Annette Caldwell Simmons School of Education & Human Development

Research in Mathematics Education

Imagination Station (Istation): Universal Screener Instrument Development for Algebra 1

Technical Report 16-04

Imagination Station (Istation): Universal Screener Instrument Development for Algebra 1

Brea Ratliff • Carole Hayata • Jason Bell • Leanne R. Ketterlin-Geller

Southern Methodist University

Summer 2016

Published by
Southern Methodist University
Department of Education Policy & Leadership
Simmons School of Education & Human Development
PO Box 750114
Dallas, TX 75275-0114
Contact information: rme@smu.edu

This research was supported by Imagination Station, Inc. Opinions expressed herein do not necessarily reflect those of Imagination Station or individuals within.

Acknowledgments: We would like to thank the following individuals for their assistance in completing this project: Lance Barasch, Jason Bell, Josh Geller, Cassandra Hatfield, Savannah Hill, and Marilea Jungman. We would also like to thank the mathematicians and teachers who carefully and thoroughly reviewed the mathematics items.

Abstract

In this technical report, we describe the development of an item bank for the Algebra 1 Universal Screener for Imagination Station (Istation). The formative assessment item bank will be used to deliver a computer-adaptive universal screener to support teachers' instructional decision-making in Algebra 1. State and national mathematics content standards informed the selection of mathematics topics assessed by the items. In this technical report, we describe: (a) the process used to define the construct (i.e., identify and sample the mathematics content), levels of cognitive complexity and difficulty assessed within each item, (b) the item writing procedures and selection criteria and qualifications of the item writers, and (c) the external item review process and how the results contribute to content-related evidence for validity.

Table of Contents

Imagination Station (Istation): Universal Screener Instrument Development for Algebra 1

Introduction

The purpose of the Algebra 1 formative assessment item bank (ALG1-IB) for the Imagination Station Indicators of Progress (ISIP) mathematics universal screener is to support Algebra 1 teachers' instructional decision making. The ALG1-IB will deliver computerized-adaptive testing (CAT) technology to identify students' risk status for meeting the expectations in Algebra 1. By administering this assessment system, teachers and administrators can use the results to answer these questions: (a) are students at risk of failure in Algebra 1, and (b) what is the degree of intensity of instructional support students need to be successful in Algebra 1? Multiple administrations of the universal screener (i.e., fall, winter, and early spring each year) provide teachers with meaningful information about students' learning over time, and support instructional decision-making over the course of the academic year. The universal screener is designed for administration to all students receiving Algebra 1 instruction.

The purpose of this technical report is to describe the development process of the ALG1-IB including: (a) the process used to define the construct (i.e., identify and sample the mathematics content) and levels of cognitive complexity and difficulty assessed, (b) the item writing procedures and selection criteria and qualifications of the item writers, and (c) the external item review process and how the results contribute to content-related evidence for validity. The test development process used to create the ALG1-IB represent best practices in test development and align with the test standards published by the American Educational Research Association (AERA), American Psychological Association (APA), and National Council on Measurement in Education (NCME) (2014).

Construct Definition

The test blueprint defines both mathematics content and levels of cognitive engagement, or independent strands promoting mathematical proficiency, (National Research Council [NRC], 2001) elicited by each item in the ALG1-IB.

Algebra and Mathematics Content

The mathematics content framework is based on the Common Core State Standards for Mathematics (CCSS-M), and mathematics content standards from the states of Texas and Virginia (See Appendix A). We began the process of developing the ALG1-IB construct by analyzing the Texas Essential Knowledge and Skills (TEKS) for Algebra 1, and a three-phase examination of each student expectation into a subset of content-related skills. The mathematical process standards from the TEKS were not considered for the construct.

The Algebra 1 content standards from the TEKS, CCSS-M, and VA SOL documents were compared and correlated in three phases as a result of the divergence in language used to define specific skills and concepts, as well as the overarching mathematical strands. While the majority of the Algebra 1 content standards could be aligned across all three documents, there were instances where a skill may only have been present in one or two documents.

In Phase 1, the subset of content-related skills derived from the Algebra 1 TEKS served as the referent document for the ALG1-IB blueprint. In Phase 2, the CCSS-M standards for High School Mathematics were re-assessed to determine alignment to the referent document. The CCSS-M standards are not associated with a particular high school mathematics course, so we utilized the Traditional Algebra 1 Pathway identified in the CCSS-M Mathematics Appendix A, *Designing High School Mathematics Courses Based on the Common Core State Standards*, as a reference document in order to determine which standards should be considered within the ALG1-IB framework. In Phase 3, the Virginia Standards of Learning (VA SOL) for Grade 8 and Algebra 1 were re-assessed in the same manner as the TEKS and CCSS-M.

Seven mathematical strands provide the structure of the ALG1-IB construct. Each content standard was assigned a new identification code for the construct using a naming convention based on these strands (See Appendix B for a description of each strand and content standard). The content standards from the TEKS, CCSS-M and VA SOL represented throughout the construct were used to determine the number of items assigned to each strand.

RME subject matter experts on the project team (See Appendix C) reviewed the construct. The assigned percentage of items for three mathematical strands was adjusted to more closely reflect the expectations expressed for mastery of Algebra 1 topics in the TEKS, CCSS-M and VA SOL. (See Appendix D).

