THEMIS SIGNAL ANALYSIS STATISTICS RESEARCH PROGRAM

SUMMARY OF RECENT WORK ON VARIABLES ACCEPTANCE SAMPLING WITH EMPHASIS ON NON-NORMALITY

by

D. B. Owen

Technical Report No. 1
Department of Statistics THEMIS Contract

September 2, 1968

Research sponsored by the Office of Naval Research
Contract N00014-68-A-0515
Project NR 042-260

DEPARTMENT OF STATISTICS
Southern Methodist University

SUMMARY OF RECENT WORK ON VARIABLES ACCEPTANCE SAMPLING

WITH EMPHASIS ON NON-NORMALITY

By

D. B. Owen
Southern Methodist University

1. Non-normal Distributions (Single Specification Limit).

Acceptance sampling based on variables instead of attributes is not widely used except possibly in the life testing area. There are many reasons for this, but the primary reason appears to be the uncertainty that the variate being measured is normally distributed, or the certainty that it is not. When a single specification limit is required, the acceptance criterion is usually based on $\bar{x} + ks$ or on $\bar{x} - ks$, where $\bar{x}$ is the sample mean and s is the sample standard deviation and k is a constant chosen to meet the reliability requirements. Even when the original variable, X, is not normal, $\bar{x}$ usually tends rapidly to normality as sample size, n, increases. Similarly $(n - 1)s^2/\sigma^2$, (where σ^2 is the population variance) tends to the chi-square distribution with increasing n for many underlying distributions for X and, in fact, $\bar{x} + ks$ tends to normality with increasing sample size, including the case where X is normal. Whether or not X is normally distributed is immaterial as long as $\bar{x}$ is normal and as long as $(n - 1)s^2/\sigma^2$ has a chi-square distribution with (n - 1) degrees of freedom. The probability statements are based on these latter assumptions, not on the normality of X.

However, there is one place in the parameterization of sampling plans where the normality assumption can be very critical. The proportion defective, p, which is usually plotted on the horizontal axis of a graph of the OC curve is obtained as follows (for an upper specification limit, U).

$$p = \Pr\{X \geq U\} = \Pr\{(X - \mu)/\sigma \geq (U - \mu)/\sigma\}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{(U - \mu)/\sigma}^{\infty} e^{-t^2/2} dt ,$$

where μ is the expected value of X. Various values of p are referred to as the AQL, LTPD, AOQL, etc. Hence, deviations which affect these special cases also, and the various sampling plans are almost without exception indexed by one or more of these quantities. In short, variables sampling plans based on the normal control the standardized deviate $(U - \mu)/\sigma$, and this is done fairly well even for small sample sizes, but the translation of this deviate into a proportion defective is very sensitive to non-normality.

Suppose we attempt to make the situation clearer by looking at a particular example. On page 37 of MIL-STD-414 an example is worked where $U = 209$, $s = 8.81$, $\bar{x} = 195$ and AQL = 1% are specified. If normality is assumed, then the deviate

$$(U - \mu)/\sigma = 2.326$$

to give the 1% AQL. But if the distribution is not normal then almost any value could be required for $(U - \mu)/\sigma$ to correspond to 1% of the population above $(U - \mu)/\sigma$. For example, suppose the population had a modified Student t-distribution with 10 degrees of freedom, modified so that the distribution had an arbitrary mean (unknown) and arbitrary variance (also unknown). All we need to know is the probability of exceeding a standardized deviate $(U - \mu)/\sigma$. This is done by calculating $(U - \mu)/\sigma = 2.7638/\sqrt{1.25} = 2.472$, (2.7638 is read from a table of Student t-distribution and the variance of the tabulated value is 10/8 = 1.25). Hence, if we ran our test based on 0.67% (reading 2.472 from a table of the normal distribution) for the AQL instead of 1% we would have a test which is very close to being correct. The only problem is that in most instances we do not know that we have the modified Student t-distribution with 10 degrees of freedom, or alternately, the form of the distribution, whatever it may be. Hence, this procedure has limited use in practical situations.

