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Imagination Station (Istation):

Universal Screener Instrument Development for Grade 6

RESEARCH IN
MATHEMATICS
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**Imagination Station (Istation):
Universal Screener Instrument Development for Grade 6**

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Abstract

In this technical report, we describe the development of the Grade 6 Formative Assessment Item Bank for the Imagination Station (Istation). The formative assessment item bank will be used to deliver a computerized adaptive universal screening assessment to support teachers' instructional decisions. The construct underlying the items is mathematics skills and knowledge in Grade 6 as defined by state and national content standards. We include a description of the process used to identify and sample the content and levels of cognitive complexity assessed in the item bank. Next, we describe the item writing procedures. Finally, we describe the process and outcomes of an external item review to document content related evidence for validity.

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Imagination Station (Istation) Universal Screener Instrument Development for Grade 6

Introduction

The purpose of the Grade 6 Formative Assessment Item Bank for the Imagination Station (Istation) is to support teachers' instructional decisions. Specifically, the formative assessment item bank is designed to serve as a computerized adaptive universal screening assessment system. By administering this assessment system, teachers and administrators can use the results to answer two questions: (1) are students at risk of failure in Grade 6 mathematics, and (2) what is the degree of intensity of instructional support students need to be successful in Grade 6 mathematics? Because the universal screener is designed for administration in fall, winter, and early spring, these decisions can be applied over the course of Grade 6. The universal screener should be administered to all students receiving grade-level instruction.

The purpose of this technical report is to describe the development of the formative assessment item bank. Included in this description is the process used to identify and sample the content and levels of cognitive complexity assessed in the item bank, the item writing process, and external review procedures and results. The test development steps used to create the Formative Assessment Item Bank are representative of the Test Standards published by the American Educational Research Association (AERA), American Psychological Association (APA), & National Council on Measurement in Education (NCME) (1999), and represent best practices in test development.

Construct Definition

The construct of an assessment is considered on two dimensions: content and cognitive engagement. The content of the Grade 4 Formative Assessment Item Bank is based on the Curriculum Focal Points (CFP) published in 2006 by National Council of Teachers of Mathematics (NCTM). Using the NCTM Curriculum Focal Points as the referent standard, we aligned the mathematics content standards published by the Common Core Standards Initiative as well as from the states of Texas, Florida, New York, California, and Virginia to the CFPs. See Appendix A for the referent documents for each state. For Grade 6, we created a fourth CFP that encompasses measurement and geometry standards assessed across the states. See Appendix B for an abbreviated description of the assessed content.

The cognitive engagement dimension of the construct refers to the level of cognitive processing through which students are expected to engage with the content. The Formative Assessment Item Bank in Mathematics relies on the taxonomy of cognitive engagement published by Kilpatrick, Swafford, and Findell (2001) for the National Research Council. The five interdependent components (or strands) of students' engagement with the content provide a framework to promote mathematical proficiency.

1. Conceptual understanding pertains to the functional grasp of mathematics that a student applies to concepts, operations, and relations. It involves

- being able to logically organize one's knowledge to integrate and understand concepts as part of a coherent whole.
2. Procedural fluency pertains to students' ability to accurately and appropriately carry out skills, including being able to select efficient and flexible approaches.
 3. Strategic competence involves one's ability to formulate a problem in mathematical terms, to represent it strategically (verbally, symbolically, graphically, or numerically), as well as to solve it effectively. It is similar to problem solving and problem formation.
 4. Adaptive reasoning involves the student's capacity to think logically about a problem, which requires reflecting on various approaches to solve a problem and deductively selecting an approach. Students who are able to do this are also able to rationalize and justify their strategy.
 5. Productive disposition refers to a student's overall ability to perceive mathematics as worthwhile and to maintain a personal belief in one's own efficacy in solving problems.

The Formative Assessment Item Bank incorporates four of the five strands; productive disposition is not assessed.

The four CFPs were assessed for each of the four levels of cognitive engagement. Conceptual understanding and procedural fluency were oversampled to accurately reflect the relative emphasis in the state standards. Easy, medium, and difficult items were written for each CFP across the four levels of cognitive engagement. The content sampling matrix is presented in Figure 1.

