone increasing function of the argument  $\frac{\delta_{it}}{q_{it}\sigma_{it}^2}$ , taking the value 0 when this argument is sufficiently small and 1 when this argument is sufficiently large.

Investigating the functional form of the monotone increasing relationship between  $\frac{\delta_{it}}{q_{it}\sigma_{it}^2}$  and the optimal proportion  $\lambda_{it}^*$  for the interval  $0 < \lambda_{it}^* < 1$  is important for empirical application of the model. If  $\lambda_{it}^*$  were a linear (or very nearly linear) function of  $\frac{\delta_{it}}{q_{it}\sigma_{it}^2}$  over that interval, then the relationship between the two would be well approximated by a piecewise-linear sigmoidal

function as shown in Figure 1 and therefore estimable using a binary choice function such as logit or probit, both of which are appropriate and well supported for empirical applications (see Ben-Akiva and Lerman 1985, pp. 67-72).<sup>7</sup>

Place Figure 1 about here

To test for linearity in the relationship, we conducted a numerical study detailed in Appendix B. For this study, we used a flexible form for the decision weight functions  $g$  and  $h$  and systematically varied these decision weight functions to investigate their impact on the relationship between  $\frac{\delta_{it}}{q_{it}\sigma_{it}^2}$  and  $\lambda_{it}^*$ . We found that for all combinations of decision weights considered, the relationship is linear or very nearly so. The virtual linearity of this relationship motivates the use of the logit form for our empirical demonstration.

Two other issues warrant discussion.

- *Price Promotions.* In some cases, the price difference for an item is known with certainty due to the retailer's feature advertising. In this case, the variance of the price difference is 0. In such cases, either  $\lambda_{it} = 1$  (when the item is cheaper at Retailer 2), or  $\lambda_{it} = 0$  (when the product is cheaper at Retailer 1).
- *Travel Routes.* An important exogenous factor in our analysis is the order in which retailers are visited. When visiting two retailers, there are two possible routes of return, each beginning with a different retailer. We do not assume the buyer begins at home, since groceries are often purchased in conjunction with other shopping activities (Dellaert, et al. 1998). However, we do assume that home is the destination after the final retailer is visited, as we anticipate that many perishable and/or frozen items would require prompt refrigeration. The two routes of return are represented in Figure 2 by solid and dashed arrows, respectively. For the bold route in Figure 2, Retailer A is the first one visited; for the dotted route, Retailer B is visited first. Observe that the setup costs for

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<sup>7</sup> Ben-Akiva and Lerman (1985) argued that "because of behaviorally unrealistic 'kinks' in the linear probability model [*i.e.*, *piecewise linear sigmoidal function*] and because of its forecasts of unrealistic, extreme probabilities in some circumstances" (p. 69), it is better approximated by logit and probit models for stochastic applications.

visiting the second retailer are not necessarily symmetric. In Figure 2, the additional setup cost of going from Retailer A to Retailer B (the route shown in bold) is greater than the additional setup cost of going from Retailer B to Retailer A (the dotted route). Retailer A is nearly “on the way home” after visiting Retailer B whereas Retailer B is “out of the way” after visiting Retailer A. Finally, note that the routing decision is dependent on the distributions of prices offered by the two retailers. The relationship between retailer pricing and the shopper’s optimal routing is examined in detail in Bhaskaran and Semple (2012). This study shows that differences in the skewness of retailers’ price distributions alone can materially affect a shopper’s expected purchase costs, resulting in different optimal routes.

Place Figure 2 about here

## **4. Empirical Demonstration**

In this section, we use actual common item purchases made by multi-store shoppers to demonstrate that some shoppers enjoy psychological benefits, in addition to the economic benefits, of saving money. Note that our objective is to provide a demonstration of the analytically-derived propositions in §3, not to conduct a generalizable test of shopping behavior.

### **4.1. Data**

We use IRI panel data from the Chicago market over 104 weeks between October 1995 and October 1997. Panelists recorded the UPCs (uniform product codes) of all packaged goods products purchased on all trips to a wide variety of retailers using in-home scanning equipment, identifying the retailer by store chain rather than by individual store. Developing the dataset requires that we select panel members, identify multi-store shopping trips and common item purchases made on those trips. We apply the following selection criteria:

- 1) We confine our attention to the two largest grocery retailers, Jewel and Dominick’s. Together, these two retailers account for 64% of all grocery purchases made by the

panelists in the dataset. If the household visited both Jewel and Dominick's on the same day (i.e., without intervening consumption), we identify these visits as multi-store shopping. The ordering of Retailer 1 and 2 is determined by the order in which panelists recorded their purchases.<sup>8</sup> 52.9% of shoppers were more likely to visit Jewel first (i.e., as Retailer 1) when multi-store shopping; 41.2% of shoppers were more likely to visit Dominick's first; 5.9% visited Jewel and Dominick's first with equal frequency. Whichever retailer the shopper was most likely to visit first, she visited that retailer first on 64.7% of her multi-store shopping trips, on average. Clearly, the multi-store shoppers in our sample did not always visit the same retailer first.

- 2) We limit the dataset to households that visited these two retailers on the same day at least five times over the duration of the dataset. In this way, we ensure that households were experienced multi-store shoppers and, consequently, that their multi-store shopping visits were planned.
- 3) The purchases in our dataset are limited to items in the fifteen categories for which item-level price data are available--1-beer & ale, 2-chocolate candy, 3-carbonated beverages, 4-salty snacks, 5-coffee, 6-facial cosmetics, 7-internal analgesics, 8-sanitary napkins, 9-shampoo, 10-vitamins, 11-cigarettes, 12-diapers, 13-dog food, 14-household cleaners, and 15-laundry detergents--and only items in those categories that were available at both retailers. Note that none of these categories are perishable, which might increase shoppers' propensity to buy them at Retailer 2 *ceteris paribus*. We also exclude items from our dataset if they were advertised at Retailer 2. This eliminates the possibility that shoppers knew the price at Retailer 2 in advance and therefore faced no uncertainty about the expected savings.
- 4) Finally, each purchase observation in the dataset must have sufficient history to compute expected savings and variance of savings, both conditioned on the observed price at the first retailer visited. These computations are detailed in Appendix C.

Applying these four criteria results in a final dataset of 873 multi-store purchases made by 51 households over the 104-week period. Though we certainly would have preferred to have more

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<sup>8</sup> Panelists were instructed to record purchases on the day and in the order that stores were visited. Our empirical testing assumes that panelists followed these rules. IRI, Inc. monitors panelist compliance, but we do not know specific monitoring policies.

households in our final dataset, we have been intentionally conservative in identifying multi-store shopping purchases that reflect the assumptions of our model. As mentioned above, limiting our dataset to experienced multi-store shoppers ensures that their shopping trips were planned in advance (Fox and Hoch 2005). Selecting common item purchases only if both stores were visited without intervening consumption ensures that the shopping list was not increased between visits. As a result of this conservative approach, the 873 purchases in the dataset are, to the extent possible, representative of our analytical model. And while our intention is simply to demonstrate that some multi-store shoppers enjoy psychological benefits, the large number of observations per household supports the reliability of our findings.<sup>9</sup>

In Table 1, we compare the frequent multi-store shoppers whose purchases were included in our dataset to the panelists whose purchases were not included. From the table, we see that frequent multi-store shoppers differ from other panelists in a number of characteristics: family size--3.25 members for multi-store shoppers vs. 2.87 members for others; working women--58.8% for multi-store shoppers vs. 64.0% for others; homeowners--92.2% for multi-store shoppers vs. 82.0% for others; and married--80.4% for multi-store shoppers vs. 68.4% for others.

