THEMIS SIGNAL ANALYSIS STATISTICS RESEARCH PROGRAM

IDENTIFICATION OF SITUATIONS WHERE COOPERATION IS PREFERABLE TO USE OF MEDIAN GAME THEORY

by

John E. Walsh

Technical Report No. 54
Department of Statistics THEMIS Contract

January 13, 1970

Research sponsored by the Office of Naval Research
Contract N00014-68-A-0515
Project NR 042-260

DEPARTMENT OF STATISTICS
Southern Methodist University

IDENTIFICATION OF SITUATIONS WHERE COOPERATION IS PREFERABLE TO USE OF MEDIAN GAME THEORY

John E. Walsh

Southern Methodist University *

ABSTRACT

Discrete two-person game theory based on median (rather than expected-value) considerations is widely applicable when the players choose their strategies separately and independently. When cooperation can occur, however, its use in choice of strategies can have advantages compared to the median approach. This holds even for some median competitive games (where both players can be simultaneously protective and vindictive in the median sense). The purpose of this paper is to identify situations where cooperation is definitely preferable, for two types of cooperation. One type is that where no side payments are made. This type of cooperation can occur for any situation where median game theory is applicable. Side payments can be made for the other type of cooperation that is considered. This type can occur for situations where all payoffs can be expressed in a common unit (and satisfy the arithmetical operations). A rule is given for deciding when cooperation is advantageous, for both types of cooperation. Some implications of this rule are discussed.

* Research partially supported by Mobil Research and Development Corporation. Also associated with ONR contract N00014-68-A-0515 and NASA Grant NGR 44-007-028.

INTRODUCTION

A form of discrete two-person game theory based on median considerations has been developed (see reference 1) for situations where the players choose their strategies separately and independently. Median game theory is found to have strong application advantages over expected-value game theory for these situations. In particular, the class of median competitive games, (see reference 2), where both players can be simultaneously protective and vindictive, is huge compared to (and includes) the corresponding class for expected-value game theory. Moreover, the median approach is applicable to games where the values in one or both payoff matrices do not satisfy the arithmetical operations (but can be ranked within each matrix). A generally applicable "optimum" solution for two-person median game theory is given in reference 3.

The usefulness of the median approach changes somewhat when the players can cooperate in their strategy choice. That is, they may be able to guarantee a game outcome (a pair of payoffs, one to each player) that is preferable, for both, to use of solutions that are median optimum for the case of no cooperation. The purpose of this paper is to identify cases where cooperation is definitely preferable to use of the median approach.

The results are based on an extension of the "relative desirability" function concept for ranking game outcomes that is introduced in reference 3. Each player chooses a procedure for deciding on his preference between any two "outcomes." For the first type of preference procedure, only the possible outcomes for the game can be considered. The payoffs are expressed in a common unit, and satisfy the arithmetical operations, for the second type of procedure. Then, all possible "outcomes" can be considered which are such that the sum of the two payoffs equals a sum that occurs for an outcome of the game.

More specifically, let (p_I, p_{II}) denote a general "outcome," where p_I is the overall amount received by player I and p_{II} is the amount received by player II. The values of p_I and p_{II} can be influenced by a payment made from one player to the other. Also, let $(p_I^{(g)}, p_{II}^{(g)})$ denote an outcome that can occur for the game. Only the $(p_I^{(g)}, p_{II}^{(g)})$ can be compared using the first type of procedure for establishing relative desirability. All (p_I, p_{II}) such that $p_I + p_{II}$ equals $p_I^{(g)} + p_{II}^{(g)}$, for some game outcome, can be compared for the second type of procedure. Of course, the $(p_I^{(g)}, p_{II}^{(g)})$ are included in the totality of the (p_I, p_{II}).

