

Technical Report 12-01

Imagination Station (Istation):

Development of Teacher Resources for Grades 3 – 8

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### Abstract

This technical report describes the development of the Imagination Station (Istation) Teacher Resources for Grades 3 through 8, research-based instructional recommendations to support students who struggle in mathematics. The grade-specific mathematics knowledge and skills presented in the Teacher Resources are informed by state and national content standards. Each resource contains two categories: (1) content standards and (2) research-based instructional recommendations. This report contains the content standards and research-based instructional recommendations used to create the Teacher Resources, the components within the Teacher Resources, and the procedures used to write and review the Teacher Resources. To document the content-related evidence for validity, this technical report details the process and outcomes of reviewing and editing the Teacher Resources.

## Imagination Station (Istation):

### Development of Teacher Resources for Grades 3 – 8

#### **Introduction**

The Teacher Resources for the Imagination Station (Istation) are research-based, standards-aligned instructional tools that support students who struggle in mathematics. The purpose of the Teacher Resources is to provide teachers with research-based instructional recommendations and accompanying sample lessons to implement with students who have been identified as struggling in mathematics by any data source. In addition, the Teacher Resources are designed to integrate evidence-based instructional recommendations with state and national content standards. Using these Teacher Resources will enable teachers to quickly identify common misconceptions students have about a mathematics topic and, as a result, anticipate the difficulties children may face and adjust their instructional plan accordingly to fit students' needs.

The purpose of this technical report is to describe the development of the Teacher Resources for Grades 3 – 8. This description includes: (a) the content standards and research-based recommendations used to create the Teacher Resources, (b) the document-writing process, and (c) the reviewing and editing procedures used to ensure content-related validity.

#### **Content and Format of Teacher Resources**

Each Teacher Resource is organized into five sections: (1) overview of the content, (2) overview of the research-based instructional practice, (3) integration of the content and research-based instructional practice, (4) sample instructional sequence, and (5) references. A description of each section follows.

#### **Overview of the Content**

This section describes the Teacher Resources content, which is based on the Curriculum

Focal Points (CFP) published in 2006 by the National Council of Teachers of Mathematics (NCTM). We aligned the mathematics content standards from the states of Texas, Florida, New York, California, and Virginia, as well as those published by the Common Core State Standards Initiative to the NCTM CFP. See Appendix A for the content standards for each state. For each grade level, we created an additional CFP (CFP4) that encompasses measurement and geometry standards assessed across all the states since there was a lack of measurement and geometry standards at every grade level. See Appendix B – G for an abbreviated description of the specific content standards used in the Teacher Resources for Grades 3 – 8. In addition, for each CFP we note several common misconceptions and error patterns students may exhibit and the concepts students will need to master by the end of their current grade level.

### **Overview of the Research-Based Instructional Practice**

This section of the Teacher Resources incorporates evidence-based instructional practices based on research reported in two documents (Gersten et al., 2009a ; Gersten et al., 2009b). The first report, published by the Institute of Education Sciences (IES), outlines eight research-based recommendations for instructional strategies that support mathematics achievement for struggling students (Gersten et al., 2009a):

- IES Recommendation #1: Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.
- IES Recommendation #2: Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8.
- IES Recommendation #3: Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving,

verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

- IES Recommendation #4: Interventions should include instruction on solving word problems that is based on common underlying structures.
- IES Recommendation #5: Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.
- IES Recommendation #6: Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.
- IES Recommendation #7: Monitor the progress of students receiving supplemental instruction and other students who are at risk.
- IES Recommendation #8: Include motivational strategies in tier 2 and tier 3 interventions.

IES Recommendation #1 and #7 are not included in the Teacher Resources since these recommendations address assessment and the Teacher Resources emphasize instruction. Each of the Teacher Resources provides an in-depth description of a single IES Recommendation (#2, #3, #4, #5, #6, or #8), strategies to improve student outcomes, justification for the strategies, and potential roadblocks to its implementation.

In the second report Gersten and his colleagues (2009b) reviewed 42 studies that used randomized control trials or quasi-experimental methods in order to identify evidence-based strategies for effective mathematics instruction. The meta-analysis included studies reporting mean effect sizes that ranged from 0.21 to 1.56. The Teacher Resources incorporate strategies

from this study such as the use of heuristics, student verbalizations, visual aids, explicit instruction, student feedback, goal setting, and peer assistance.

### **Integration of the Content and Research-Based Instructional Practice**

This section describes how the content and research discussed in the second section can be applied to instructional practice and learning outcomes. This section presents instructional strategies that integrate the content from the CFP with the research-based IES Recommendations, a rationale for each instructional approach, and how they can be used to improve student outcomes. In addition, we include several types of difficulties students may face and how using the instructional strategies supports and promotes future mathematics learning.

### **Sample Instructional Sequence**

This section includes a description of an instructional sequence that teachers might use to implement the research-based instructional practice when teaching the content. This instructional sequence contains multiple parts that together form a sample lesson, showcasing how to integrate the CFP with the research-based instructional practice. First, we provide teachers with an overview of a sample lesson consisting of lesson objectives, prerequisite skills and knowledge, mathematical vocabulary with accurate yet developmentally appropriate definitions of the terms, and materials and resources to be used in the lesson. Next, we provide teachers with tiered intervention strategies. Lastly, this section includes suggestions about how to modify and extend the lesson to support students with different learning needs.

The instructional sequence contains a complete sample lesson, including teacher modeling, verbalizations, and notes. The lesson contains examples of explicit instruction, teacher modeling, and questioning strategies. Teacher notes, included throughout the lesson plan, provide teachers with additional information about student misconceptions, possible student

responses, and materials. Beyond content-related examples and practice problems, the sample lesson includes additional problems for guided and independent practice. The last section of the instructional sequence provides suggestions for re-teaching and extensions, evaluation and feedback, and closure. Each section in the lesson has research-based components incorporated in order to increase the practical significance of the lesson. See Appendix H for the lesson template.