Place Table 1 about here

In Table 2, we present descriptive statistics for the frequent multi-store shoppers whose purchases are included in our dataset both when they are shopping at multiple stores on the same day and when they are not. The table shows the average number of store visits over the duration of our dataset and average expenditure per trip. We see that the multi-store shoppers in our dataset made almost 31% ( $= 43.80 / 143.45$ ) of their store visits on days when they visited multiple stores. These shoppers also spent almost as much on each visit when multi-store shopping (\$56.06) as when visiting only one store (\$62.73). The same statistics for panelists not

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<sup>9</sup> Note that we performed two robustness checks by applying different selection criteria, which are detailed in §4.5.

included in our dataset are presented for comparison, and it appears that they shop considerably less than the multi-store shoppers in our dataset. We observe that the frequent multi-store shoppers in our dataset made 187.25 (= 43.80 + 143.45) total store visits while the other shoppers made only 64.02 (= 2.40 + 61.62) total store visits--nearly three times fewer.

Place Table 2 about here

#### 4.2. Variable Definitions

We carry forward the notation from the analytical model; however, our empirical analysis is conducted at the individual level so we will add a subscript for household. Accordingly, the dependent variable in our econometric model is the probability  $\pi_{hit}$  that household  $h$  purchases common item  $i$  on trip  $t$  at Retailer 1 (rather than deferring purchase to Retailer 2), where  $h = 1, 2, \dots, H$  denotes households,  $t = 1, 2, \dots, T_h$  denotes trips and  $i = 1, 2, \dots, n_{ht}$  denotes common items.<sup>10</sup> Observe that this dependent variable is the dual of  $\lambda^*$ , the optimal proportion to be deferred to Retailer 2, and that the signs of response parameters should be consistent with this dependent variable (i.e., negative algebraic sign).

The expected utility that a multi-store shopper receives from purchasing a common item at Retailer 1 (rather than deferring purchase to Retailer 2) is a function of the rational economic benefits (**REB**) and psychological benefits (**PB**) from saving money by deferring purchase to Retailer 2. As asserted in *Proposition 1*, the **REB** shopper's decision to defer purchase to Retailer 2 is an increasing function of the contribution to expected savings. For household  $h$  purchasing item  $i$ , the predictor is  $q_{hit}\delta_{hit}$ . *Proposition 2* asserts that, for the **PB+REB** shopper (who derives psychological benefits, in addition to economic benefits, from saving money), the

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<sup>10</sup> Recall from §3 that the shopper's decision to defer purchases to Retailer 2 is, strictly speaking, a proportion of item purchases. Practically, however, purchases are discrete and units of the same common item are seldom bought at both retailers (fewer than 1.2% of purchases in our data) so the dependent variable in our empirical specification is a probability.



decision to defer purchase to Retailer 2 is an increasing function of the contribution to expected savings, divided by the product of the quantity and the contribution to variance of savings. For household  $h$  purchasing item  $i$ , the predictor is  $\frac{\delta_{hit}}{q_{hit}\sigma_{hit}^2}$ . The two quantities  $q_{hit}\delta_{hit}$  and  $\frac{\delta_{hit}}{q_{hit}\sigma_{hit}^2}$  will play a critical role in our empirical analysis.

We assume that the shopper's information about price savings comes from previous multi-store trips on which the household made purchases in the category. During those trips, the shopper would have access to comparative pricing information for common items in the category. Because they depend on shopping history, contributions to expectation and variance of savings for each common item are household-specific, hence the  $h$  subscript. Moreover, because the shopper observes the prices of common items at Retailer 1 before making deferral decisions, we assume that she uses this information to condition contributions to expected savings (for example, observing a discounted price at Retailer 1 reduces the expected savings of deferring purchase to Retailer 2). The computations of conditional contributions to expectation and variance are detailed in Appendix C. To simplify the exposition, however, we will omit the conditioning arguments in the remainder of this section.

It is important to acknowledge that there may be non-price reasons for multi-store shoppers to purchase specific categories at Retailer 1. Consequently, while it is not possible to capture all of the non-price factors separately, we construct a category-specific store loyalty measure for each household as a proxy (cf. Bell, et al. 1998):

$$Loy_{htc} = \frac{\sum_{i \in c} \sum_{w=1}^{t-1} y_{hiw}^{(j)}}{\sum_{i \in c} \sum_{w=1}^{t-1} y_{hiw}^{(j)} + \sum_{i \in c} \sum_{w=1}^{t-1} (1 - y_{hiw}^{(j)})} \quad (8)$$

where  $y_{hiw}^{(j)}$  is an indicator variable which takes the value 1 if item  $i$  from household  $h$ 's shopping list for trip  $w$  was purchased at Retailer  $j$ ;  $j=1,2$ ; 0 otherwise. Note that category-specific store loyalty is indexed by trip because it reflects the proportion of all category purchases made by household  $h$  at each retailer prior to the current trip  $t$ .

### 4.3. Modeling Approach

The analytical development in §3 predicts differences in purchase decisions for common items between **REB** shoppers, motivated only by the rational economic benefits of saving money, and **PB+REB** shoppers, motivated by both the economic and psychological benefits of saving money. For **REB** shoppers, we write the deterministic component of the indirect utility as follows

$$U_{hit} = \beta_0 + \gamma Loy_{hct} + \beta_E (q_{hit} \delta_{hit}), \quad (9)$$

where  $\beta_E$  denotes a decision weight that governs the extent to which the multi-store shopper is motivated by economic benefits. Note that we expect  $\beta_E < 0$ , given the way in which we computed the contribution to expected savings in §4.2. *Category-specific store loyalty* to Retailer 1 for item  $i \in c$ ,  $Loy_{hct}$ , is specified as a covariate with associated parameter  $\gamma$ . Similarly, for **PB+REB** shoppers who are motivated by both the psychological and economic benefits of saving money, we write the deterministic component of indirect utility as follows

$$U_{hit} = \beta_0 + \gamma Loy_{hct} + \beta_P \left( \frac{\delta_{hit}}{q_{hit} \sigma_{hit}^2} \right), \quad (10)$$

where  $\beta_P$  denotes a decision weight that indicates the extent to which the multi-store shopper is motivated by both psychological and economic benefits. Here too, we expect  $\beta_P < 0$  and again specify  $Loy_{hct}$  for item  $i \in c$  as a covariate with associated parameter  $\gamma$ .

**4.3.1. Model Forms.** The decision weights  $\beta_E$  and  $\beta_P$  play a critical role in our analysis. Consider the following possible scenarios:

- $\beta_E \neq 0$  but  $\beta_P = 0$ ; in this case, all shoppers are motivated by the economic benefits, but not psychological benefits, of the expected saving money--the probability of deferring purchase to the second retailer will increase as the contribution to expected savings increases.
- $\beta_P \neq 0$  but  $\beta_E = 0$ ; in this case, all shoppers are motivated by both the psychological and economic benefits of the saving money--the probability of deferring purchase to the second retailer will increase as the ratio of contribution to expected savings divided by contribution to variance of savings increases.

