Probability Sampling: An Application

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Introduction

- General Probability Sampling
  - Definition of a Probability Sample
  - Cost Function and Constraints
  - Sample Design
  - Objective Function
  - Tools
- Application
  - Background Information
  - Development of Cost Function, Constraints
  - Development of Sample Design
  - Development of Objective Function
  - Implementation
- Morals of the Story

Probability Sample

- Definition: A sample selected in such a way that every member of the sample has a known chance of selection (and every member of the population has a nonzero chance of selection)
- This definition does not require EQUAL probability of selection, as long as we use some sort of randomness to make our decisions.
- Sampling Units
  - Primary Sampling Unit (PSU)
  - Secondary Sampling Unit (SSU)

Probability Sample

- Advantages
  - Reduces risk of selection bias (allows for unbiased estimators of population parameters)
  - Produces results that are generalizable to the U.S. population (allow for known margins of error from population parameters)

Cost Function and Constraints

- Cost Function Formula for a Balanced Survey:

$$c_{total} = c_1 n + c_2 mn$$

n is the total number of PSUs

m is the total number of SSUs

- When the survey is unbalanced, replace m with $\bar{m}$
- Constraints can be
  - Financial
  - Time
  - Resources

Sample Design

- Depends on what information you have and what information you want…
  - Simple Random Sample
  - Cluster Sample
- Requires the use of auxiliary variable:
  - Stratified Sample
  - Probability Proportional to Size Sampling

Objective Function

- Helps us choose optimal sample size with two goals:
  - Minimize the cost with respect to a certain variance
  - Minimize the variance with respect to a certain cost
  - Both reduce to the same answer!
- Either way, we need to know how to calculate the variance of our statistic of interest.
  - Total
  - Mean
  - Proportion
  - Regression

Tools for Design

- Excel
  - Solver
    - Allows for customizable objective function
    - Allows for additional constraints
    - Extremely useful for choosing sample sizes
    - Can solve problems numerically
  - "Using Spreadsheet Solvers in Sample Design"
    - (Stokes & Plummer, 2004).

Tools for Analysis

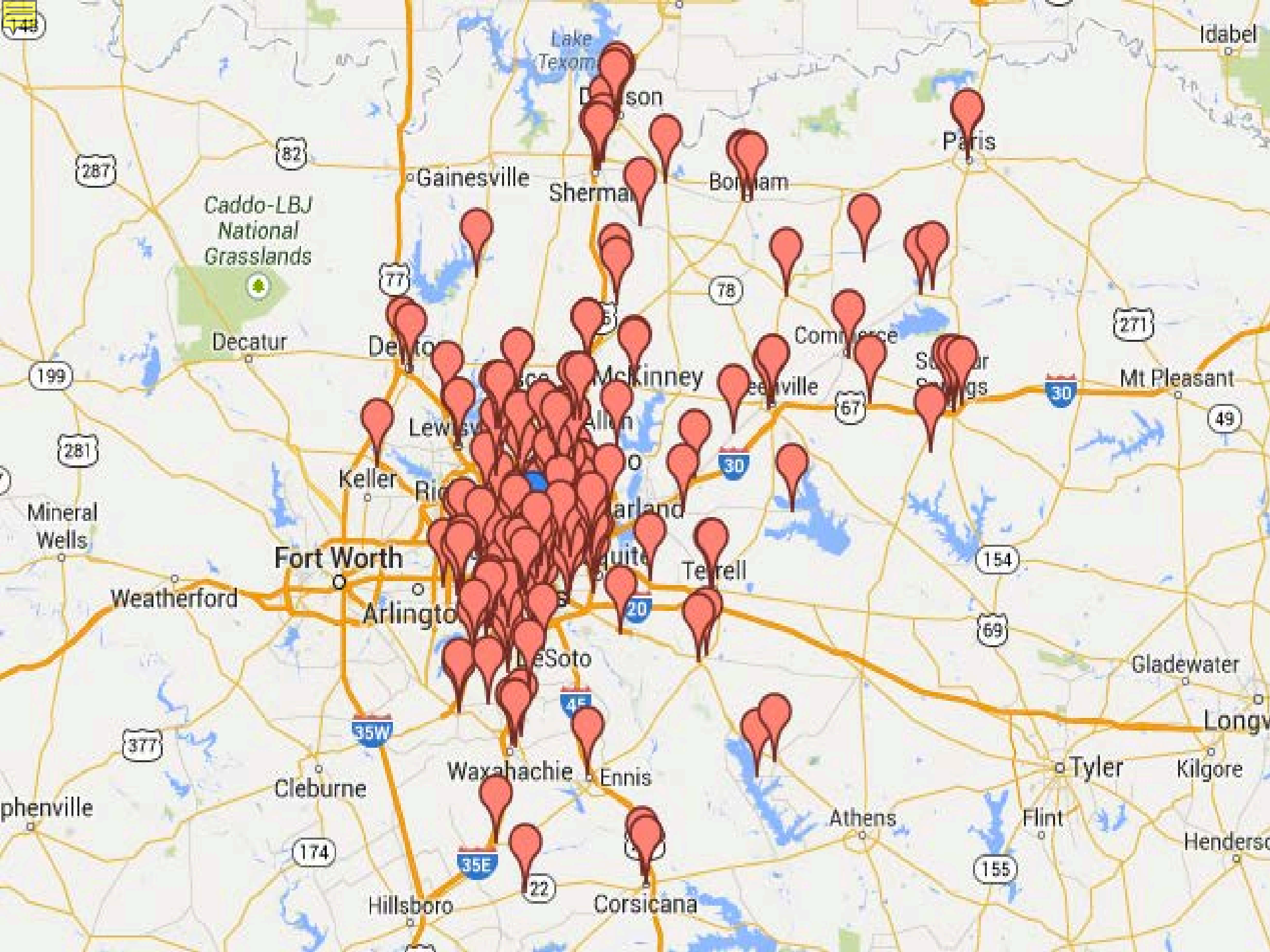
- SAS
  - Selection: SURVEYSELECT
  - Analysis of Means and Totals: SURVEYMEANS
  - Analysis of Regression: SURVEYREG
  - Analysis of Generalized Regression: SURVEYLOGISTIC
  - Analysis of Frequency Tables: SURVEYFREQ
  - Easily allows use of sampling unit weights
- Do not ignore weights when analyzing data!