Levels of Cognitive Engagement

The cognitive engagement component refers to the level of cognitive processing through which students are expected to engage with the content. The ALG1-IB uses the taxonomy of cognitive engagement that consists of five interdependent strands that promote mathematical proficiency: (a) conceptual understanding, (b) procedural fluency, (c) strategic competence, (d) adaptive reasoning, and (e) productive disposition. Items in the formative assessment item bank assess student understanding of the content at four levels of cognitive engagement. Productive disposition was not assessed because it refers to a student's overall perception of mathematics, and personal belief in one's own efficacy in solving problems (NRC, 2001). A brief description of each assessed level of cognitive processing is as follows:

- *Conceptual understanding* pertains to the functional grasp of mathematics that a student applies to concepts, operations, and relations. It involves being able to logically organize one's knowledge to integrate and understand concepts as part of a coherent whole.

- *Procedural fluency* pertains to students' ability to accurately and appropriately carry out skills, including being able to select efficient and flexible approaches.

- *Strategic competence* involves one's ability to formulate a problem in mathematical terms, to represent it strategically (verbally, symbolically, graphically, or numerically), as well as to solve it effectively. It is similar to problem solving and problem formation.

- *Adaptive reasoning* involves the student's capacity to think logically about a problem, which requires reflecting on various approaches to solve a problem and deductively selecting an approach. Students who are able to do this are also able to rationalize and justify their strategy.

Each content standard was examined to determine if it could be assessed at the four levels of cognitive engagement. Conceptual understanding and procedural fluency were oversampled to represent a balanced approach to mathematics proficiency as described by the CCSS-M.

Levels of Difficulty

In addition to content and levels of cognitive engagement, a level of difficulty designation was assigned. Easy, medium, and difficult items were written for each standard across the four levels of cognitive engagement. The level of difficulty of each item is a relative description that is subject to change with empirical analyses.

See Table 1 for the content sampling matrix that specifies the number of items written for each level of cognitive engagement by difficulty level throughout the ALG1-IB.

Item Writing

Item Specifications

Approximately 800 items were written for the ALG1-IB. Multiple-choice items were created for efficiency in the computer delivery system. The item stem included text and/or graphics. Whenever possible, plain language and simple, straightforward statements were incorporated into the items. Readability statistics were captured and indicated an overall Flesch-Kincaid grade level of 7.2 and Flesch Reading Ease rating of 63.7. Graphics were used in instances where they explained the problem, provided a visual clue to clarify the context, or were integral to the stem or answer choices. Irrelevant graphics were not included. Each item had three distractors and one correct answer. Items were written for dichotomous scoring as either correct or incorrect. The distractors represent plausible developmental misconceptions or procedural errors.

The assessment items were written according to the principles of universal design for assessment (Ketterlin-Geller, 2005, 2008) and are amenable to accommodations. As delivered, the formative assessment system will include a read-aloud feature to support item readability. This ensures that students are being assessed on their mathematics knowledge and skills as opposed to their reading skills.

The CAT is designed for individual or group administration in an untimed setting.

Item Writer Biographies

Thirteen item writers, including the four RME subject matter experts on the project team (Appendix C), contributed items to the Algebra 1 formative assessment item bank.

Item Writer 1 holds a Master's degree in Education Administration and Teaching, as well as a Bachelor's degree in Mathematics. She has nine years of combined experience in education as a middle school and high school teacher, an elementary school mathematics instructional coach, and an item writer and developer for multiple mathematics tests. She is currently pursuing a doctoral degree in Curriculum and Instruction.

Item Writer 2 holds a Master's degree in in Curriculum and Instruction, a Bachelor's degree in Mathematics, and has credentials in Secondary Mathematics (6-12) and Secondary Physics (6-12). She has a total of 16 years of experience as a high school mathematics teacher.

Item Writer 3 holds a Master's degree in Curriculum and Instruction and a Bachelor's degree in Mathematics. She has 10 years of experience in mathematics education at the high school level and is currently working as a high school math specialist.

Item Writer 4 holds a Bachelor's degree in Education and is working towards a Master's degree in Curriculum and Instruction with a concentration in Mathematics. He has been in education for 22 years and is currently working as a high school mathematics strategist, writing district assessments.

Item Writer 5 holds a Master's degree in Mathematics Education, a Bachelor's degree in Education and Social Policy, and also holds an ESL Endorsement. She has 12 years of experience at the middle and high school level, and is currently working as a high school teacher.

Item Writer 6 hold's a Master's degree in Mathematics Education and Bachelor's degree in Mathematics. He has been in education for 21 years and is an active member in the National Council of Teachers of Mathematics where he presents regularly at conferences. He is currently working as an 8th grade math teacher.

Item Writer 7 holds a Master's degree in School Administration, a Bachelor's degree in Bioenvironmental Sciences, and credentials in Mathematics (4-8) and Mathematics (8-12). She has 10 years of combined experience in education as a high school math teacher, curriculum writer, and instructional specialist. She currently works as a STEM instructional specialist designing online curriculum.

Item Writer 8 holds a Master of Business Administration degree in Human Resource Management as well as a Bachelor's degree in Management. He is certified in both middle school and high school mathematics and has been teaching mathematics for 10 years. He currently works as a high school math teacher.

Item Writer 9 holds a Doctoral degree in Curriculum and Instruction with an emphasis in Mathematics Education, a Master's degree in Mathematics, and a Bachelor's degree in Mathematics. She has 22 years of experience teaching at the secondary and postsecondary levels. She is currently a professor of mathematics.

