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RESEARCH IN MATHEMATICS EDUCATION

Imagination Station (Istation):

Universal Screener Instrument Development for Grade 7

Technical Report 12-02

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Universal Screener Instrument Development for Grade 7

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Abstract

In this technical report, we describe the development of the Grade 7 formative assessment item bank for Imagination Station (Istation). The formative assessment item bank will be used to deliver a computerized adaptive universal screening assessment to support teachers' instructional decision-making. State and national mathematics content standards for Grade 7 inform the construct underlying the items. In this technical report, we include a description of the process used to identify and sample the mathematics content and levels of cognitive complexity assessed in the item bank. Next, we describe the item writing procedures. Finally, we describe how the external item review process and outcomes impact content-related evidence for validity.

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Imagination Station (Istation): Universal Screener Instrument Development for Grade 7

Introduction

The purpose of the Grade 7 formative assessment item bank for Imagination Station (Istation) is to support teachers' instructional decision-making. The formative assessment item bank is a computerized adaptive universal screening assessment system to monitor student progress with fundamental mathematics skills and grade level standards. By administering this assessment system, teachers and administrators can use the results to answer two questions: (1) are students at risk of failure in Grade 7 mathematics, and (2) what is the degree of intensity of instructional support students need to be successful in Grade 7 mathematics? Multiple administrations of the universal screener (i.e., fall, winter, and early spring each year) provide teachers with meaningful information about student progress to support instructional decision-making over the course of Grade 7. The universal screener is designed for administration to all students receiving grade-level instruction.

The purpose of this technical report is to describe the development of the formative assessment item bank. This description includes (1) the process used to identify and sample the mathematics content assessed in the item bank, (2) the item writing process, and (3) the external review process and results. The test development steps used to create the formative assessment item bank represent best practices in test development and the Test Standards published by the American Educational Research Association (AERA), American Psychological Association (APA), and National Council on Measurement in Education (NCME) (1999).

Construct Definition

The assessed construct consists of (1) mathematics content and (2) level of cognitive engagement. The mathematics content of the Grade 7 formative assessment item bank is based on the Curriculum Focal Points (CFP) (National Council of Teachers of Mathematics [NCTM], 2006), mathematics content standards published by the Common Core Standards Initiative, and state standards from Texas, Florida, New York, California, and Virginia. See Appendix A for the state content standards. We aligned the Common Core State Mathematics standards and state mathematics content standards to the CFP. We created a fourth CFP to include two content standards that were assessed across the states but was not represented in the NCTM focal points: representing and interpreting data; and geometry and measurement (e.g., currency, temperature, and time).. See Appendix B for an abbreviated description of the assessed content.

The cognitive engagement dimension of the construct refers to the level of cognitive processing at which students are expected to engage an assessment item. The formative assessment item bank uses the taxonomy of cognitive engagement in mathematics published by Kilpatrick, Swafford, and Findell (2001) for the National Research Council. The taxonomy consists of five interdependent strands that promote mathematical proficiency: (1) conceptual understanding, (2) procedural fluency, (3) strategic competence, (4) adaptive reasoning, and (5) productive disposition. The formative assessment item bank assesses student understanding of the content at varying levels of cognitive engagement. A brief description of each level follows:

1. *Conceptual understanding* pertains to the functional grasp of mathematics that a student applies to concepts, operations, and relations. It involves being able to logically organize one's knowledge to integrate and understand concepts as part of a coherent whole.

2. *Procedural fluency* pertains to students' ability to accurately and appropriately carry out skills, including being able to select efficient and flexible approaches.

3. *Strategic competence* involves one's ability to formulate a problem in mathematical terms, to represent it strategically (verbally, symbolically, graphically, or numerically), as well as to solve it effectively. It is similar to problem solving and problem formation.

4. *Adaptive reasoning* involves the student's capacity to think logically about a problem, which requires reflecting on various approaches to solve a problem and deductively selecting an approach. Students who are able to do this are also able to rationalize and justify their strategy.

5. *Productive disposition* refers to a student's overall ability to perceive mathematics as worthwhile and to maintain a personal belief in one's own efficacy in solving problems.