Item Writing

Item Specifications

Approximately 400 items were written for Grade 4. Multiple-choice items were created for efficiency in the computer delivery. Each item had three distractors and one correct answer. The distractors represented plausible misconceptions or errors. Errors included computational errors, conceptual errors, procedural errors, and strategy errors. The stem included text and/or graphics. Graphics were used in instances where they explained the problem, provided a visual clue to clarify the context, or were integral to the stem or answer choices. Irrelevant graphics were not included. The reading level of the items was intentionally constrained to the 4th grade level; however, readability statistics were not calculated for each item. Whenever possible, plain language and simple, straight-forward statements were incorporated into the items.

Items are written following the principles of universal design for assessment (c.f., Ketterlin-Geller, 2005; 2008), and are amendable to accommodations. Items are scored dichotomously.

The computerized adaptive test can be administered individually or in a group in an untimed setting.

Item Writers

Four item writers contributed items to the Grade 6 Formative Assessment Item Bank.

Item Writer 1. Item Writer 1 obtained a Bachelor of Public Administration degree from Texas State University – San Marcos. She worked as a long-term substitute on various high school campuses across the State of Texas teaching mathematics. Upon completing her degree she worked for the Office of the Attorney General for the State of Texas as a Child Support Officer. After almost four years working in the public sector, she was accepted into the New York City Teaching Fellows Program where she taught high school mathematics to special education students in an inner city school in Manhattan. During this time, Item Writer 1 completed a Master of Science degree in Special Education with Honors from the City College of New York. While teaching, her main focus was in a co-teaching environment instructing students with disabilities in the areas of Algebra and Geometry. After two years teaching in New York City, she left the classroom to focus on research, specifically in the area of test development for students with disabilities. She has worked on several projects and nationally funded grants in the area of educational research and is currently completing the requirements for a Ph.D. in Educational Research from Southern Methodist University.

Item Writer 2. Item Writer 2 holds a Bachelors of Science and a Masters degree in Special Education. He has been a research assistant, project coordinator and independent contractor for federally funded grants and state contracts since 2001. He assisted in the creation of a web-delivered math assessment, researching effectiveness of accommodations. Additionally, he was involved in the development of the alternate assessment for students with significant cognitive disabilities at the elementary, middle, and high school levels between 2002 and 2009. Item Writer 2 also was a member of an item writing team for creating items for math screening tests and assisted in development of accommodated versions of math items. He has been a part of several research teams conducting multi-state research projects examining comparability of performance on different alternate assessments.

Item Writer 3. Item Writer 3 earned a Master of Science in Mathematics Education and an Honors Bachelor of Science in Mathematics, both from Oregon State University. Throughout her six years of teaching, she has taught math at all levels from middle school through high school and community college. In addition to teaching, she currently works as a mathematics coach for her district. Within this position, she focuses on improving math instruction across the district by coordinating and developing curriculum that is aligned to state mathematics standards. Her interest with assessments led her to writing mathematics assessment items.

Item Writer 4. Item Writer 4 is a school psychologist with expertise in mathematics education. She earned a Ph.D. in Educational Leadership with a focus on assessment and measurement. She has been the lead author on a district-wide mathematics formative assessment given to all first through eighth grade students three times per year. Her work on this project also included vertical equating and scaling tests. Since graduating, she worked for a nonprofit organization assisting in the design, development, and data collection of evaluations of education programs and improvement initiatives. Most recently, she served as a school psychologist where she conducted comprehensive psycho-educational evaluations to determine if students are eligible for special services and inform interventions.

Item Writing Training

All item writers were previously trained to write items that aligned with the content expectations and met the item specifications. The training included review of the Item Writing Training Manual, as well as participating in a training conference call with the researchers and project staff. The Item Writing Training Manual provides a detailed description of the principles of universal design for assessment. In addition, information was provided on the elements of high quality test design. Guidelines for writing selected response items were provided from recognized experts in item design. Moreover, sample items were provided to illustrate important components of effective items. A glossary of useful terms and a list of relevant websites were provided. The Item Writing Training Manual also included logistical information about formatting, reviewing, and submitting items.