Item Writing Training

All item writers were trained to write items that aligned with the content expectations and item specifications. Training included review of the RME Style Guide (2015) and the Istation Universal Screener Item Writer's Guide (Research in Mathematics [RME], 2015), as well as participation in two virtual training sessions with project staff. The RME style guide provides explanations, examples, and non-examples of stylistic expectations of items to support the item writers' efforts to write high-quality mathematics items. The Istation Universal Screener Item Writer's Guide provides an overview of the project, detailed information about the procedures for submission of items and the review process. It also includes information on the cognitive levels of engagement and describes the principles of universal design for assessment.

The two, one-hour virtual training sessions were held before the item writing process began. The first training consisted of these topics:

- Response to Intervention and an overview of the ISIP Mathematics universal screener assessment system and its intended use by teachers;
- a review of elements of high quality test design as it relates to validity, reliability, and fairness in testing;
- universal design and universal design for assessment; and
- the levels of cognitive engagement.

In developing the ALG1-IB, item writers were trained on the importance of using a universal screener as part of an RtI process to identify whether or not students need additional support in Algebra 1. The data resulting from the screener is designed to assist teachers in identifying the level of targeted instructional support, and must be reliable, valid, and fair. In order to ensure the items within the ALG1-IB meet these criteria, item writers were trained on the critical elements of universal design and universal design for assessment. Item writers were encouraged to write items that allow students to better access the intended constructs in the assessment without bias, through the lens of the cognitive engagement component.

The second training consisted of these topics:

- the SMU Honor Code for writing original items;
- the RME Style Guide; and
- guidelines for writing selected response items.

During this training, item writers were asked to develop original items and refrain from repurposing any work previously developed. The training included information about the RME internal review process, which included items being submitted through a plagiarism prevention service to ensure originality. Item writers received training focused on the ability to construct multiple-choice items in alignment with the elements of universal design, and appropriate distractors that address the misconceptions and procedural errors common to students in Algebra 1.

Item writers continued to receive training from RME subject matter experts throughout the duration of the project to ensure the expectations for writing were being met. RME subject matter experts were able to provide one-on-one virtual training over the levels of cognitive engagement, and established weekly communication with each item writer to provide coaching on how to develop appropriate items.

Item Writing Process

After completing the training and attending a project conference call, item writers were given the item writing template to create items. Upon completion of the items, items were submitted to RME project staff for review. An assigned staff reviewer provided feedback for each item. Reviewers evaluated items for:

- mathematical accuracy,
- alignment with the content standards,
- age-appropriateness of language and graphics for students in Grade 8, and
- compliance with universal design principles.

Reviewer comments were returned to the item writers to revise and resubmit the item for approval. All finalized items were cross-referenced to the test blueprint to ensure a corresponding item represented each content standard and the specified levels of cognitive engagement.

Once items written by the item writers were reviewed and accepted, item level information was entered into an Item Database. The Istation graphic design team created all graphics. The finalized items with graphics were reviewed for grammatical errors as well as visual spacing and alignment within the interface by RME project staff and Istation staff. *Figure 1* outlines the item writing process.

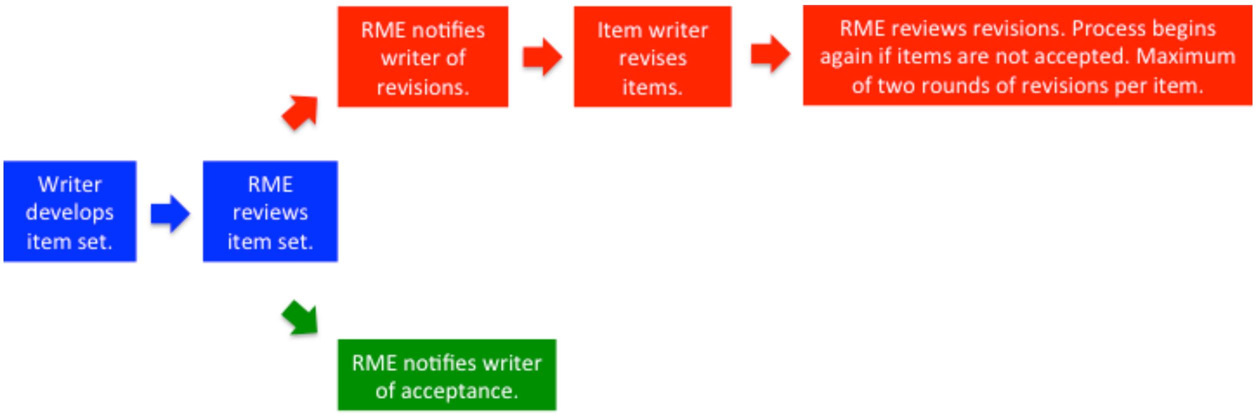

Figure 1
Illustrates the RME Item Writing and Review Process for the Istation formative assessment item bank.

Content-Related Evidence for Validity

The finalized items in the ALG1-IB were reviewed by mathematics teacher educators and high school mathematics teachers or content specialists. Each reviewer was required to evaluate and rate all items across several key criteria to ensure the appropriateness and accuracy of the content in the ALG1-IB.