The formative assessment item bank incorporates four of the five strands; productive disposition is not assessed.

Each CFP was assessed at the four levels of cognitive engagement. Conceptual understanding and procedural fluency were oversampled to accurately reflect the relative emphasis in the state standards. Easy, medium, and difficult items were written for each CFP across the four levels of cognitive engagement. The content sampling matrix is presented in Figure 1.

Item Writing

Item Specifications

Approximately 400 items were written for Grade 7. Multiple-choice items were created for efficiency in the computer delivery system. Each item had three distractors and one correct answer. Items were scored dichotomously as either correct or incorrect. The distractors represent plausible misconceptions or errors in computation, procedure, conceptual understanding, and strategy.

The item stem included text and/or graphics. The language used in all text was intentionally constrained to the 7th grade level; however, readability statistics were not calculated for each item. Whenever possible, plain language and simple, straightforward statements were incorporated into the items. Graphics were used in instances where they explained the problem, provided a visual clue to clarify the context, or were integral to the stem or answer choices. Irrelevant graphics were not included.

The assessment items were written according to the principles of universal design for assessment (See Ketterlin-Geller, 2005; 2008) and are amendable to accommodations. As delivered, the formative assessment system will include a read aloud feature to support item readability. This ensures that mathematics ability is tested, rather than students' reading ability.

The computerized-adaptive test can be administered individually or in a group in an untimed setting.

Item Writers

Five item writers contributed items to the Grade 7 formative assessment item bank.

Item Writer 1. Item Writer 1 obtained a Bachelor of Public Administration degree from Texas State University—San Marcos. She worked as a long-term substitute for various high school mathematics classes in Texas. After almost four years working in the public sector, she entered the New York City Teaching Fellows Program and taught high school algebra and geometry to special education students in an inner city school in Manhattan. During this time, Item Writer 1 completed a Master of Science degree in Special Education with Honors from the City College of New York. After two years of teaching she left the classroom to pursue research in test development for students with disabilities. She has worked on several education research projects and nationally funded grants and is currently pursuing a doctoral degree in Educational Research from Southern Methodist University.

Item Writer 2. Item Writer 2 holds a Bachelors of Science and a Masters degree in Special Education. He has been a research assistant, project coordinator and independent contractor for federally funded grants and state contracts since 2001. He assisted in the creation of a web-delivered math assessment researching effectiveness of accommodations. He was also involved in developing an alternate assessment for elementary, middle, and high school students with significant cognitive disabilities between 2002 and 2009. He also helped to write and create items for mathematics screening tests, as well as to develop accommodated versions of items. He has been a part of several research teams conducting multi-state research projects examining comparability of performance on alternate assessments.

Item Writer 3. Item Writer 3 earned a Bachelor of Science in Mathematics and a Master of Science in Mathematics Education from Oregon State University. She taught mathematics for six years at the middle, high school, and community college level. In addition to teaching, she currently works as a mathematics coach in her school district. In this position, she focuses on improving instruction across the district by developing curriculum that is aligned to state mathematics standards. Her interest in assessments led her to become an item-writer for mathematics assessments.

Item Writer 4. Item Writer 4 is a school psychologist with expertise in mathematics education. She earned a Ph.D. in Educational Leadership with a focus on assessment and measurement. She was the lead author on a district-wide mathematics formative assessment administered three times yearly to all students in Grades 1-8. Her work on this project also included vertical equating and scaling tests. Since graduating, she worked for a nonprofit organization assisting in the design, development, and data collection of evaluations of education programs and improvement initiatives. Most recently, she served as a school psychologist where she conducted comprehensive psycho-educational evaluations to determine student eligibility for special services and to further assist teachers in implementing instructional interventions to meet student needs.

Item Writer 5. Item Writer 5 is a research associate at the University of Oregon. She earned a Ph.D. in School Psychology from the same university. Prior to that, she obtained a Master of Arts degree in Special Education at San Francisco State University. She has served as a resource teacher and education specialist for both middle and high school in math and science.