The sample lesson was purposefully designed using evidence-based components of effective lessons such as “think a-louds” (Ellis & Larkin, 1998), guided practice, and differentiated instruction (Gresham & Little, 2012). For example, lessons use guided practice as a scaffolding strategy to model and think through new information presented during the lesson, providing a foundation upon which they can apply burgeoning skills until they can do so independently (Gresham & Little, 2012). The included lessons encourage teachers to model the cognitive processes needed to solve problems by using “think a-louds”, followed by gradual release of responsibility from tasks being completed in whole or small groups, to students working independently to demonstrate mastery (Ellis & Larkin, 1998). We incorporate differentiated instruction strategies tailored to the learning environment and the needs of individual learners by varying interests and readiness levels, and by using manipulatives and visuals aids (Gresham & Little, 2012).

## **References**

This section includes a list of helpful websites, articles, and/or books used to develop the Teacher Resources, as well as those relevant to the sample lesson.

## Document-Writing

### Document Specifications

There are 144 Teacher Resource documents across grades 3 – 8, a total of 24 per grade level (i.e., each document integrates one of four NCTM CFPs with one of six IES instructional recommendations). Figure 1 depicts the matrix of 24 documents created for one grade level.

The Teacher Resources are written in a manner that is both accessible to teachers and developmentally rigorous for students. All research references included in the Teacher Resources achieve readability while maintaining research integrity. When creating examples and practice problems for teachers to use during instruction, document writers carefully considered the developmental age of the target student population. All student examples use language that is accessible, yet mathematically precise. In addition, sample lessons include graphics as needed in order to convey important mathematical concepts.

### Document Writers

Six writers contributed to the development of the Teacher Resources.

**Writer 1.** Writer 1 received her B.A. in Mathematics from the University of Texas at Austin with the UTeach program, and her M.Ed. in Educational Leadership and Policy Studies from the University of Texas at Arlington. She taught elementary and middle school mathematics for four years. She also served at the Texas Education Agency for three years in a variety of roles, including the Assistant Director of Mathematics and Mathematics Curriculum Specialist in the Curriculum Division and Mathematics Assessment Specialist in the Student Assessment Division. She is currently pursuing her doctoral degree at Southern Methodist University.

**Writer 2.** Writer 2 earned her B.A. in Biochemistry and Mathematics from Austin

College. She then earned a M.S. degree in Biochemistry from Baylor College of Medicine. She has taught high school algebra and geometry and has tutored middle school, high school, and college level math. She has a M.Ed. degree from Southern Methodist University and is currently pursuing her doctoral degree at the same university. Her interests are in the field of mathematics measurement and assessments.

**Writer 3.** Writer 3 graduated from Texas Christian University with a B.S. in Mathematics Education. She then taught high school Geometry and Pre-Calculus for three years. While teaching, she earned her M.Ed. degree from The University of Texas at Arlington in Mathematics Curriculum and Instruction. She has also privately tutored students in all levels of mathematics courses ranging from eighth grade mathematics to Pre-Calculus. She is currently the professional development coordinator for the Research in Mathematics Education unit at Southern Methodist University in Dallas, Texas.

**Writer 4.** Writer 4 has a B.A. in German from the University of Texas at Austin. She has taken multiple courses in Early Childhood Education at the University of Texas at Dallas and Eastfield Community College. She obtained her alternative teacher certification from Texas A&M University. She has taught pre-school and elementary school and has 20 years of teaching experience.

**Writer 5.** Writer 5 received her B.A. in Psychology from University of California, Davis, and her M.S. in Education with an emphasis in Mild/Moderate Special Education from California State University. She then obtained her Ph.D. in Educational Leadership with an emphasis in Learner Assessment/Systems from the University of Oregon. She has worked as a resource specialist at the elementary school level to provide individualized instruction to students, and has written and implemented Individualized Educational Plans (IEPs). She is currently

working as a postdoctoral research associate at the University of Oregon.

**Writer 6.** Writer 6 graduated from Northwestern University with a B.A. in Comparative Literary Studies. She also has a M.A. degree in Math Education from DePaul University. She has her middle school and high-school math certification and has taught 7<sup>th</sup> and 8<sup>th</sup> grade algebra. She has also previously worked as a content specialist to plan and implement campaigns for education materials. She currently works as a content developer to develop, write, and review interactive whiteboard presentation, practice, and assessment content for mathematics and reading lessons.

### **Document-Writing Training**

All document writers received training to write the Teacher Resources in alignment with targeted content expectations and IES Recommendations. A face-to-face training was conducted to provide the document writers with: (a) information on the content and research-based instructional practices, (b) the Teacher Resources format, and (c) procedures for writing, submitting, and reviewing the Teacher Resources. As part of this training, writers reviewed and discussed a sample lesson plan that illustrated clear and accessible language, relevant graphics, developmentally appropriate content, and precise mathematical terminology. Writers received an Istation Teacher Resources Training Manual with procedural information, a review checklist, a list of mathematics resources and websites, and research articles to reference during lesson development.

### **Document-Writing Process**

After completing the training, document writers received a template to create the Teacher Resources. Document writers referenced research articles, books, and websites to develop ideas for lessons and strategies to integrate the IES Recommendations with the CFP-specific

mathematics content. The document writers completed approximately two Teacher Resources each week.

### **Content-Related Evidence for Validity**

To evaluate the accuracy and appropriateness of the content of the Teacher Resources, the Teacher Resources underwent a two-tier review process, beginning internally with the document writers and followed by project manager reviews. To begin, two document writers reviewed each Teacher Resource to provide feedback to the original document writer. The first reviewer provided comments, feedback, and edits on the Teacher Resource and then sent it to the second reviewer, who also provided comments, feedback, and edits. Each reviewer utilized the Istation Math Teacher Resource Review Checklist included in the Istation Teacher Resources Training Manual. The criteria used to review the Teacher Resources are described below:

- **Content alignment:** Is the mathematical content aligned to the NCTM CFP? Does the instructional practice align with the IES Recommendation?
- **Appropriateness of content:** Are the materials appropriate for the specified grade level? Are the materials free from bias based on gender, ethnicity, race, social class, disability, geography, or experiences unique to a subset of students?
- **Mathematical precision:** Is the mathematical content accurate and precise?
- **Language:** Is the language consistent across Teacher Resources? Is the language clear, concise, complete, and grammatically correct? Is the language in the lesson materials grade-appropriate for students? Are the directions unambiguous?
- **Graphics:** Are the graphics relevant to the mathematical concepts?

