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Quantification of energy absorption capability in abrasive water jet machining

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The paper contains a mathematical model for the estimation of the energy absorption capability of materials during abrasive water jet machining based on an energy balance inside the workpiece. A parameter $\chi(h)$ is defined to describe and calculate the energy absorption capability. A method for the estimation of this parameter is developed based on a parabolic striation model. It is shown that the energy absorption depends on the depth of cut following a second-order equation. The relation between the relative depth of cut h/h_{max} and the energy absorption capability $\chi(h)$ can also be described by a second-order equation. For such materials as aluminium, cast iron and stainless steel a critical point of abrasive water jet energy absorption is detected at a depth of cut of $h = 0.52h_{max}$, which corresponds to a striation angle of about 75°.

Key words: energy absorption capability, abrasive water jet machining, energy balance

NOTATION

a, b, c	parabola regression parameters
A, B, C	regression parameters
$A(h)$	area, occupied by the striation
A_0	area, occupied by the striation at $h = h_{max}$
E_{AB}	energy, absorbed by the material
E_C	material threshold energy
E_{IN}	AWJ input energy
E_L	AWJ exit energy
h	depth of cut
h_{max}	maximum depth of cut
K	parabola function parameter
$\dot{m}_p$	abrasive flowrate
p_C	material threshold pressure
s	stand-off distance
v	traverse rate
x	cutting ordinate
α	striation angle
ξ	energy absorption intensity parameter
Φ	relative depth of cut h/h_{max}
χ	energy absorption parameter

1 INTRODUCTION

As a new manufacturing process, abrasive water jet (AWJ) cutting has been very effective in machining difficult-to-machine materials. This cutting technique is one of the most recently introduced machining methods. AWJs are used for cutting a wide range of materials, including ceramics (1, 2), and composite materials (3, 4). As shown by Laurinat *et al.* (5), AWJs also have the potential for milling operations and three-dimensional machining. A typical commercial AWJ system consists of a pressure generator, typically an intensifier pump, an abrasive cutting head, an *xyz* positioning system and a catcher. On the basis of jet generation, abrasive water jets can generally be categorized as injection jets or suspension jets. For practical applications, injection jets are more commonly used. For this type of jet, the pressure ranges between 100 and 400 MPa. An injection AWJ is formed by accelerating small abrasive particles (garnet, aluminium oxide, silica carbide) through contact with a high-velocity plain water jet. A typical abrasive grain diameter is 400 μm. The plain water jet is formed in an orifice on top of the abrasive cutting head. The abrasives enter the cutting head at a separate entry. The mixing between abrasive, water and air takes place in a mixing chamber, and the acceleration process occurs in an acceleration tube, or abrasive waterjet nozzle. The abrasive particles leave this nozzle at velocities of several hundred metres per second. A high number of abrasives (10^5 per second) leads to a high-frequency impingement on the materials being processed. This process is illustrated in Fig. 1. The intensity and the efficiency of the cutting process depend on several process parameters, such as pump pressure, orifice diameter, traverse rate, stand-off distance, abrasive flowrate, abrasive type and mixing chamber geometry.

AWJs are stream-like tools, similar to lasers and electron beams, which are characterized by an unsteady material removal process. The most pronounced characteristic of AWJ-generated surfaces is the presence of striation marks which transpire below a region of relatively smooth surface finish as shown in Fig. 2. Detailed observations in transparent materials by Hashish (6) and Blickwedel (7) and in opaque materials by Kovacevic *et al.* (8) have revealed that the material removal in an AWJ cutting process occurs in two stages. In the first stage the abrasive particles strike the surface at shallow angles, producing a relatively smooth surface. The secondary stage, which yields unsteady cutting with striation marks, is controlled by erosive wear due to