For player I (II), relative desirability is a nonincreasing function of p_I (p_{II}) for a fixed value of p_{II} (p_I). This restriction seems resonable and helps assure that the procedures result in preference rankings of the $(p_I^{(g)}, p_{II}^{(g)})$ and of the (p_I, p_{II}). A condition on both types of preference procedures is that a preference ranking is obtained for the outcomes that can be considered (no circularity in definite preferences). Otherwise, the preference procedure used by a player can be of any nature. In particular, the requirement that the outcomes of set I (set II) have maximum preference to player I (II), a condition of the "relative desirability" function of reference 3, need not hold. Actually, the procedure for one player can be arbitrarily different from that used by the other player. For deciding on the advantage of cooperation, however, the preference rankings for both players are assumed to be known.

Two types of cooperation are considered. No payments from one player to the other are made for the first type, which is associated with the first type of preference procedure. Since the amounts received are not influenced by side payments, only the $(p_I^{(g)}, p_{II}^{(g)})$ are considered. Cooperation of the first type can occur for any situation where the median approach is usable. That is, the only requirement on the kind of payoffs is that, separately, they can be ranked in the payoff matrix for each player.

The second type of cooperation can involve side payments but imposes a condition on the kind of payoffs that can occur. That is, the payoffs are expressible in a common unit and satisfy the arithmetical operations. Occurrence of a side payment in combination with specified $(p_I^{(g)}, p_{II}^{(g)})$ results in amounts that still sum to $p_I^{(g)} + p_{II}^{(g)}$. Thus, all the (p_I, p_{II}) with $p_I + p_{II} = p_I^{(g)} + p_{II}^{(g)}$, for some game outcome, can be considered for the second type of cooperation.

Cooperation is beleived to have a definite advantage, compared to the median solution, when both players can gain through agreement in the selection of an outcome. A rule for deciding when cooperation is advantageous, and some discussion, is given in the next section. Also, some implications of this rule for competitive games (reference 1), for the special case of generated median competitive games given in reference 1, and for the general median competitive games of reference 2 are examined in the final section.

THE RULE AND SOME DISCUSSION

Let the game outcomes $(p_I^{(g)}, p_{II}^{(g)})$ be ranked according to increasing preference separately by each player. There is a smallest subset S_I (S_{II}), consisting of most preferable game outcomes to player I, (II), such that player I (II) can assure an outcome of S_I (S_{II}) with probability at least 1/2. These sets provide the basis for the rule used to decide when cooperation is preferable to this median approach. Incidentally, this always usable optimum solution for median games is an extension of the solution given in reference 3, and is itself a subject for research. The extension allows a less restricted form of "relative desirability" function.

General Rule: Cooperation is definitely advantageous to both players when an achievable outcome exists that is at least as desirable to player I as one or more outcomes in S_I and also is at least as desirable to player II as one or more outcomes in S_{II}.

This rule follows from the consideration that an outcome with at least the minimum desirability for both S_I and S_{II} can be guaranteed by cooperation. The median approach only assures, with probability at least 1/2, that player I obtains an outcome with desirability at least equal to the minimum for S_I and player II gets an outcome with desirability at least the minimum for S_{II}.

Of course, only the $(p_I^{(g)}, p_{II}^{(g)})$ are achievable for the first type of cooperation, and all the (p_I, p_{II}) that satisfy $p_I + p_{II} = p_I^{(g)} + p_{II}^{(g)}$, for some game outcome, are achievable for the second type of cooperation.

The general rule can be simplified. For the first type, cooperation is definitely advantageous when the same game outcome value (for example, the same outcome) occurs in both S_I and S_{II}. This could happen, for example, when an outcome occurs with $p_I^{(g)}$ large compared to nearly all of the payoffs to player I, with $p_{II}^{(g)}$ large compared to nearly all the payoffs to player II, and where an increase in payoff to a player is noticeably more desirable than a comparable decrease in the payoff to the other player.

For the second type of cooperation, a simplification occurs when there is a game outcome value that occurs in both S_I and S_{II}. A simplification also occurs for the rather common situation where an equal increase in the amount received by each player is preferable (including equal preference) to both players. Then, cooperation is always advantageous if there is an achievable (p_I, p_{II}), not necessarily a game outcome, such that p_I is at least as large as the smallest $p_I^{(g)}$ in S_I and also p_{II} is at least as large as the smallest $p_{II}^{(g)}$ in S_{II}.

