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RESEARCH IN MATHEMATICS EDUCATION

Imagination Station (Istation):

Universal Screener Instrument Development for Grade 8

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Universal Screener Instrument Development for Grade 8

Pooja Shivraj • Leanne R. Ketterlin-Geller • Diane Gifford

Southern Methodist University

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Department of Education Policy & Leadership Simmons School of Education & Human Development PO Box 750114
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Contact information: lkgeller@smu.edu

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Abstract

In this technical report, we describe the development of the Grade 8 formative assessment item bank for Imagination Station (Istation). The formative assessment item bank will be used to deliver a computerized adaptive universal screening assessment to support teachers' instructional decision-making. State and national mathematics content standards for Grade 8 inform the construct underlying the items. In this technical report, we include a description of the process used to identify and sample the mathematics content and levels of cognitive complexity assessed in the item bank. Next, we describe the item writing procedures. Finally, we describe how the external item review process and outcomes impact content-related evidence for validity.

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Imagination Station (Istation) Universal Screener Instrument Development for Grade 8

Introduction

The purpose of the Grade 8 formative assessment item bank for the Imagination Station (Istation) is to support teachers' instructional decision-making. The formative assessment item bank is a computerized adaptive universal screening assessment system to monitor student progress with fundamental mathematics skills and grade level standards. By administering this assessment system, teachers and administrators can use the results to answer two questions: (1) are students at risk of failure in Grade 8 mathematics, and (2) what is the degree of intensity of instructional support students need to be successful in Grade 8 mathematics? Multiple administrations of the universal screener (i.e., fall, winter, and early spring each year) provide teachers with meaningful information about student progress to support instructional decision-making over the course of Grade 8. The universal screener is designed for administration to all students receiving grade-level instruction.

The purpose of this technical report is to describe the development of the formative assessment item bank. This description includes (1) the process used to identify and sample the mathematics content assessed in the item bank, (2) the item writing process, and (3) the external review process and results. The test development steps used to create the formative assessment item bank represent best practices in test development and the Test Standards published by the American Educational Research Association (AERA), American Psychological Association (APA), and National Council on Measurement in Education (NCME) (1999).

Construct Definition

The assessed construct consists of (1) mathematics content and (2) level of cognitive engagement. The mathematics content of the Grade 8 formative assessment item bank is based on the Curriculum Focal Points (CFP) (National Council of Teachers of Mathematics [NCTM], 2006), mathematics content standards published by the Common Core Standards Initiative, and state standards from Texas, Florida, New York, California, and Virginia. See Appendix A for the state content standards. We aligned the Common Core State Mathematics standards and state content standards to the Curriculum Focal Points (CFP). We created a fourth CFP to include two content standards that were assessed across the states but was not represented in the NCTM focal points: representing and interpreting data; and geometry and measurement (e.g., currency, temperature, and time). See Appendix B for an abbreviated description of the assessed content.

The cognitive engagement dimension of the construct refers to the level of cognitive processing through which students are expected to engage an assessment item. The formative assessment item bank uses the taxonomy of cognitive engagement in mathematics published by Kilpatrick, Swafford, and Findell (2001) for the National Research Council. The taxonomy consists of five interdependent strands that promote mathematical proficiency: (1) conceptual understanding, (2)

procedural fluency, (3) strategic competence, (4) adaptive reasoning, and (5) productive disposition. The formative assessment item bank assesses student understanding of the content at varying levels of cognitive engagement. A brief description of each level follows:

1. *Conceptual understanding* pertains to the functional grasp of mathematics that a student applies to concepts, operations, and relations. It involves being able to logically organize one's knowledge to integrate and understand concepts as part of a coherent whole.

2. *Procedural fluency* pertains to students' ability to accurately and appropriately carry out skills, including being able to select efficient and flexible approaches.