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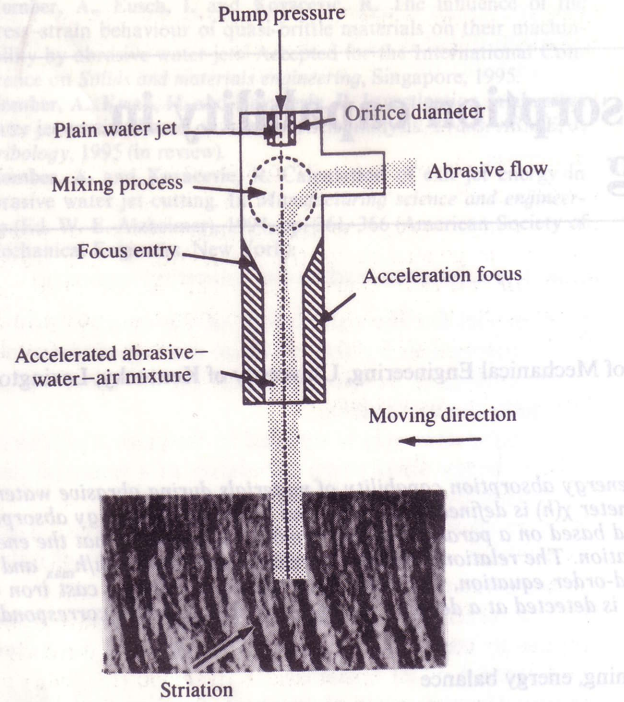

Fig. 1 Principles of injection of abrasive water jet generation and performance

particles impacting at large angles of attack. Arola and Ramulu (3) suggest that the striation formation is related only to the available AWJ energy at a certain depth of cut. During the cutting process a certain amount of AWJ energy is absorbed by the material, which must be considered in the development of cutting models and in the abrasive water jet cutting process interpretation. In contrast, Chao and Geskin (9) assumed that the striation formation is independent of the material removal process, and observed a strong relation between machine vibrations and the structure

Fig. 2 Striation formation during abrasive water jet machining (sample: aluminium)

of the generated striations. This may be questionable, because investigations of Hashish (10), who used a material with very high erosion resistance to filter external effects, showed that external effects do not influence the striation formation.

A part of the AWJ input energy is consumed by the workpiece and contributes to the material destruction. As shown by Momber *et al.* (11, 12), about 1.0 per cent of the AWJ energy is used to remove the material. The rest of the energy may be dissipated by different mechanisms, such as friction and heat generation (8). An important part of the AWJ energy is occupied by the mixture of abrasive, water and wear particles after it leaves the workpiece (13).

Capello and Groppetti (14) considered the energy loss during cutting in their energetic model and approximated it by using a negative power function. However, this assumption was not verified by experiments. Also, this function leads to a total energy loss only for an infinite cutting thickness. Serious investigations on the energy absorption capability during AWJ cutting have been carried out by Zeng *et al.* (15) and Momber and Kovacevic (13). Both groups modelled the shape of the striations using a parabolic function. Based on this model, Zeng *et al.* (15) estimated the angle between the workpiece and the exiting AWJ and used it as a process control parameter. Momber and Kovacevic (13) developed a calculation method for the estimation of the AWJ exit energy after cutting a workpiece, which is based on an energy balance and considers the influence of the depth of cut on the energy absorption during the process.

The objectives of the present paper are the development of a model for the quantitative estimation of the energy absorption capability of the materials during AWJ machining and the mathematical description of the influence of the depth of cut on the energy absorption process.

2 DEVELOPMENT OF THE ENERGY ABSORPTION MODEL

2.1 Energy balance during AWJ cutting

The general energy situation in cutting a material with an AWJ is shown in Fig. 3. According to this situation the energy which is absorbed during the cutting process, E_{AB}, can be estimated using

$$E_{AB} = E_{IN} - E_L \tag{1}$$

where E_{IN} is the input energy of the AWJ and E_L is the amount of energy in the jet when it leaves the workpiece. Many investigations in jet cutting have shown that a critical minimum energy E_C (critical pump pressure, critical traverse rate) is necessary to destroy a material and to initiate the cutting process. A review on this problem is given by Momber (16). An example which is based on measurements on stainless steel is shown in Fig. 4. It is assumed that this critical situation will also be found at the bottom of the cut in a workpiece ($h = h_{max}$), when the AWJ does not have enough energy to penetrate the material. This simple assumption leads to