Mathematics Teacher Educator Reviewer Biographies

Five mathematics teacher educators reviewed the items in the ALG1-IB. The mathematics teacher educators were selected based on current research and teaching in mathematics or mathematics education at the university level.

Mathematics Educator Reviewer 1 holds a Master's degree in Mathematics and a Bachelor's degree in Mathematics. He has seven years of experience working in education. He is currently a research assistant and doctoral student, but also has over four years of experience teaching at the college level.

Mathematics Educator Reviewer 2 holds a Doctoral degree in Mathematics Education, a Master's degree in Mathematics, and a Bachelor's degree in Mathematics. She has nine years of experience teaching undergraduate mathematics courses in Elementary College Mathematics, Calculus I, II, III, and Introduction to Combinatorial Analysis.

Mathematics Educator Reviewer 3 holds a Doctoral degree in Curriculum and Instruction with an emphasis on Mathematics Education, a Master's degree in Curriculum and Instruction, a

Bachelor’s degree in Mathematics, and a teaching credential in Mathematics. She currently serves as an associate professor.

Mathematics Educator Reviewer 4 holds a Doctoral degree in Mathematics, a Master's degree in Mathematics, and a Bachelor's degree in Mathematics. He is currently a professor of Mathematics and Statistics and has eighteen teaching Mathematics, Mathematical Sciences, and Statistics.

Mathematics Educator Reviewer 5 holds a Doctoral degree in Mathematics, a Master’s degree in the Mathematics Education, and a Bachelor’s degree in Mathematics with a minor in Religious Studies. She is currently an assistant professor of mathematics.

Mathematics Teacher Educators Expert Review

The mathematics educators were each required to review approximately 200 items and evaluate the (a) mathematical accuracy of the item, (b) precision of the language, (c) appropriateness of the response options, (d) alignment with the identified cognitive engagement level, and (e) alignment with the assigned standard. The mathematics educators were also required to verify the correct response option. The criteria used for item evaluation were as follows:

- *Precise Language*: Does the item contain unnecessary language or information that students could find distracting?

- *Mathematical Accuracy*: Is adequate information provided to enable the student to solve the item? Are the images used accurate and appropriate? Are formulas provided appropriately?

- *Appropriateness of the Responses*: Some students use a process of elimination to narrow their options in the context of multiple-choice questions. The purpose of selecting appropriate distractors is to reduce the likelihood of students with misconceptions from choosing a correct answer in the elimination process. Does the item limit the student’s ability to use the elimination process? Does the item use distractors that distinguish between students who know and did not know the skills/concepts? Are the distractors appropriate for the item? Are the distractors mathematically plausible misconceptions?

Items and distractors were evaluated on a 4-point scale for each criterion. A rating of 1 indicated that the item was not accurate, precise, or appropriate; a rating of 2 indicated that the item was somewhat accurate, precise, or appropriate; a rating of 3 indicated that the item was mostly accurate, precise, or appropriate; and a rating of 4 indicated the item was extremely accurate, precise, or appropriate. In instances where the reviewer assigned a score of 1 or 2 for any criterion, recommendations were solicited that would aid in revision.

Items that received ratings of 2 or 1 in mathematical accuracy, response options, or language precision, and/or received a comment related to the response, cognitive process, or standard alignment, were designated for a priority review by the RME project staff. Items that received 4’s and 3’s were reviewed based on reviewer comments. The ratings and comments from the reviewers were used to further refine the existing items for consistency (e.g., language, standards alignment, etc.).

The items received ratings above 85% in each criterion evaluated by the reviewers. In addition, 89% of the items included the correct response, 93% were assigned the correct level of cognitive response, and 94.6% were found to align with the designated standard. Items that were designated for priority review and comments from the reviewers on all items prompted further revisions. See Table 2 for full results from mathematics education expert review.

Mathematics Teacher Reviewer Biographies

Four mathematics teachers reviewed the items in the ALG1-IB. The high school mathematics teachers and content specialists selected as external reviewers are state-certified educators who are experts in their knowledge of middle and high school mathematics content, particularly Algebra 1.

Mathematics Teacher Reviewer 1 holds a Master's degree in Educational Administration and a Bachelor's degree in Chemical Engineering. She has experience as a math teacher, instructional coach, and a math methods teacher. She currently works as a graduate research assistant, an instructional specialist, and an adjunct instructor.

Mathematics Teacher Reviewer 2 holds Master's degree in Industrial and Organizational Psychology and a Bachelor's degree in Secondary Mathematics Education. She has nineteen years experience in education and is currently a mathematics department chair, instructional coach, and Algebra 1 teacher.

Mathematics Teacher Reviewer 3 holds a Master's degree in Secondary Education with an emphasis in Mathematics Education and a Bachelor's degree in Computer Science with a minor in German. She has experience as a mathematics teacher and as an instructional mathematics coach.

Mathematics Teacher Reviewer 4 holds a Doctoral degree in Education, Curriculum, and Instruction and Master's degree in Mathematics. She has 21 years experience as a math teacher and high school math instructional specialist. She is currently a secondary mathematics coordinator.

Mathematics Teacher Review

The mathematics teachers were each required to review approximately 200 items and evaluate the (a) appropriateness of language, (b) appropriateness of mathematical vocabulary, (c) appropriateness of visual representations, and (d) language bias. The mathematics teachers were also required to verify the correct response option. The criteria used for item evaluation were as follows:

- *Use of appropriate language*: Is the language used in the item appropriate for students in grades 8, 9 and 10? Are the question and response options written in a clear manner?