3. *Strategic competence* involves one's ability to formulate a problem in mathematical terms, to represent it strategically (verbally, symbolically, graphically, or numerically), as well as to solve it effectively. It is similar to problem solving and problem formation.

4. *Adaptive reasoning* involves the student's capacity to think logically about a problem, which requires reflecting on various approaches to solve a problem and deductively selecting an approach. Students who are able to do this are also able to rationalize and justify their strategy.

5. *Productive disposition* refers to a student's overall ability to perceive mathematics as worthwhile and to maintain a personal belief in one's own efficacy in solving problems.

The formative assessment item bank incorporates four of the five strands. Productive disposition is not assessed.

Each CFP was assessed at the four levels of cognitive engagement. Conceptual understanding and procedural fluency were oversampled to accurately reflect the relative emphasis in the state standards. Easy, medium, and difficult items were written for each CFP across the four levels of cognitive engagement. The content sampling matrix is presented in Figure 1.

Item Writing

Item Specifications

Approximately 400 items were written for Grade 8. Multiple-choice items were created for efficiency in the computer delivery system. Each item had three distractors and one correct answer. Items were scored dichotomously as either correct or incorrect. The distractors represent plausible misconceptions or errors in computation, procedure, conceptual understanding, and strategy.

The item stem included text and/or graphics. The language used in all text was intentionally

constrained to the 8th grade level; however, readability statistics were not calculated for each item. Whenever possible, plain language and simple, straightforward statements were incorporated into the items. Graphics were used in instances where they explained the problem, provided a visual clue to clarify the context, or were integral to the stem or answer choices. Irrelevant graphics were not included.

The assessment items were written according to the principles of universal design for assessment (See Ketterlin-Geller, 2005; 2008) and are amendable to accommodations. As delivered, the formative assessment system will include a read aloud feature to support item readability. This ensures that mathematics ability is tested, rather than students' reading ability.

The computerized-adaptive test can be administered individually or in a group in an untimed setting.

Item Writers

Four item writers contributed items to the Grade 8 formative assessment item bank.

Item Writer 1. Item Writer 1 obtained a Bachelor of Public Administration degree from Texas State University—San Marcos. She worked as a long-term substitute for various high school mathematics classes in Texas. After almost four years working in the public sector, she entered the New York City Teaching Fellows Program and taught high school algebra and geometry to special education students in an inner city school in Manhattan. During this time, Item Writer 1 completed a Master of Science degree in Special Education with Honors from the City College of New York. After two years of teaching she left the classroom to pursue research in test development for students with disabilities. She has worked on several education research projects and nationally funded grants and is currently pursuing a doctoral degree in Educational Research from Southern Methodist University.

Item Writer 2. Item Writer 2 is a research associate at the University of Oregon. She earned a Ph.D. in School Psychology from the same university. Prior to that, she obtained a Master of Arts degree in Special Education at San Francisco State University. She has served as a resource teacher and education specialist for both middle and high school math and science.

Item Writer 3. Item Writer 3 earned a Bachelor of Science in Mathematics and a Master of Science in Mathematics Education from Oregon State University. She taught mathematics for six years at the middle, high school, and community college level. In addition to teaching, she currently works as a mathematics coach in her school district. In this position, she focuses on improving instruction across the district by developing curriculum that is aligned to state mathematics standards. Her interest in assessments led her to become an item-writer for

mathematics assessments.

Item Writer 4. Item Writer 4 is a school psychologist with expertise in mathematics education. She earned a Ph.D. in Educational Leadership with a focus on assessment and measurement. She was the lead author on a district-wide mathematics formative assessment administered three times yearly to all students in Grades 1-8. Her work on this project also included vertical equating and scaling tests. Since graduating, she worked for a nonprofit organization assisting in the design, development, and data collection of evaluations of education programs and improvement initiatives. Most recently, she served as a school psychologist where she conducted comprehensive psycho-educational evaluations to determine student eligibility for special services and to further assist teachers in implementing instructional interventions to meet student needs.