$$E_{AB}(h = h_{max}) = E_{IN} - E_C \tag{2}$$

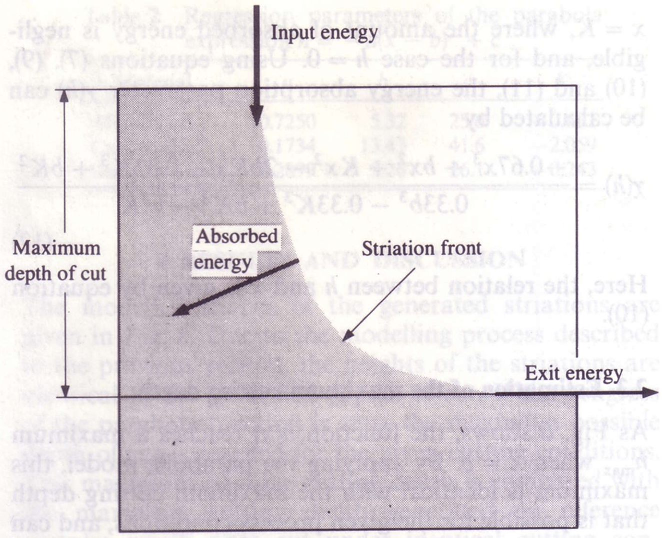

Fig. 3 Energy balance during abrasive water jet cutting

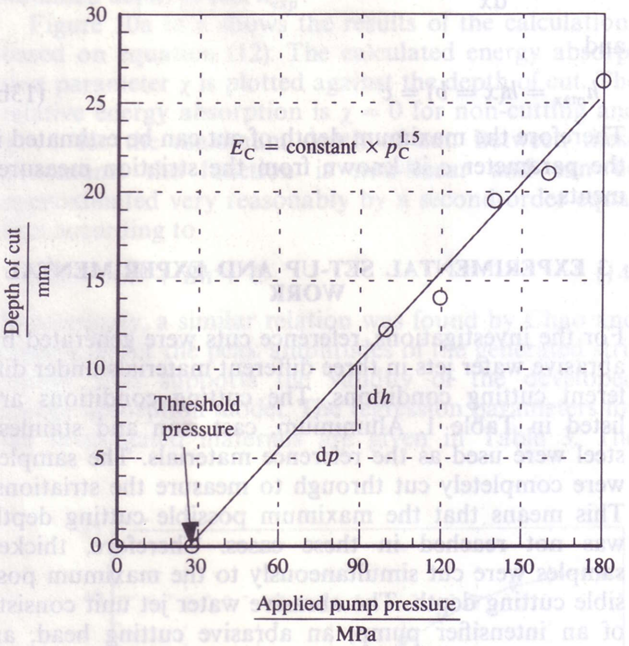

Fig. 4 Typical relation between pump pressure and cutting depth in abrasive water jet cutting (material: stainless steel): $v = 0.42$ mm/s, $s = 7.0$ mm, $\dot{m}_p = 3.4$ g/s, abrasive: garnet

Another critical condition exists in the case $h = 0$. Here no energy is absorbed by the material and the total amount of the AWJ energy goes into the exit jet. This situation leads to

$$E_{AB}(h = 0) = 0 \tag{3}$$

By introducing a parameter $\chi(h)$ which depends on the depth of cut and using equation (1), the situation can be described by equation (4) for all cases between these extremums:

$$E_{AB}(h) = \chi(h)(E_{IN} - E_C), \qquad 0 < h < h_{max} \tag{4}$$

For the conditions in equation (2), $\chi(h = h_{max}) = 1$ is obtained, whereas equation (3) leads to the case $\chi(h = 0) = 0$. Thus the parameter $\chi(h)$ considers the energy absorption of the material during cutting. Section 2.2 describes how to determine this parameter from the striation measurements.

2.2 Estimation of the energy absorption parameter $\chi(h)$

Figure 5 shows two simplifications of the cutting process with and without energy losses due to absorption during cutting. As Fig. 5a suggests, a rectangular area A_1 is generated as the jet moves in the traverse direction without energy loss. This is an ideal case. Realistically a striation would be found as shown in Fig. 5b, which occupies an area A_0. This area is used to characterize the energy loss in real cutting. The calculation of this area is given by

$$A_0 = \int_0^x h(x)\,dx \tag{5}$$

Zeng *et al.* (15) have assumed a parabolic curve to describe the shape of a striation. A similar relation—a second-order polynome—was found by Chao and Geskin (9) for the relation between the striation peak amplitude and kerf depth. These findings were developed further by Momber and Kovacevic (13), who used the following equation to estimate the function $h(x)$:

$$h(x) = -a(x - b)^2 + c \tag{6}$$

Here a, b and c are constants which can be evaluated by experiments. The role of these parameters is shown in Fig. 6. As the figure shows, for the case $x = b$ the area