Application

- Background Information:
  - Predict food-insecurity in the Greater North Texas Area
  - The grant, from the Hunger Center of Dallas, Subsidiary of North Texas Food Bank, focuses on
    - Financial Literacy
    - Social Network
    - Educational Level
  - Reaches beyond the unidirectional definition of poverty
- Sampling Units
  - 172 Food Banks (PSU)
  - Approximately 1000 Clients (SSU)

Application

- Additional Information on Food Banks
  - ID Number
  - Name
  - Number of clients served in the last three months
  - Mileage
  - Housing in America Survey Info
  - County
  - City
  - Zip Code
  - IBSCO Map
  - Unemployment Rate by County – Proxy Variable


Lake Texoma
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Sherman
Bonham
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Decatur
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Commerce
Mt Pleasant
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Mineral Wells
Fort Worth
Weatherford
Arlington
Terrell
DeSoto
Gladewater
Waxahachie
Ennis
Cleburne
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Athens
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155

Development of Cost Function

- Constraints:
  - Financial
  - Cost per survey: $50 per respondent
    - $20 incentive, given to the respondent
    - $15 paycheck, given to the interviewer
    - $10 paycheck, given for data entry
    - $5 reimbursement for gas and travel time
  - For N=1000, total cost is $50,000

Development of Cost Function

- Constraints
  - Time
    - Time per survey: 40 minutes
    - 35 minutes for survey
    - 5 minutes for travel time
  - For N=1000, total time is 660 hours
- Interviewers
  - 4 surveyors, contributing 165 hours each
  - If they spend 10 hours per week interviewing, the survey will take 16 weeks to complete.
  - Plan: conduct surveys November – February (4 months total)

Development of Cost Function

- Other practical issues:
  - Interviewers could work for a maximum in one day.
  - Interviews could potentially take longer than 30 minutes.
  - Interviewers were sent in pairs for safety reasons.

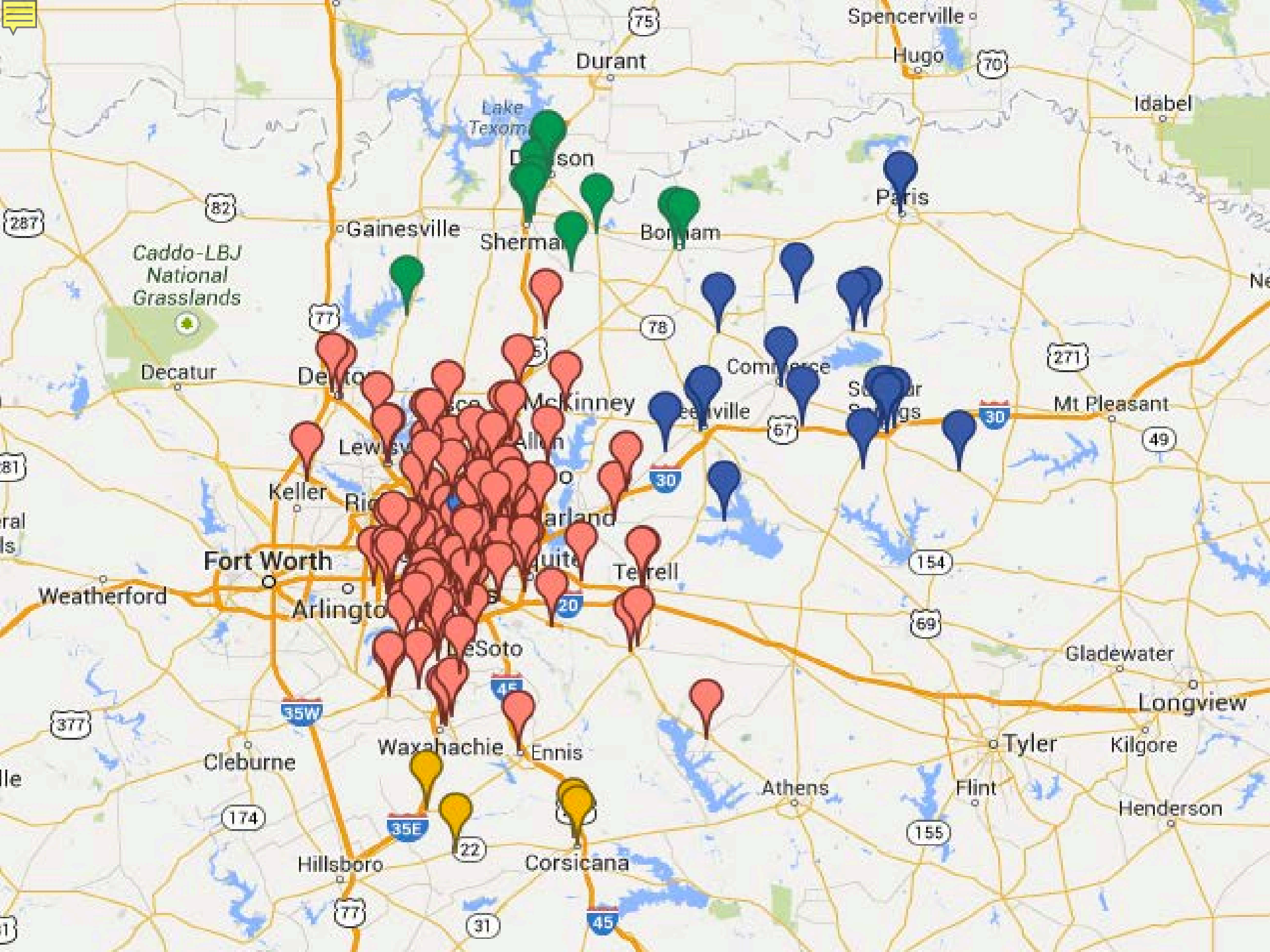
Development of Cost Function

- Assume that interviewers could conduct a constant number of interviews per day, and that the financial cost was constant up to travel reimbursement.
- Travel reimbursement is proportional to the miles the interviewers drive.

Cost = Miles x (Reimbursement + Hourly Wage/Average Speed)

Development of Sample Design

- Stratification
  - Ensures representation of both urban and rural areas
  - No objective definition of urban and rural
  - Instead, we ensured that food banks from the northern, southern, and eastern regions would be represented.
  - Four Geographical Strata
    - Metropolitan Region
    - North Region
    - South Region
    - East Region


Durant
Spencerville
Hugo
Idabel
Lake Texoma
Paris
Gainesville
Sherman
Bonham
Caddo-LBJ National Grasslands
Decatur
McKinney
Commerce
Mt Pleasant
Keller
Fort Worth
Weatherford
Arlington
Terrell
DeSoto
Gladewater
Longview
Tyler
Kilgore
Cleburne
Waxahachie
Ennis
Athens
Flint
Henderson
Hillsboro
Corsicana
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Development of Sample Design

- PPS Sampling
  - Assigned probabilities to each bank proportional to the number of clients served
  - Larger banks have a higher probability of selection
  - Sampling the same number of clients from each bank ensures approximately equal probability of selection for clients (makes analysis easier)