Reviewers returned comments to the original document writers, who then made revisions based on the feedback. Next, the Teacher Resources were sent to the two project managers for

review. Project Manager 1 is a research specialist and a PhD candidate in the Education Department at Southern Methodist University. She has over 18 years of experience as a classroom teacher and administrator. Project Manager 2 has a Ph.D. in Educational Leadership from the University of Oregon. She works as an Associate Professor at the Education Policy and Leadership Department at Southern Methodist University, and is currently also the Director of the Research in Mathematics Education Unit at SMU. The two project managers reviewed the Teacher Resources based on the criteria outlined above. Document writers made final revisions based on the project managers' feedback and suggestions.

### **Conclusions**

The purpose of this technical report was to describe the development of the Teacher Resources. We described the content standards and the research-based recommendations that were used to create the Teacher Resources, the qualifications of the document writers, the document writing process itself, and finally, the reviewing and editing procedures to document content-related evidence for validity.

## References

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- Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009a). *Assisting students struggling with mathematics: Response to Intervention (RTI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from <http://ies.ed.gov/ncee/wwc/publications/practiceguides/>
- Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009b). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, 79(3), 1202–1242.
- Gresham, R. H., & Little, M. E. (2012). *RTI and mathematics: Practical tools for teachers in K-8 classrooms*. Upper Saddle River, NJ: Pearson Education, Inc.



## Appendix A: State Content Standards Referent Sources

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

## Florida

Florida's Next Generation Sunshine State Math Standards (adopted 2007) were retrieved on July 5, 2012 from <http://www.floridastandards.org/Standards/FLStandardSearch.aspx>.

## California

California's Math Content Standards (adopted 1997) were retrieved on July 5, 2012 from <http://www.cde.ca.gov/be/st/ss/documents/mathstandards.pdf>. California Green Dot Standards are the selected standards that appear 85% of the time on California state tests.

## Common Core Standards

The Common Core Standards in Mathematics were retrieved on July 5, 2012 from <http://www.corestandards.org/the-standards/mathematics> . These standards were published in 2010. They were developed as part of an initiative led by National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

## New York

The New York State Standards (revised 2005) were retrieved on July 5, 2012 from <http://www.p12.nysed.gov/ciai/mst/math/standards/core.html>.

## Texas

The Texas State Standards for Math (Version 2.1; revised 2010) were retrieved on July 5, 2012 from <http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html>. The Texas Education Agency (TEA) released a 2010 document entitled *Texas Response to*

*Curriculum Focal Points: Kindergarten through Grade 8 Mathematics* that included coordinating TEKS.

#### Virginia

Virginia's Standards for Learning Document for Mathematics (adopted 2009 for full implementation in 2011-12) were retrieved on July 5, 2012 from

[http://www.doe.virginia.gov/testing/sol/standards\\_docs/mathematics/index.shtml](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/index.shtml).

Appendix B: Grade 3 Content Description

<b>GRADE 3 MATHEMATICS CURRICULUM FOCAL POINTS</b>	
<b><i>CFP 1: Number and Operations and Algebra</i></b>	
<b>Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts</b>	
<b>3.1.A.1</b>	Students understand the <b>meanings of multiplication and division of whole numbers through the use of representations</b> (e.g., equal-sized groups, arrays, area models, and equal “jumps” on number lines for multiplication and successive subtraction, partitioning, and sharing for division).
<b>3.1.B.1</b>	Identify examples of the <b>identity and commutative properties</b> for addition and multiplication.
<b>3.1.B.2</b>	Students <b>use properties of addition and multiplication</b> [e.g., commutativity, associativity, distributive property (including identities and zero properties for New York) to multiply whole numbers].
<b>3.1.C.1</b>	<b>Develop fluency</b> with single-digit multiplication facts.
<b>3.1.C.2</b>	Demonstrate fluency and <b>apply</b> single-digit division facts.
<b>3.1.C.3</b>	Students apply increasingly sophisticated strategies based on properties of multiplication and division to <b>solve multiplication and division problems involving basic facts</b> [multiply and divide within 100]
<b>3.1.D.4</b>	By comparing a variety of solution strategies, students relate <b>multiplication and division as inverse operations</b> .
<b><i>CFP 2: Number and Operations</i></b>	
<b>Developing an understanding of fractions and fraction equivalence</b>	
<b>3.2.A.1</b>	Students develop an <b>understanding of the meanings and uses of fractions</b> to represent parts of a whole, parts of a set, or points or distances on a number line.
<b>3.2.A.2</b>	Understand and recognize the <b>meaning of numerator and denominator</b> in the symbolic form of a fraction.
<b>3.2.B.1</b>	Students understand that the <b>size of a fractional part is relative to the size of the whole</b> , and they use fractions to represent numbers that are equal to, less than, or greater than 1.
<b>3.2.C.1</b>	<b>Compare and order unit fractions</b> ( $\frac{1}{2}$ , $\frac{1}{3}$ , $\frac{1}{4}$ ) and find their approximate locations on a number line.
<b>3.2.C.2</b>	Students solve problems that involve <b>comparing and ordering fractions</b> by using models, benchmark fractions, or common numerators and denominators
<b>3.2.D.1</b>	<b>Use concrete materials and pictures to model</b> at least halves, thirds, fourths, eighths, tenths, and twelfths.
<b>3.2.D.2</b>	<b>Represent a given fraction or mixed number, using concrete materials, pictures, and symbols.</b> For example,