However, let us pursue it just a little further, for the occasional instance where the form of the non-normal distributions is known, but tables for sampling plans based on these distributions are not available. Assume we have a chi-square distribution with 10 degrees of freedom, again modified so the distribution has arbitrary mean and variance (both unknown). The upper 1% value corresponds to $(U - \mu)/\sigma = (23.209 - 10)/\sqrt{20} = 2.954$ and hence an AQL of 0.16% instead should be read in the tables in applying the normal sampling plans. [The mean of a chi-square distribution equals the degrees of freedom and the variance is twice the degrees of freedom.] The procedure is exactly the same if an LTPD or an AOQL is specified instead of the AQL. That is, the entire OC curve can be plotted this way.

If the distribution is not known exactly, but it is known to have a monotone hazard rate then some tables given by Barlow and Marshall (1965) may be used to give bounds on the proportion in the tail. For example, if the distribution has an increasing hazard rate (IHR) with $\mu_1 = 1$, $\mu_2' = 1.25$ so that $\sigma^2 = 1.25 - 1$, and $\mu = 1$,

then reading from Barlow and Marshall's table IV, page 880, we find $t = 2.90$ or $(U - \mu)/\sigma = (2.9-1)/0.5 = 3.8$ corresponds to an upper bound of 1% in the upper tail of an increasing hazard rate distribution. In this case, then, the corresponding normal sampling plan is one with only 0.007% defectiveness allowed in the upper tail of the distribution.

There are two problems with making practical use of this. First, in order to get this bound we had to assume μ and σ^2 were known, although we only needed to know them within a scaling factor, but if we knew μ and σ^2 that well, there would be better acceptance sampling tests to run than those based on the usual normal procedures. We would probably be interested in characteristics not measured by $\bar{x}$ and s, but measured by some higher moments. Second, tables of normal acceptance plans ordinarily do not have entries corresponding to percentage defective as small as 0.007%. The only use of this approach might be to give the user a rough estimate of a restricted "worst" case.

Hanson and Koopmans (1964) give a procedure for establishing one sided tolerance limits for the class of distributions with increasing hazard rates. The procedure given there is easily extended to the sampling situation where, say, an upper specification limit, U, is given. Suppose, for example, that we have an IHR distribution and a sample of size n is taken. The largest value in the sample is $X_{(n)}$ and the next to the largest is $X_{(n-1)}$. Then Hanson and Koopmans show that

$$\Pr\{X_{(n-1)} + b\left(X_{(n)} - X_{(n-1)}\right) \leq U\} \leq \gamma,$$

or equivalently,

$$\Pr\{F[X_{(n-1)} + b\left(X_{(n)} - X_{(n-1)}\right)] \geq 1-p\} = \gamma.$$

Hence, to use their procedure as an acceptance sampling plan, we would usually take $\gamma = 0.90$, let p = maximum proportion defectiveness we would allow and read b from their table. Then we accept the lot if $X_{(n-1)} + b\left(X_{(n)} - X_{(n-1)}\right) \leq U$ and reject if otherwise. For example, we can be at least 90% sure that there is no more than 5% of an IHR population above U if in a sample of size 20: $X_{(n-1)} + 5.077\left(X_{(n)} - X_{(n-1)}\right) \leq U$. The value 5.077 is read from the Hanson-Koopmans table. Note that slightly different order statistics are sometimes used and these are indicated by asterisks in the table.

Undoubtedly more could be done with this approach. We leave it here with the remark that Barlow and Gupta (1966) have used the ideas of hazard rates to construct some distribution-free life test sampling plans. Sobel and Tischendorf (1959), Gupta and Groll (1961), Gupta (1962), and Qureishi, Nabavian and Alanen (1965), all give life test sampling plans based on non-normal as well as normal distributions. Life testing is, of course, a different objective than the one considered here of controlling the proportion defective. The effect of non-normality has been considered by Srivastava (1961) and Das and Mitra (1964) by examining Edgeworth and Gram Charlier series, respectively. Singh (1966) considers non-normality in a closely related problem.