A training conference call was conducted to review the content standards and levels of cognitive complexity for Grade 6. Project staff provided a detailed description of the content. Example items for each level of cognitive complexity were disseminated and discussed. The material in the Item Writing Training Manual was reviewed and discussed until the item writers were confident they understood the content and objectives of the project.

Item Writing Process

After completing the training and attending a project conference call, item writers were provided with the item writing template for creating items. Items were submitted and reviewed by the researchers and project staff. At least two internal reviewers provided feedback for each item. Dimensions of the review included the mathematical accuracy of the item, alignment with the content standards, appropriateness of language and graphics for students in Grade 6, and compliance with the principles of universal design. Comments were returned to the item writers; revisions were made and resubmitted for approval.

Once items were accepted, item level information was entered into an Item Database. The Istation graphic design team created all graphics. The finalized items were copy edited and reviewed by SMU researchers and Istation staff.

Content-Related Evidence for Validity

To evaluate the accuracy and appropriateness of the content of the Formative Assessment Item Bank for students in Grade 6, mathematicians and teachers reviewed all items.

Mathematician Review

Three mathematicians reviewed all items in Grade 6. Two reviewers were professors of mathematics at universities in Texas and had undergraduate and graduate degrees in mathematics. The third mathematician was a post-doctoral fellow at a doctoral-degree granting university in California with prior experience as a mathematics textbook author for K-12 education. The years teaching and researching in mathematics ranged from 6-17 years. Two reviewers were female; one reviewer was male.

The mathematicians were asked to review each item and evaluate the accuracy of the content, precision of the vocabulary, and effectiveness of distractors. The criteria were further described as follows:

- **Mathematical accuracy of content:** Each item was written to reflect an integration of knowledge and skills identified by the NCTM Curriculum Focal Points. Is the item mathematically accurate?
- **Precision of mathematical vocabulary:** Is the mathematical vocabulary used accurately? Is the mathematical vocabulary precise?
- **Appropriateness of the distractors:** Most students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions from choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Are the distractors mathematically plausible misconceptions?

Items and distractors were evaluated on a 4-point scale for each criterion. A rating of 1 indicated that the item was not accurate, precise, or the distractors were not effective; a rating of 2 indicated that the item was somewhat accurate, precise, or the distractors were somewhat effective; rating of 3 indicated that the item was somewhat accurate, precise, or the distractors were mostly effective; and a rating of 4 indicated the item was somewhat accurate, precise, or the distractors were extremely effective. In instances where the reviewer assigned a score of 1 or 2 on a category, recommendations were solicited that would aid in revision.

Overall, the mathematicians rated the items as mostly accurate, precise, and effective. For 64 items, the mathematicians recommended revisions. One reviewer noted the following issues on 19 items: the possibility of multiple plausible answer choices, inappropriate use of the term “solve” when the term “simplify” should be used, some distractors may not match key misconceptions, and the need to reference a “constant speed” when calculating time or distance traveled. Another reviewer noted the following issues on 37 items: items in which the correct answer was missing and the possibility of multiple plausible answer choices. The reviewer also

suggested wording of the stem of several items to improve communication of mathematical concepts. Finally, the last reviewer noted the following issues on eight items: missing or inaccurate graphics and missing definitions for variables in expressions/equations.

We revised all items in response to the recommendations. In instances where the mathematician did not provide a suitable suggestion, we revised the item and requested an additional review from an independent mathematician.

Teacher Review

Three teachers with experience teaching Grade 6 mathematics reviewed the items. One reviewer was a Caucasian female with over 31 years of teaching middle school mathematics. Another reviewer was a Hispanic female who had been teaching Grade 6 for more than 10 years. The final reviewer was a female Caucasian teacher with two years experience teaching middle school mathematics. All reviewers were certified to teach middle school mathematics by the state of Texas.