These scenarios raise the possibility that shoppers may differ in their motivations, the way they process information, and the way they make decisions. Accordingly, we will allow for heterogeneity by estimating a constrained finite-mixture formulation which assumes that there are  $s = 1, \dots, S$  segments of multi-store shoppers such that, within each segment, shoppers are motivated by the economic and, perhaps, also the psychological benefits of saving money.<sup>11</sup> We will always allow  $\gamma$  to be freely estimable which is consistent with the role of category-specific store loyalty as a covariate. The finite mixture models allow us to determine whether some multi-store shoppers are motivated by psychological benefits in deciding where to purchase the common items on their shopping lists. Thus, we rewrite the deterministic component of household  $h$ 's utility of purchasing item  $i$  at the first retailer on trip  $t$  as

$$U_{hit} = \beta_{0s} + \gamma_s Loy_{hct} + \beta_{Es} (q_{hit} \delta_{hit}) + \beta_{Ps} \left( \frac{\delta_{hit}}{q_{hit} \sigma_{hit}^2} \right), \quad (11)$$

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<sup>11</sup> We could have chosen to utilize a continuous mixture formulation to account for heterogeneity instead, but we are interested in the size of the segments.

where the decision weights and category-specific store loyalty parameters now vary by segment. However, as stated above, we will never allow both  $\beta_E$  and  $\beta_P$  to be non-zero within a given segment. Rather, we align each segment with a specific motivation by setting one of these parameters to zero.

**4.3.2. Estimation.** We model the probability that household  $h$  chooses to buy common item  $i$  from her shopping list on trip  $t$  at Retailer 1, the first retailer visited on that multi-store shopping trip, as

$$\pi_{hit} = \frac{e^{U_{hit}}}{1 + e^{U_{hit}}}. \quad (12)$$

As discussed in §3.2, because the purchase decision function is approximately piecewise-linear, the logit is an appropriate model form for estimating the probability (for a detailed discussion, see Ben-Akiva and Lerman 1985, pp. 67-72).

In general, the unconditional likelihood for a multi-store shopper with common item purchase vector  $y_h$  can be written as

$$L(y_h) = \int L(y_h | \theta) dF(\theta), \quad (13)$$

where  $L(y_h|\theta)$  is the conditional likelihood with parameters  $\theta$  ( $=\gamma, \beta_E,$  and  $\beta_P$ ), and  $F(\bullet)$  is the mixing distribution. It can be shown that a continuous mixing distribution function  $F(\bullet)$  can be consistently estimated with a finite number of  $S$  mass points (cf. Simon 1976), i.e.,

$$L(y_h) = \sum_{s=1}^S \alpha_s L(y_h | \theta_s), \quad (14)$$

where  $\theta_s$  is the vector of parameters, and  $\alpha_s$  is the mixing proportion or segment share for segment  $s$ , such that  $0 \leq \alpha_s \leq 1$  and  $\alpha_1 + \alpha_2 + \dots + \alpha_S = 1$ . Parameters are estimated using both the EM algorithm and the Newton-Raphson method. To decrease the chance of local maxima solutions, we use multiple sets of random start values. Within each set of random start values, we perform a number of iterations and continue with the best solution until convergence.

#### 4.4. Empirical Results

Table 3 provides a description of alternative model forms along with goodness of fit statistics and hit rates. Table 4 provides parameter estimates for the best fitting model. We use AIC3 (Andrews and Currim 2003) and BIC (Schwartz 1978) to compare model fits.<sup>12</sup>

Table 3 is divided into two sections. The first section considers model forms in which all shoppers are assumed to be homogeneous, so no explicit segments are specified. Specifically, models  $M0\_1$  and  $M0\_2$  assume that *all* shoppers are either motivated by economic benefits alone, i.e., *expected contribution to savings* (Model  $M0\_1$ ) or by both economic and psychological benefits, i.e., the ratio of *expected contribution to savings* to *variance in contribution to savings* (Model  $M0\_2$ ), with store *loyalty* as a covariate. We present these models as baselines to compare the heterogeneous models. The second section considers model forms that assume shoppers are not all alike, the nature of heterogeneity being characterized by the number of segments specified and the constraints placed on the  $\beta_E$  and  $\beta_P$  parameters.

Place Table 3 about here

From the table we see that all of the heterogeneous model forms fit better than either of the homogeneous models  $M0\_1$  and  $M0\_2$ . Among the heterogeneous model forms, models  $M3\_1$ - $M3\_4$  generally provide better fits than the others. Coincidentally, models  $M3\_1$ - $M3\_4$  are structural mixtures in that they include both **REB** and **PB+REB** shopper segments. The uniformly superior fits of these structural mixtures compared to models  $M0\_1$  and  $M0\_2$ , the homogeneous model forms, suggests that multi-store shoppers are in fact heterogeneous in their motivations for saving money.

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<sup>12</sup> There is some evidence to suggest that the AIC3 is superior to the other information theoretic statistics for the purpose of deciding on the number of segments to retain in finite mixture models (Andrews and Currim 2003; Dias 2004).

Model  $M3\_1$  is the most parsimonious of the structural mixture models, with one **REB** segment and one **PB+REB** segment. This model performs best on both information criteria and yields the highest hit rate; thus, it offers the best balance of fit and parsimony. From Table 4 we see that all  $M3\_1$  parameter estimates are statistically significant and have the expected algebraic sign. A greater *contribution to expected savings* at Retailer 2 (for the **REB** segment) and a greater ratio of *contribution to expected savings* over *contribution to variance in savings* (for the **PB+REB** segment) lead to lower purchase probabilities at Retailer 1; for both segments, loyalty to Retailer 1 leads to higher purchase probabilities at Retailer 1. Thus, after controlling for category-specific store loyalty, we find evidence that multi-store shoppers can be “typed” based on whether they experience psychological benefits from saving money. Interestingly, we infer from model  $M3\_1$  that, in our sample, 62% multi-store shoppers are motivated by economic benefits alone, while 38% are motivated by both psychological and economic benefits.

Place Table 4 about here

#### **4.5. Robustness Checks**

In this section, we investigate the sensitivity of our results to model specification and the criteria used to design our dataset.

**4.5.1. Model Forms with Additive Risk Term.** While our utility specification for **PB+REB** shoppers clearly implies risk aversion (see §3.2), we now estimate a more common specification of risk aversion. Analysis of risky returns in finance uses an additive term to capture the premium required for an investor to be indifferent between the risky return and a risk-free return. This “risk premium” reflects the risk aversion commonly observed in financial decisions. The riskiness of a return is measured by the standard deviation of that return.

Because the price savings that a shopper may realize by deferring common item purchases is effectively a risky return, we estimate an alternative model for this decision which includes an additive risk term. Specifically, we assume that multi-store shoppers' decisions to defer purchase of common items to Retailer 2 depend uniformly on 1) *store loyalty*, 2) *contribution to expected savings*, and 3) the *standard deviation of contribution to savings*.<sup>13</sup> If shoppers are generally risk-averse, then we would expect the parameter for the standard deviation of contribution to savings, or risk parameter (see footnote 15), to be positive. In other words, a greater risk of realizing the expected savings at Retailer 2 would result in a concomitantly greater probability of purchasing at Retailer 1. We find that the estimated risk parameter is in fact positive (0.6379) but the *t-value* of 1.00, is not statistically significant at the  $p < .05$  level. In addition, we find that goodness-of-fit criteria do not favor this model with an additive risk term compared to the model without it (model form  $M0\_1$ ).