	write the symbol for one-fourth and represent it with concrete materials and/or pictures.
<b>3.2.D.4</b>	Students understand and use models, including the number line, to <b>identify equivalent fractions.</b>
<b>CFP 3: Geometry</b>	
<b>Describing and analyzing properties of two-dimensional shapes</b>	
<b><i>Data Analysis Connection to the Focal Point</i> includes students using addition, subtraction, multiplication, and division of whole numbers come into play as students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems.</b>	
<b>3.3.A.2</b>	Students describe, analyze, compare, and classify <b>two-dimensional shapes by their sides and angles.</b>
<b>3.3.B.1</b>	Students <b>connect attributes of two-dimensional shapes</b> to their definitions.
<b>3.3.C.1</b>	Students investigate, describe, and reason about <b>decomposing, combining, and transforming polygons</b> to make other polygons.
<b>3.3.D.1</b>	Through building, drawing, and analyzing two-dimensional shapes, students <b>understand attributes and properties of two-dimensional space.</b>
<b>3.3.E.3</b>	Students use <b>attributes and properties</b> of two-dimensional shapes in solving problems, including applications <b>involving congruence.</b>
<b>3.3.F.4</b>	Students use <b>attributes and properties</b> of two-dimensional shapes in solving problems, including applications <b>involving symmetry.</b>
<b>CFP 4: Other</b>	
<b><i>Measurement Connections to Focal Points.</i> Students in grade 3 strengthen their understanding of fractions as they confront problems in linear measurement that calls for more precision than the whole unit allowed them in their work in grade 2. They develop their facility in measuring with fractional parts of linear units. Students develop measurement concepts and skills through experience in analyzing attributes and properties of two-dimensional objects. They form an understanding of perimeter as a measurable attribute and select appropriate units, strategies, and tools to solve problems involving perimeter.</b>	
<b>3.4.D.1</b>	Students <b>measure with fractional parts of linear units.</b>
<b>3.4.G.1</b>	Students select appropriate <b>units, strategies, and tools to solve problems involving perimeter.</b>
<b>3.4.I.2</b>	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l) [and English units for volume and weight].

<b>3.4.L.1</b>	Recognize area as an attribute of plane figures and understand concepts of area measurement...A square with side length 1 unit, called “a unit square” is said to have “one square unit” of area, and can be used to measure area.
<b>3.4.M.1</b>	Recognize area as an attribute of plane figures and understand concepts of area measurement. A plane figure that can be covered without gaps or overlaps by $n$ unit squares is said to have an area of $n$ square units.
<b>3.4.N.2</b>	Measure areas by counting unit squares (square <i>cm</i> , square <i>m</i> , square <i>in</i> , square <i>ft</i> , and improvised units).
<b>3.4.O.1</b>	Relate area to the operations of multiplication and addition...Find the area of a rectangle with whole-number side lengths by tilting it, and show that the area is the same as would be found by multiplying the side lengths.
<b>3.4.P.1</b>	Relate area to the operations of multiplication and addition...Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
<b>3.4.Q.1</b>	Relate area to the operations of multiplication and addition...Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b + c$ is the sum of $a \times b$ and $a \times c$ . Use area models to represent the distributive property in mathematical reasoning.
<b>3.4.R.1</b>	Relate area to the operations of multiplication and addition...Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of non-overlapping parts, applying this technique to solve real-world problems.

Appendix C: Grade 4 Content Description

GRADE 4 MATHEMATICS CURRICULUM FOCAL POINTS	
<b><i>CFP 1: Number and Operations and Algebra</i></b>	
<b>Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication.</b>	
<b>4.1A.1</b>	Students use understandings of multiplication to develop <b>quick recall of the basic multiplication facts</b> and related division facts.
<b>4.1B.1</b>	Students apply their understanding of <b>models for multiplication (i.e., equal-sized groups, arrays, area models, equal intervals on the number line)</b> as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multi-digit whole numbers.
<b>4.1B.2</b>	Students apply their understanding of models for <b>place value</b> as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multi-digit whole numbers.
<b>4.1B.3</b>	Students apply their understanding of models for <b>properties of operations</b> (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multi-digit whole numbers.
<b>4.1C.1</b>	Students select appropriate methods and apply them accurately to <b>estimate products or calculate them mentally</b> , depending on the context and numbers involved.
<b>4.1D.1</b>	Students develop <b>fluency with efficient procedures</b> , including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.
<b>A4.CFP1.1</b>	Evaluate and express relationships using open sentences with one operation
<b>A4.CFP1.2</b>	Use the symbols $<$ , $>$ , $=$ , and $\neq$ (with and without the use of a number line) to compare whole numbers and unit fractions and decimals (up to hundredths)
<b>A4.CFP1.6</b>	Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given
<b><i>CFP 2: Number and Operations</i></b>	
<b>Developing an understanding of decimals, including the connections between fractions and decimals</b>	
<b>4.2A.1</b>	Students <b>understand decimal notation</b> as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on.
<b>4.2B.1</b>	Students <b>read and write fractions or decimals that are greater than or less than 1</b> in problem solving.
<b>4.2B.2</b>	Students <b>identify equivalent fractions or decimals</b> in problem solving.
<b>4.2B.3</b>	Students <b>compare and order fractions or decimals</b> in problem solving.
<b>4.2C.1</b>	Students <b>connect equivalent fractions and decimals</b> by comparing models to symbols and locating equivalent symbols on the

	number line.
<b>4.2G.1</b>	By working with decimals, students extend their ability to recognize equivalent fractions.
<b>A4.CFP2.7</b>	Add and subtract proper fractions with common denominators
<p><b><i>CFP 3: Measurement</i></b>  <b>Developing an understanding of area and determining the areas of two-dimensional shapes</b></p> <p><b><i>Data Analysis Connections to the Focal Point</i></b> includes students making frequency tables, bar graphs, picture graphs, line plots, and stem-and-leaf plots.</p>	
<b>4.3A.1</b>	Students recognize <b>area as an attribute</b> of two-dimensional regions.
<b>4.3B.1</b>	Students learn that they can <b>quantify area</b> by finding the total number of same-sized units of area that cover the shape without gaps or overlaps.
<b>4.3C.1</b>	Students understand that a <b>square</b> , which is 1 unit on a side, is the <b>standard unit</b> for measuring area.
<b>4.3D.1</b>	Students select appropriate <b>units, strategies</b> (e.g., decomposing shapes), <b>and tools</b> for solving problems that involve <b>estimating or measuring</b> area.
<b>4.3E.1</b>	Students <b>connect area measure to the area model</b> that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle.
<b>4.3F.1</b>	Students extend their understanding of properties of two-dimensional shapes as they find the <b>areas of polygons</b> .
<b>4.3G.1</b>	Students build on their earlier work with symmetry and congruence in grade 3 to encompass <b>transformations</b> , including those that produce line and rotational symmetry.
<p><b><i>CFP 4: Other</i></b></p> <p><b><i>Measurement and Geometry Connections to the Focal Point</i></b>  <b>Students build on their earlier work with symmetry and congruence in grade 3 to encompass transformations, including those that produce line and rotational symmetry. Students extend their understanding of properties of two-dimensional shapes as they find the areas of polygons.</b>  <b>Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</b></p>	
<b>Measurement</b>	
<b>A4.CFP3.1</b>	Students select appropriate units, strategies, and tools for solving problems that involve estimating measurements or measuring various attributes.