Teachers analyzed each item for grade-level appropriateness in terms of understandability of language and vocabulary, content or concepts, graphics, potential bias in language and/or content, clarity of directions and answers, and effectiveness of distractors. The criteria were further described as follows:

- Appropriateness of language: Is the language used in the item appropriate for students in your grade level? Are the question and response options written so that students in your grade level can understand the meaning of the problem?
- Appropriateness of mathematical vocabulary: Is the mathematical vocabulary representative of pre-requisite or instructional expectations in your grade level?
- Appropriateness of content or concepts: Is the task representative of pre-requisite or instructional expectations in your grade level?
- Appropriateness of visual representation: Is the visual representation (i.e., graphic, table, image) used in the item appropriate for students in your grade level? Can students in your grade level understand the meaning of the visual representation? Is the visual representation of the item clear?
- Bias in language or content: Does the item require background knowledge unrelated to the concept being tested that would differ for students with different backgrounds? Is the language sensitive to students from diverse backgrounds, students with limited English proficiency and students with special needs? Example: “*What is the most appropriate measurement unit for the length of a sub or hoagie?*” may be unfair for students in certain geographic regions and students with diverse background who are unfamiliar with the terms “sub or hoagie.”

- Effectiveness of the distractors: Some students use an eliminating process to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions choosing a correct answer in the elimination process. Are the distractors appropriate for the item? Do the distractors discriminate between students with specific misconceptions?

The items and distractors were rated on a scale of 1 to 4 for each criterion. A rating of 1 indicated that the item/distractors were not at all appropriate based on the criterion (or very biased); a rating of 2 indicated that the item/distractors were somewhat appropriate based on the criterion (or somewhat biased); rating of 3 indicated that the item/distractors were appropriate based on the criterion (or not biased); and a rating of 4 indicated that the item/distractors were extremely appropriate based on the criterion (or not biased *and* has multi-cultural components to it). In instances where the teachers provided a rating of 2 or lower, they were asked to provide additional suggestions and comments to improve the item.

Overall, the teachers rated the items as mostly to always appropriate in regards to language, vocabulary content, visual representation, bias, and effectiveness of distractors. The teachers recommended revising 114 items, primarily due to the difficulty of the items. For over 90 items, the teachers perceived of the items as too difficult for students in grade 6. These reviewers were referencing the state content standards when making this determination, instead of the NCTM Curriculum Focal Points. For each of the items in which a reviewer expressed concern, alignment with the NCTM focal points was verified.

Approximately 20 items were identified by the teacher reviewers as having confusing language; teachers recommended clarifying the language of some items to clearly articulate the mathematical concept for grade 6 students. The graphics for two items were noted as being confusing or difficult to read. The distractors on one item were thought to be too obviously incorrect. Finally, the symbols in the equations were not displaying correctly for one item. The research team reviewed all suggestions and made revisions based on teacher feedback.

Conclusions

The purpose of this technical report was to describe the development of the formative assessment item bank. We described the construct underlying the items in reference to the content standards and levels of cognitive complexity and described the process for sampling the content assessed in the item bank. Next, we described the item writing procedures and provided the qualifications for the item writers. Finally, we documented the process and outcomes of an external item review by mathematicians and teachers to document content related evidence for validity.

References

American Educational Research Association (AERA), American Psychological Association (APA), & National Council on Measurement in Education (NCME). (1999). *Standards for educational and psychological testing*. Washington, DC: American Psychological Association.

Ketterlin-Geller, L.R. (2005). Knowing what all students know: Procedures for developing universally designed assessments. *Journal of Technology, Learning, and Assessment*, 4(2). Available from <http://escholarship.bc.edu/jtla/vol4/2>

Ketterlin-Geller, L.R. (2008). Testing students with special needs: A model for understanding the interaction between assessment and student characteristics in a universally designed environment. *Educational Measurement: Issues and Practice*, 27(3), 3-16. DOI: 10.1111/j.1745-3992.2008.00124.x

Kilpatrick, J., Swafford, J. & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Figure 1