**4.5.2. Sensitivity to Sample Selection Criteria.** Recall that household panel members' common item purchases were included in our analysis dataset based on specific selection criteria. The selection criteria were applied to ensure that multi-store shoppers and their common item purchases were consistent with the assumptions underlying our analytical model. We now investigate the robustness of our empirical results by selecting two alternative purchase samples using different selection criteria.

*Robustness Sample A:* This sample was restricted to households that made at least ten (10) multi-store shopping trips to the two retailers during the data collection period as opposed to five; thus, only the most experienced multi-store shoppers

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<sup>13</sup> Under this specification, the deterministic component of utility of a multi-store shopper purchasing at Retailer 1 is

$$U_{hit} = \beta_0 + \gamma Loy_{hct} + \beta_E (q_{hit} \delta_{hit}) + \beta_R (q_{hit} \sigma_{hit}) \quad (15)$$

where  $\sigma_{hit} = \sqrt{\sigma_{hit}^2}$  reflects the risk of realizing the expected savings by deferring purchase to Retailer 2 and  $\beta_R$  is the associated decision weight. Thus, in this specification, utility is an additive function of risk.

were included. We selected this sample to test whether our empirical results depend on households' multi-store shopping expertise.

*Robustness Sample B:* This sample reflects a different assumption about how shoppers develop expectations about items' contribution to price savings. Adopting a *rational expectations* approach, this sample incorporates the entire history of price differences between retailers, regardless of which prices the household might have observed. The implication is that multi-store shoppers' common item purchase decisions are made *as if* the shoppers know the entire history of price differences for common items.

In the case of both robustness samples, we fit all model forms described in Table 3. For both robustness samples,  $M3\_1$  was again the best-fitting model in terms of both goodness-of-fit and hit rate. Interestingly, some patterns in the parameter estimates are noteworthy. For *Robustness Sample A*, which imposed a stricter multi-store shopping experience criterion, the relationships were stronger, fits were better and parameter estimates were statistically significant with tighter confidence intervals. These findings suggest that, the more experienced the multi-store shopper, the more consistent their common item decision-making is with our proposed model.

For *Robustness Sample B*, which was less restrictive by virtue of assuming rational expectations for expected price savings, we found that parameter estimates  $\beta_E$  and  $\beta_P$  had wider confidence intervals, signaling weaker relationships. This finding suggests that the fit of the empirical model does depend on one's assumption about how expectations about price savings are developed. This is not surprising because, by assuming that shoppers know the entire history of price differences between stores (which is practically impossible), *Robustness Sample B* systematically understates shoppers' sampling variation and hence their true beliefs about the variance of price savings. As a result, the ratio of contribution to expected savings (which is



unbiased) over contribution to variance in savings (which has a downward bias) is overstated, with some highly influential observations due to the small denominator. This explains the weaker relationships found in *Robustness Sample B*.

## 5. Discussion, Limitations and Future Research

We now return to the multi-store shopper's problem of deciding where to purchase the items on her shopping list, given visits to two stores on a predetermined route. Both **REB** and **PB+REB** shoppers benefit from reducing uncertainty about prices at the second store that they visit. For the **REB** shopper, the sum of savings realized over the entire basket is maximized when prices at both stores are known, because savings is maximized for each item. For the **PB+REB** shopper, the probability of saving money on each item in the basket is also maximized when prices at the second store are known. In either case, both retailers sell only items that they price lower than their competitor.

The multi-store shopper's primary method of reducing price uncertainty is to study retailer ads, thereby eliminating uncertainty about price savings for some items.<sup>14</sup> The retailer can also affect shoppers' uncertainty about savings, thereby affecting where multi-store shoppers will buy. For items priced below competition, the retailer can maximize its sales to multi-store shoppers by reducing the uncertainty of expected savings. This can be accomplished by consistently pricing a fixed amount below competition. Such a strategy requires that the retailer's timing and depth of discounts match competition, resulting in a high correlation in price between the two retailers. For products priced above competition, the retailer can maximize its sales to multi-store shoppers by discounting independent of competition (thereby preventing shoppers from using the retailer's own discounts to reduce uncertainty about competitor prices).

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<sup>14</sup> Recall from §2.1 that Urbany, et al. (1991) and Urbany, et al. (2000) found multi-store shoppers are more likely to study retailer ads.

The primary limitation of this research is our inability to identify all multi-store shopping visits in the empirical data. We have applied a restrictive criterion, i.e., visits to multiple grocery retailers without intervening consumption, which clearly understates the amount of multi-store shopping. This limitation is an obstacle to actually determining the frequency of multi-store shopping. Moreover, our empirical demonstration implicitly assumes that shoppers behave the same way when visiting multiple grocery stores in a single day as they would if splitting their purchases with an intervening break. We have also limited our demonstration to the two largest grocery chains in a duopolistic market, and therefore do not consider the possibility of multi-store shopping across three or more retailers (though according to Fox and Hoch 2005 and Talukdar, et al. 2010, this occurs very infrequently). Additionally, we have implicitly assumed that consumer decision processes and preferences do not vary over time. Though this assumption is common in models of consumer choice, it is possible that consumers' motivations and preferences for saving money are non-stationary.

Our theory of multi-store shoppers' purchase decisions for common items introduces the ratio of expected savings to the variance of savings. The manner in which we have modeled the formation of items' contributions to expectation and variance of savings, however, is only one possible operationalization. How expectations and variances are actually constructed is open to competing possibilities. This represents a significant opportunity for future research.

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## Appendix A

**Proof of Theorem 1.** Because  $\sum_{i \in I_t^+} q_{it} \delta_{it} - k > 0$ , consider the solution  $\lambda_{it} = 1$  for all  $i \in I_t^+$ , and  $\lambda_{it} = 0$  for  $i \notin I_t^+$ .  $\sum_{i \in I_t^+} \lambda_{it} q_{it} \delta_{it} - k > 0$  for this solution. Observe that a solution with positive expected savings dominates all solutions with non-positive expected savings, i.e., both  $g$  and  $h$  are larger with positive expected savings. We may therefore assume that  $\sum_{i \in I_t^+} \lambda_{it}^* q_{it} \delta_{it} - k > 0$  in any optimal solution vector  $\lambda_t^*$ .

Now we show that  $\lambda_{it}^* = 0$  for all  $i \notin I_t^+$ . Suppose this were not the case, and  $\lambda_{it}^* > 0$  for some  $i \notin I_t^+$ . Then construct a new vector  $\lambda_t'$  as follows:  $\lambda_{jt}' = \lambda_{jt}^*$  for  $j \neq i$ ,  $\lambda_{jt}' = 0$  for  $j = i$ . Observe (a)  $\lambda_t'^T Q_t \delta_t > \lambda_t^{*T} Q_t \delta_t$ , (b)  $k - \lambda_t'^T Q_t \delta_t < k - \lambda_t^{*T} Q_t \delta_t < 0$ , and (c)  $\lambda_t'^T Q_t \Sigma_t Q_t \lambda_t' < \lambda_t^{*T} Q_t \Sigma_t Q_t \lambda_t^*$ . Observation (a) implies  $g$  will increase for the new solution  $\lambda_t'$ . Observations (b) and (c) imply the argument of  $\Phi$  will decrease, and so  $h$  will increase. This contradicts the optimality of  $\lambda_t^*$  and implies  $\lambda_{it}^* = 0$  for all  $i \notin I_t^+$ . This proves part 1 of the theorem.