Development of Objective Function

- Goal: Predict food insecurity using decision trees
  - Closest Statistic: Proportion
- Objective: Minimize variance with respect to cost
- Variance:

$$Var_{total} = \sum [Var_{within} + Var_{between}]$$

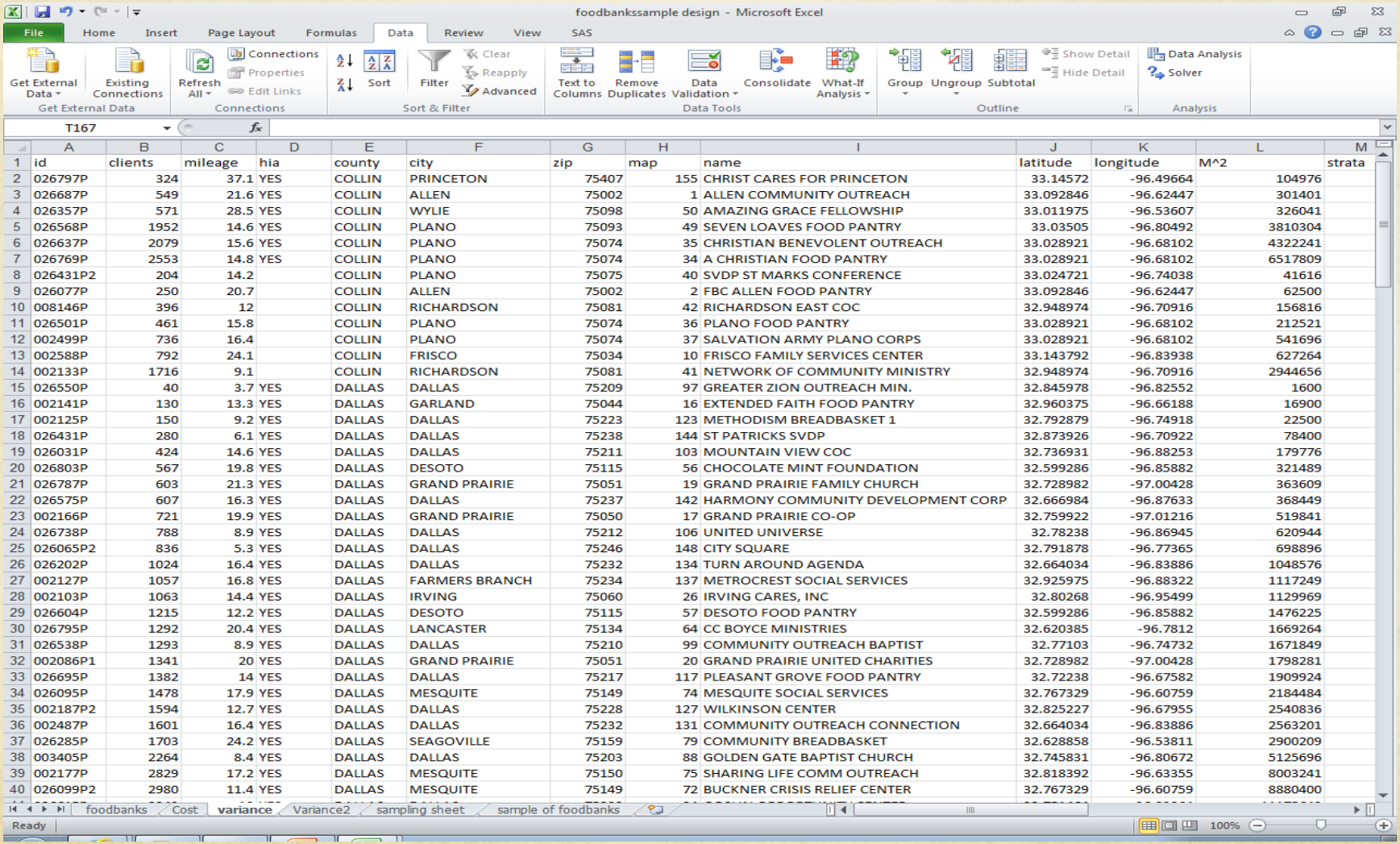
Objective Function

$$Var_{between} = N(1-\frac{n}{N})(\frac{\sum m^2(\pi(2\alpha-1)-E(\alpha))^2}{n-1})$$

$$Var_{within} = \frac{N}{n}\sum m^2(1-\frac{\min(m, N/n)}{m})V(\alpha)$$

where α represents the proxy variable, unemployment rate,
π represents the sampling weight based on the number of clients per bank,
and m represents the number of clients served at the bank.