Item Writing Training

All item writers were trained to write items that aligned with the content expectations and item specifications. Training included review of the Item Writing Training Manual and participation in a training conference call with the researchers and project staff. The Item Writing Training Manual provides a detailed description of the principles of universal design for assessment and logistical information about formatting, reviewing, and submitting items. Reviewers received guidelines for writing selected response items, written by recognized experts in item design, and information on the elements of high quality test design. Moreover, reviewers were given sample items illustrating important components of effective items. A glossary of useful terms and a list of relevant websites were provided.

A training conference call was conducted with the item writers to review the content standards and levels of cognitive complexity of the items for Grade 7. Project staff first provided a detailed description of the content by reviewing each CFP for the grade level. Item writers were then provided with the blueprint for Grade 7 Universal Screener, which delineated the number of items to be written for each CFP and the number of associated cognitive complexity levels to be addressed in item development. Example items for each CFP and respective levels of cognitive complexity were disseminated and discussed. Finally, any additional material in the Item Writing Training Manual was reviewed and discussed until the item writers were confident they understood the content and objectives of the project.

Item Writing Process

After completing the training and attending a project conference call, item writers were given the item writing template to create items. Upon completion of the items, reviewers submitted items to researchers and project staff for review. At least two internal reviewers provided feedback for each item. Reviewers evaluated items for (1) mathematical accuracy, (2) alignment with the content standards, (3) age-appropriateness of language and graphics for students in Grade 7, and (4) compliance with universal design principles. Reviewer comments were returned to the item writers to revise and resubmit for approval. All finalized items were cross-referenced to the test blueprint and specifically to the content standard to ensure that each standard had a corresponding item. When standards were found without items, items were written.

Once items were accepted, item level information was entered into an Item Database. The Istation graphic design team created all graphics. The finalized items were copy-edited and reviewed by SMU researchers and Istation staff.

Content-Related Evidence for Validity

Mathematicians and mathematics teachers evaluated all items for accuracy and appropriateness of the content written for the formative assessment item bank for students in Grade 7.

Mathematician Review

Three mathematicians reviewed all items in Grade 7. Two reviewers were professors of mathematics at universities in Texas and held undergraduate and graduate degrees in mathematics. The third mathematician was a recent doctoral graduate in mathematics education who was working as an assistant professor at a university in Texas. Their experience in mathematics education and research ranged from 6-22 years. Two reviewers were female; one reviewer was male.

The mathematicians were asked to review each item and evaluate the accuracy of the content, precision of the vocabulary, and effectiveness of distractors. The criteria used for item evaluation are as follows:

- Mathematical accuracy of content: Each item was written to reflect an integration of knowledge and skills identified by the NCTM Curriculum Focal Points. Is the item mathematically accurate?

- Precision of mathematical vocabulary: Is the mathematical vocabulary used accurately? Is the mathematical vocabulary precise?

- Appropriateness of the distractors: Most students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions from choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Are the distractors mathematically plausible misconceptions?

Items and distractors were evaluated on a 4-point scale for each criterion. A rating of 1 indicated that the item was not accurate, precise, or appropriate; a rating of 2 indicated that the item was somewhat accurate, precise, or appropriate; a rating of 3 indicated that the item was mostly accurate, precise, or appropriate; and a rating of 4 indicated the item was extremely accurate, precise, or appropriate. In instances where the reviewer assigned a score of 1 or 2 for any criterion, recommendations were solicited that would aid in revision.

Overall, the mathematicians rated the items as mostly accurate, precise, and effective. The mathematicians recommended revisions for 82 items. One reviewer noted the following issues on 56 items and offered several suggestions: the use of complex vocabulary, inappropriate distractors, and mathematical inaccuracies in item stems and item responses. The second reviewer recommended revisions for 22 items primarily to improve the effectiveness of distractors and in fewer instances to increase mathematical accuracy. In addition, the reviewer made recommendations to improve the mathematical precision of vocabulary in some of the other items. The third reviewer suggested improving clarity of communication and using precise vocabulary on 4 items.

We revised all items in response to the recommendations. In instances where the mathematician did not provide a suitable suggestion, we revised the item and requested an additional review from an independent mathematician.