We now show that  $\lambda_{it}^* > 0$  for all  $i \in I_t^+$ . Because  $\sum_{i \in I_t^+} \lambda_{it}^* q_{it} \delta_{it} - k > 0$ , at least one component of the optimal solution is positive. Let that component be  $\lambda_{it}^* > 0$   $i \in I_t^+$ . Now suppose  $\lambda_{jt}^* = 0$  for some (other) item  $j \in I_t^+$ . We will construct a strictly better solution that satisfies  $\lambda_{jt}^* > 0$ . Consider the vector  $\lambda_t'$ ,  $\lambda_{it}' = \lambda_{it}^* + \varepsilon$ ,  $\lambda_{jt}' = \frac{q_{it} \delta_{it}}{q_{jt} \delta_{jt}} \varepsilon$ , and  $\lambda_{kt}' = \lambda_{kt}^*$  for all  $k \neq i, j$  where  $\varepsilon > 0$  is a small positive perturbation. This new solution is feasible for  $\varepsilon$  taken sufficiently close to 0. Moreover, by design, the solution preserves expected savings for all values of  $\varepsilon$ , i.e.,

$$\sum_{i \in I_t^+} \lambda_{it}' q_{it} \delta_{it} = \sum_{i \in I_t^+} \lambda_{it}^* q_{it} \delta_{it} \text{ for all } \varepsilon.$$

This means the value of  $g$  is identical for both  $\lambda_t'$  and  $\lambda_t^*$ . We now show  $h$  increases.

Observe that  $h$ 's argument is  $1 - \Phi \left( \frac{k - \lambda_i^T Q_i \delta_i}{\sqrt{\lambda_i^T Q_i \Sigma_i Q_i \lambda_i}} \right)$ . The term  $k - \lambda_i^T Q_i \delta_i$  in the numerator

of  $\Phi$ 's argument is identical for  $\lambda^i$  and  $\lambda^*$  (it's the negative of expected savings). However, the denominator of  $\Phi$ 's argument does change. In fact, straightforward algebra reveals the expression under the root changes by a net amount of

$$\left(-2\lambda_{it}^* \varepsilon + \varepsilon^2\right) q_{it}^2 \sigma_{it}^2 + \left(\frac{q_{it} \delta_{it}}{q_{jt} \delta_{jt}} \varepsilon\right)^2 q_{jt}^2 \sigma_{jt}^2.$$

For sufficiently small  $\varepsilon > 0$ , this expression is negative because the linear term (in  $\varepsilon$ ) dominates the quadratic term (in  $\varepsilon$ ). This means  $\lambda_i^T Q_i \Sigma_i Q_i \lambda_i^i < \lambda_i^{*T} Q_i \Sigma_i Q_i \lambda_i^*$  and so the argument of  $\Phi$  decreases as well (recall the numerator satisfies  $k - \lambda_i^T Q_i \delta_i = k - \lambda_i^{*T} Q_i \delta_i < 0$ ). This means  $1 - \Phi$  increases, and so  $h$  does as well. Thus  $\lambda_{it}^* > 0$  for all  $i \in I^+$ . This proves part 2.

To prove part 3, consider any optimal solution vector  $\lambda^*$  having  $\lambda_{it}^* < 1$ ,  $\lambda_{jt}^* < 1$ , and define a new vector  $\lambda^i$  such that  $\lambda_{it}^i = \lambda_{it}^* - \varepsilon$ ,  $\lambda_{jt}^i = \lambda_{jt}^* + \frac{q_{it} \delta_{it}}{q_{jt} \delta_{jt}} \varepsilon$ , and  $\lambda_{kt}^i = \lambda_{kt}^*$  for all  $k \neq i, j$  where  $\varepsilon$  is again a small perturbation, but this time it can be of either sign because  $\lambda_{it}^*$  and  $\lambda_{jt}^*$  are interior solutions ( $< 1$  by assumption,  $> 0$  by part 2). For small enough values of  $\varepsilon$  near 0, the solution remains feasible. As in the proof of part 2, this constructed solution maintains the same expected savings and so it preserves the value of  $g$ . Also as in the proof of part 2, the vector  $\lambda^i$  preserves the value in the numerator of  $\Phi$ 's argument. Consequently, the denominator of  $\Phi$ 's argument must increase or stay the same so that optimality is preserved. After some algebra, the net change in the expression under the root in the denominator of  $\Phi$ 's argument is

$$\left(-2\lambda_{it}^* \varepsilon + \varepsilon^2\right) q_{it}^2 \sigma_{it}^2 + \left[ 2\lambda_{jt}^* \frac{q_{it} \delta_{it}}{q_{jt} \delta_{jt}} \varepsilon + \left(\frac{q_{it} \delta_{it}}{q_{jt} \delta_{jt}} \varepsilon\right)^2 \right] q_{jt}^2 \sigma_{jt}^2.$$

This expression must be nonnegative to preserve optimality of  $\lambda^*$ . The linear term dominates the quadratic terms as  $\varepsilon$  becomes small, but the linear term is also symmetric in  $\varepsilon$ , and so we must have

$$-2\lambda_{it}^* q_{it}^2 \sigma_{it}^2 \varepsilon + 2\lambda_{jt}^* \frac{q_{it} \delta_{it}}{q_{jt} \delta_{jt}} q_{jt}^2 \sigma_{jt}^2 \varepsilon = 0$$

to preserve optimality. Dividing this expression by  $\varepsilon \neq 0$  and rearranging the remaining terms yields the expression in part 3.

To prove part 4, consider an optimal solution with  $\lambda_{jt}^* < \lambda_{it}^*$ , and use the same perturbation as in part 3, with the exception now that because  $\lambda_{it}^* \leq 1$  we can only consider one-sided perturbations  $\varepsilon > 0$  (to ensure feasibility if  $\lambda_{it}^* = 1$ ). The optimality of  $\lambda_{it}^*$  implies the net change in the expression under the root in  $\Phi$ 's argument must satisfy the inequality

$$\left(-2\lambda_{it}^* \varepsilon + \varepsilon^2\right) q_{it}^2 \sigma_{it}^2 + \left[2\lambda_{jt}^* \frac{q_{it} \delta_{it}}{q_{jt} \delta_{jt}} \varepsilon + \left(\frac{q_{it} \delta_{it}}{q_{jt} \delta_{jt}} \varepsilon\right)^2\right] q_{jt}^2 \sigma_{jt}^2 \geq 0.$$

Dividing this by  $\varepsilon > 0$ , taking the limit as  $\varepsilon \rightarrow 0$ , and rearranging the remaining terms yields

$$\frac{\lambda_{jt}^*}{\lambda_{it}^*} \geq \frac{q_{it}^2 \sigma_{it}^2}{q_{jt}^2 \sigma_{jt}^2} \cdot \frac{q_{jt} \delta_{jt}}{q_{it} \delta_{it}}.$$

But  $\frac{\lambda_{jt}^*}{\lambda_{it}^*} < 1$ , hence  $1 > \frac{q_{it}^2 \sigma_{it}^2}{q_{jt}^2 \sigma_{jt}^2} \cdot \frac{q_{jt} \delta_{jt}}{q_{it} \delta_{it}}$ , which implies part 4.