<b>A4.CFP3.8</b>	Use a ruler to measure to the nearest standard unit (whole, $\frac{1}{2}$ and $\frac{1}{4}$ inches, whole feet, whole yards, whole centimeters, and whole meters)
<b>A4.CFP3.15</b>	Calculate elapsed time in days and weeks, using a calendar
<b>Geometry</b>	
<b>A4.CFP3.20</b>	Identify lines that are parallel and perpendicular
<b>A4.CFP3.23</b>	Know the definitions of a right angle, an acute angle, and an obtuse angle. Understand that $90^\circ$ , $180^\circ$ , $270^\circ$ , and $360^\circ$ are associated, respectively, with $\frac{1}{4}$ , $\frac{1}{2}$ , $\frac{3}{4}$ , and full turns
<b>A4.CFP3.4</b>	Identify and build a three-dimensional object from a two-dimensional representation of that object and vice versa.
<b>A4.CFP3.5</b>	Identify points and line segments when drawing a plane figure
<b>A4.CFP3.2</b>	Students use concrete models of standard cubic units to estimate and measure volume.

Appendix D: Grade 5 Content Description

<b>GRADE 5 MATHEMATICS CURRICULUM FOCAL POINTS</b>	
<b><i>CFP 1: Number and Operations and Algebra</i></b>	
<b>Developing an understanding of and fluency with division of whole numbers.</b>	
<b>5.1A.1</b>	Students apply their understanding of <b>models for division</b> as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends.
<b>5.1A.2</b>	Students apply their understanding of <b>place value</b> as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends.
<b>5.1A.4</b>	Students apply their understanding of <b>the relationship of division to multiplication</b> as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends.
<b>5.1B.1</b>	Students select appropriate methods and apply them accurately to <b>estimate quotients or calculate them mentally</b> , depending on the context and numbers involved.
<b>5.1C.1</b>	Students <b>develop fluency</b> with efficient procedures, including the standard algorithm, for dividing whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.
<b>5.1D.1</b>	Students consider the context in which a problem is situated to <b>select the most useful form of the quotient</b> for the solution, and they interpret it appropriately.
<b>5.1E.1</b>	Students use patterns, models, and relationships as contexts for <b>writing and solving simple equations and inequalities</b> .
<b><i>CFP 2: Number and Operations</i></b>	
<b>Developing an understanding of and fluency with addition and subtraction of fractions and decimals</b>	
<b>A5.CFP2.8</b>	<b>Compare and order fractions</b> including unlike denominators (with and without the use of a number line) Note: Commonly used fractions such as those that might be indicated on ruler, measuring cup, etc.
<b>A5.CFP2.14</b>	Simplify fractions to <b>lowest terms</b>
<b>5.2A.1</b>	Students apply their understandings of fractions and fraction models to represent the <b>addition and subtraction of fractions</b> with unlike denominators as equivalent calculations with like denominators.
<b>5.2B.1</b>	Students apply their understandings of <b>decimal models, place value, and properties</b> to add and subtract decimals.
<b>5.2D.1</b>	Students make reasonable <b>estimates of fraction and decimal</b> sums and differences.
<b>5.2E.1</b>	Students <b>add and subtract fractions and decimals</b> to solve problems, including problems involving measurement.
<b>A5.CFP2.12</b>	Identify the <b>factors</b> of a given number
<b>A5.CFP2.13</b>	Find the common <b>factors</b> and <b>greatest common factor</b> of two numbers

<b>A5.CFP2.16</b>	Round numbers to the nearest hundredth and up to 10,000
<b><i>CFP 3: Geometry and Measurement and Algebra</i></b>	
<b>Describing three-dimensional shapes and analyzing their properties, including volume and surface area.</b>	
<b><i>Data Analysis Connections to the Focal Point</i> includes students applying their understanding of whole numbers, fractions, and decimals as they construct and analyze double-bar and line graphs and use ordered pairs on coordinate grids.</b>	
<b>5.3A.1</b>	Students <b>relate two-dimensional shapes to three-dimensional shapes</b> and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces.
<b>5.3B.1</b>	Students recognize <b>volume as an attribute</b> of three-dimensional space.
<b>5.3C.1</b>	Students understand that they can <b>quantify volume</b> by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps.
<b>5.3D.1</b>	Students understand that a cube that is <b>1 unit</b> on an edge is the <b>standard unit for measuring volume</b> .
<b>5.3E.1</b>	Students select appropriate units, strategies, and tools for solving problems that involve <b>estimating or measuring volume</b> .
<b>5.3F.1</b>	Students decompose three-dimensional shapes and <b>find surface areas and volumes of prisms</b> .
<b>5.3J.1</b>	Students solve problems that require attention to both <b>approximation and precision of measurement</b> (including lengths, angles, and other forms of measurement).
<b><i>CFP 4: Other</i></b>	
<b>Measurement</b>	
<b>A5.CFP3.15</b>	Identify customary equivalent units of length
<b>A5.CFP3.17</b>	Convert measurement within a given system
<b>A5.CFP3.19</b>	Calculate elapsed time in hours and minutes
<b>Geometry</b>	
<b>A5.CFP3.2</b>	Identify pairs of similar triangles
<b>A5.CFP3.3</b>	Identify the ratio of corresponding sides of similar triangles
<b>A5.CFP3.7</b>	Know that the sum of the interior angles of a triangle is 180 degrees
<b>A5.CFP3.8</b>	Find a missing angle when given two angles of a triangle
<b>A5.CFP3.11</b>	Identify and draw lines of symmetry of basic geometric shapes
<b>A5.CFP3.12</b>	Identify and plot points in the first quadrant