Excel Solver



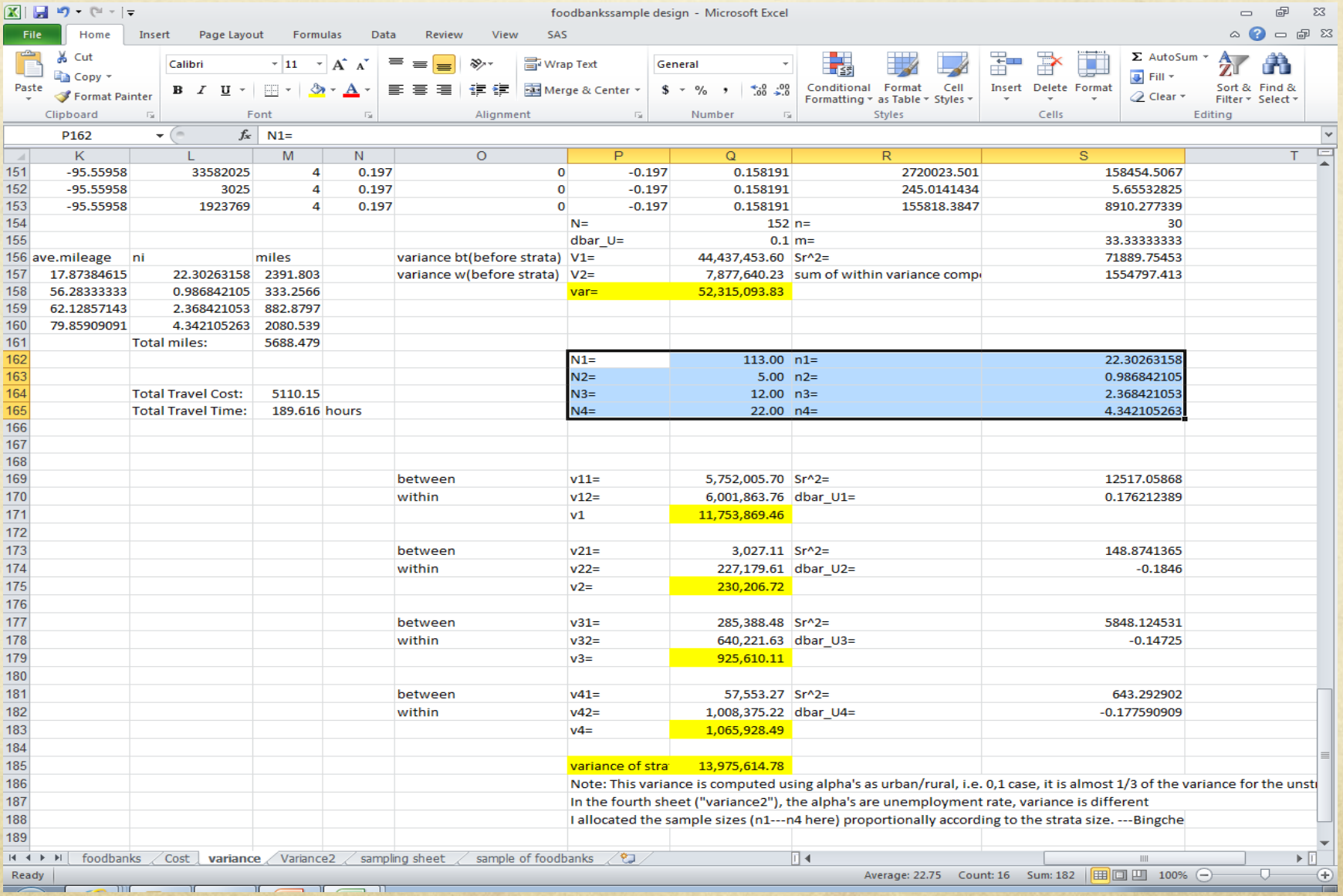
	A	B	C	D	E	F	G	H	I	J	K	L	M
1	id	clients	mileage	hia	county	city	zip	map	name	latitude	longitude	M^2	strata
2	026797P	324	37.1	YES	COLLIN	PRINCETON	75407	155	CHRIST CARES FOR PRINCETON	33.14572	-96.49664	104976	
3	026687P	549	21.6	YES	COLLIN	ALLEN	75002	1	ALLEN COMMUNITY OUTREACH	33.092846	-96.62447	301401	
4	026357P	571	28.5	YES	COLLIN	WYLIE	75098	50	AMAZING GRACE FELLOWSHIP	33.011975	-96.53607	326041	
5	026568P	1952	14.6	YES	COLLIN	PLANO	75093	49	SEVEN LOAVES FOOD PANTRY	33.03505	-96.80492	3810304	
6	026637P	2079	15.6	YES	COLLIN	PLANO	75074	35	CHRISTIAN BENEVOLENT OUTREACH	33.028921	-96.68102	4322241	
7	026769P	2553	14.8	YES	COLLIN	PLANO	75074	34	A CHRISTIAN FOOD PANTRY	33.028921	-96.68102	6517809	
8	026431P2	204	14.2		COLLIN	PLANO	75075	40	SVDP ST MARKS CONFERENCE	33.024721	-96.74038	41616	
9	026077P	250	20.7		COLLIN	ALLEN	75002	2	FBC ALLEN FOOD PANTRY	33.092846	-96.62447	62500	
10	008146P	396	12		COLLIN	RICHARDSON	75081	42	RICHARDSON EAST COC	32.948974	-96.70916	156816	
11	026501P	461	15.8		COLLIN	PLANO	75074	36	PLANO FOOD PANTRY	33.028921	-96.68102	212521	
12	002499P	736	16.4		COLLIN	PLANO	75074	37	SALVATION ARMY PLANO CORPS	33.028921	-96.68102	541696	
13	002588P	792	24.1		COLLIN	FRISCO	75034	10	FRISCO FAMILY SERVICES CENTER	33.143792	-96.83938	627264	
14	002133P	1716	9.1		COLLIN	RICHARDSON	75081	41	NETWORK OF COMMUNITY MINISTRY	32.948974	-96.70916	2944656	
15	026550P	40	3.7	YES	DALLAS	DALLAS	75209	97	GREATER ZION OUTREACH MIN.	32.845978	-96.82552	1600	
16	002141P	130	13.3	YES	DALLAS	GARLAND	75044	16	EXTENDED FAITH FOOD PANTRY	32.960375	-96.66188	16900	
17	002125P	150	9.2	YES	DALLAS	DALLAS	75223	123	METHODISM BREADBASKET 1	32.792879	-96.74918	22500	
18	026431P	280	6.1	YES	DALLAS	DALLAS	75238	144	ST PATRICKS SVDP	32.873926	-96.70922	78400	
19	026031P	424	14.6	YES	DALLAS	DALLAS	75211	103	MOUNTAIN VIEW COC	32.736931	-96.88253	179776	
20	026803P	567	19.8	YES	DALLAS	DESOTO	75115	56	CHOCOLATE MINT FOUNDATION	32.599286	-96.85882	321489	
21	026787P	603	21.3	YES	DALLAS	GRAND PRAIRIE	75051	19	GRAND PRAIRIE FAMILY CHURCH	32.728982	-97.00428	363609	
22	026575P	607	16.3	YES	DALLAS	DALLAS	75237	142	HARMONY COMMUNITY DEVELOPMENT CORP	32.666984	-96.87633	368449	
23	002166P	721	19.9	YES	DALLAS	GRAND PRAIRIE	75050	17	GRAND PRAIRIE CO-OP	32.759922	-97.01216	519841	
24	026738P	788	8.9	YES	DALLAS	DALLAS	75212	106	UNITED UNIVERSE	32.78238	-96.86945	620944	
25	026065P2	836	5.3	YES	DALLAS	DALLAS	75246	148	CITY SQUARE	32.791878	-96.77365	698896	
26	026202P	1024	16.4	YES	DALLAS	DALLAS	75232	134	TURN AROUND AGENDA	32.664034	-96.83886	1048576	
27	002127P	1057	16.8	YES	DALLAS	FARMERS BRANCH	75234	137	METROCREST SOCIAL SERVICES	32.925975	-96.88322	1117249	
28	002103P	1063	14.4	YES	DALLAS	IRVING	75060	26	IRVING CARES, INC	32.80268	-96.95499	1129969	
29	026604P	1215	12.2	YES	DALLAS	DESOTO	75115	57	DESOTO FOOD PANTRY	32.599286	-96.85882	1476225	
30	026795P	1292	20.4	YES	DALLAS	LANCASTER	75134	64	CC BOYCE MINISTRIES	32.620385	-96.7812	1669264	
31	026538P	1293	8.9	YES	DALLAS	DALLAS	75210	99	COMMUNITY OUTREACH BAPTIST	32.77103	-96.74732	1671849	
32	002086P1	1341	20	YES	DALLAS	GRAND PRAIRIE	75051	20	GRAND PRAIRIE UNITED CHARITIES	32.728982	-97.00428	1798281	
33	026695P	1382	14	YES	DALLAS	DALLAS	75217	117	PLEASANT GROVE FOOD PANTRY	32.72238	-96.67582	1909924	
34	026095P	1478	17.9	YES	DALLAS	MESQUITE	75149	74	MESQUITE SOCIAL SERVICES	32.767329	-96.60759	2184484	
35	002187P2	1594	12.7	YES	DALLAS	DALLAS	75228	127	WILKINSON CENTER	32.825227	-96.67955	2540836	
36	002487P	1601	16.4	YES	DALLAS	DALLAS	75232	131	COMMUNITY OUTREACH CONNECTION	32.664034	-96.83886	2563201	
37	026285P	1703	24.2	YES	DALLAS	SEAGOVILLE	75159	79	COMMUNITY BREADBASKET	32.628858	-96.53811	2900209	
38	003405P	2264	8.4	YES	DALLAS	DALLAS	75203	88	GOLDEN GATE BAPTIST CHURCH	32.745831	-96.80672	5125696	
39	002177P	2829	17.2	YES	DALLAS	MESQUITE	75150	75	SHARING LIFE COMM OUTREACH	32.818392	-96.63355	8003241	
40	026099P2	2980	11.4	YES	DALLAS	MESQUITE	75149	72	BUCKNER CRISIS RELIEF CENTER	32.767329	-96.60759	8880400	