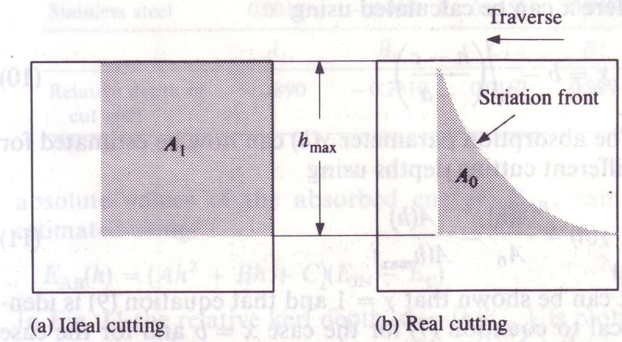

Fig. 5 Comparison between ideal cutting without energy losses and real cutting using abrasive water jet

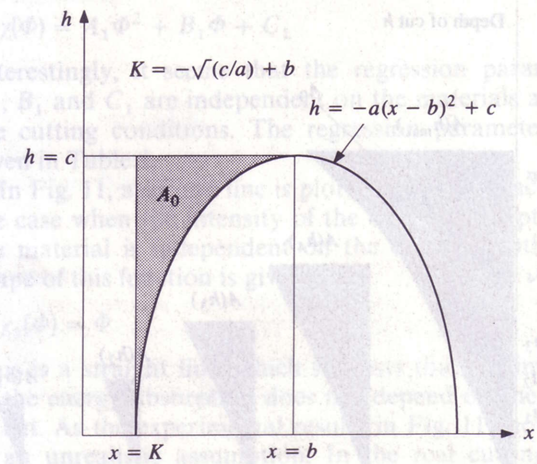

Fig. 6 Parabolic model of the abrasive water jet cutting process

A_0 can be calculated by

$$A_0 = (b - K)c - \int_K^b h(x)\,dx$$
$$= a(0.33b^3 - 0.33K^3 + bK^2 - b^2K) \quad (7)$$

The value K can be estimated for the case $h(x) = 0$ using

$$K = b - \sqrt{\left(\frac{c}{a}\right)} \quad (8)$$

It is assumed that A_0 includes all energy losses during cutting of the material up to a depth of $h = h_{max}$, which is the maximum cutting depth under the given process conditions. A change of the striation parameters a, b and c with the cutting depth is neglected. A_0 is not considered to have the unit of energy, but it represents the energy losses due to absorption. If this parameter is related to other area values from different depths of cut, it can be used to describe the dependence between energy absorption and depth of cut $\chi(h)$. Figure 7 shows that certain areas $A(h)$ exist on certain depth levels. The area $A(h)$ which is occupied by the striation during a certain cutting depth $h(x)$ is shown in Fig. 7 and can be described by

$$A(h) = (x - K)h(x) - \int_K^x h(x)\,dx$$
$$= a(-0.67x^3 + bx^2 + Kx^2 - 2bKx - 0.33K^3 + bK^2) \quad (9)$$

Here x can be calculated using

$$x = b - \sqrt{\left(\frac{h - c}{-a}\right)} \quad (10)$$

The absorption parameter $\chi(h)$ can now be estimated for different cutting depths using

$$\chi(h) = \frac{A(h)}{A_0} = \frac{A(h)}{A(h_{max})} \quad (11)$$

It can be shown that $\chi = 1$ and that equation (9) is identical to equation (7) for the case $x = b$ and for the case $h = h_{max}$. Also, it is easy to see that $\chi = 0$ for the case $x = K$, where the amount of absorbed energy is negligible, and for the case $h = 0$. Using equations (7), (9), (10) and (11), the energy absorption parameter $\chi(h)$ can be calculated by

$$\chi(h) = \frac{-0.67x^3 + bx^2 + Kx^2 - 2bKx - 0.33K^3 + bK^2}{0.33b^3 - 0.33K^3 + bK^2 - b^2K} \quad (12)$$

Here, the relation between h and x is given by equation (10).

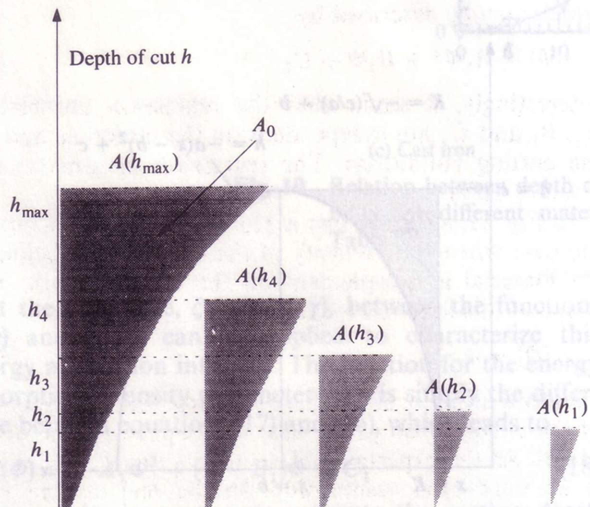

Fig. 7 Structure and partition of the model area

2.3 Estimation of the maximum cutting depth

As Fig. 6 shows, the function $h(x)$ reaches a maximum h_{max} when $x = b$. By applying the parabolic model, this maximum is identical with the maximum cutting depth that is possible for the given process conditions, and can be calculated using

