

THEMIS SIGNAL ANALYSIS STATISTICS RESEARCH PROGRAM

GENERALLY APPLICABLE SOLUTIONS FOR
TWO-PERSON MEDIAN GAME THEORY

by

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INTRODUCTION AND DISCUSSION

The case of two players with finite numbers of strategies is considered. Each player selects his strategy separately and independently of the strategy choice by the other player. Median game theory and its application advantages over expected-value game theory are discussed in ref. 1 (also see ref. 2).

The class of median competitive games, and of one player median

competitive (OPMC) games, are identified in ref. 1. These identifications are stated in terms of the pairs of payoffs that correspond to the strategy combinations for the players (called I and II). A game is OPMC for player I (II) if and only if he can assure, with probability at least $1/2$, that a pair in set I (set II) occurs. Set I (set II) consists of the pairs such that both the payoff to player I (II) is at least $P_I(P_{II})$ and the payoff to player II (I) is at most $P_I(P_{II})$. Here $P_I(P_{II})$ is determined as the largest value that player I (II), acting as protectively as possible, can assure himself with probability at least $1/2$. Also, a smallest value $P_I(P_{II})$ occurs in the payoff matrix for player I (II) such that vindictive player II (I) can assure, with probability at least $1/2$, that player I (II) receives at most this amount. A way of evaluating

P_I, P_{II}, P_I, P_{II} , and of deciding whether a game is OPMC for a player, is given in ref. 1. A game is median competitive if and only if it is OPMC for both players.

When a game is OPMC for a player, the median optimum solution given

in ref. 1 would seem satisfactory for that player. However, the over-

whelmingly large class of games is that where an OPMC situation does

not occur for at least one player.

Since the game is not OPMC for the player considered, say player I, a pair of set I cannot be assured with probability at least $1/2$. However, pairs not in set I can be ordered according to their desirability to player I.

Consider the payoff matrix for player I. The rows of this matrix correspond to the strategies for player I while the columns are the strategies for the other player. First mark the payoffs in this matrix that occur for the pairs of set I. Then, according to decreasing desirability, mark the payoffs for the other pairs, where all pairs of equal desirability are marked at the same time. Continue until the first time that marks in all columns can be obtained from two or fewer of the rows. Now, remove the marks for the least desirable pair(s) of those that received marks. Then, by the following method, determine whether some one of the remaining marked pairs can be assured with probability at least $1/2$. The method is to replace every marked value in the matrix by unity and all others by zero. The resulting matrix of ones and zeroes is considered to be for a zero-sum game with an expected-value basis and is solved for the value of the game to player I. Some one of the remaining marked pairs can be obtained with probability at least $1/2$ if and only if this game value is at least $1/2$.

Suppose that player I cannot assure a remaining marked pair with probability at least $1/2$. Then the maximum level of desirability that can be assured with probability at least $1/2$ is the level corresponding to a pair that just had its markings removed. Otherwise, remove the

RESULTS

REFERENCES

1. Walsh, John E., "Median two-person game theory for median competitive games." Report 45, Thesis, Statistics Department, Southern Methodist University, Dallas, Texas, U.S.A., 1969, 13 pp. Submitted to Journal of the Operations Research Society of Japan.
2. Walsh, John E., "Discrete two-person game theory with median payoff criterion," Opsearch, Vol. 6 (1969), pp. 83-97.