- *Use of appropriate mathematical vocabulary*: Is the mathematical vocabulary representative of pre-requisite or instructional expectations in grades 8, 9, and 10?

- *Appropriateness of visual representations*: Is the visual representation (i.e., graphic, table, image) used in the item appropriate for students in grades 8, 9, and 10? Can students understand the meaning of the visual representation? Is the visual representation easily read?

- *Language bias*: Does the item require background knowledge unrelated to the concept being tested that would differ for students with different backgrounds? Is the language sensitive to students from diverse backgrounds, students with limited English proficiency and students with special needs?

The items and distractors were rated on a scale of 1 to 4 for each criterion. A rating of 1 indicated that the item/distractors were not at all appropriate based on the criterion (or very biased); a rating of 2 indicated that the item/distractors were somewhat appropriate based on the criterion (or somewhat biased); rating of 3 indicated that the item/distractors were appropriate based on the criterion (or not biased); and a rating of 4 indicated that the item/distractors were extremely appropriate based on the criterion (i.e., not biased and with multicultural components). In instances where the teachers provided a rating of 2 or lower, they were asked to provide additional suggestions and comments to improve the item.

Items that received ratings of 2 or 1 in mathematical accuracy, response options, or language precision, and/or did not perform favorably in regards to response, cognitive process, or standard alignment, were designated for a priority review by the RME project staff. Items that received 4's and 3's were reviewed based on comments. The ratings and comments from the reviewers' were used to further refine the existing items for consistency (e.g., language, mathematical vocabulary, visual representations, etc.)

Items that were designated for priority review and comments from the reviewers on all items prompted further revisions. For example, reviewer comments such as, "Students will struggle with this one—a lot will choose the last answer because of the way it's worded." and "Instead of 'how long' should it read 'to determine the time the ball took to reach the ground'?" indicated that all items should be read through carefully for ambiguous language and revised if necessary.

The items received above 94% in each criterion evaluated by the reviewers. In addition, 98.7% were considered to have no bias and 94.1% had a correct response. See Table 3 for full results from mathematics education expert review.

Conclusion

The purpose of this technical report is to describe the development of the Algebra 1 Formative Assessment Item Bank. We described the process used to define the construct, levels of cognitive complexity and difficulty assessed within each item. We detailed the iterative training process established for item writers, and the intensity of support provided for each content writer. Next, we outlined the item writing process for sampling the content assessed in the item bank. Finally, we documented the process and outcomes of an external item review by mathematics educators and mathematics teachers to document content-related evidence for validity.

References

American Educational Research Association (AERA), American Psychological Association (APA), & National Council on Measurement in Education (NCME). (2014). *Standards for educational and psychological testing*. Washington, DC: American Psychological Association.

Haladyna, T. M., Downing, S. M., Rodriguez, M.C. (2004). A review of multiple-choice item-writing guidelines for classroom assessment. *Applied Measurement in Education*, 15, 309-333.

Ketterlin-Geller, L.R. (2005). Knowing what all students know: Procedures for developing universally designed assessments. *Journal of Technology, Learning, and Assessment, 4(2).* Available from http://www.jtla.org

Ketterlin-Geller, L.R. (2008). Testing students with special needs: A model for understanding the interaction between assessment and student characteristics in a universally designed environment. *Educational Measurement: Issues and Practice, 27*(3), 3-16. DOI: 10.1111/j. 1745-3992.2008.00124.x

National Research Council (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academy Press.

Research in Mathematics Education (2015). *Istation Universal Screener Item Writer's Guide.* Dallas, TX: Southern Methodist University.

Research in Mathematics Education (2015). *Research in Mathematics Education Item Writing Style Guide.* Dallas, TX: Southern Methodist University.

Table 1

Content Sampling Matrix

Cognitive Engagement Level	Easy	Medium	Difficult	Totals
Procedural fluency	87	75	48	210
Conceptual understanding	102	92	90	284
Strategic competence	55	75	59	189
Adaptive reasoning	32	45	41	118
Total	276	287	238	801

Table 2

Mathematics Education Expert Review Ratings

	Rating (Accurate/Appropriate)			
Criteria	**4-Extremely**	**3-Mostly**	**2-Somewhat**	**1-Not at All**
Mathematical Accuracy	77.5%	10.5%	6.8%	5.1%
Response Options	72.8%	14.7%	9.4%	2.9%
Use of Precise Language	75.9%	14.5%	7.0%	2.4%
	Yes	**No**		
Does the item include the correct response?	89%	11%		
Is the level of cognitive process accurate?	93%	7%		
Does the item align with the designated standard?	94.6%	5.4%		

Note: This table represents the results from 3 individual reviewers for 802 ALG1-IB items.

Table 3

Mathematics Teacher Review Ratings

	Rating (Accurate/Appropriate)			
Criteria	**4-Extremely**	**3-Mostly**	**2-Somewhat**	**1-Not at All**
Overall Language	80%	14.6%	4%	1.4%
Mathematics Vocabulary	80.7%	14.9%	2.9%	1.5%
Visual Representations	83.7%	12.3%	1.9%	2.1%
	Yes	**No**		
Does the Item contain bias in language or content??	1.3%	98.7%		
Does the Item Contain One Correct Response?	94.1%	5.9%		

Note: This table represents the results from 3 individual reviewers for 802 ALG1-IB items.