In summary, if you have a non-normal distribution with unknown mean and/or variance and you are asked to prepare an acceptance sampling plan (for controlling the proportion defective), we recommend that you look into the possibility of using one of the normal plans with an adjustment (as outlined above) made in the percentage defective controlled. This adjustment depends on your being able to find the probability that the quantity being measured is greater than $(U - \mu)/\sigma$. For some distributions the parameters enter in such a way that this computation cannot be made. This discussion has been limited to one-sided plans, that is, where only one tail of the distribution is to be controlled. It would be best, of course, to use sampling plans tailor-made to the particular distribution you have, but these are seldom available.

2. Control of the Sum of the two Tails of the Distribution (Double Specification Limits).

In Owen (1967) it is pointed out that the user may want to control both tails of a distribution simultaneously and separately or he may just want to control the sum of the two tails. The control of the sum of the two tails allows the situation, for instance, where all of the defectiveness is in one tail, shifts gradually over to an even split in each tail and then ends up with all the defectiveness in the other tail. When the sum of the two tails is controlled, the OC becomes a band with the sum of the two proportions in the tails usually plotted on the horizontal and the vertical axis is used to plot the probability of acceptance. If the sample size exceeds 1.2n, where n is read from table I of Owen (1967), the top of the band corresponds to an even split in the two tail proportions and the bottom corresponds to all defectiveness in one tail.

For double specification limits the effect of non-normality on the sum of the proportions in the tails of the distribution may not be as severe as it was for the single specification limit discussed above. Walsh (1956) and (1958) has shown that for many "practical type" distributions the sum of the proportions in the tails is about the same as for the normal, provided that the limits set up are symmetrical about the mean. In the application here, this means non-normality generally should not be a problem when controlling the sum of two tails if the sample size exceeds 1.2n in table I of Owen (1967). Walsh (1958) makes a point which, when translated to the situation at hand, suggests that if the sum of the proportions in the tails is .0833, then the effects of non-normality are further reduced. In other words the OC curve is not too adversely affected as long as the main interest centers on that part of the band corresponding to an equal split in the two tails. If the split is not equal then problems such as those discussed in Section 1 can take over.

3. Control of Each Tail at Separate Levels (Double Specification Limits).

There are many instances where there are different criteria for criticality for the two tails of a distribution. For example, if a battery's voltage is being measured it may be important that the voltage exceed a lower specification limit, L, in order for a circuit to operate, but there may be parallel circuits to build up the reliability. On the other hand, if an upper specification limit, U, is exceeded then all circuits are burned out. Hence, in this case, one might want to specify that no more than 10% of the batteries be below L, and simultaneously no more than 1% of the batteries exceed U, with probability 0.90. This is just the place to use the procedure for the control of each tail of the distribution separately. The procedure, then, is to accept the lot if both $\bar{x} - 2.066s \geq L$ and $\bar{x} + 3.532s \leq U$ based on a sample of size 10 and to reject the lot if otherwise. The constants, 2.066 and 3.532, are read from Table III of Owen (1967). The OC curve in this case is a three dimensional surface. The proportion in each tail is plotted on two of the axes and the third axis corresponds to the probability of acceptance.

In the case considered in this section the effects of non-normality may be inferred from the OC curves of the two preceding sections. That is, each of the two planes corresponding to zero proportions in the tails are just the same as for the one-sided situation and the plane corresponding to an equal split in the two proportions may be hardly affected at all. The rest of the surface is a mixture of these two effects.

If, instead of proportions defective, one plotted standardized deviates $(U - \mu)/\sigma$ and $(L - \mu)/\sigma$ (where L is a lower specification limit) then for most practical situations the effect of non-normality would not be great, but this is usually not what is really wanted. There may be some instances, however, where standardized deviates can be used as effectively as proportions defective.