Teacher Review

Three teachers with experience teaching Grade 7 mathematics reviewed the items. One reviewer was a Caucasian female with 18 years of experience teaching middle school mathematics. Another reviewer was a Hispanic female who taught middle school for 20 years. The final reviewer was a Caucasian female with 22 years of teaching experience in grades 1-8, but primarily in grades 6 and 7. All reviewers were certified to teach middle school mathematics by the state of Texas.

Teachers analyzed each item for appropriate grade-level language and vocabulary, content or concepts, graphics, potential bias in language and/or content, clarity of directions and answers, and effectiveness of distractors. The criteria presented for item evaluation are as follows:

- Appropriateness of language: Is the language used in the item appropriate for students in your grade level? Are the question and response options written so that students in your grade level can understand the meaning of the problem?
- Appropriateness of mathematical vocabulary: Is the mathematical vocabulary representative of pre-requisite or instructional expectations in your grade level?
- Appropriateness of content or concepts: Is the task representative of pre-requisite or instructional expectations in your grade level?
- Appropriateness of visual representation: Is the visual representation (i.e., graphic, table, image) used in the item appropriate for students in your grade level? Can students in your grade level understand the meaning of the visual

representation? Is the visual representation of the item clear?

- Bias in language or content: Does the item require background knowledge unrelated to the concept being tested that would differ for students with different backgrounds? Is the language sensitive to students from diverse backgrounds, students with limited English proficiency and students with special needs? Example: "*What is the most appropriate measurement unit for the length of a sub or hoagie?"* may be unfair for students in certain geographic regions and students with diverse background who are unfamiliar with the terms "sub or hoagie."

- Effectiveness of the distractors: Some students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Do the distractors discriminate between students with specific misconceptions?

The items and distractors were rated on a scale of 1 to 4 for each criterion. A rating of 1 indicated that the item/distractors were not at all appropriate based on the criterion (or very biased); a rating of 2 indicated that the item/distractors were somewhat appropriate based on the criterion (or somewhat biased); rating of 3 indicated that the item/distractors were appropriate based on the criterion (or not biased); and a rating of 4 indicated that the item/distractors were extremely appropriate based on the criterion (or not biased *and* has multicultural components to it). In instances where the teachers provided a rating of 2 or lower, they were asked to provide additional suggestions and comments to improve the item.

Overall, the teachers rated the items as mostly to always appropriate in regard to language, vocabulary content, visual representation, bias, and effectiveness of distractors. The teachers recommended revising 43 items. One reviewer recommended changes to seven items—six needed further clarification for students to better understand the question and the answer choices for one item did not match the stem. The second reviewer noted the following types of changes to 16 items: the need to increase the size of images, indicate multiplication with a "dot" instead of an "x", and further clarify the vocabulary used in the items. The third reviewer suggested changes for 20 items. She recommended rewriting two questions to simplify the language and increasing the effectiveness of the distractors by changing the answer choices. The research team reviewed all suggestions and made revisions based on teacher feedback.

Conclusions

The purpose of this technical report was to describe the development of the formative assessment item bank. We described the construct underlying the items in reference to the content standards and levels of cognitive complexity. In addition, we described the process for sampling the content assessed in the item bank. Next, we described the item writing procedures and provided the qualifications for the item writers. Finally, we documented the process and outcomes of an external item review by mathematicians and mathematics teachers to document content-related evidence for validity.

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Figure 1

Content Sampling Matrix

CFP	Procedural fluency			Conceptual understanding			Strategic competence			Adaptive reasoning		
	Easy	Medium	Difficult	Easy	Medium	Difficult	Easy	Medium	Difficult	Easy	Medium	Difficult
1	10	10	10	10	10	10	7	6	7	7	6	7
2	10	10	10	10	10	10	7	6	7	7	6	7
3	10	10	10	10	10	10	7	6	7	7	6	7
4	10	10	10	10	10	10	7	6	7	7	6	7
Total By Difficulty	40	40	40	40	40	40	28	24	28	28	24	28

Appendix A: State Content Standards Referent Sources

National Council of Teachers of Mathematics (NCTM) Curricular Focal Points

The National Council of Teachers of Mathematics (NCTM) Curricular Focal Points were retrieved from http://www.nctmmedia.org/cfp/front_matter.pdf on April 20, 2010. Additional information was also retrieved on April 20, 2010 from: www.nctm.org/focalpoints .