Item Writing Training

All item writers were trained to write items that aligned with the content expectations and item specifications. Training included review of the Item Writing Training Manual and participation in a training conference call with the researchers and project staff. The Item Writing Training Manual provides a detailed description of the principles of universal design for assessment and logistical information about formatting, reviewing, and submitting items. Reviewers received guidelines for writing selected response items, written by recognized experts in item design, and information on the elements of high quality test design. Moreover, reviewers were given sample items illustrating important components of effective items. A glossary of useful terms and a list of relevant websites were provided.

A training conference call was conducted with the item writers to review the content standards and levels of cognitive complexity of the items for Grade 8. Project staff first provided a detailed description of the content by reviewing each CFP for the grade level. Item writers were then provided with the blueprint for the Grade 8 Universal Screener, which delineated the number of items to be written for each CFP and the number of associated cognitive complexity levels to be addressed in item development. Example items for each CFP and respective levels of cognitive complexity were disseminated and discussed. Finally, any additional material in the Item Writing Training Manual was reviewed and discussed until the item writers were confident they understood the content and objectives of the project.

Item Writing Process

After completing the training and attending a project conference call, item writers were given the item writing template to create items. Upon completion of the items, reviewers submitted items to researchers and project staff for review. At least two internal reviewers provided feedback for each item. Reviewers evaluated items for (1) mathematical accuracy, (2) alignment with the content standards, (3) age-appropriateness of language and graphics for students in Grade 8, and (4) compliance with universal design principles. Reviewer comments were returned to the item

writers to revise and resubmit for approval. All finalized items were cross-referenced to the test blueprint and specifically to the content standard to ensure that each standard had a corresponding item. When standards were found without items, items were written.

Once items were accepted, item level information was entered into an Item Database. The Istation graphic design team created all graphics. The finalized items were copy-edited and reviewed by SMU researchers and Istation staff.

Content-Related Evidence for Validity

Mathematicians and mathematics teachers evaluated all items for accuracy and appropriateness of the content written for the formative assessment item bank for students in Grade 8.

Mathematician Review

Three mathematicians reviewed all items in Grade 8. Two reviewers were professors of mathematics at universities in Texas and held undergraduate and graduate degrees in mathematics. The third mathematician was a recent doctoral graduate in mathematics education who was working as an assistant professor at a university in Texas. Their experience in mathematics education and research ranged from 6-22 years. Two reviewers were female; one reviewer was male.

The mathematicians were asked to review each item and evaluate the accuracy of the content, precision of the vocabulary, and effectiveness of distractors. The criteria used for item evaluation are as follows:

- Mathematical accuracy of content: Each item was written to reflect an integration of knowledge and skills identified by the NCTM Curriculum Focal Points. Is the item mathematically accurate?
- Precision of mathematical vocabulary: Is the mathematical vocabulary used accurately? Is the mathematical vocabulary precise?
- Appropriateness of the distractors: Most students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions from choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Are the distractors mathematically plausible misconceptions?

Items and distractors were evaluated on a 4-point scale for each criterion. A rating of 1 indicated that the item was not accurate, precise, or appropriate; a rating of 2 indicated that the item was somewhat accurate, precise, or appropriate; a rating of 3 indicated that the item was mostly accurate, precise, or appropriate; and a rating of 4 indicated the item was extremely accurate, precise, or appropriate. In instances where the reviewer assigned a score of 1 or 2 for any criterion, recommendations were solicited that would aid in revision.

Overall, the mathematicians rated the items as mostly accurate, precise, and effective. The mathematicians recommended revisions for 64 items. One reviewer noted the following issues on 35 items and offered several suggestions: the use of complex vocabulary, inappropriate distractors, and mathematical inaccuracies in item stems and item responses. The reviewer offered several suggestions including several possible distractors and alternate questions. The second reviewer noted issues on 16 items. The reviewer suggested improving the clarity of the image on three items, increasing the mathematical accuracy on two other items, and making the mathematical vocabulary more precise on three other items. The reviewer also suggested new distractors for 8 items. The third reviewer recommended changes on 13 items, primarily making the mathematical vocabulary more precise. The reviewer also suggested improving the clarity of the images of two items and improving the mathematical accuracy of two other items.