$$\frac{d[-a(x - b)^2 + c]}{dx} = 0 \quad (13a)$$

and

$$h_{max} = h(x = b) = c \quad (13b)$$

Therefore the maximum depth of cut can be estimated if the parameter c is known from the striation measurements.

3 EXPERIMENTAL SET-UP AND EXPERIMENTAL WORK

For the investigations, reference cuts were generated by abrasive water jets in three different materials under different cutting conditions. The cutting conditions are listed in Table 1. Aluminium, cast iron and stainless steel were used as the reference materials. The samples were completely cut through to measure the striations. This means that the maximum possible cutting depth was not reached in these cases. Therefore, thicker samples were cut simultaneously to the maximum possible cutting depth. The abrasive water jet unit consists of an intensifier pump, an abrasive cutting head, an abrasive storage and metering system, a catcher and a CNC (computer numerical controlled) positioning system. The position of the cutting head was controlled using an xyz positioning table.

After cutting, the shapes of the striations on the cut walls (see Fig. 2) were measured using a video camera for magnification. After this the striations were modelled by using the parabolic approximation given in equation (6). The parameters of these regressions are listed in Table 2.

Table 1 Cutting parameters used in the experimental work

Parameter	Unit	Interval
Pump pressure	MPa	276
Traverse rate	mm/s	0.42–1.06
Abrasive flowrate	g/s	3.4
Abrasive material		Garnet 80
Stand-off distance	mm	9.0
Impact angle	deg	90
Orifice diameter	mm	0.254

Table 2 Regression parameters of the parabola expression $h = -a(x - b)^2 + c$

Material	a	b	c	K
Aluminium	0.7250	5.32	25.6	−0.622
Cast iron	0.1734	13.43	41.6	−2.059
Stainless steel	0.2899	9.26	26.1	−0.243

4 RESULTS AND DISCUSSION

The modelled shapes of the generated striations are given in Fig. 8. Due to the modelling process described in the previous section, the heights of the striations are identical to the generated depths of cut. If the progress of the parabola function is zero, the maximum possible depth of cut is reached for the given cutting conditions. This maximum possible cutting depth is compared with the maximum cutting depth generated on reference samples which were cut under identical cutting conditions. As Fig. 9 shows, the modelled depth of cut, $h_{max} = c$, is in good agreement with the experimentally obtained depth of cut, h_{exp}.

Figure 10a to c shows the results of the calculations based on equation (12). The calculated energy absorption parameter χ is plotted against the depth of cut. The relative energy absorption is $\chi = 0$ for non-cutting and $\chi = 1$ for the maximum depth of cut. Between these extremums the function is non-linear and can be approximated very reasonably by a second-order equation according to

$$\chi(h) = Ah^2 + Bh + C \tag{14}$$

Interestingly, a similar relation was found by Chao and Geskin (9) for the peak amplitudes of the generated striations. This supports the validity of the developed energy absorption model. The regression parameters for the investigated materials are given in Table 3. The

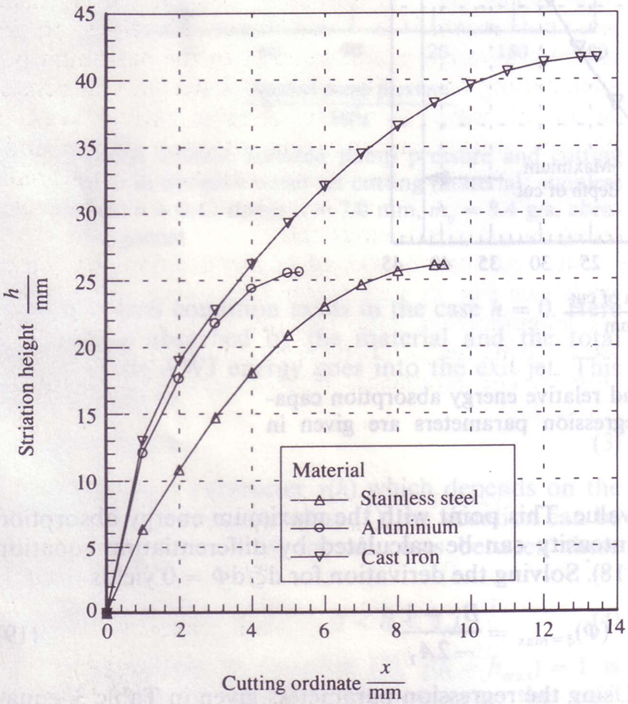

Fig. 8 Parabolic models of the generated striations (regression parameters are given in Table 2)