Appendix A – State Content Standards Referent Sources

Texas
The Texas Essential Knowledge and Skills (adoption 2012) were retrieved from:
http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html

Common Core Standards
The Common Core Standards in Mathematics were retrieved on September 15, 2015 from http://www.corestandards.org/Math/. These standards were published in 2010. They were developed as part of an initiative led by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

Virginia
Virginia's Standards for Learning Document for Mathematics (adopted 2009 for full implementation in 2011-2012) were retrieved from
http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/

Appendix B – ALG1-IB Assessed Standards

Strand 1: Linear Functions, Equations, and Inequalities

A.LFE.02A	Determine the domain of a linear function in mathematical problems.
A.LFE.02B	Determine the range of a linear function in mathematical problems.
A.LFE.02C	Determine reasonable continuous domain values for real-world situations.
A.LFE.02D	Determine reasonable continuous range values for real-world situations.
A.LFE.02E	Determine reasonable discrete domain values for real-world situations.
A.LFE.02F	Determine reasonable discrete range values for real-world situations.
A.LFE.02G	Represent domain using inequalities.
A.LFE.02H	Represent range using inequalities.
A.LFE.02J	Write linear equations in two variables in the form $y = mx + b$, given one point and the slope.
A.LFE.02K	Write linear equations in the form $y = mx + b$, given two points.
A.LFE.02L	Write linear equations in two variables in the form $\mathrm{A}x + \mathrm{B}y = \mathrm{C}$, given one point and the slope.
A.LFE.02M	Write linear equations in two variables in the form $\mathrm{A}x + \mathrm{B}y = \mathrm{C}$, given two points.
A.LFE.02N	Write linear equations in two variables in the form $y - y_1 = m(x - x_1)$ given one point and the slope.
A.LFE.02P	Write linear equations in two variables in the form $y - y_1 = m(x - x_1)$ given two points.
A.LFE.02Q	Write linear equations in two variables given a table of values.
A.LFE.02R	Write linear equations in two variables given a graph.
A.LFE.02S	Write linear equations in two variables given a verbal description.
A.LFE.02T	Write equations involving direct variation.
A.LFE.02U	Solve equations involving direct variation.

A.LFE.02V	Write the equation of a line that contains a given point and is parallel to a given line.
A.LFE.02W	Write the equation of a line that contains a given point and is perpendicular to a given line.
A.LFE.02X	Write an equation of a line that is parallel to the x-axis.
A.LFE.02Y	Determine whether the slope of a line parallel to the x-axis is zero or undefined.
A.LFE.02AA	Write an equation of a line that is parallel to the y-axis.
A.LFE.02BB	Determine whether the slope of a line parallel to the y-axis is zero or undefined.
A.LFE.02CC	Write an equation of a line that is perpendicular to the x-axis.
A.LFE.02DD	Determine whether the slope of a line perpendicular to the x-axis is zero or undefined.
A.LFE.02EE	Write an equation of a line that is perpendicular to the y-axis.
A.LFE.02FF	Determine whether the slope of a line perpendicular to the y-axis is zero or undefined.
A.LFE.02GG	Write linear inequalities in two variables given a table of values.
A.LFE.02HH	Write linear inequalities in two variables given a graph.
A.LFE.02JJ	Write linear inequalities in two variables given a verbal description
A.LFE.02KK	Write systems of two linear equations given a table of values.
A.LFE.02LL	Write systems of two linear equations given a graph.
A.LFE.02MM	Write systems of two linear equations given a verbal description.
A.LFE.03A	Determine the slope of a line given a table of values.
A.LFE.03B	Determine the slope of a line given a graph.
A.LFE.03C	Determine the slope of a line given two points on the line.
A.LFE.03D	Determine the slope of a line given an equation written in the form $y = mx + b$.
A.LFE.03E	Determine the slope of a line given an equation written in the form $Ax + By = C$.

A.LFE.03F	Determine the slope of a line given an equation written in the form $y - y_1 = m(x - x_1)$
A.LFE.03G	Calculate the rate of change of a linear function represented tabularly in context of mathematical problems.
A.LFE.03H	Calculate the rate of change of a linear function represented graphically in context of mathematical problems.
A.LFE.03J	Calculate the rate of change of a linear function represented algebraically in context of mathematical problems.
A.LFE.03K	Calculate the rate of change of a linear function represented tabularly in context of real-world problems.
A.LFE.03L	Calculate the rate of change of a linear function represented graphically in context of real-world problems.
A.LFE.03M	Calculate the rate of change of a linear function represented algebraically in context of real-world problems.
A.LFE.03N	Graph linear functions on the coordinate plane and identify slope in real-world problems.
A.LFE.03P	Graph linear functions on the coordinate plane and identify slope in mathematical problems.
A.LFE.03Q	Graph linear functions on the coordinate plane and identify the *y*-intercept in mathematical problems.
A.LFE.03R	Graph linear functions on the coordinate plane and identify the *y*-intercept in real-world problems.
A.LFE.03S	Graph linear functions on the coordinate plane and identify the *x*-intercept in real-world problems.
A.LFE.03T	Graph linear functions on the coordinate plane and identify the *x*-intercept in mathematical problems.
A.LFE.03U	Graph linear functions on the coordinate plane and identify zeros in mathematical problems.
A.LFE.03V	Graph linear functions on the coordinate plane and identify zeros in real-world problems.
A.LFE.03W	Graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

A.LFE.03X Determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $f(x - c)$ for specific values of c.