Content Sampling Matrix

CFP	Procedural fluency			Conceptual understanding			Strategic competence			Adaptive reasoning		
	Easy	Medium	Difficult	Easy	Medium	Difficult	Easy	Medium	Difficult	Easy	Medium	Difficult
1	10	10	10	10	10	10	7	6	7	7	6	7
2	10	10	10	10	10	10	7	6	7	7	6	7
3	10	10	10	10	10	10	7	6	7	7	6	7
4	10	10	10	10	10	10	7	6	7	7	6	7
Total By Difficulty	40	40	40	40	40	40	28	24	28	28	24	28

Appendix A - State Content Standards Referent Sources

National Council of Teachers of Mathematics (NCTM) Curricular Focal Points

The National Council of Teachers of Mathematics (NCTM) Curricular Focal Points were retrieved from http://www.nctmmedia.org/cfp/front_matter.pdf on 4/20/2010. Additional information was also retrieved on 4/20/2010 from: www.nctm.org/focalpoints. The coding system for the NCTM Critical Focal Points can be found under Part II.

Florida

Florida's Next Generation Sunshine State Math Standards (adopted 2007) were retrieved on 4/20/2010 from <http://www.floridastandards.org/Standards/FLStandardSearch.aspx>. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from Florida Department of Education. Big Ideas for each of the grade levels were also verified.

California

California's Math Content Standards (adopted 1997) were retrieved on 4/24/2010 from <http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf>. California Green Dot Standards are the selected standards (as of 2006) that appear 85% of the time on California state tests. These green dot standards were retrieved on 4/24/2010 from http://caworldclassmath.com/high_ca_standards.html and etc.usf.edu/flstandards/math/california.ppt. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the California State Board of Education.

New York

The New York State Standards (revised on March 15, 2005) were retrieved on 4/21/2010 from: <http://www.bootstrapworld.org/standards/ny/NYMathematicsCoreCurriculum.pdf>. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the New York State Board of Education.

Texas

The Texas State Standards for Math (Version 2.1; revised 2010) were retrieved on 4/21/2010 from: <http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html>. Verification of accuracy and currency of the standards was obtained on 5/5/2010 from the Texas State Board of Education. The Texas Education Agency (TEA) released a 2010 document entitled *Texas Response to Curriculum Focal Points: Kindergarten through Grade 8 Mathematics* that included coordinating TEKS.

Common Core Standards

The Common Core Standards in Mathematics were retrieved on June 10, 2011 from <http://www.corestandards.org/the-standards/mathematics>. These standards were published in 2010.

They were developed as part of an initiative led by National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

Virginia

Virginia's Standards for Learning Document for Mathematics (adopted 2009 for full implementation in 2011-12) were retrieved on June 10, 2011 from www.doe.virginia.gov/testing/sol/standards_docs/mathematics/review.shtml. Verification of accuracy and currency of the standards was obtained from Istation on June 10, 2011. The Curriculum Frameworks documents were referenced to determine the essential knowledge and skills students are expected to learn for each grade.

Appendix B - Content Description

GRADE 6 MATHEMATICS CURRICULUM FOCAL POINTS

CFP 1: Number and Operations

Developing an understanding of and fluency with multiplication and division of fractions and decimals.

- 6.1A.1** Students use the meanings of **fractions** to make sense of procedures for **multiplying and dividing fractions** and explain why they work.
- 6.1A.2** Students use the meanings of **multiplication and division** to make sense of procedures for **multiplying and dividing fractions** and explain why they work.
- 6.1A.3** Students use the **inverse relationship between multiplication and division** to make sense of procedures for multiplying and dividing fractions and explain why they work.
- 6.1B.1** Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for **multiplying and dividing decimals**.
- 6.1C.1** Students use common procedures to **multiply and divide fractions and decimals** efficiently and accurately.
- 6.1D.1** Students multiply and divide fractions and decimals to **solve problems**, including multistep problems and problems involving measurement.
- 6.1E.1** Students' work in dividing fractions shows them that they can express the result of **dividing two whole numbers as a fraction** (viewed as parts of a whole).
- 6.1F.1** Students then extend their work in grade 5 with division of whole numbers to give **mixed number and decimal solutions to division problems** with whole numbers.
- A6.CFP1.4** Represent fractions as **terminating or repeating decimals**

- A6.CFP1.1 Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators.
- A6.CFP1.2 Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators.
- A6.CFP1.3 Students make reasonable estimates of fraction and decimal sums and differences and add and subtract fractions and decimals to solve problems and judge the reasonableness of the results
- A6.CFP3.2 Compare and order fractions, decimals, and percents, including finding their approximate location on a number line.