Excel Solver

- Solver is an available add-in to MS Excel that handles sampling design
  - Objective Variable
  - Changing Variables
  - Constraints

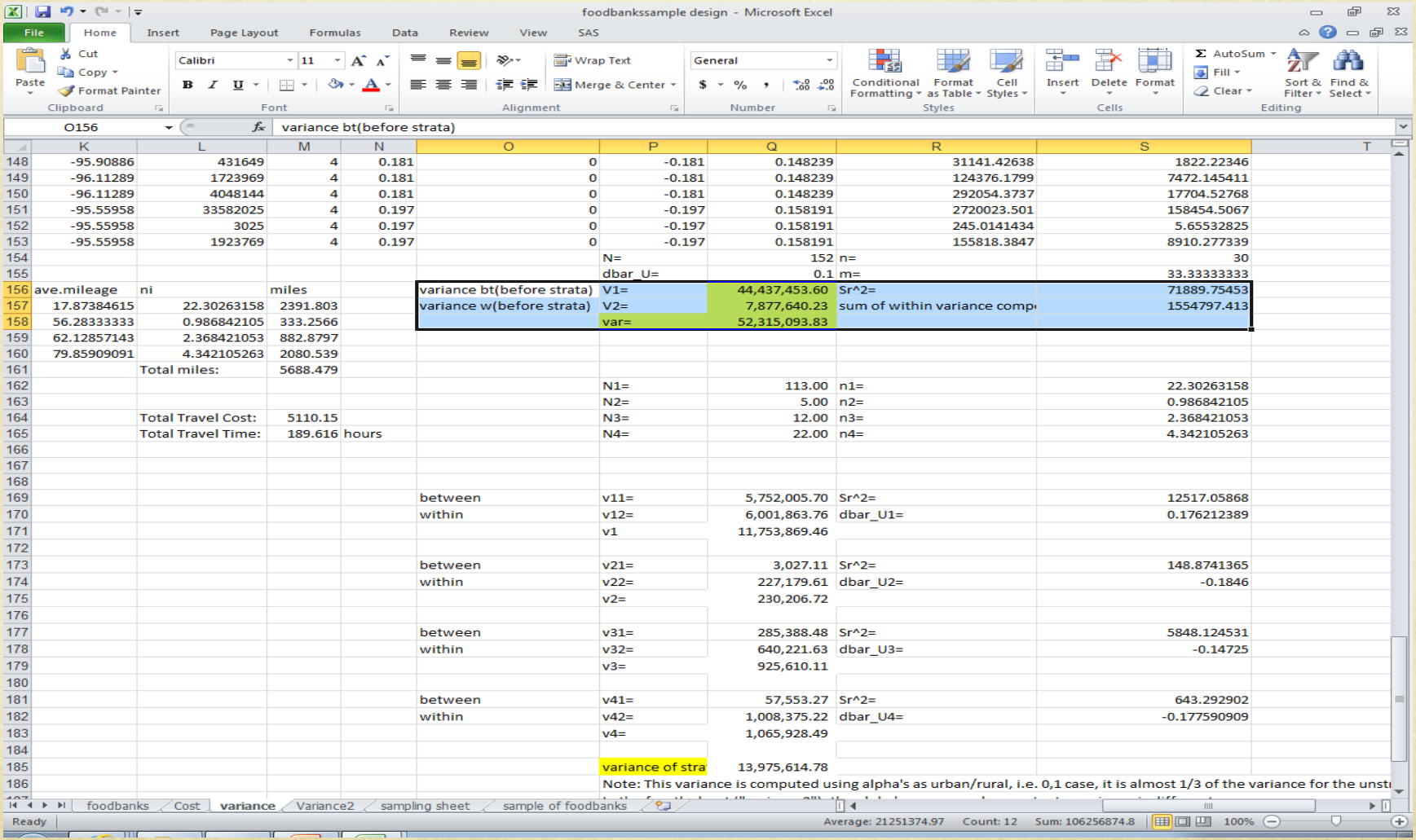
- Obtains an optimal solution when it exists