We revised all items in response to the recommendations. In instances where the mathematician did not provide a suitable suggestion, we revised the item and requested an additional review from an independent mathematician.

Teacher Review

Three teachers with experience teaching Grade 8 mathematics reviewed the items. One reviewer was a Caucasian female with 18 years of experience teaching middle school mathematics. Another reviewer was a Caucasian female who had been teaching middle school for 2 years. The final reviewer was a Caucasian female teacher with 14 years of experience teaching high school.

Teachers analyzed each item for appropriate grade-level language and vocabulary, content or concepts, graphics, potential bias in language and/or content, clarity of directions and answers, and effectiveness of distractors. The criteria presented for item evaluation are as follows:

- Appropriateness of language: Is the language used in the item appropriate for students in your grade level? Are the question and response options written so that students in your grade level can understand the meaning of the problem?
- Appropriateness of mathematical vocabulary: Is the mathematical vocabulary representative of pre-requisite or instructional expectations in your grade level?
- Appropriateness of content or concepts: Is the task representative of pre-requisite or instructional expectations in your grade level?
- Appropriateness of visual representation: Is the visual representation (i.e., graphic, table, image) used in the item appropriate for students in your grade level? Can students in your grade level understand the meaning of the visual representation? Is the visual representation of the item clear?
- Bias in language or content: Does the item require background knowledge unrelated to the concept being tested that would differ for students with different backgrounds? Is the language sensitive to students from diverse backgrounds, students with limited English proficiency and students with special needs?

Example: "*What is the most appropriate measurement unit for the length of a sub or hoagie?"* may be unfair for students in certain geographic regions and students with diverse background who are unfamiliar with the terms "sub or hoagie."

- Effectiveness of the distractors: Some students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Do the distractors discriminate between students with specific misconceptions?

The items and distractors were rated on a scale of 1 to 4 for each criterion. A rating of 1 indicated that the item/distractors were not at all appropriate based on the criterion (or very biased); a rating of 2 indicated that the item/distractors were somewhat appropriate based on the criterion (or somewhat biased); a rating of 3 indicated that the item/distractors were appropriate based on the criterion (or not biased); and a rating of 4 indicated that the item/distractors were extremely appropriate based on the criterion (or not biased *and* has multicultural components to it). In instances where the teachers provided a rating of 2 or lower, they were asked to provide additional suggestions and comments to improve the item.

Overall, the teachers rated the items as mostly to always appropriate in regard to language, vocabulary content, visual representation, bias, and effectiveness of distractors. The teachers recommended revising 48 items. One reviewer recommended changes to seven items—one was related to the precision of the mathematical vocabulary used and the other was a suggestion for a possible distractor. The second reviewer noted the need to improve the precision of mathematics vocabulary for 5 items. The third reviewer made suggestions to improve distractors for 41 items, primarily addressing the clarity of the images and improving the precision of mathematics vocabulary. The research team reviewed all suggestions and made revisions based on teacher feedback.

Conclusions

The purpose of this technical report was to describe the development of the formative assessment item bank. We described the construct underlying the items in reference to the content standards and levels of cognitive complexity as well as explained the process for sampling the content assessed in the item bank. Next, we described the item writing procedures and provided the qualifications for the item writers. Finally, we documented the process and outcomes of an external item review by mathematicians and mathematics teachers to document content-related evidence for validity.