CFP 2: Number and Operations

Connecting ratio and rate to multiplication and division.

- 6.2A.1 Students use simple reasoning about multiplication and division to **solve ratio and rate problems** (e.g., “If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”).
- 6.2B.1 By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students **extend whole number multiplication and division to ratios and rates.**
- 6.2C.1 Students expand the repertoire of problems that they can solve by using multiplication and division, and they build on their **understanding of fractions to understand ratios.**
- 6.2D.1 Students **solve a wide variety of problems involving ratios and rates.**
- 6.2E.1 Students recognize that **ratio tables** not only derive from rows in the multiplication table but also connect with equivalent fractions.
- 6.2F.1 Students **distinguish multiplicative comparisons** from additive comparisons.
- A6.CFP2.3 Read and write whole numbers to trillions
- A6.CFP2.17 Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction)

- A6.CFP2.1** Use proportions to solve problems (e.g., determine the value of N if $\frac{4}{7} = \frac{N}{21}$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse
- A6.CFP2.5** Verify the proportionality using the product of the means equals the product of the extremes
- A6.CFP2.2** Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips
- A6.CFP2.6** Read, write, and identify percents of a whole (0% to 100%)
- A6.CFP2.15** Estimate a percent of quantity (0% to 100%)
- A6.CFP2.7** Define absolute value and determine the absolute value of rational numbers (including positive and negative)
- A6.CFP2.8** Locate rational numbers on a number line (including positive and negative) and/or order rational numbers (including positive and negative)
- A6.CFP2.16** Solve addition, subtraction, multiplication, and division problems, IN CONCRETE SITUATIONS ONLY, which use positive and negative integers and combinations of these operations.
- A6.CFP2.11** Evaluate numerical expressions using order of operations (may include exponents of one, two and three)
- A6.CFP2.12** Represent repeated multiplication in exponential form and exponential form as repeated multiplication
- A6.S.10** List possible outcomes for compound events (CA: Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and use this to determine the probabilities of events when the outcomes have equal probability)
- A6.S.11** Determine the probability of dependent events
- A6.S.20** Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, 1 - P is the probability of an event not occurring
- A6.S.21** Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities
- A6.S.22** Understand the difference between independent and dependent events

CFP 3: Algebra

Writing, interpreting, and using mathematical expressions and equations.

***Data Analysis Connection to the Focal Point* includes students using and interpreting measures of central tendency for a given set of data. They do this through the use of mathematical expressions and equations. They also apply their understanding to construct and analyze a variety of ways (i.e., tables, diagrams, graphs) in which to display the data.**

- 6.3A.1** Students **write** mathematical expressions and equations that correspond to given situations, they **evaluate** expressions, and they **use expressions and formulas** to solve problems.
- 6.3B.1** Students **understand that variables represent numbers** whose exact values are not yet specified, and they use variables appropriately.
- 6.3C.1** Students understand that **expressions in different forms can be equivalent**, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information).
- 6.3D.1** Students know that the solutions of an equation are the **values of the variables that make the equation true**.
- 6.3E.1** Students **solve simple one-step equations** by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation.
- 6.3F.1** Students **construct and analyze tables** (e.g., to show quantities that are in equivalent ratios), and they use equations to describe simple relationships (such as $3x = y$) shown in a table.
- 6.3G.1** Students **use** the commutative, associative, and distributive **properties** to show that two expressions are equivalent.
- A6.CFP2.4** **Define and identify the multiplicative inverse** (reciprocal) of a number and the zero property of multiplication
- 6.3H.1** Students also **illustrate properties of operations** by showing that two expressions are equivalent in a given context (e.g., determining the area in two different ways for a rectangle whose dimensions are $x + 3$ by 5).
- 6.3I.1** **Sequences**, including those that arise in the context of finding possible rules for patterns of figures or stacks of objects, provide opportunities for students to **develop formulas**.
- A6.CFP3.1** **Work backward with two-step function rules** to undo expressions.