A.LFE.03Y Determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $f(bx)$ for specific values of b.

A.LFE.03AA Determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$ for specific values of a.

A.LFE.03BB Determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $f(x) + d$ for specific values of d.

A.LFE.03CC Graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist.

A.LFE.03DD Estimate graphically the solutions to systems of two linear equations with two variables in real-world problems.

A.LFE.03EE Graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

A.LFE.04A Interpret the correlation coefficient as a measure of the strength of the linear association.

A.LFE.04B Calculate, using technology, the correlation coefficient between two quantitative variables.

A.LFE.04C Compare and contrast association and causation in real-world problems.

A.LFE.04D Write, with technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

A.LFE.04E Write, without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

A.LFE.05A Solve linear equations in one variable for which the application of the distributive property is necessary.

A.LFE.05B Solve linear equations in one variable for which variables are included on both sides.

A.LFE.05C Solve linear inequalities in one variable for which the application of the distributive property is necessary.

A.LFE.05D Solve linear inequalities in one variable for which variables are included on both sides.

A.LFE.05E Solve systems of two linear equations with two variables for mathematical problems.

A.LFE.05F	Solve systems of two linear equations with two variables for real-world problems.
A.LFE.A	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A.LFE.B	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Strand 2: Quadratic Functions and Equations

A.QF.06A	Determine the domain of quadratic functions.
A.QF.06B	Determine the range of quadratic functions.
A.QF.06C	Represent the domain of quadratic functions using inequalities.
A.QF.06D	Represent the range of quadratic functions using inequalities.
A.QF.06E	Write equations of quadratic functions given the vertex and another point on the graph.
A.QF.06F	Write equations of quadratic functions in vertex form $f(x) = a(x - h)^2 + k$.
A.QF.06G	Rewrite equations of quadratic functions from vertex form to standard form $f(x) = ax^2 + bx + c$.
A.QF.06H	Write quadratic functions when given real solutions and graphs of their related equations.
A.QF.07A	Graph quadratic functions on the coordinate plane.
A.QF.07B	Use the graph of a quadratic function to identify the x-intercept.
A.QF.07C	Use the graph of a quadratic function to identify the y-intercept.
A.QF.07D	Use the graph of a quadratic function to identify zeros.
A.QF.07E	Use the graph of a quadratic function to identify the maximum value.
A.QF.07F	Use the graph of a quadratic function to identify the minimum value.
A.QF.07G	Use the graph of a quadratic function to identify the vertex.

A.QF.07H	Use the graph of a quadratic function to identify the equation of the axis of symmetry.
A.QF.07J	Describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions.
A.QF.07K	Determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$ for specific values of a.
A.QF.07L	Determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $f(x) + d$ for specific values of d.
A.QF.07M	Determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $f(x - c)$ for specific values of c.
A.QF.07N	Determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $f(bx)$ for specific values of b.
A.QF.08A	Solve quadratic equations having real solutions by factoring.
A.QF.08B	Solve quadratic equations having real solutions by taking square roots.
A.QF.08C	Solve quadratic equations having real solutions by completing the square.
A.QF.08D	Solve quadratic equations having real solutions by applying the quadratic formula.
A.QF.08E	Write, using technology, quadratic functions that provide a reasonable fit to data to estimate solutions for real-world problems.
A.QF.08F	Write, using technology, quadratic functions that provide a reasonable fit to data to make predictions for real-world problems.

Strand 3: Exponential Functions and Equations

A.EF.A	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
A.EF.09A	Determine the domain of exponential functions of the form $f(x) = ab^x$.
A.EF.09B	Represent the domain of exponential functions of the form $f(x) = ab^x$ using inequalities.
A.EF.09C	Determine the range of exponential functions of the form $f(x) = ab^x$.
A.EF.09D	Represent the range of exponential functions of the form $f(x) = ab^x$ using inequalities.

A.EF.09E	Interpret the meaning of the values of a in exponential functions of the form $f(x) = ab^x$ in real-world problems.
A.EF.09F	Interpret the meaning of the values of b in exponential functions of the form $f(x) = ab^x$ in real-world problems.
A.EF.09G	Write exponential functions that model growth in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical situations.
A.EF.09H	Write exponential functions that model growth in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from real-world situations.
A.EF.09J	Write exponential functions that model decay in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical situations.
A.EF.09K	Write exponential functions that model growth in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from real-world situations.
A.EF.09L	Graph exponential functions that model growth and identify the y-intercept in mathematical problems.
A.EF.09M	Graph exponential functions that model growth and identify the y-intercept in real-world problems.
A.EF.09N	Graph exponential functions that model growth and identify the asymptote in mathematical problems.
A.EF.09P	Graph exponential functions that model growth and identify the asymptote in real-world problems.
A.EF.09Q	Graph exponential functions that model decay and identify the y-intercept in mathematical problems.
A.EF.09R	Graph exponential functions that model decay and identify the y-intercept in real-world problems.
A.EF.09S	Graph exponential functions that model decay and identify the asymptote in mathematical problems.
A.EF.09T	Graph exponential functions that model decay and identify the asymptote, in real-world problems.
A.EF.09U	Write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.