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Figure 1

Content Sampling Matrix

CFP	Procedural fluency			Conceptual understanding			Strategic competence			Adaptive reasoning		
	Easy	Medium	Difficult	Easy	Medium	Difficult	Easy	Medium	Difficult	Easy	Medium	Difficult
1	10	10	10	10	10	10	7	6	7	7	6	7
2	10	10	10	10	10	10	7	6	7	7	6	7
3	10	10	10	10	10	10	7	6	7	7	6	7
4	10	10	10	10	10	10	7	6	7	7	6	7
Total By Difficulty	40	40	40	40	40	40	28	24	28	28	24	28

Appendix A - State Content Standards Referent Sources

National Council of Teachers of Mathematics (NCTM) Curricular Focal Points

The National Council of Teachers of Mathematics (NCTM) Curricular Focal Points were retrieved from http://www.nctmmedia.org/cfp/front_matter.pdf on 4/20/2010. Additional information was also retrieved on 4/20/2010 from: www.nctm.org/focalpoints . The coding system for the NCTM Critical Focal Points can be found under Part II.

Florida

Florida's Next Generation Sunshine State Math Standards (adopted 2007) were retrieved on 4/20/2010 from http://www.floridastandards.org/Standards/FLStandardSearch.aspx. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from Florida Department of Education. Big Ideas for each of the grade levels were also verified.

California

California's Math Content Standards (adopted 1997) were retrieved on 4/24/2010 from http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf . California Green Dot Standards are the selected standards (as of 2006) that appear 85% of the time on California state tests. These green dot standards were retrieved on 4/24/2010 from http://caworldclassmath.com/high_ca_standards.html and etc.usf.edu/flstandards/math/california.ppt . Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the California State Board of Education.

New York

The New York State Standards (revised on March 15, 2005) were retrieved on 4/21/2010 from: http://www.bootstrapworld.org/standards/ny/NYMathematicsCoreCurriculum.pdf .Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the New York State Board of Education.

Texas

The Texas State Standards for Math (Version 2.1; revised 2010) were retrieved on 4/21/2010 from: http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the Texas State Board of Education. The Texas Education Agency (TEA) released a 2010 document entitled *Texas Response to Curriculum Focal Points: Kindergarten through Grade 8 Mathematics* that included coordinating TEKS.

Common Core Standards

The Common Core Standards in Mathematics were retrieved on June 10, 2011 from http://www.corestandards.org/the-standards/mathematics . These standards were published in 2010.

They were developed as part of an initiative led by National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

Virginia

Virginia's Standards for Learning Document for Mathematics (adopted 2009 for full implementation in 2011-12) were retrieved on June 10, 2011 from www.doe.virginia.gov/testing/sol/standards_docs/mathematics/review.shtml . Verification of accuracy and currency of the standards was obtained from Istation on June 10, 2011. The Curriculum Frameworks documents were referenced to determine the essential knowledge and skills students are expected to learn for each grade.

Appendix B: Content Description

GRADE 8 MATHEMATICS CURRICULUM FOCAL POINTS	
CFP 1: Algebra **Analyzing and representing linear functions and solving linear equations and systems of linear equations**	
8.1A.1	Students **use linear functions, linear equations, and systems** of linear equations to represent, analyze, and solve a variety of problems.
8.1B.1	Students recognize a **proportion** (y/x = k, or y = kx) as a **special case of a linear equation** of the form y = mx + b, understanding that the constant of proportionality (k) is the slope and the resulting graph is a line through the origin.
8.1C.1	Students understand that the **slope** (m) of a line is a **constant rate of change**, so if the input, or x-coordinate, changes by a specific amount, a, the output, or y-coordinate, changes by the amount 'ma'
8.1D.1	Students translate among verbal, tabular, graphical, and algebraic **representations of functions** (recognizing that tabular and graphical representations are usually only partial representations), and they describe how such aspects of a function as slope and y-intercept appear in different representations.
8.1E.1	Students solve **systems of two linear equations in two variables** and relate the systems to pairs of lines that intersect, are parallel, or are the same line, in the plane. Students use linear equations, systems of linear equations, linear functions, and their understanding of the slope of a line to analyze situations and solve problems.
8.1F.1	Students encounter some **nonlinear functions** (such as the inverse proportions that they studied in grade 7 as well as basic quadratic and exponential functions) whose rates of change contrast with the constant rate of change of linear functions and they distinguish between linear and nonlinear equations graphically.
8.1G.1	Students view **arithmetic sequences**, including those arising from patterns or problems, **as linear functions** whose inputs are counting numbers.