- A6.S.1** **Determine** the measures of central tendency (mean, median, mode) and variability (range) for a given set of data.
- A6.S.2** **Select and analyze** the measures of central tendency or variability to represent, describe, analyze, and/or summarize a data set for the purposes of answering questions appropriately.
- A6.S.13** Understand how the inclusion or exclusion of outliers affects measures of central tendency
- A6.S.15** Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample
- A6.S.16** Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.
- A6.S.17** Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached
- A6.S.18** Identify data that represent sampling errors and explain why the sample (and the display) might be biased
- A6.S.4** Record data in a frequency table
- A6.S.5** Construct Venn diagrams to sort data
- A6.S.6** Determine and justify the most appropriate graph to display a given set of data (pictograph, bar graph, line graph, histogram, or circle graph)
- A6.S.8** Read and interpret graphs
- A6.S.9** Justify predictions made from data
- A6.S.19** Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims

Measurement and Geometry Connections to Focal Points

Students convert like measurement units within a given measurement system. Problems that involve areas and volumes, calling on students to find areas or volumes from lengths or to find lengths from volumes or areas and lengths, are especially appropriate. These problems extend students' work in grade 5 on area and volume and provide a context for applying new work with equations. They solve problems that require attention to both approximation and precision of measurement. Students will classify two-dimensional figures into categories based on their properties.

Measurement

- A6.G.18** Measure capacity and calculate volume of a rectangular prism
- A6.G.19** Identify customary units of capacity (cups, pints, quarts, and gallons) and equivalent customary units of capacity (cups to pints, pints to quarts, and quarts to gallons)
- A6.G.21** Identify metric units of capacity (liter and milliliter) and equivalent metric units of capacity milliliter to liter and liter to milliliter)
- A6.G.23** Determine the tool and technique to measure with an appropriate level of precision: capacity
- A6.G.26** Determine personal references for capacity

Geometry

- A6.G.6** Determine a missing dimension of a plane figure or prism given its area or volume and some of the dimensions, or determine the area or volume given the dimensions.
- A6.G.7** Calculate the length of corresponding sides of similar triangles, using proportional reasoning
- A6.G.8** Determine the area of triangles and quadrilaterals (squares, rectangles, rhombi, and trapezoids) and develop formulas. Use a variety of strategies to find the area of regular and irregular polygons.
- A6.G.10** Determine the volume of rectangular prisms by counting cubes and develop the formula

- A6.G.27** Students identify characteristics of and evaluate formulas for two- and three-dimensional figures or objects that can be measured, either directly or indirectly, including angle measure, perimeter, circumference, area, volume, capacity, and weight.
- A6.G.28** Students select appropriate units, tools, and formulas to estimate and measure or calculate these characteristics and use them to solve problems.
- A6.G.2** Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms
- A6.G.3** Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle)
- A6.G.5** Find the perimeters and areas of composite two-dimensional figures, including non-rectangular figures (such as semicircles) using various strategies.
- A6.G.1** Understand the concept of a constant such as π ; know the formulas for the circumference and area of a circle; know common estimates of π (3.14; $22/7$) and use these values to estimate and calculate the circumference and the area of circles.
- A6.G.11** Identify radius, diameter, chords and central angles of a circle
- A6.G.12** Understand the relationship between the diameter and radius of a circle and the relationship between the circumference and the diameter of a circle
- A6.G.13** Determine the area and circumference of a circle, using the appropriate formula
- A6.G.14** Calculate the area of a sector of a circle, given the measure of a central angle and the radius of the circle
- A6.G.16** Identify and plot points in all four quadrants
- A6.G.17** Calculate the area of basic polygons drawn on a coordinate plane (rectangles and shapes composed of rectangles having sides with integer lengths)
- A6.G.24** Estimate volume, area, and circumference and justify reasonableness of estimates