Excel Solver

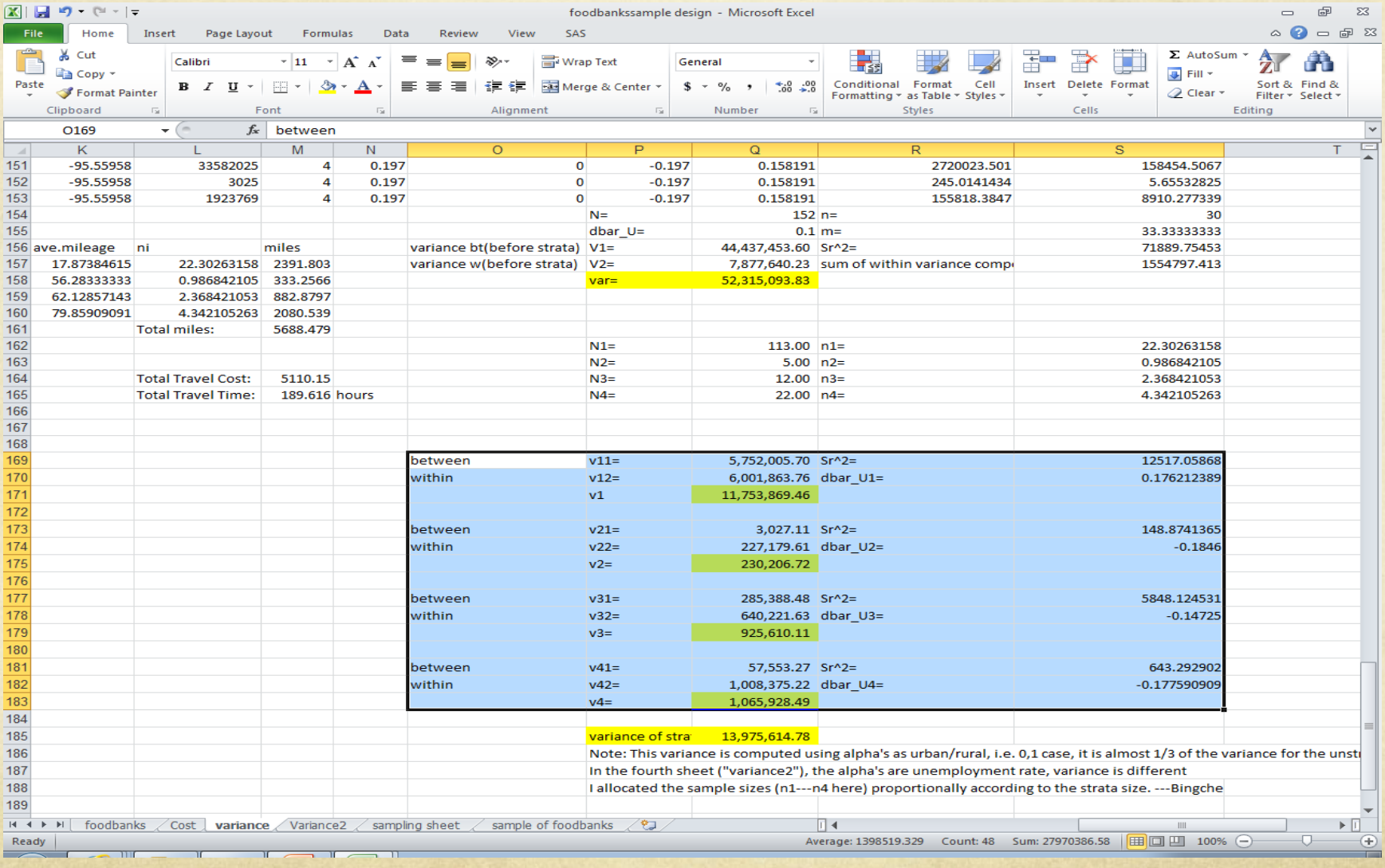


	K	L	M	N	O	P	Q	R	S	T
151	-95.55958	33582025	4	0.197	0	-0.197	0.158191	2720023.501	158454.5067	
152	-95.55958	3025	4	0.197	0	-0.197	0.158191	245.0141434	5.65532825	
153	-95.55958	1923769	4	0.197	0	-0.197	0.158191	155818.3847	8910.277339	
154						N=	152	n=	30	
155						dbar_U=	0.1	m=	33.33333333	
156	ave.mileage	ni	miles		variance bt(before strata)	V1=	44,437,453.60	Sr^2=	71889.75453	
157	17.87384615	22.30263158	2391.803		variance w(before strata)	V2=	7,877,640.23	sum of within variance comp	1554797.413	
158	56.28333333	0.986842105	333.2566			var=	52,315,093.83			
159	62.12857143	2.368421053	882.8797							
160	79.85909091	4.342105263	2080.539							
161		Total miles:	5688.479							
162						N1=	113.00	n1=	22.30263158	
163						N2=	5.00	n2=	0.986842105	
164		Total Travel Cost:	5110.15			N3=	12.00	n3=	2.368421053	
165		Total Travel Time:	189.616	hours		N4=	22.00	n4=	4.342105263	
166										
167										
168										
169					between	v11=	5,752,005.70	Sr^2=	12517.05868	
170					within	v12=	6,001,863.76	dbar_U1=	0.176212389	
171						v1	11,753,869.46			
172										
173					between	v21=	3,027.11	Sr^2=	148.8741365	
174					within	v22=	227,179.61	dbar_U2=	-0.1846	
175						v2=	230,206.72			
176										
177					between	v31=	285,388.48	Sr^2=	5848.124531	
178					within	v32=	640,221.63	dbar_U3=	-0.14725	
179						v3=	925,610.11			
180										
181					between	v41=	57,553.27	Sr^2=	643.292902	
182					within	v42=	1,008,375.22	dbar_U4=	-0.177590909	
183						v4=	1,065,928.49			
184										
185						variance of stra	13,975,614.78			
186						Note: This variance is computed using alpha's as urban/rural, i.e. 0,1 case, it is almost 1/3 of the variance for the unst				
187						In the fourth sheet ("variance2"), the alpha's are unemployment rate, variance is different				
188						I allocated the sample sizes (n1---n4 here) proportionally according to the strata size. ---Bingche				
189										

Excel Solver - Variance



Excel Solver - Variance



foodbankssample design - Microsoft Excel

File Home Insert Page Layout Formulas Data Review View SAS

Cut Copy Format Painter Paste — Clipboard
Calibri 11 — Font
Wrap Text Merge & Center — Alignment
General — Number
Conditional Formatting Format as Table Cell Styles — Styles
Insert Delete Format — Cells
AutoSum Fill Clear Sort & Filter Find & Select — Editing