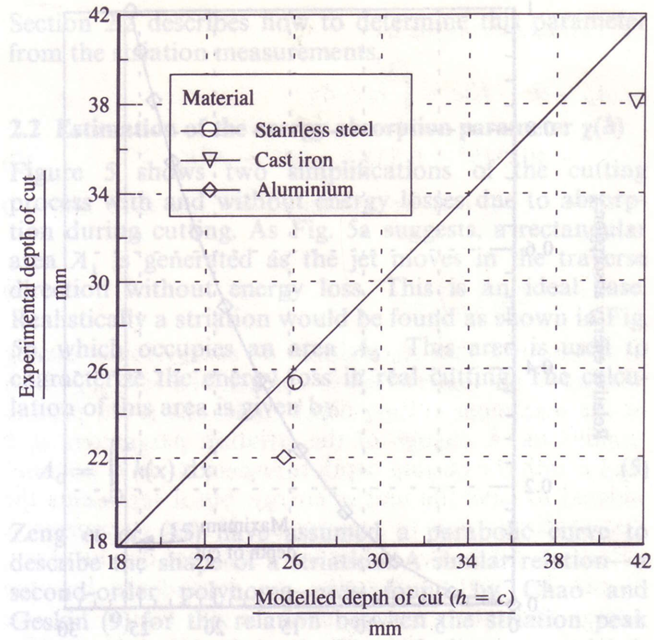

Fig. 9 Comparison between the calculated depth of cut using equation (13) and the experimentally estimated depth of cut

Table 3 Regression parameters for the approximation $\chi(h) = Ah^2 + Bh + C$

Material	A	B	C	R^2
Aluminium	0.0018	−0.0087	0.0156	0.999
Cast iron	0.0007	−0.0057	0.0191	0.999
Stainless steel	0.0018	−0.0088	0.0150	0.999
	$\bar{A}_1$	$\bar{B}_1$	$\bar{C}_1$	R^2
Relative depth of cut $\chi(\Phi)$	1.1890	−0.2310	0.0167	0.999

absolute values of the absorbed energy, E_{AB}, can be estimated using

$$E_{AB}(h) = (Ah^2 + Bh + C)(E_{IN} - E_C) \tag{15}$$

In Fig. 11 the relative kerf depth, $\Phi = (h/h_{max})$, is plotted against the relative energy absorption, $\chi(h)$, for the used materials and the different cutting conditions. It is seen that all plotted points can be fitted by the same line which may be described by

$$\chi(\Phi) = A_1\Phi^2 + B_1\Phi + C_1 \tag{16}$$

Interestingly, it seems that the regression parameters A_1, B_1 and C_1 are independent on the materials and on the cutting conditions. The regression parameters are given in Table 3.

In Fig. 11, a second line is plotted which characterizes the case when the intensity of the energy absorption of the material is independent on the cutting depth. The shape of this function is given by

$$\chi_1(\Phi) = \Phi \tag{17}$$

This is a straight line, which suggests that the intensity of the energy absorption does not depend on the depth of cut. As the experimental results in Fig. 11 show, this is an unrealistic assumption. In the real cutting with AWJ the intensity of the energy absorption changes with the depth of the generated cut. It is assumed now