Florida

Florida's Next Generation Sunshine State Math Standards (adopted 2007) were retrieved on July 5, 2012 from http://www.floridastandards.org/Standards/FLStandardSearch.aspx.

California

California's Math Content Standards (adopted 1997) were retrieved on July 5, 2012 from http://www.cde.ca.gov/be/st/ss/documents/mathstandards.pdf. California Green Dot Standards are the selected standards that appear 85% of the time on California state tests.

Common Core Standards

The Common Core Standards in Mathematics were retrieved on July 5, 2012 from http://www.corestandards.org/the-standards/mathematics . These standards were published in 2010. They were developed as part of an initiative led by National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

New York

The New York State Standards (revised 2005) were retrieved on July 5, 2012 from http://www.p12.nysed.gov/ciai/mst/math/standards/core.html.

Texas

The Texas State Standards for Math (Version 2.1; revised 2010) were retrieved on July 5, 2012 from http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html. The Texas Education Agency (TEA) released a 2010 document entitled *Texas Response to Curriculum Focal Points: Kindergarten through Grade 8 Mathematics* that included coordinating TEKS.

Virginia

Virginia's Standards for Learning Document for Mathematics (adopted 2009 for full implementation in 2011-12) were retrieved on July 5, 2012 from http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml.

GRADE 7 MATHEMATICS CURRICULUM FOCAL POINTS

CFP 1: Number and Operations* and *Algebra* and *Geometry
Developing an understanding of and applying proportionality, including similarity.

***Probability Connection to the Focal Point* includes students understanding that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Students use theoretical probability and proportions to make approximate predictions.**

7.1A.1	Students extend their work with ratios to develop an understanding of **proportionality** that they apply to solve single and multistep problems in numerous contexts.
7.1B.1	Students use ratio and proportionality to solve a wide variety of **percent** problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease.
A7.CFP2.13	Calculate and compare **unit price** using proportions
7.1C.1	Students also solve problems about similar objects (including figures) by using **scale factors** that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects.
A7.CFP2.23	Construct and read **drawings and models made to scale**
7.1D.1	Students **graph proportional relationships** (linear functions) and identify the unit rate as the slope of the related line, noting that the vertical change (change in y- value) per unit of horizontal change (change in x- value) is always the same and know that the ratio ("rise over run") is called the slope of a graph
7.1E.1	Students **distinguish proportional relationships** ($y/x = k$, or $y = kx$) from other relationships, including inverse proportionality ($xy = k$, or $y = k/x$).

7.1F.1	Students now use division to express any **fraction as a decimal**, including fractions that they must represent with infinite decimals.
7.1H.1	Students connect their work with dividing fractions to **solving equations** of the form ax = b, where a and b are fractions.
7.1I.1	Students use proportions to **make estimates** relating to a population on the basis of a sample and evaluate the reasonableness
7.1J.1	Students **apply percentages** to make and interpret histograms and circle graphs
7.1K.1	Students **use theoretical probability and proportions** to make approximate predictions
A7.S.11	Interpret data to provide the basis for predictions and to establish experimental probabilities
A7.S.12	Determine the validity of sampling methods to predict outcomes
A7.S.13	Determine the outcome of an experiment and predict which events are likely and unlikely, and if the experiment is fair or unfair
A7.S.14	Design and conduct an experiment to test predictions
A7.S.15	Compare actual results to predicted results
A7.CFP3.26	Students select, justify, and use appropriate symbolic representations for given situations.
A7.CFP3.28	Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly
A7.CFP3.35	Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A)
A7.CFP1.3	Solve one-step inequalities (positive coefficients only) (See 7.G.10) and/or graph the solution set of an inequality (positive coefficients only) on a number line
A7.CFP1.5	Draw the graphic representation of a pattern from an equation or from a table of data and interpret the meaning of a specific part of a graph in the situation by the graph
A7.CFP1.6	Create algebraic patterns using charts/tables, graphs, equations, and expressions

A7.CFP1.8 Write an equation to represent a function from a table of values

A7.CFP2.31 Identify and plot ordered pairs in all four quadrants of the coordinate plane.