## Appendix B

**Numerical Study.** Using a numerical study, we investigate the relationship between the ratio of expected savings to variance in savings (scaled by quantity) and the optimal proportion to be purchased at Retailer. We are particularly interested in how this relationship might be impacted by different decision weighting functions  $g$  and  $h$  in the maximization (7), shown below for convenience

$$\text{Max}_{\lambda_i} g(\lambda_i^T Q_i \delta_i - k) + h \left[ 1 - \Phi \left( \frac{k - [\lambda_i^T Q_i \delta_i]}{\sqrt{\lambda_i^T Q_i \Sigma_i Q_i \lambda_i}} \right) \right]$$

Recall that the decision weighting functions  $g$  and  $h$  are required only to be non-negative and strictly monotone increasing. We have therefore adopted a flexible specification  $\mu_m + \omega_m (\bullet)^{v_m}$ ;  $m = g, h$  to explore these weighting functions, where  $\mu$  is an intercept,  $\omega$  is a slope,  $v$  is an exponent,  $(\bullet)$  is the relevant argument and  $m$  indexes the decision weighting function  $g$  or  $h$  (a similar specification was used in Chambers, et al. 2006). While this specification does not capture all possible functional forms, it is nonetheless quite general, allowing for non-linearity, concavity and convexity.

We selected three different slopes for each weighting function:  $\omega_g = .01, .02, .05$  and  $\omega_h = .25, .50$  and  $1.00$ . Note that the scales are quite different for  $\omega_g$  and  $\omega_h$ . This is because the argument of the  $g$  function sums savings over all items, given the decision vector  $\lambda$ . Thus, the argument is unbounded from above and increases with the number of items on the shopping list. The argument of the  $h$  function is a probability and therefore bounded  $[0,1]$ , regardless of the number of items on the list.

We selected three exponents which were applied to the two weighting functions:  $v_m = .5, 1,$  and  $2$ . This enables us to investigate concavity, linearity and convexity for both weighting functions. Note that when the unbounded argument of the  $g$  function has a convex weighting function (in our analysis  $v_g = 2$ ), together with a larger shopping list and correspondingly larger savings, the first term in the maximization function becomes very large relative to the second term. The result is very few optimal proportions between 0 and 1.

The intercept terms  $\mu_m$  enter the maximization (7) as additive constants and consequently do not affect the resulting optimal decision vector. Without loss of generality, then, we set  $\mu_g =$

$\mu_h = 0$ . The diagonal matrix of quantities  $Q$  simply scales the expected savings and variance of savings terms. It is set to the identity matrix.

In conducting our numerical study, we systematically varied  $\omega_g$ ,  $\omega_h$ ,  $\nu_g$  and  $\nu_h$ . For each combination of slopes and exponents, we assumed a shopping list of fifty items. The contributions to expected savings by purchasing these items at Retailer 2 were randomly drawn from a uniform distribution,  $\delta_i \sim U(-1,1)$ ; the contributions to variances of savings were also randomly drawn from a uniform distribution,  $\sigma_i^2 \sim U(0,1)$ .<sup>15</sup> The ratio of contribution to expected savings over contribution to variance in savings for item  $i$ ,  $\frac{\delta_i}{\sigma_i^2}$ , could therefore vary from  $-\infty$  to  $\infty$ . In order to test for a linear relationship between this ratio and the corresponding optimal proportion  $\lambda_{ii}^*$  for  $0 < \lambda_{ii}^* < 1$  we required that at least 3 of the 50 items on the list have optimal proportions in this range. Using a rejection sampling approach, we i) drew the random vectors  $\delta_i$  and  $\sigma_i^2$ , ii) maximized (7) using the Newton-Raphson method to find the vector of optimal proportions  $\lambda_i^*$ ; iii) if at least 3 of the 50 optimal proportions  $\lambda_{ii}^*$  were between 0 and 1, then we regressed the ratios  $\frac{\delta_{ii}}{\sigma_{ii}^2}$  on the corresponding proportions  $\lambda_{ii}^*$  and recorded the  $R^2$  of that regression. If fewer than 3 of the 50 products were found to have optimal proportions in this range, we rejected that sample and repeated the process. We continued sampling until 20 conforming samples were reached for each combination of slopes and exponents.

The results of our numerical study are shown in panels A and B of Table B1. We find that the relationship is virtually linear, with the average  $R^2$  for 20 conforming samples at least 0.9987 for every combination of slopes and exponents except one ( $\omega_g = .2$ ,  $\omega_h = .25$ ,  $\nu_g = .5$  and  $\nu_h = .5$  yielded an  $R^2$  of 0.9621). Overall, the mean of the average  $R^2$ s across all 81 combinations of slopes and exponents is 0.9994.

We believe that this numerical study provides sufficient support to treat the relationship between the ratio  $\frac{\delta_{ii}}{\sigma_{ii}^2}$  and the corresponding optimal proportion  $\lambda_{ii}^*$  as approximately linear for  $\lambda_{ii}^*$  between 0 and 1 so that:

- for all values of the ratio less than or equal to 0, the optimal proportion is 0

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<sup>15</sup> Note that the diagonal matrix of quantities  $\Sigma$  was assumed to be an identity matrix.

- for all values of the ratio above a certain critical threshold, the optimal proportion is 1
- in the interval between, the optimal proportion is approximately a linear function of the ratio

Insert Table B1 about here

## Appendix C

The conditional contribution to expected price savings is computed as follows:

$$E\left(p_{it}^{(1)} - p_{it}^{(2)} \mid p_{it}^{(1)}\right) = E\left(p_{it}^{(1)} - p_{it}^{(2)}\right) + \left(p_{it}^{(1)} - E\left(p_{it}^{(1)}\right)\right) \left( \frac{\text{Cov}\left(p_{it}^{(1)}, p_{it}^{(1)} - p_{it}^{(2)}\right)}{\text{Var}\left(p_{it}^{(1)}\right)} \right), \quad (\text{C.1})$$

where

$E\left(p_{it}^{(1)} - p_{it}^{(2)}\right)$  is the unconditional contribution to expected savings,

$E\left(p_{it}^{(1)}\right)$  is the expected price at Retailer 1,

$\text{Cov}\left(p_{it}^{(1)}, p_{it}^{(1)} - p_{it}^{(2)}\right)$  is the covariance between the price at Retailer 1 and the contribution to savings from deferring to Retailer 2, and

$\text{Var}\left(p_{it}^{(1)}\right)$  is the variance of prices at Retailer 1.

Similarly, the conditional contribution to variance of price savings is computed as follows

$$\text{Var}\left(p_{it}^{(1)} - p_{it}^{(2)} \mid p_{it}^{(1)}\right) = \text{Var}\left(p_{it}^{(1)} - p_{it}^{(2)}\right) - E\left(p_{it}^{(1)} - p_{it}^{(2)}\right)^2 \left( \frac{\text{Cov}\left(p_{it}^{(1)}, p_{it}^{(1)} - p_{it}^{(2)}\right)^2}{\text{Var}\left(p_{it}^{(1)}\right)} \right), \quad (\text{C.2})$$

where  $\text{Var}\left(p_{it}^{(1)} - p_{it}^{(2)}\right)$  is the unconditional contribution to variance of savings. For our dataset, the conditional contributions to expectation and variance of savings were computed iteratively using the Expectation-Maximization (EM) algorithm until the estimates converged (see, for example, Johnson and Wichern 2007, Ch. 5).