Appendix E: Grade 6 Content Description

<b>GRADE 6 MATHEMATICS CURRICULUM FOCAL POINTS</b>	
<b><i>CFP 1: Number and Operations</i></b>	
<b>Developing an understanding of and fluency with multiplication and division of fractions and decimals.</b>	
<b>6.1A.1</b>	Students use the meanings of <b>fractions</b> to make sense of procedures for <b>multiplying and dividing fractions</b> and explain why they work.
<b>6.1A.2</b>	Students use the meanings of <b>multiplication and division</b> to make sense of procedures for <b>multiplying and dividing fractions</b> and explain why they work.
<b>6.1A.3</b>	Students use the <b>inverse relationship between multiplication and division</b> to make sense of procedures for multiplying and dividing fractions and explain why they work.
<b>6.1B.1</b>	Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for <b>multiplying and dividing decimals</b> .
<b>6.1C.1</b>	Students use common procedures to <b>multiply and divide fractions and decimals</b> efficiently and accurately.
<b>6.1D.1</b>	Students multiply and divide fractions and decimals to <b>solve problems</b> , including multistep problems and problems involving measurement.
<b>6.1E.1</b>	Students’ work in dividing fractions shows them that they can express the result of <b>dividing two whole numbers as a fraction</b> (viewed as parts of a whole).
<b>6.1F.1</b>	Students then extend their work in grade 5 with division of whole numbers to give <b>mixed number and decimal solutions to division problems</b> with whole numbers.
<b><i>CFP 2: Number and Operations</i></b>	
<b>Connecting ratio and rate to multiplication and division.</b>	
<b>6.2A.1</b>	Students use simple reasoning about multiplication and division to <b>solve ratio and rate problems</b> (e.g., “If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”).
<b>6.2B.1</b>	By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students <b>extend whole number multiplication and division to ratios and rates</b> .
<b>6.2C.1</b>	Students expand the repertoire of problems that they can solve by using multiplication and division, and they build on their <b>understanding of fractions to understand ratios</b> .

<b>6.2D.1</b>	Students <b>solve a wide variety of problems involving ratios and rates.</b>
<b>6.2E.1</b>	Students recognize that <b>ratio tables</b> not only derive from rows in the multiplication table but also connect with equivalent fractions.
<b>A6.CFP2.1</b>	Use proportions to solve problems (e.g., determine the value of N if $\frac{4}{7} = \frac{N}{21}$ , find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse
<p><b><i>CFP 3: Algebra</i></b>  <b>Writing, interpreting, and using mathematical expressions and equations.</b></p> <p><b><i>Data Analysis Connection to the Focal Point</i></b> includes students using and interpreting measures of central tendency for a given set of data. They do this through the use of mathematical expressions and equations. They also apply their understanding to construct and analyze a variety of ways (i.e., tables, diagrams, graphs) in which to display the data.</p>	
<b>6.3A.1</b>	Students <b>write</b> mathematical expressions and equations that correspond to given situations, they <b>evaluate</b> expressions, and they <b>use expressions and formulas</b> to solve problems.
<b>6.3B.1</b>	Students <b>understand that variables represent numbers</b> whose exact values are not yet specified, and they use variables appropriately.
<b>6.3C.1</b>	Students understand that <b>expressions in different forms can be equivalent</b> , and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information).
<b>6.3D.1</b>	Students know that the solutions of an equation are the <b>values of the variables that make the equation true.</b>
<b>6.3E.1</b>	Students <b>solve simple one-step equations</b> by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation.
<b>6.3F.1</b>	Students <b>construct and analyze tables</b> (e.g., to show quantities that are in equivalent ratios), and they use equations to describe simple relationships (such as $3x = y$ ) shown in a table.
<b>6.3G.1</b>	Students <b>use</b> the commutative, associative, and distributive <b>properties</b> to show that two expressions are equivalent.
<b>6.3H.1</b>	Students also <b>illustrate properties of operations</b> by showing that two expressions are equivalent in a given context (e.g., determining the area in two different ways for a rectangle whose dimensions are $x + 3$ by $5$ ).
<p><b><i>CFP 4: Other</i></b></p> <p><b><i>Measurement and Geometry Connections to Focal Points</i></b>  <b>Students convert like measurement units within a given measurement system. Problems that involve areas and volumes,</b></p>	

<p><b>calling on students to find areas or volumes from lengths or to find lengths from volumes or areas and lengths, are especially appropriate. These problems extend students’ work in grade 5 on area and volume and provide a context for applying new work with equations. They solve problems that require attention to both approximation and precision of measurement. Students will classify two-dimensional figures into categories based on their properties.</b></p>	
<p><b>Measurement</b></p>	
<b>A6.G.18</b>	Measure capacity and calculate volume of a rectangular prism
<p><b>Geometry</b></p>	
<b>A6.G.8</b>	Determine the area of triangles and quadrilaterals (squares, rectangles, rhombi, and trapezoids) and develop formulas. Use a variety of strategies to find the area of regular and irregular polygons.
<b>A6.G.10</b>	Determine the volume of rectangular prisms by counting cubes and develop the formula
<b>A6.G.27</b>	Students identify characteristics of and evaluate formulas for two- and three-dimensional figures or objects that can be measured, either directly or indirectly, including angle measure, perimeter, circumference, area, volume, capacity, and weight.
<b>A6.G.28</b>	Students select appropriate units, tools, and formulas to estimate and measure or calculate these characteristics and use them to solve problems.
<b>A6.G.3</b>	Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle)
<b>A6.G.5</b>	Find the perimeters and areas of composite two-dimensional figures, including non-rectangular figures (such as semicircles) using various strategies.
<b>A6.G.16</b>	Identify and plot points in all four quadrants
<b>A6.G.17</b>	Calculate the area of basic polygons drawn on a coordinate plane (rectangles and shapes composed of rectangles having sides with integer lengths)