8.1H1	Students apply ideas about linear functions and apply algebraic techniques to **solve problems involving rates** such as motion at a constant speed, work problems, and percent mixture problems
A8.CFP1.1	Students select and use expressions, equations, and functions used to represent and **solve problems involving rational numbers**, some of which are computationally and conceptually challenging
A8.CFP1.2	**Solve one-step and multi-step inequalities** in one or two variables (including word problems) and graph the solution set (e.g. on a number line, etc.)
A8.CFP1.17	Students understand the **concepts of a relation and a function**, determine whether a given relation defines a function, and give pertinent information about given relations and functions
A8.CFP1.14	Understand that numerical information can be represented in multiple ways: arithmetically, algebraically, tabularly, and graphically
A8.CFP1.15	Find a set of ordered pairs to satisfy a given linear numerical pattern (expressed algebraically); then plot the ordered pairs and draw the line
A8.CFP1.24	Students are able to find the equation of a line perpendicular to a given line that passes through a given point.
A8.CFP1.35	Students use correct terminology and determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
A8.CFP1.6	Use physical models to perform operations with polynomials
A8.CFP1.7	Multiply and divide monomials.
A8.CFP1.8	Add and subtract polynomials (integer coefficients)
A8.CFP1.9	Multiply a binomial by a monomial or a binomial (integer coefficients)
A8.CFP1.10	Divide a polynomial by a monomial (integer coefficients) Note: The degree of the denominator is less than or equal to the degree of the numerator for all variables.
A8.CFP1.11	Students apply basic factoring techniques (i.e., factoring algebraic expressions using the GCF) to second- and simple third-degree polynomials.

A8.CFP1.28	Students applying basic factoring techniques will also include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.
A8.CFP1.29	Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.
A8.CFP1.32	Students solve a quadratic equation (a trinomial in the form $ax^2 + bx + c$; a=1 and c having no more than three sets of factors) by factoring or completing the square.
A8.CFP1.36	Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.
A8.CFP1.37	Students graph quadratic functions and know that their roots are the x-intercepts.
A8.CFP1.38	Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.
A8.CFP2.18	Recognize the characteristics of quadratics in tables, graphs, equations, and situations
A8.CFP1.46	Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.
CFP 2: Geometry* and *Measurement **Analyzing two- and three-dimensional spaces and figures by using distance and angle**	
8.2A.1	Students use fundamental facts about distance and angles to describe and **analyze figures and situations in two- and three-dimensional space** and to solve problems, including those with multiple steps.
8.2B.1	Students prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines.
8.2C.1	Students apply this **reasoning about similar triangles** to solve a variety of problems, including those that ask them to find heights and distances.
8.2D.1	Students use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and they apply this fact about triangles to find unknown measures of angles