Strand 4: Number and Algebraic Methods

A.NA.A	Interpret parts of an expression, such as terms, factors, and coefficients.
A.NA.B	Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.
A.NA.C	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
A.NA.10A	Add polynomials of degree one.
A.NA.10B	Add polynomials of degree two.
A.NA.10C	Subtract polynomials of degree one.
A.NA.10D	Subtract polynomials of degree two.
A.NA.10E	Multiply polynomials of degree one.
A.NA.10F	Multiply polynomials of degree two.
A.NA.10G	Determine the quotient of a polynomial of degree one when divided by a polynomial of degree one when the degree of the divisor does not exceed the degree of the dividend.
A.NA.10H	Determine the quotient of a polynomial of degree two when divided by a polynomial of degree one when the degree of the divisor does not exceed the degree of the dividend.
A.NA.10K	Determine the quotient of a polynomial of degree two when divided by a polynomial of degree two.
A.NA.10L	Rewrite polynomial expressions of degree one in equivalent forms using the distributive property.
A.NA.10M	Rewrite polynomial expressions of degree two in equivalent forms using the distributive property.
A.NA.10N	Factor, if possible, trinomials with real factors in the form ax^2+bx+c, including perfect square trinomials of degree two.
A.NA.10P	Decide if a binomial can be written as the difference of two squares.
A.NA.10R	Use the structure of a difference of two squares to rewrite a binomial.

A.NA.11A	Simplify numerical radical expressions involving square roots.
A.NA.11B	Simplify numeric expressions with integral exponents using the laws of exponents.
A.NA.11C	Simplify numeric expressions with rational exponents using the laws of exponents.
A.NA.11D	Simplify algebraic expressions with integral exponents using the laws of exponents.
A.NA.11E	Simplify algebraic expressions with rational exponents using the laws of exponents.
A.NA.12A	Decide whether relations represented verbally define a function.
A.NA.12B	Decide whether relations represented tabularly define a function.
A.NA.12C	Decide whether relations represented graphically define a function.
A.NA.12D	Decide whether relations represented symbolically define a function.
A.NA.12E	Evaluate functions, expressed in function notation, given one or more elements in their domains.
A.NA.12F	Identify terms of arithmetic sequences when the sequences are given in function form using recursive processes.
A.NA.12G	Identify terms of geometric sequences when the sequences are given in function form using recursive processes.
A.NA.12H	Write a formula for the n^{th} term of arithmetic sequences, given the value of several of their terms.
A.NA.12J	Write a formula for the n^{th} term of geometric sequences, given the value of several of their terms.
A.NA.12K	Solve mathematic formulas for a specified variable.
A.NA.12L	Solve scientific formulas for a specified variable.
A.NA.12M	Solve literal equations (excluding mathematic and scientific formulas) for a specified variable.

Strand 5: Building Functions

A.BF.A	Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
A.BF.B	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.

Strand 6: Interpreting Functions

A.F.A	Compare and contrast absolute value, step and piecewise- defined functions with linear, quadratic, and exponential functions.
A.F.B	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Strand 7: Interpreting Categorical and Quantitative Data

A.CQ.A	Represent data with plots on the real number line (dot plots, histograms, and box plots).
A.CQ.B	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
A.CQ.C	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
A.CQ.D	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
A.CQ.E	Informally assess the fit of a function by plotting and analyzing residuals.

Appendix C – RME Subject Matter Experts Biographies

Subject Matter Expert 1 holds a Master’s degree in Mathematics Curriculum and Instruction, a Bachelor’s degree in Mathematics Education, and holds credentials in teaching mathematics to students in Grades 4-8 and 8-12. She has three years of experience teaching Geometry and Pre-calculus. She is currently the Professional Development Coordinator for a mathematics research unit.

Subject Matter Expert 2 holds a Master's degree in Educational Leadership and Policy Studies, a Bachelor's degree in Mathematics Education, and holds credentials in teaching mathematics to students in Grades 1-12. She has eleven years of experience teaching elementary, middle, and high school mathematics, as a mathematics instructional specialist, and as a district administrator.

Subject Matter Expert 3 holds a Doctorate in Curriculum & Instruction with a focus on Mathematics Education, a Master’s degree in Mathematics, and a Bachelor’s degree in Mathematics. She has eighteen years of experience as a classroom teacher at the elementary level, a mathematics teacher and beginning teacher support and assessment support provider at the secondary level, and is an assistant professor of mathematics.

Subject Matter Expert 4 holds a Bachelor’s degree in Mathematics, and with his certification to teach mathematics to students in Grades 8-12. He has six years of experience teaching high school mathematics.

Appendix D: Item Assignment by Strand

Strand	CCSS-M	VA SOL	TEKS	Overall Average (%)	Assigned Percentage	Number of Items
	% of total content standards					
Linear functions, equations and inequalities	30	46	47	41	**35**	**280**
Quadratic functions and equations	22	22	16	20	**21**	**168**
Exponential functions and equations	17	0	10	9	**13**	**104**
Number and algebraic methods	20	19	27	22	**22**	**176**
Building functions	3	3	0	2	**2**	**16**
Interpreting functions	2	3	0	2	**2**	**16**
Interpreting categorical and quantitative data	6	8	0	5	**5**	**40**
						800