Note that conditioning the contribution to expectation and variance of savings on current prices at Retailer 1 is a special case of the set of conditioning arguments assumed in our analytical development in §3. It is also important to note that we do not claim that shoppers actually perform the computations in equations (C.1) and (C.2); rather, we argue that they make decisions *as if* they had performed them.

Exhibit 1

<b>Summary of Notation</b>	
<i>Symbol</i>	<i>Meaning</i>
$p_t^{(1)} = (p_{1t}^{(1)}, p_{2t}^{(1)}, \dots, p_{nt}^{(1)})$	Item-level stochastic prices for Retailer 1 on trip $t$
$p_t^{(2)} = (p_{1t}^{(2)}, p_{2t}^{(2)}, \dots, p_{nt}^{(2)})$	Item-level stochastic prices for Retailer 2 on trip $t$
$\lambda_t = (\lambda_{1t}, \lambda_{2t}, \dots, \lambda_{nt})$	Proportion of item purchases at Retailer 2 on trip $t$
$Q_t = \text{diag}(q_{1t}, q_{2t}, \dots, q_{nt})$	Diagonal matrix of required quantities on trip $t$
$d_t = (d_{1t}, d_{2t}, \dots, d_{nt})$	Item-level contributions to savings by deferring purchases to Retailer 2 on trip $t$
$\delta_t = (\delta_{1t}, \delta_{2t}, \dots, \delta_{nt})$	Expected item-level contributions to savings by deferring purchases to Retailer 2 on trip $t$
$\Sigma_t$	Variance/covariance matrix of random error terms from shopper's model of price savings on trip $t$
$K$	Fixed cost of purchasing common items at Retailer 2

Figure 1  
The Optimal Proportion of an Item to Buy at the Second Retailer

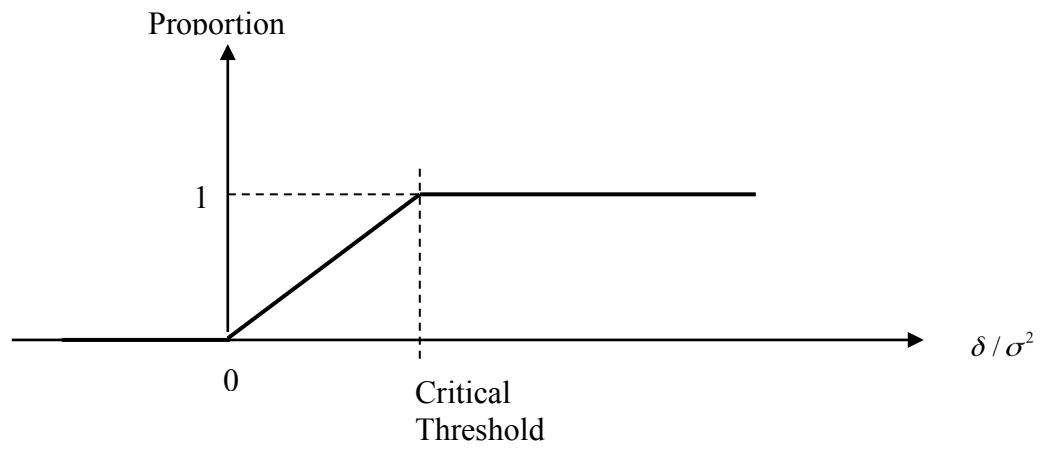


Figure 2  
Routes of Return Beginning with Different Retailers

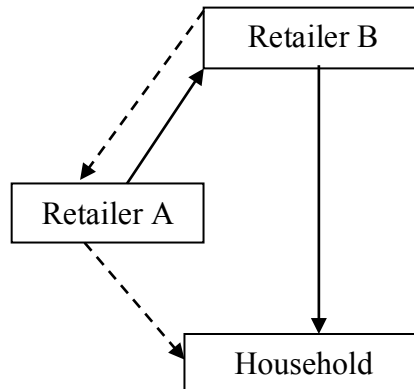


Table 1  
**Panelist Demographics**

	Multi-Store Shoppers Included in Dataset (n=51)		Other Shoppers Not in Dataset (n=485)	
	mean	std dev	mean	std dev
Family Size	3.25	1.44	2.87	1.44
Household Income (x\$1,000)	53.4	25.9	51.3	26.3
Working Adult Female	0.588	0.497	0.640	0.480
College Education	0.216	0.415	0.202	0.402
Home Owner	0.922	0.272	0.820	0.385
Married	0.804	0.401	0.684	0.465

Table 2  
**Panelist Multi-Store Shopping vs. Single-Store Shopping Behavior**

		n	mean	std dev
<b>Multi-Store Shoppers Included in Dataset</b>				
When Multi-Store Shopping	# Store Visits	51	43.80	30.32
	\$ / Store Visit	2336	\$56.06	\$43.61
When Single-Store Shopping	# Store Visits	51	143.45	65.16
	\$ / Store Visit	7316	\$62.73	\$51.34
<b>Other Shoppers Not in Dataset</b>				
When Multi-Store Shopping	# Store Visits	485	2.40	4.69
	\$ / Store Visit	1164	\$50.28	\$43.66
When Single-Store Shopping	# Store Visits	485	61.62	59.66
	\$ / Store Visit	29888	\$70.72	\$57.37