8.2E.1	Students explain **why the Pythagorean theorem is valid** by using a variety of methods – for example, by decomposing a square in two different ways.
8.2F.1	Student **use square roots** when they apply the Pythagorean theorem.
8.2G.1	Students apply the **Pythagorean theorem to find distances** between points in the Cartesian coordinate plane to measure lengths and analyze polygons and polyhedra.
8.2H.1	Given a line in a coordinate plane, students understand that all **"slope triangles**"—triangles created by a vertical "rise" line segment (showing the change in y), a horizontal "run" line segment (showing the change in x), and a segment of the line itself—are similar. They also understand the relationship of these similar triangles to the constant slope of a line.
CFP 3: Data Analysis* and *Numbers and Operations* and *Algebra **Analyzing and summarizing data sets**	
8.3A.1	Students use descriptive statistics, including **mean, median, and range,** to summarize and compare data sets, and they organize and display data to pose and answer questions.
8.3B.1	Students compare the information provided by the mean and the median and investigate the different effects that **changes in data values have on these measures of center**.
8.3C.1	In addition to the median, students determine the **25th and 75th percentiles** (1st and 3rd quartiles) to obtain information about the spread of data. They may use box-and-whisker plots to convey this information.
8.3D.1	Students **make scatterplots** to display bivariate data, and they informally estimate lines of best fit to make and test conjectures.
8.3E.1	Students understand that a measure of center alone does not thoroughly describe a data set because **very different data sets can share the same measure of center**.
8.3F.1	Students **select the mean or the median** as the appropriate measure of center for a given purpose.
8.3G.1	Students **use exponents and scientific notation** to describe very large and very small numbers.

A8.CFP3.4	Read, write, and identify percents less than 1% and greater than 100%
A8.CFP3.5	Apply percents to: • Tax • Percent increase/decrease• Simple interest • Sale price • Commission • Interest rates • Gratuities
A8.CFP3.6	Estimate a percent of quantity, given an application, and justify its reasonable
A8.CFP1.3	Simplify real number expressions using the laws of exponents.
A8.CFP1.4	Make reasonable approximations of square roots and mathematical expressions that include square roots, and use them to estimate solutions to problems and to compare mathematical expressions involving real numbers and radical expressions.
A8.CFP1.5	Perform operations on real numbers (including integer exponents, radicals, percents, scientific notation, absolute value, rational numbers, and irrational numbers) using multi-step and real world problems
A8.CFP1.21	Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power.
A8.CFP1.19	Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable.
A8.CFP1.39	Students use and know simple aspects of a logical argument.
A8.CFP1.40	Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.
A8.CFP1.41	Students identify the hypothesis and conclusion in logical deduction.
A8.CFP1.42	Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.
A8.CFP1.43	Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements.
A8.CFP1.44	Students use properties of numbers to demonstrate whether assertions are true or false and to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.
A8.CFP1.45	Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step and according to their knowledge of data representation and analysis

Measurement **and** ***Geometry Standards and their Connections to Focal Points***

Students convert measurement units between different measurement systems. They use properties and geometric proofs to reason through more advanced problems. Students further their understanding of the Pythagorean theorem and use trigonometric functions to solve for unknown lengths of right triangles.

	Measurement
A8.CFP2.2	Compare, contrast, and convert units of measure between different measurement systems (US customary or metric (SI)) and dimensions including temperature, area, volume, and derived units to solve problems.
	Geometry
A8.CFP2.6	Describes transformations in a coordinate plane, and recognizes the image of a figure under a translation, a dilation, a reflection over a given line, and rotations of 90 and 180 degree.
A8.CFP2.11	Identifies the properties preserved and not preserved under a translation, dilation, reflection, rotation,
A8.CFP2.20	Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.
A8.CFP2.21	Students use geometric proofs, including proofs by contradiction.
A8.CFP2.23	Students commit to memory the formulas for prisms, pyramids, and cylinders.
A8.CFP2.24	Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.
A8.CFP2.25	Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.
A8.CFP2.26	Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.
A8.CFP2.5	Calculate the missing angle measurements when given two intersecting lines and an angle
A8.CFP2.27	Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles (e.g., they identify pairs of vertical angles as congruent)

A8.CFP2.28	Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line.
A8.CFP2.29	Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.
A8.CFP2.30	Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle and are able to use elementary relationships between them (e.g., tan(x) = sin(x)/cos(x), (sin(x))^2 + (cos(x)) ^2 = 1.
A8.CFP2.32	Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.
A8.CFP2.33	Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles.
A8.CFP2.34	Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.