Appendix F: Grade 7 Content Description

<b>GRADE 7 MATHEMATICS CURRICULUM FOCAL POINTS</b>	
<b><i>CFP 1: Number and Operations and Algebra and Geometry</i></b>	
<b>Developing an understanding of and applying proportionality, including similarity.</b>	
<b>7.1A.1</b>	Students extend their work with ratios to develop an understanding of <b>proportionality</b> that they apply to solve single and multistep problems in numerous contexts.
<b>7.1B.1</b>	Students use ratio and proportionality to solve a wide variety of <b>percent</b> problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease.
<b>7.1C.1</b>	Students also solve problems about similar objects (including figures) by using <b>scale factors</b> that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects.
<b>7.1D.1</b>	Students <b>graph proportional relationships</b> (linear functions) and identify the unit rate as the slope of the related line, noting that the vertical change (change in y- value) per unit of horizontal change (change in x- value) is always the same and know that the ratio ("rise over run") is called the slope of a graph
<b>7.1E.1</b>	Students <b>distinguish proportional relationships</b> ( $y/x = k$ , or $y = kx$ ) from other relationships, including inverse proportionality ( $xy = k$ , or $y = k/x$ ).
<b><i>CFP 2: Measurement and Geometry and Algebra</i></b>	
<b>Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes</b>	
<b>7.2A.1</b>	By decomposing two- and three-dimensional shapes into smaller, component shapes, students <b>find surface areas</b> and develop and justify formulas for the surface areas and volumes of prisms and cylinders.
<b>7.2B.1</b>	As students decompose prisms and cylinders by slicing them, they develop and <b>understand formulas for their volumes</b> (Volume = Area of base $\times$ Height)
<b>7.2C.1</b>	Students apply these formulas in problem solving to determine <b>volumes of prisms and cylinders</b>
<b>7.2D.1</b>	Students see that the <b>formula for the area of a circle</b> is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram.
<b>A7.CFP1.4</b>	Students <b>evaluate formulas</b> for given input values (surface area, rate, and density problems)
<b>7.2E.1</b>	Students select appropriate two- and three-dimensional shapes to model real-world situations and <b>solve a variety of problems</b> (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders, including estimating surface area and calculating the radius or diameter given the circumference or area of a circle.
<b>7.2F.1</b>	Students connect their work on proportionality with their work on area and volume by <b>investigating similar objects</b> .

7.2G.1	Students understand that if a <b>scale factor</b> describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related.
7.2H.1	Students apply their work on proportionality to <b>measurement in different contexts</b> , including converting among different units of measurement to solve problems involving rates such as motion at a constant speed.
<p><b><i>CFP 3: Numbers and Operations and Algebra</i></b>  <b>Developing an understanding of operations on all rational numbers and solving linear equations</b></p> <p><b><i>Data Analysis Connection to the Focal Point</i> includes students using proportions to make estimates relating to a population on the basis of a sample. They apply percentages to make and interpret histograms and circle graphs.</b></p>	
7.3A.1	Students extend understandings of <b>addition, subtraction, multiplication, and division</b> , together with their properties, to all rational numbers, including negative integers (with and without the use of a number line)
7.3B.1	By applying properties of arithmetic and considering <b>negative numbers</b> in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense.
7.3C.1	Students use the arithmetic of rational numbers as they formulate and <b>solve linear equations</b> in one variable and use these equations to solve problems.
7.3D.1	Students make strategic choices of procedures to <b>solve linear equations</b> in one variable and implement them efficiently, understanding that when they <b>use the properties of equality</b> to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.
A7.CFP3.37 Students use <b>models</b> to formulate and solve linear equations in one variable.	
A7.CFP3.1 Distinguish between the various subsets of real numbers (counting/natural numbers, whole numbers, integers, rational numbers, and irrational numbers)	
A7.CFP3.5 Read and write numbers in scientific notation (both positive and negative powers of 10)	
A7.CFP3.6 Translate numbers from scientific notation into standard form	
A7.S.9 Read and interpret data represented graphically (pictograph, bar graph, histogram, line graph, double line/bar graphs or circle graph)	
A7.S.10 Identify and explain misleading statistics and graphs	
<b><i>CFP 4: Other</i></b>	

<p><b><i>Measurement and Geometry Standards and their Connections to Focal Points</i></b></p> <p><b>Students convert measurement units between different measurement systems. They connect what they know about two-dimensional figures and apply that knowledge to three-dimensional figures. They further their understanding of right triangles and use the Pythagorean theorem to find the lengths of missing sides.</b></p>	
<p><b>Geometry</b></p>	
<b>A7.CFP2.2</b>	Identify the two-dimensional shapes that make up the faces and bases of three-dimensional shapes (prisms, cylinders, cones, and pyramids)
<b>A7.CFP2.5</b>	Find a missing angle when given angles of a quadrilateral
<b>A7.CFP1.7</b>	Build a pattern to develop a rule for determining the sum of the interior angles of polygons
<b>A7.CFP2.16</b>	Draw central angles in a given circle using a protractor (circle graphs)
<b>A7.CFP2.26</b>	Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections

Appendix G: Grade 8 Content Description

<b>GRADE 8 MATHEMATICS CURRICULUM FOCAL POINTS</b>	
<b><i>CFP 1: Algebra</i></b>	
<b>Analyzing and representing linear functions and solving linear equations and systems of linear equations</b>	
<b>8.1A.1</b>	Students <b>use linear functions, linear equations, and systems</b> of linear equations to represent, analyze, and solve a variety of problems.
<b>8.1B.1</b>	Students recognize a <b>proportion</b> ( $y/x = k$ , or $y = kx$ ) as a <b>special case of a linear equation</b> of the form $y = mx + b$ , understanding that the constant of proportionality ( $k$ ) is the slope and the resulting graph is a line through the origin.
<b>8.1C.1</b>	Students understand that the <b>slope</b> ( $m$ ) of a line is a <b>constant rate of change</b> , so if the input, or $x$ -coordinate, changes by a specific amount, $a$ , the output, or $y$ -coordinate, changes by the amount ' $ma$ '
<b>8.1D.1</b>	Students translate among verbal, tabular, graphical, and algebraic <b>representations of functions</b> (recognizing that tabular and graphical representations are usually only partial representations), and they describe how such aspects of a function as slope and $y$ -intercept appear in different representations.
<b>8.1E.1</b>	Students solve <b>systems of two linear equations in two variables</b> and relate the systems to pairs of lines that intersect, are parallel, or are the same line, in the plane. Students use linear equations, systems of linear equations, linear functions, and their understanding of the slope of a line to analyze situations and solve problems.
<b><i>CFP 2: Geometry and Measurement</i></b>	
<b>Analyzing two- and three-dimensional spaces and figures by using distance and angle</b>	
<b>8.2A.1</b>	Students use fundamental facts about distance and angles to describe and <b>analyze figures and situations in two- and three-dimensional space</b> and to solve problems, including those with multiple steps.
<b>8.2B.1</b>	Students prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines.
<b>8.2C.1</b>	Students apply this <b>reasoning about similar triangles</b> to solve a variety of problems, including those that ask them to find heights and distances.
<b>8.2D.1</b>	Students use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and they apply this fact about triangles to find unknown measures of angles
<b>8.2E.1</b>	Students explain <b>why the Pythagorean theorem is valid</b> by using a variety of methods – for example, by decomposing a square in two different ways.
<b>8.2F.1</b>	Student <b>use square roots</b> when they apply the Pythagorean theorem.
<b>8.2G.1</b>	Students apply the <b>Pythagorean theorem to find distances</b> between points in the Cartesian coordinate plane to measure lengths

	and analyze polygons and polyhedra.
<b>CFP 3: Data Analysis and Numbers and Operations and Algebra</b>	
<b>Analyzing and summarizing data sets</b>	
<b>8.3A.1</b>	Students use descriptive statistics, including <b>mean, median, and range</b> , to summarize and compare data sets, and they organize and display data to pose and answer questions.
<b>8.3B.1</b>	Students compare the information provided by the mean and the median and investigate the different effects that <b>changes in data values have on these measures of center</b> .
<b>8.3C.1</b>	In addition to the median, students determine the <b>25th and 75th percentiles</b> (1st and 3rd quartiles) to obtain information about the spread of data. They may use box-and-whisker plots to convey this information.
<b>8.3E.1</b>	Students understand that a measure of center alone does not thoroughly describe a data set because <b>very different data sets can share the same measure of center</b> .
<b>8.3F.1</b>	Students <b>select the mean or the median</b> as the appropriate measure of center for a given purpose.
<b>8.3G.1</b>	Students <b>use exponents and scientific notation</b> to describe very large and very small numbers.
<b>A8.CFP3.5</b>	Apply percents to: • Tax • Percent increase/decrease• Simple interest • Sale price • Commission • Interest rates • Gratuities
<b>A8.CFP1.3</b>	Simplify real number expressions using the laws of exponents.
<b>A8.CFP1.4</b>	Make reasonable approximations of square roots and mathematical expressions that include square roots, and use them to estimate solutions to problems and to compare mathematical expressions involving real numbers and radical expressions.
<b>A8.CFP1.5</b>	Perform operations on real numbers (including integer exponents, radicals, percents, scientific notation, absolute value, rational numbers, and irrational numbers) using multi-step and real world problems
<b>CFP 4: Other</b>	
<b>Measurement and Geometry Standards and their Connections to Focal Points</b>	
<b>Students convert measurement units between different measurement systems. They use properties and geometric proofs to reason through more advanced problems. Students further their understanding of the Pythagorean theorem and use trigonometric functions to solve for unknown lengths of right triangles.</b>	
<b>Geometry</b>	
<b>A8.CFP2.6</b>	Describes transformations in a coordinate plane, and recognizes the image of a figure under a translation, a dilation, a reflection over a given line, and rotations of 90 and 180 degree.
<b>A8.CFP2.11</b>	Identifies the properties preserved and not preserved under a translation, dilation, reflection, rotation,

<b>A8.CFP2.23</b>	Students commit to memory the formulas for prisms, pyramids, and cylinders.
<b>A8.CFP2.26</b>	Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.

Appendix H: Sample Lesson Template

<b>Sample Instructional Sequence</b>	
Lesson Writer:	
Grade Level:	Curriculum Focal Point:

**Purpose of Section:** This section will include a description of how a teacher might implement the research-based instructional practice when teaching the content. Suggestions will be provided for introducing the content, teacher-guided modeling, guided practice, and independent practice.

- Overview
  - Lesson objectives that are measurable and observable
  - Prerequisite skills/knowledge
  - Mathematical vocabulary for pre-teaching and reinforcement (definitions should be mathematically accurate, yet developmentally appropriate)
  - Materials and resources needed to operationalize the lesson
- Ideas about grouping for differentiation based on tiered levels of support; explicit reference to tiered level of instructional support; identify how the practice can be intensified to support students with more significant needs
- Teacher models including introduction to the lessons and sample teacher verbalizations
- Guided practice opportunities including sample prompts to encourage student think-alouds
- Independent practice with suggestions for corrective feedback
- Reteaching plans
- Closure

**Length of Section:** 2-3 pages

**Name of Lesson/Concept Covered**

**Lesson Overview**

*Lesson Objective*

Stated as a measureable and observable objective.

*Prerequisite Skills*

- Skill 1
- Skill 2

*Vocabulary*

- Term 1
- Term 2

*Materials and Resources*

- Materials 1
- Materials 2

**Ideas for Tiered Instruction and/or Grouping**

General description of how lesson can be adjusted for struggling students (i.e., Tier 2 and 3 adjustments).

**Sample Lesson**

Introduction:

Description worded for the teacher. Reference the content standard (e.g., The objective of the lesson is for students to use patterns, models, and relationships as contexts for writing and solving simple equations.)

Model:

Lesson worded for the student. In this section, when content is directed to the teacher, use brackets [Teacher note: xxxx]. If a short statement, such as an answer to a student-question, use brackets only.

**Guided and Independent Practice**

General description worded for the teacher. Provide examples or several sample problems.

**Reteaching and/or Extensions**

General description worded for the teacher.

**Evaluation/Feedback**

General description worded for the teacher.

**Closure**

General description worded for the teacher.