Table 3  
Description of Models Estimated

Model	Description	No. of Segments	Parameters	LL	AIC3	BIC	Hit Rate
M0_1	Purchasing at Retailer 1 function of <i>loyalty</i> and <i>expected price savings</i> : $\gamma \neq 0, \beta_E \neq 0, \beta_P = 0$	1	2	-651.34	1305.68	1306.34	57.48%
M0_2	Purchasing at Retailer 1 function of <i>loyalty</i> and <i>expected price savings/variance</i> : $\gamma \neq 0, \beta_E = 0, \beta_P \neq 0$	1	2	-660.45	1323.90	1324.56	54.51%
<i>Heterogeneity Models</i>							
M1_1	Heterogeneity in <i>loyalty</i> and <i>expected price savings</i> : $\gamma_1 \neq 0, \beta_{E1} \neq 0, \beta_{P1} = 0; \gamma_2 \neq 0, \beta_{E2} \neq 0, \beta_{P2} = 0$	2	5	-637.55	1290.10	1293.42	63.57%
M1_2	Heterogeneity in <i>loyalty</i> and <i>expected price savings</i> : $\gamma_1 \neq 0, \beta_{E1} \neq 0, \beta_{P1} = 0; \gamma_2 \neq 0, \beta_{E2} \neq 0, \beta_{P2} = 0; \gamma_3 \neq 0, \beta_{E3} \neq 0, \beta_{P3} = 0$	3	8	-634.21	1292.42	1297.73	64.88%
M2_1	Heterogeneity in <i>loyalty</i> and <i>expected price savings/variance</i> : $\gamma_1 \neq 0, \beta_{E1} = 0, \beta_{P1} \neq 0; \gamma_2 \neq 0, \beta_{E2} = 0, \beta_{P2} \neq 0$	2	5	-641.95	1298.90	1302.22	54.51%
M2_2	Heterogeneity in <i>loyalty</i> and <i>expected price savings/variance</i> : $\gamma_1 \neq 0, \beta_{E1} = 0, \beta_{P1} \neq 0; \gamma_2 \neq 0, \beta_{E2} = 0, \beta_{P2} \neq 0; \gamma_3 \neq 0, \beta_{E3} = 0, \beta_{P3} \neq 0$	3	8	-639.16	1302.32	1307.63	54.10%
M3_1	Heterogeneity in <i>loyalty</i> , <i>expected price savings</i> and <i>expected price savings/variance</i> : $\gamma_1 \neq 0, \beta_{E1} \neq 0, \beta_{P1} = 0; \gamma_2 \neq 0, \beta_{E2} = 0, \beta_{P2} \neq 0$	2	5	-636.54	1288.08	1291.40	66.70%
M3_2	Heterogeneity in <i>loyalty</i> , <i>expected price savings</i> and <i>expected price savings/variance</i> : $\gamma_1 \neq 0, \beta_{E1} \neq 0, \beta_{P1} = 0; \gamma_2 \neq 0, \beta_{E2} = 0, \beta_{P2} \neq 0; \gamma_3 \neq 0, \beta_{E3} \neq 0, \beta_{P3} = 0$	3	8	-633.01	1290.02	1295.33	66.42%
M3_3	Heterogeneity in <i>loyalty</i> , <i>expected price savings</i> and <i>expected price savings/variance</i> : $\gamma_1 \neq 0, \beta_{E1} \neq 0, \beta_{P1} = 0; \gamma_2 \neq 0, \beta_{E2} = 0, \beta_{P2} \neq 0; \gamma_3 \neq 0, \beta_{E3} = 0, \beta_{P3} \neq 0$	3	8	-634.19	1292.38	1297.69	65.16%
M3_4	Heterogeneity in <i>loyalty</i> , <i>expected price savings</i> and <i>expected price savings/variance</i> : $\gamma_1 \neq 0, \beta_{E1} \neq 0, \beta_{P1} = 0; \gamma_2 \neq 0, \beta_{E2} = 0, \beta_{P2} \neq 0; \gamma_3 \neq 0, \beta_{E3} \neq 0, \beta_{P3} = 0; \gamma_4 \neq 0, \beta_{E4} = 0, \beta_{P4} \neq 0$	4	11	-631.60	1296.20	1303.50	60.45%



Table 4  
**Model M3\_1 Parameter Estimates**

	<i>Segment 1</i>				<i>Segment 2</i>			
	<i>Economic Benefit(62.36%)</i>				<i>Psychological Benefit (37.64%)</i>			
<i>Variable</i>	<i>Estimate</i>	<i>Std Error</i>	<i>t-Value</i>	<i>p-level</i>	<i>Estimate</i>	<i>Std Error</i>	<i>t-Value</i>	<i>p-level</i>
Loyalty	1.3996	0.1099	12.73521	<0.001	0.7885	0.0656	12.01982	<0.001
Expected Price Savings	-0.2925	0.0142	-20.5986	<0.001	<i>C</i>	<i>NA</i>	<i>NA</i>	<i>NA</i>
Expected Price Savings/ Variance in Price Savings	<i>C</i>	<i>NA</i>	<i>NA</i>	<i>NA</i>	-0.0516	0.0098	-5.26531	<0.01

Table B1  
Decision Weighting Function Numerical Study

<i>Panel A</i>					<i>Panel B</i>				
<i>g</i>	<i>h</i>		<i>R</i> <sup>2</sup>		<i>g</i>	<i>h</i>		<i>R</i> <sup>2</sup>	
slope ( <i>w<sub>g</sub></i> )	exp ( <i>u<sub>g</sub></i> )	slope ( <i>w<sub>h</sub></i> )	exp ( <i>u<sub>h</sub></i> )		slope ( <i>w<sub>g</sub></i> )	exp ( <i>u<sub>g</sub></i> )	slope ( <i>w<sub>h</sub></i> )	exp ( <i>u<sub>h</sub></i> )	
0.01	0.5	0.25	0.5	0.998792	0.01	1	0.25	2	0.999858
0.02	0.5	0.25	0.5	0.962059	0.02	1	0.25	2	0.999960
0.05	0.5	0.25	0.5	0.999723	0.05	1	0.25	2	0.999980
0.01	0.5	0.5	0.5	0.999825	0.01	1	0.5	2	0.999998
0.02	0.5	0.5	0.5	0.998936	0.02	1	0.5	2	0.999885
0.05	0.5	0.5	0.5	0.999795	0.05	1	0.5	2	0.999997
0.01	0.5	1	0.5	0.999842	0.01	1	1	2	0.999998
0.02	0.5	1	0.5	0.999069	0.02	1	1	2	0.999999
0.05	0.5	1	0.5	0.999678	0.05	1	1	2	0.999994
0.01	0.5	0.25	1	0.999116	0.01	2	0.25	0.5	0.999953
0.02	0.5	0.25	1	0.998968	0.02	2	0.25	0.5	0.999841
0.05	0.5	0.25	1	0.999832	0.05	2	0.25	0.5	0.999804
0.01	0.5	0.5	1	0.999857	0.01	2	0.5	0.5	0.999995
0.02	0.5	0.5	1	0.999914	0.02	2	0.5	0.5	0.999996
0.05	0.5	0.5	1	0.999735	0.05	2	0.5	0.5	0.999993
0.01	0.5	1	1	0.999956	0.01	2	1	0.5	0.999987
0.02	0.5	1	1	0.999972	0.02	2	1	0.5	0.999974
0.05	0.5	1	1	0.999991	0.05	2	1	0.5	0.999962
0.01	0.5	0.25	2	0.999971	0.01	2	0.25	1	0.999999
0.02	0.5	0.25	2	0.999988	0.02	2	0.25	1	0.999717
0.05	0.5	0.25	2	0.999761	0.05	2	0.25	1	0.999986
0.01	0.5	0.5	2	0.999988	0.01	2	0.5	1	0.999999
0.02	0.5	0.5	2	0.999977	0.02	2	0.5	1	0.999989
0.05	0.5	0.5	2	0.999997	0.05	2	0.5	1	0.999968
0.01	0.5	1	2	0.999998	0.01	2	1	1	0.999999
0.02	0.5	1	2	0.999993	0.02	2	1	1	1.000000
0.05	0.5	1	2	0.999994	0.05	2	1	1	0.999999
0.01	1	0.25	0.5	0.999763	0.01	2	0.25	2	0.999998
0.02	1	0.25	0.5	0.999902	0.02	2	0.25	2	0.999968
0.05	1	0.25	0.5	0.999939	0.05	2	0.25	2	0.999998
0.01	1	0.5	0.5	0.999778	0.01	2	0.5	2	1.000000
0.02	1	0.5	0.5	0.999953	0.02	2	0.5	2	0.999985
0.05	1	0.5	0.5	0.999980	0.05	2	0.5	2	0.999997
0.01	1	1	0.5	0.999827	0.01	2	1	2	1.000000
0.02	1	1	0.5	0.999714	0.02	2	1	2	1.000000
0.05	1	1	0.5	0.999881	0.05	2	1	2	1.000000
0.01	1	0.25	1	0.999814					
0.02	1	0.25	1	0.999975					
0.05	1	0.25	1	0.999956					
0.01	1	0.5	1	0.999865					
0.02	1	0.5	1	0.999936					
0.05	1	0.5	1	0.999961					
0.01	1	1	1	0.999977					
0.02	1	1	1	0.999872					
0.05	1	1	1	0.999996					