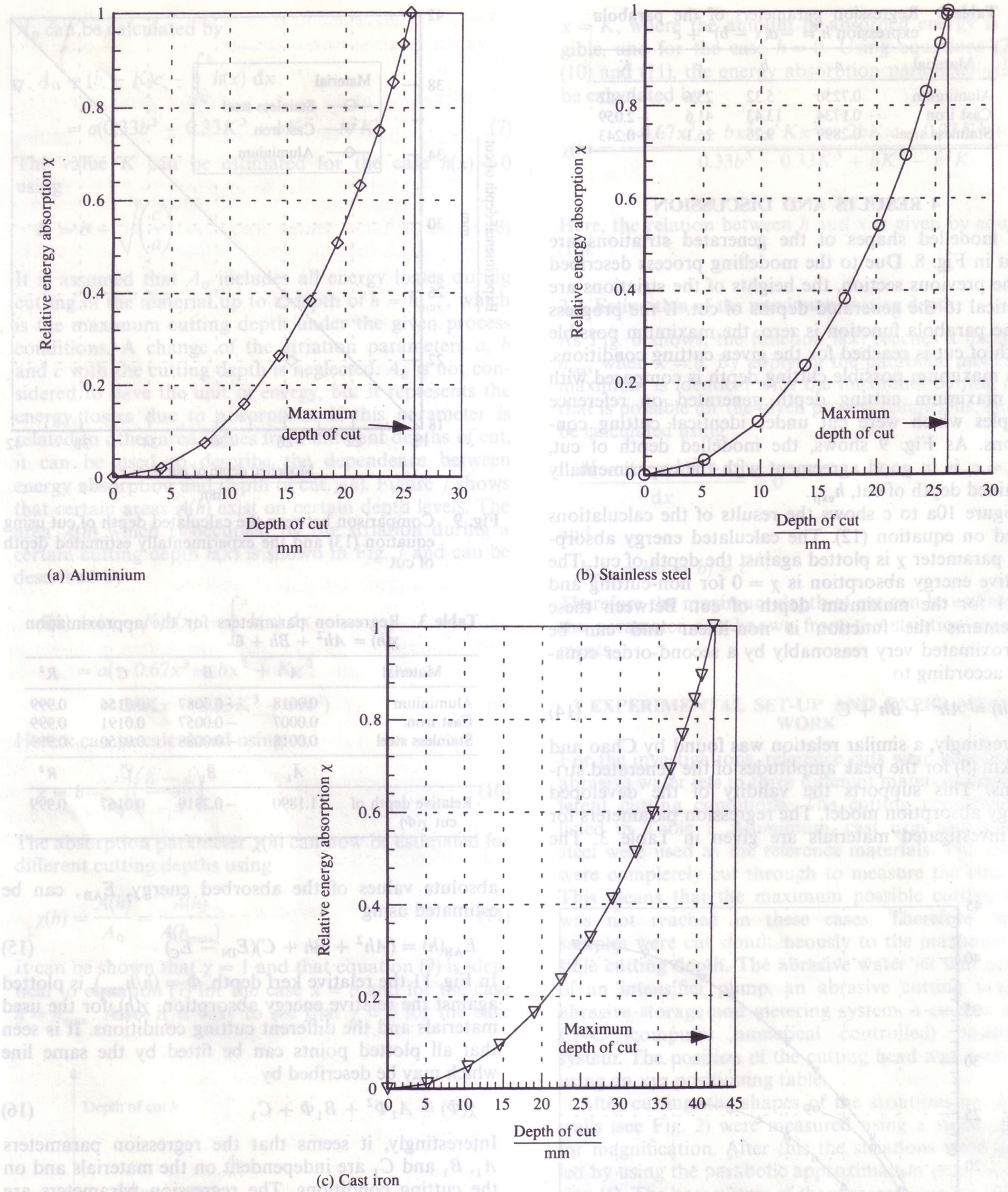

Fig. 10 Relation between depth of cut and relative energy absorption capability for different materials (regression parameters are given in Table 3)

that the difference, $\xi = (\chi_1 - \chi)$, between the functions $\chi(\Phi)$ and $\chi_1(\Phi)$ can be applied to characterize this energy absorption intensity. The function for the energy absorption intensity parameter $\xi(\Phi)$ is simply the difference between equations (17) and (16), which leads to

$$\xi(\Phi) = -A_1\Phi^2 + (1 - B_1)\Phi - C_1 \tag{18}$$

It may be interesting to estimate the cutting depth where the energy absorption intensity has a maximum value. This point with the maximum energy absorption intensity can be calculated by differentiating equation (18). Solving the derivation for $d\xi/d\Phi = 0$ yields

$$(\Phi)_{\xi=\max} = \frac{B_1 - 1}{-2A_1} \tag{19}$$

Using the regression parameters given in Table 3, equation (19) gives the solution $\Phi = 0.52$. Equation (19) can also be solved graphically, as shown in Fig. 12. It is