CFP 2: Measurement* and *Geometry* and *Algebra
Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes

7.2A.1 By decomposing two- and three-dimensional shapes into smaller, component shapes, students **find surface areas** and develop and justify formulas for the surface areas and volumes of prisms and cylinders.

7.2B.1 As students decompose prisms and cylinders by slicing them, they develop and **understand formulas for their volumes** (Volume = Area of base × Height)

7.2C.1 Students apply these formulas in problem solving to determine **volumes of prisms and cylinders**

7.2D.1 Students see that the **formula for the area of a circle** is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram.

A7.CFP1.4 Students **evaluate formulas** for given input values (surface area, rate, and density problems)

7.2E.1 Students select appropriate two- and three-dimensional shapes to model real-world situations and **solve a variety of problems** (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders, including estimating surfacte area and calculating the radius or diameter given the circumference or area of a circle.

7.2F.1 Students connect their work on proportionality with their work on area and volume by **investigating similar objects.**

7.2G.1 Students understand that if a **scale factor** describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related.

A7.CFP2.24 Relate the **changes in measurement with a change of scale to the units used** (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft^2] = [144 in^2], 1 cubic inch is approximately 16.38 cubic centimeters or [1 in^3] = [16.38 cm^3])

7.2H.1 Students apply their work on proportionality to **measurement in different contexts**, including converting among different units of measurement to solve problems involving rates such as motion at a constant speed.

7.2I.1 Students also **apply proportionality** when they work with the circumference, radius, and diameter of a circle; when they find the area of a sector of a circle; and when they make scale drawings.

CFP 3: Numbers and Operations **and** ***Algebra***
Developing an understanding of operations on all rational numbers and solving linear equations

Data Analysis Connection to the Focal Point **includes students using proportions to make estimates relating to a population on the basis of a sample. They apply percentages to make and interpret histograms and circle graphs.**

7.3A.1 Students extend understandings of **addition, subtraction, multiplication, and division**, together with their properties, to all rational numbers, including negative integers (with and without the use of a number line)

A7.CFP3.27 Simplify numerical expressions by applying **properties of rational numbers** (e.g., identity, inverse, distributive, associative, commutative) and justify the process used

A7.CFP3.36 Students extend their **application of rational numbers** to include converting measurement units, describing probability, and describing arithmetic sequences

7.3B.1 By applying properties of arithmetic and considering **negative numbers** in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense.

7.3C.1 Students use the arithmetic of rational numbers as they formulate and **solve linear equations** in one variable and use these equations to solve problems.

7.3D.1 Students make strategic choices of procedures to **solve linear equations** in one variable and implement them efficiently, understanding that when they **use the properties of equality** to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.

A7.CFP3.37 Students use **models** to formulate and solve linear equations in one variable.

A7.CFP3.1	Distinguish between the various subsets of real numbers (counting/natural numbers, whole numbers, integers, rational numbers, and irrational numbers)
A7.CFP3.25	Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers
A7.CFP3.2	Recognize the difference between rational and irrational numbers
A7.CFP3.3	Place rational and irrational numbers (approximations) on a number line and justify the placement of the number
A7.CFP3.17	Classify irrational numbers as non-repeating/non-terminating decimals
A7.CFP3.8	Find the common factors and greatest common factor of two or more numbers
A7.CFP3.9	Determine multiples and least common multiple of two or more numbers
A7.CFP3.10	Determine the prime factorization of a given number and write in exponential form
A7.CFP3.22	Add and subtract fractions by using factoring to find common denominators
A7.CFP3.4	Develop the laws of exponents for multiplication and division (e.g., Multiply and divide expressions involving exponents with a common base, including negative whole-number exponents)
A7.CFP3.30	Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse; Simplify and evaluate expressions that include exponents
A7.CFP3.23	Multiply, divide, and simplify rational numbers by using exponent rules
A7.CFP3.5	Read and write numbers in scientific notation (both positive and negative powes of 10)
A7.CFP3.6	Translate numbers from scientific notation into standard form
A7.CFP3.7	Compare numbers written in scientific notation
A7.CFP3.11	Simplify expressions using order of operations (Note: Expressions may include absolute value and/or integral exponents greater than 0.)