O169 between

	K	L	M	N	O	P	Q	R	S	T
151	-95.55958	33582025	4	0.197	0	-0.197	0.158191	2720023.501	158454.5067	
152	-95.55958	3025	4	0.197	0	-0.197	0.158191	245.0141434	5.65532825	
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158	56.28333333	0.986842105	333.2566			var=	52,315,093.83			
159	62.12857143	2.368421053	882.8797							
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162						N1=	113.00	n1=	22.30263158	
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164		Total Travel Cost:	5110.15			N3=	12.00	n3=	2.368421053	
165		Total Travel Time:	189.616	hours		N4=	22.00	n4=	4.342105263	
166										
167										
168										
169					between	v11=	5,752,005.70	Sr^2=	12517.05868	
170					within	v12=	6,001,863.76	dbar_U1=	0.176212389	
171						v1	11,753,869.46			
172										
173					between	v21=	3,027.11	Sr^2=	148.8741365	
174					within	v22=	227,179.61	dbar_U2=	-0.1846	
175						v2=	230,206.72			
176										
177					between	v31=	285,388.48	Sr^2=	5848.124531	
178					within	v32=	640,221.63	dbar_U3=	-0.14725	
179						v3=	925,610.11			
180										
181					between	v41=	57,553.27	Sr^2=	643.292902	
182					within	v42=	1,008,375.22	dbar_U4=	-0.177590909	
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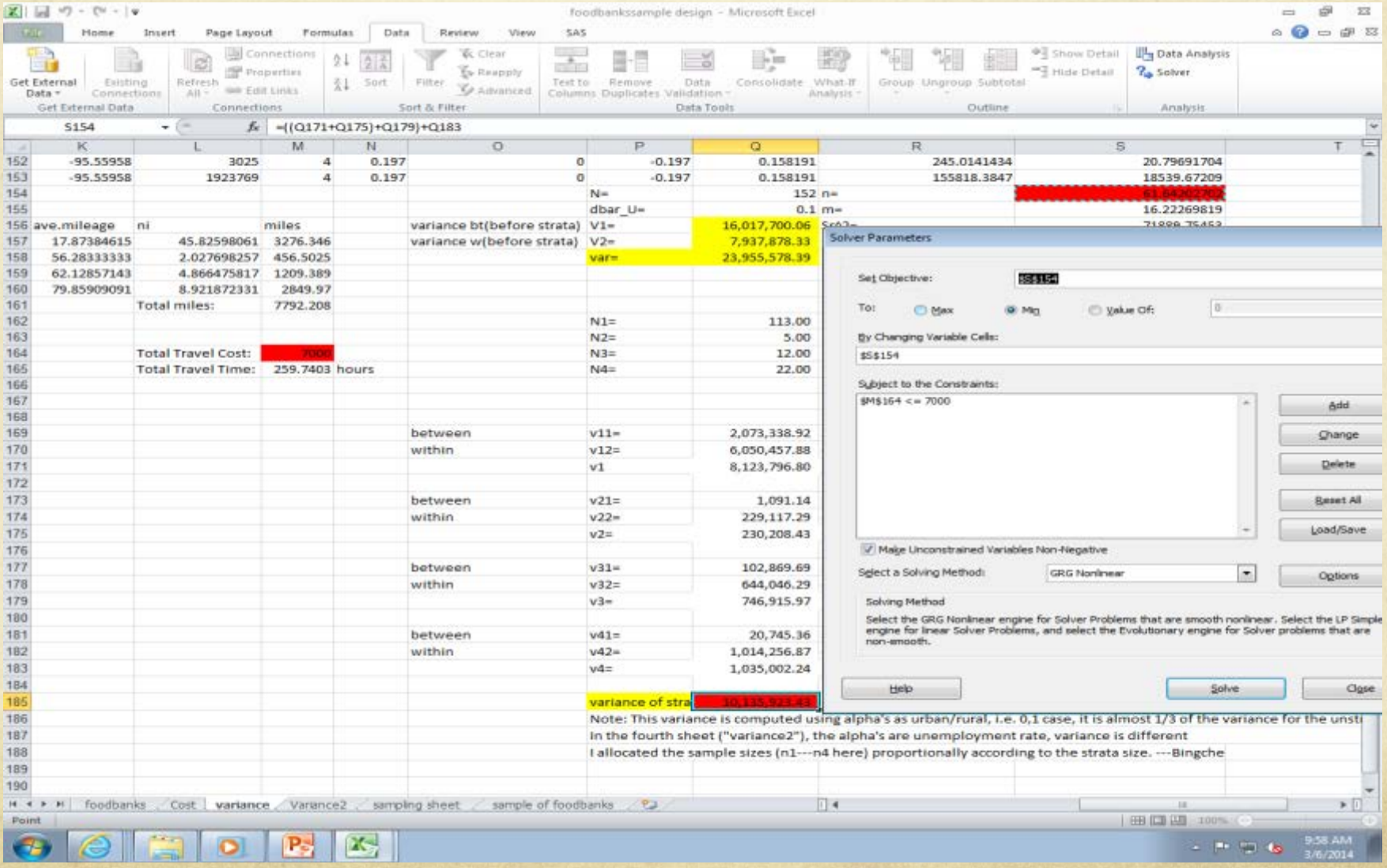
189										

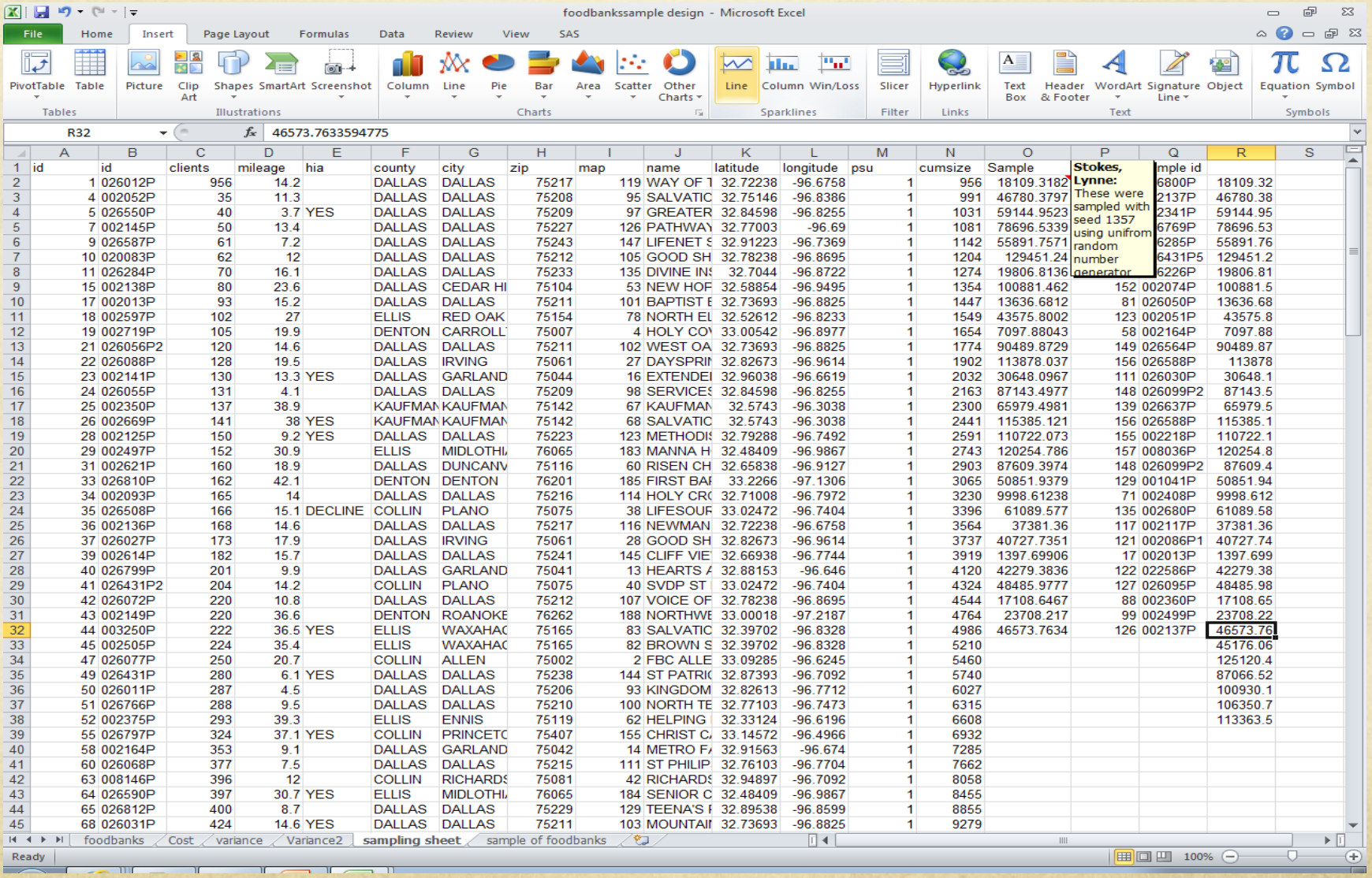
foodbanks Cost variance Variance2 sampling sheet sample of foodbanks