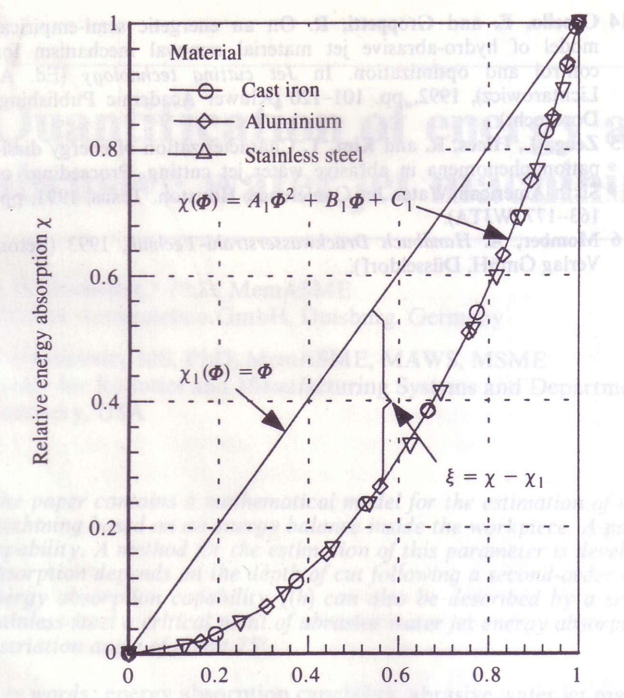

Fig. 11 Relation between relative depth of cut, $\Phi = h/h_{max}$, and relative energy absorption parameter, $\chi(\Phi)$, for different materials (regression parameters are given in Table 3)

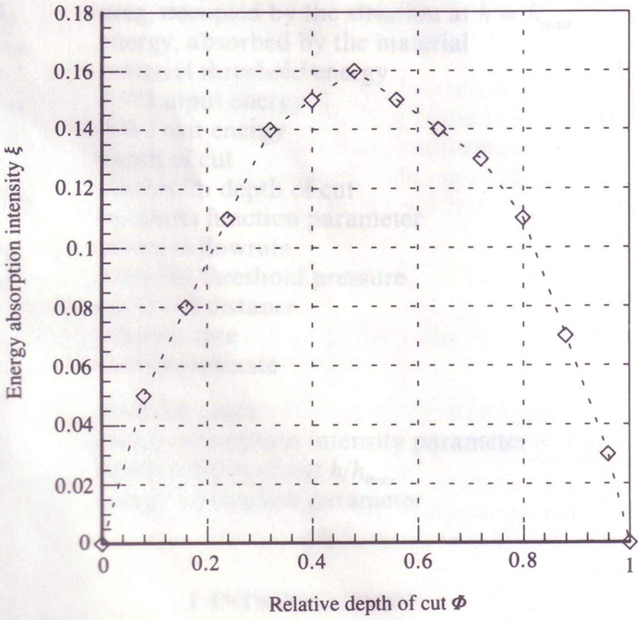

Fig. 12 Relation between relative depth of cut, $\Phi = h/h_{max}$, and energy absorption intensity, $\xi(\Phi)$

evident that a point of maximum energy absorption intensity exists at a depth of cut of $h = 0.52h_{max}$. These results suggest that a critical point of maximum energy absorption intensity may exist. It can be assumed that this point is related to a critical striation angle α. Using the findings from Section 2 it is possible to calculate the angle α of the striations at any particular cutting depth by

$$\alpha = \arctan\left[\frac{\mathrm{d}\{-a(x-b)^2 + c\}}{\mathrm{d}x}\right] \tag{20}$$

For the given case $h = 0.52h_{max}$, it was found that the striation angle α varies in a very narrow range between 73° and 78° for all processed materials and all cutting conditions. This result needs further assistance by additional measurements, especially on non-metallic materials.

5 SUMMARY

The investigations lead to the following summary:

1. A model for the calculation of the energy losses due to absorption by the material during abrasive water jet cutting is developed based on an energy balance inside the workpiece.
2. A parameter $\chi(h)$ is defined to describe and calculate the energy absorption capability of a material during cutting by an abrasive water jet.
3. A method which is based on a parabolic striation model is developed to estimate the energy absorption parameter $\chi(h)$.
4. The relation between the cutting depth h and the energy capability parameter $\chi(h)$ can be described by second-order equation.
5. For the materials aluminium, cast iron and stainless steel a critical point of abrasive water jet energy absorption was detected at a cutting depth of $h = 0.52h_{max}$, which corresponds to a striation angle of about 75°.

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