To verify the above statement, consider an outcome (p_I', p_{II}') in S_I that has smallest $p_I^{(g)}$ and an outcome (p_I'', p_{II}'') in S_{II} with smallest $p_{II}^{(g)}$. A game outcome occurs such that the sum of its payoffs is at least equal to $p_I' + p_{II}''$. This implies that, perhaps by a side payment, an achievable (p_I, p_{II}) occurs of the form

$$p_I = p_I' + 2a, \quad p_{II} = p_{II}'' + 2b,$$

with a and b nonnegative. In addition to the side payment to obtain (p_I, p_{II}), suppose that player I makes a side payment of a-b to player II. Then, choice of a game outcome whose payoffs sum to $p_I + p_{II}$, and use of the stated side payments, gives

$$(p_I - a+b,\ p_{II} + a-b) = (p_I' + a+b,\ p_{II}' + a-b)$$

which is preferable to (p_I', p_{II}') for player I, since p_I' was increased at least as much as p_{II}'. Similarly,

$$(p_I - a+b,\ p_{II} + a-b) = (p_I'' - a+b,\ p_{II}'' + a+b),$$

which is preferable to (p_I'', p_{II}'') for player II.

COMPETITIVE, MEDIAN COMPETITIVE CASES

For a competitive game, the totality of game outcomes can be arranged so that the payoffs to player I are nondecreasing and also the payoffs to player II are nonincreasing. Then, S_I and S_{II} are mutually exclusive, since the arrangement is according to nondecreasing preference for player I and nonincreasing preference for player II (due to the restrictions on preference procedures). However, the same outcome value could possibly occur in both S_I and S_{II}. This can happen only when the least preferable outcome of S_I has the same value as the least preferable outcome of S_{II}. Moreover, no combination of side payment and cooperative choice would necessarily be beneficial to both players when compared to use of the median approach. Thus, cooperation (of either type) is seldom useful for competitive games.

The situation is different for median competitive games that are not competitive. First, consider the kind of median competitive game that is generated by a competitive game (reference 1). In S_I, also in S_{II}, for the competitive game, consider the payoffs to player I and the payoffs to player II. Within each of S_I and S_{II}, let the outcomes for another game (the actual game) be obtained by combining payoffs for the two players. In this way, the game outcomes are divided into the set generated by S_I, the set generated by S_{II}, and the remainder (if any). The situation is like that for competitive games if the preference procedures for the players yield for the new S_I the outcomes generated by S_I for the competitive game, and for the new S_{II} the outcomes generated by S_{II} for the competitive game. Then, the new S_I and S_{II} are mutually exclusive. Also, the payoffs to player I (II), in S_I (S_{II}) are at least as large as those to player I (II) in S_{II} (S_I). However, many preference procedures do not result in this situation. Some procedures that do yield this situation are those that, for player I (II), assign maximum preference to any outcome with $p_I^{(g)} \geq P_I$ and $p_{II}^{(g)} \leq P_{II}'$ ($p_I^{(g)} \leq P_I'$ and $p_{II}^{(g)} \geq P_{II}$). The payoffs P_I, P_{II}, P_I', P_{II}' are defined in reference 1 (also see reference 2). These are some of the properties that are required for the "relative desirability" function of reference 3.

Finally, consider general median competitive games (reference 2). Cooperation is often advantageous for these games. An exception is when some form of the "relative desirability" function of reference 3 is used by each player.

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UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
SOUTHERN METHODIST UNIVERSITY	UNCLASSIFIED
	2b. GROUP: UNCLASSIFIED

3. REPORT TITLE
Identification of Situations Where Cooperation is Preferable to Use of Median Game Theory

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Technical Report

5. AUTHOR(S) (First name, middle initial, last name)
John E. Walsh

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
January 13, 1970	9	3

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
N00014-68-A-0515	54
b. PROJECT NO. NR 042-260	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Office of Naval Research

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DD FORM 1 NOV 65 1473 (PAGE 1)
S/N 0101-807-6811

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Security Classification

A-31408