Ready — Average: 1398519.329 Count: 48 Sum: 27970386.58 — 100%

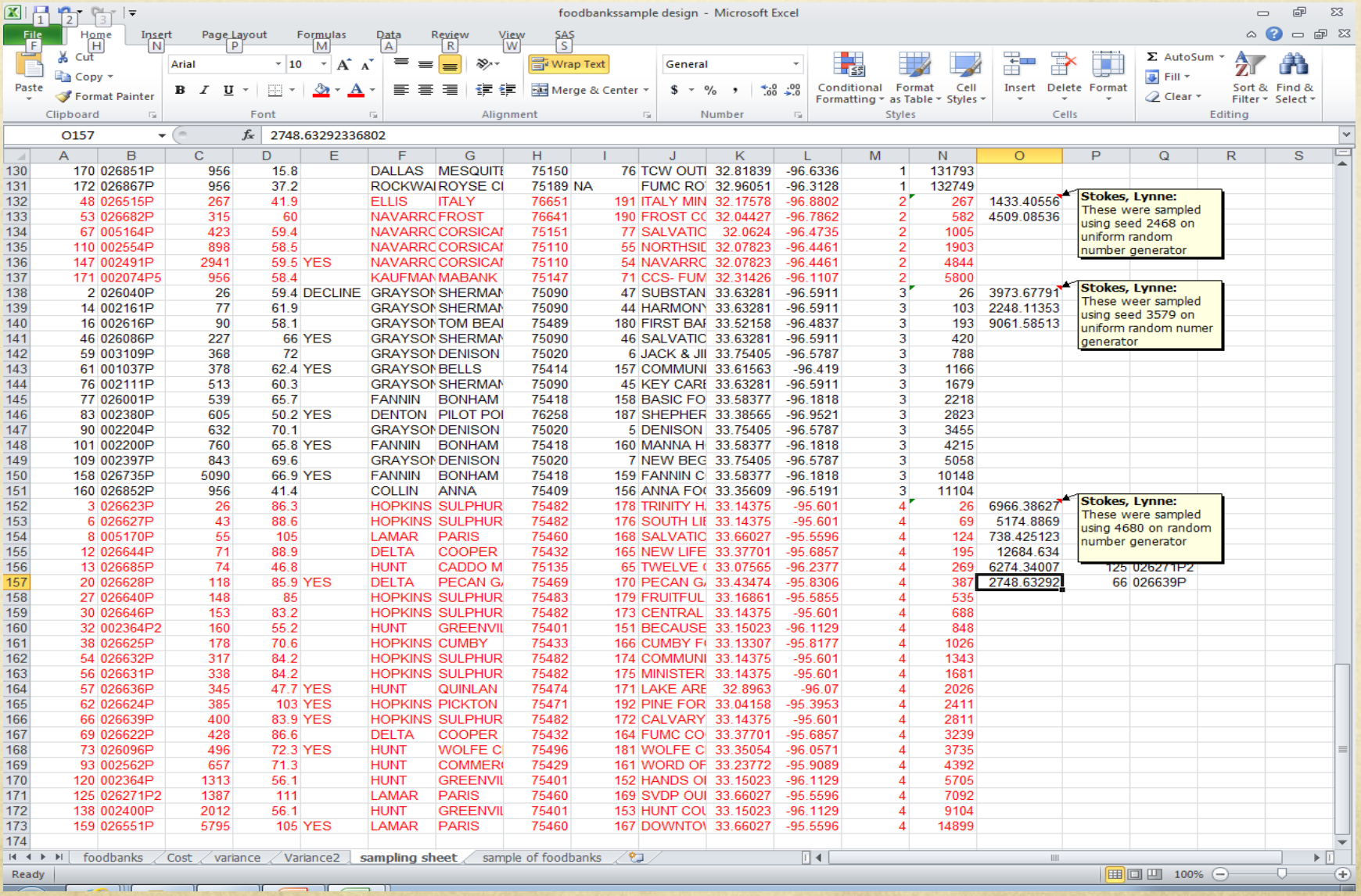
Excel Solver – Minimize Sample Size Based on Variance



Excel Solver – Final Sample



Excel Solver – Final Sample



Final Sample

- Eight Interviews per Day
  - 42 Food Banks
  - 24 Clients per Food Bank
- Twelve Interviews per Day
  - 30 Food Banks
  - 33 Clients per Food Bank

Other Practical Issues

- How do we select clients once we're actually inside the food banks?
  - As randomly as possible
  - No possible frame
  - Choose them systematically, i.e., every tenth person
- Non-response: Clients may choose not to participate

Concluding Remarks

- When in doubt, it's helpful to think of what we would do if we had more data.
- It's also helpful to start from a simple scenario where you know all the formulas and analysis and work your way up to a more complicated scenario.

Concluding Remarks

- This is not a math problem! We rarely have enough information to find the optimal solution, and if we do, it may be too complicated. Our goal is to do something reasonable.
- Part of doing something reasonable is to take a look at different scenarios and see how the results may be affected.
- If you spend time making a complicated design, you need to make sure the analysis is also done the right way.

References

- Stokes, L., & Plummer, J. (2004). Using Spreadsheet Solvers in Sample Design. *Computational Statistics & Data Analysis* , *44* (3), 527-546.

Special thanks to the STAT 6380 students, especially Chelsea Allen and Charles South, for helping finish this project and providing materials for this presentation.