4. Other Developments.

Kao (1966) considers mixed attributes, variables sampling plans as have Gregory and Resnikoff (1955). Continuous sampling plans are prepared by White (1966) and Hillier (1964). Dodge and Stephens (1965) study chain sampling inspection. Hald (1968) designs attribute sampling plans for continuous prior distributions. Zeigler and Tietjen (1968) examine double sampling plans based on the variance. Strange (1960) and (1966), Freeman and Weiss (1964), Flehinger and Miller (1964), Campling (1968) and Lieberman (1965) consider other aspects of acceptance sampling inspection.

Folks, Pierce and Stewart (1965), among other things, give an estimator of the proportion, p , mentioned earlier in this paper. Wheeler (1968) shows that this estimator is equivalent to the one given by Bowker and Goode (1952) and by Lieberman and Resnikoff (1955), and gives an additional form of the estimator and discusses its variance. Ellison (1964) gives another derivation of the Bowker and Goode (1952) estimator.

Theodorescu and Vaduva (1967) give a procedure for the control of several variables simultaneously based on the generalized range.

In summary, there is activity on theoretical problems of variables acceptance sampling, but it is not as great as one might expect when one considers the potential usefulness of the techniques which can be developed.

REFERENCES

1. Barlow, R. E. and S. S. Gupta, 1966. "Distribution-free life test sampling plans," Technometrics, 8, 591-613.

2. Barlow, R. E. and A. W. Marshall, 1965. "Tables of bounds for distributions with monotone hazard rate," Journal of the American Statistical Association, 60, 872-890.

3. Bowker, A. H. and H. P. Goode, 1952. Sampling Inspection by Variables. McGraw-Hill, New York.

4. Campling, G. E. G., 1968. "Serial sampling acceptance schemes for large batches of items where the mean quality has a normal prior distribution," Biometrika, 55, 393-399.

5. Das, N. G. and S. K. Mitra, 1964. "The effect of non-normality on sampling inspection," Sankhyā, 26A, 169-176.

6. Dodge, H. F. and K. S. Stephens, 1966. "Some new chain sampling inspection plans," Industrial Quality Control, 23, 61-67.

7. Ellison, B. E., 1964. "Two theorems for inference about the normal distribution with applications in acceptance sampling," Journal of the American Statistical Association, 59, 89-95.

8. Flehinger, Betty J. and J. Miller, 1964. "Incentive contracts and price differential acceptance tests," Journal of the American Statistical Association, 59, 149-159.

9. Folks, J. L., D. A. Pierce, and C. Stewart, 1965. "Estimating the fraction of acceptable product," Technometrics, 7, 43-50.

10. Freeman, D. and L. Weiss, 1964. "Sampling plans which approximately minimize the maximum expected sample size," Journal of the American Statistical Association, 59, 67-88.

11. Gerhardt, K. 1966. "The application of plan games for the elaboration of standards for sampling inspection," Standordisierung, Teil 1, 12, 381-389.

12. Gregory, G. and G. J. Resnikoff, 1955. "Some notes on mixed variables and attributes sampling plans," Stanford University Applied Mathematics and Statistics Laboratory Technical Note, #10.

13. Gupta, S. S., 1962. "Life test sampling plans for the normal and lognormal distributions," Technometrics, 4, 151-175.

14. Gupta, S. S., and P. A. Groll, 1961. "Gamma distribution in acceptance sampling based on life tests," Journal of the American Statistical Association, 56, 942-970.

15. Hald, A., 1968. "Bayesian single sampling attribute plans for continuous prior distributions," Technometrics, 10, 667-683.

16. Hanson, D. L. and L. H. Koopmans, 1964. "Tolerance limits for the class of distributions with increasing hazard rates," Annals of Mathematical Statistics, 35, 1561-1570.

17. Hillier, F. S., 1964. "Continuous sampling plans under destructive testing," Journal of the American Statistical Association, 59, 376-401.

18. Kao, J. H. K., 1966. "Single-sample attri-vari plans for item-variability in percent defective," Annual Tech. Conf. Trans., American Society for Quality Control, 743-758.

19. Lieberman, Gerald J., 1965. "Statistical process control and the impact of automatic process control," Technometrics, 7, 283-292.