A7.CFP3.14	Develop a conceptual understanding of negative and zero exponents with a base of ten and relate to fractions and decimals (e.g., 10^{-2} = .01 = 1/100)
A7.CFP3.24	Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root less than 225 lies (with and without the use of a number line) and explain why
A7.CFP3.15	Recognize and state the value of the square root of a perfect square (up to 225)
A7.CFP3.16	Determine the square root of non-perfect squares using a calculator
A7.CFP1.1	Add and subtract monomials with exponents of one
A7.CFP3.31	Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent
A7.CFP1.2	Identify a polynomial as an algebraic expression containing one or more terms
A7.S.4	Identify and collect data using a variety of methods
A7.S.6	Convert raw data into double bar graphs and double line graphs
A7.S.7	Calculate the range for a given set of data
A7.S.8	Select the appropriate measure of central tendency
A7.S.9	Read and interpret data represented graphically (pictograph, bar graph, histogram, line graph, double line/bar graphs or circle graph)
A7.S.10	Identify and explain misleading statistics and graphs
A7CFP2.18	Identify the relationships between relative error and magnitude when dealing with large numbers (e.g., money, population)
A7.S.16	Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data

A7.S.17 Represent two numerical variables on a scatter plot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level)

Measurement* and *Geometry Standards and their Connections to Focal Points

Students convert measurement units between different measurement systems. They connect what they know about two-dimensional figures and apply that knowledge to three-dimensional figures. They further their understanding of right triangles and use the Pythagorean theorem to find the lengths of missing sides.

Measurement

A7.CFP2.32 Compare, contrast, and convert units of measure between different measurement systems (US customary or metric (SI)), dimensions, and derived units to solve problems.

A7.CFP2.10 Convert capacities and volumes within a given system

A7.CFP2.11 Identify customary and metric units of mass and convert mass within a given system

A7.CFP2.20 Determine personal references for customary /metric units of mass

A7.CFP2.17 Determine the tool and technique to measure with an appropriate level of precision: mass

A7.CFP2.21 Justify the reasonableness of the mass of an object

A7.CFP2.15 Convert money between different currencies with the use of an exchange rate table and a calculator

A7.CFP2.22 Compare and calculate weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, distance using a map scale, cubic inches to cubic centimeters)

A7.CFP2.34 Solve multi step problems involving rate, average speed, distance, and time or a direct variation

A7.CFP2.19 Justify the reasonableness of answers using estimation

Geometry

A7.CFP2.2 Identify the two-dimensional shapes that make up the faces and bases of three-dimensional shapes (prisms, cylinders, cones, and pyramids)

A7.CFP2.3	Identify the right angle, hypotenuse, and legs of a right triangle
A7.CFP2.4	Explore the relationship between the lengths of the three sides of a right triangle to develop the Pythagorean Theorem
A7.CFP2.7	Determine whether a given triangle is a right triangle by applying the Pythagorean Theorem and using a calculator (i.e., use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement)
A7.CFP2.5	Find a missing angle when given angles of a quadrilateral
A7.CFP1.7	Build a pattern to develop a rule for determining the sum of the interior angles of polygons
A7.CFP2.16	Draw central angles in a given circle using a protractor (circle graphs)
A7.CFP2.25	Identify and construct basic elements of geometric figures (e.g., altitudes, mid-points, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge
A7.CFP2.27	Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures
A7.CFP2.28	Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones
A7.CFP2.29	Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect)
A7.CFP2.32	Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths)
A7.CFP2.26	Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections
A7.CFP2.30	Predict the results of transformations, and draw transformed figures with and without the coordinate plane.