20. Lieberman, Gerald J. and George J. Resnikoff, 1955. "Sampling plans for inspection by variables," Journal of the American Statistical Association, 50, 457-516.

21. Office of the Assistant Secretary of Defense (Supply and Logistics), 1957. "Sampling procedures and tables for inspection by variables for percent defective," MIL-STD-414.

22. Owen, D. B., 1967. "Variables sampling plans based on the normal distribution," Technometrics, 9, 417-423.

23. Qureishi, A. A., K. J. Nabavian, and J. D. Alanen, 1965. "Sampling inspection plans for discriminating between two Weibull processes," Technometrics, 7, 589-601.

24. Singh, H. R., 1966. "Producer and consumer risks in non-normal populations," Technometrics, 8, 335-343.

25. Sobel, M. and J. A. Tischendorf, 1959. Acceptance sampling with new life test objectives. Proc. of fifth national symposium on reliability and quality control, Philadelphia.

26. Srivastava, A. B. L., 1961. "Variables sampling inspection for non-normal samples," J. Sci. Engrg. Res., 5, 145-152.

27. Stange, K., 1966. "On graphical determination of sequential designs for sampling by variables with known variance of production," Biom. Zeit., 8, 55-74.

28. Stange, K. 1960. "Variables inspection design," Qualitatskontrolle, 5, 2-7, 29-34, 75-80 and 155-161.

29. Summers, R. D., 1965. "An inequality for the sample coefficient of variation and an application of variables sampling," Technometrics, 7, 67-68.

30. Theodorescu, R. and I. Vaduva, 1967. "Statistical quality control for several simultaneous characteristics, the method of the range, Wiss. Zeit. Humboldt Univ., Berlin, 16, 105-110.

31. White, L. S., 1966. "The evaluation of H106 continuous sampling plans under the assumption of worst conditions," Journal of the American Statistical Association, **61**, 833-841.

32. Walsh, J. E., 1956. "Validity of approximate normality values for $\mu \pm k\sigma$ areas of practical type continuous populations," Annals of the Institute of Stat. Math., (Tokyo), **8**, 79-86.

33. Walsh, J. E., 1958. "Further consideration of normality values for $\mu \pm k\sigma$ areas of continuous populations," Annals of the Institute of Stat. Math., (Tokyo), **9**, 127-129.

34. Wheeler, D. J., 1968. "The variance of an estimator in variables sampling," submitted to Technometrics.

35. Zeigler, R. K. and G. L. Tietjen, 1968. "Double sampling plans where the acceptance criterion is the variance," Technometrics, **10**, 99-106.

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) SOUTHERN METHODIST UNIVERSITY	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP UNCLASSIFIED

3. REPORT TITLE

Summary of Recent work on Variables Acceptance Sampling with Emphasis on Non-normality

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report

5. AUTHOR(S) (First name, middle initial, last name)

Donald B. Owen

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 2, 1968	12	35

8a. CONTRACT OR GRANT NO. N00014-68-A-0515 b. PROJECT NO. NR 042-260 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 1. 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited. Reproduction in whole or in part is permitted for any purpose of the United States Government.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Office of Naval Research

13. ABSTRACT

Acceptance sampling plans based on the normal distribution have been available since 1955 and before. Yet reports of potential users indicate a general lack of enthusiasm for their application. There is the uncertainty of the assumption of the normal distribution, but the difficulties users have encountered are attributable more to the translation from the standardized deviate to proportion defective than with the probabilities involved. Some possible ways of adjusting for this are discussed.

Sampling plans for truncated life tests based on the exponential, normal lognormal, gamma, Weibull, etc., distributions are also available, as are distribution-free life test plans based on increasing or decreasing failure rate distributions. These are extremely useful and futher extensions of these ideas are in the offing.

For the normal distribution, plans have been devised which control each tail of the distribution to separate levels. These plans are useful in the very common situation where defectiveness measured on the high side has one kind of effect while defectiveness on the low side has another kind of effect. Again questions on the appropriateness of the normality assumption are raised. A discussion of various ways of meeting this assumption is given.

DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification