THEMIS SIGNAL ANALYSIS STATISTICS RESEARCH PROGRAM

MEASURING YOUNG'S MODULUS IN PHOTOELASTIC MATERIALS

by

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Southern Methodist University

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Symbols and Definitions

A - Cross-sectional area, inches^2.

c - Velocity of wave propagation, inches/second.

E - Young's modulus of elasticity, pounds/inch^2.

t - Time, seconds.

$\left.\begin{matrix} u \\ v \\ w \end{matrix}\right\}$ - Particle displacement in the $\left\{\begin{matrix} x \\ y \\ z \end{matrix}\right\}$ directions, respectively.

$\left.\begin{matrix} x \\ y \\ z \end{matrix}\right\}$ - Orthogonal coordinate axes.

ϵ - Linearized strain, $\Delta l/l$.

ρ - Mass density, pounds second^2/inches^4.

λ, μ - Lames constants, $\mu \equiv G$ (shear modulus)

$\bar{\omega}_x$ - Distortional vector due shearing stress, $\bar{\omega}_x = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$

σ - Stress, pounds/inch^2.

Δ - Dilatation; change in volume due to extensional strains.

∇^2 - The dot product of the vector operator $\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)$ operating on itself.

MEASURING YOUNG'S MODULUS

IN

PHOTOELASTIC MATERIALS

by J.E. Tepera

INTRODUCTION

The rate at which a stress pulse propagates through an isotropic elastic material is related to the material properties through the generalized form of Hooke's Law. Using this relation it is possible to determine the modulus of elasticity, as well as other properties, for this class of materials by measuring this propagation rate. This has been done by many investigators and hence, the "speed of sound" for a large number of materials is readily available in various references.

The purpose of this experiment is to show the applicability of this procedure in determining Young's modulus in low modulus photoelastic materials.

THEORY

Utilizing Newton's second law and the generalized form of Hooke's Law, the equations of motion (1) for a system of forces acting on a differential volume are

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial x} + \mu \nabla^2 u$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial y} + \mu \nabla^2 v \qquad 1$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu) \frac{\partial \Delta}{\partial z} + \mu \nabla^2 w$$

where Δ is the differential change in the volume due to extensional strain, $\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$. Noting that the extensional stress-strain relations are independent of the distortional terms and vice-versa in Hooke's Law, and carrying this concept to the equations of motion, then equation 1 can be separated into

$$\frac{\partial^2 u}{\partial t^2} = \left[(\lambda + 2\mu)/\rho \right] \nabla^2 u \qquad 2$$

for dilatational effects and

$$\frac{\partial^2 \bar{\omega}_x}{\partial t^2} = \left[\mu/\rho \right] \nabla^2 \bar{\omega}_x \qquad 3$$

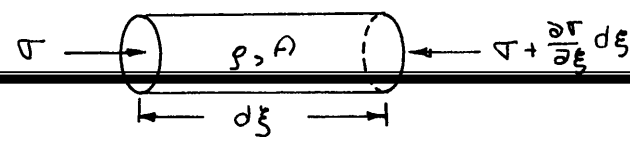
for distortional effects (realizing that there are similar relations for the remaining orthogonal directions).

These differential equations express the motion of a wave moving through an isotropic elastic material at the velocities of

$(\lambda + 2\mu)/\rho$ and μ/ρ . The constants λ and μ are Lames constants and ρ is the mass density of the material. Equations 2 and 3 are the differential equations representing wave motion.

In a more specific example, consider an element of material of constant cross-section A and **length** $d\xi$. The equation of motion for a plane wave traveling with its normal parallel to the axis of symmetry becomes

$$\frac{\partial \sigma}{\partial \xi} = \rho \frac{\partial^2 u}{\partial t^2} \tag{4}$$



For the case of plane stress $\sigma = E\epsilon = E\frac{\partial u}{\partial \xi}$ and

$$\frac{\partial \sigma}{\partial \xi} = E \frac{\partial^2 u}{\partial \xi^2} \tag{5}$$

Substitution of equation 5 into equation 4 yields the familiar wave equation

$$E \frac{\partial^2 u}{\partial \xi^2} = \rho \frac{\partial^2 u}{\partial t^2} \tag{6}$$

or

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial \xi^2} \tag{7}$$

where $c^2 = E/\rho$, defined to be the wave velocity.

DISCUSSION

The concepts used in this study are based on the birefringent properties of photoelastic materials. If a stress pulse is introduced into any elastic material by some means, a corresponding stress wave is established at the point of inception. A photoelastic material will display an equivalent fringe pattern. As the wave moves from point to point, the shift in light intensity will occur simultaneously. Hence, the wave will travel with a fringe pattern superimposed, so to speak, and will be detectable using light sensitive equipment.

If a uniaxial stress pulse has a wave length which is much larger than the major dimension parallel to the wave front, then the plane wave assumption will hold; consequently, the bar velocity relationship with Young's modulus will be valid.

Since the velocity of propagation in a medium of this nature is a constant, then all that is necessary is to establish the time lapse occurring as the wave passes two predetermined points of known seperation. To accomplish this, a rectangular speciman of commercial grade of urethane rubber (PSM-4) was mounted in a test rig with a narrow beam of light passing through two points. This light source is a low power laser, which in itself

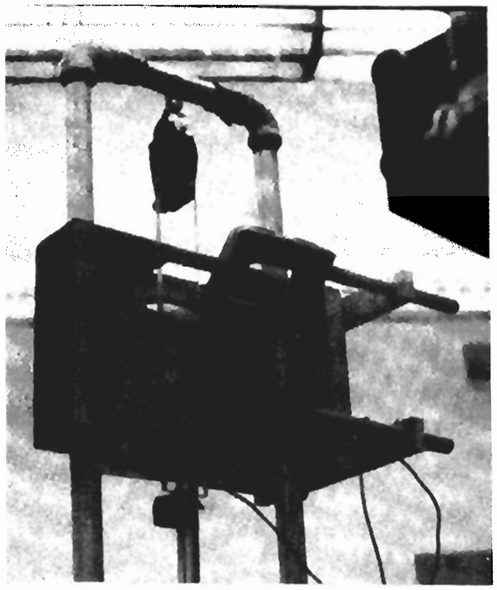


FIGURE 1a (left). PSM-4 speciman suspended in the loading rig, showing the location of the beam splitter (upper reflective surface) and the laser (upper right).

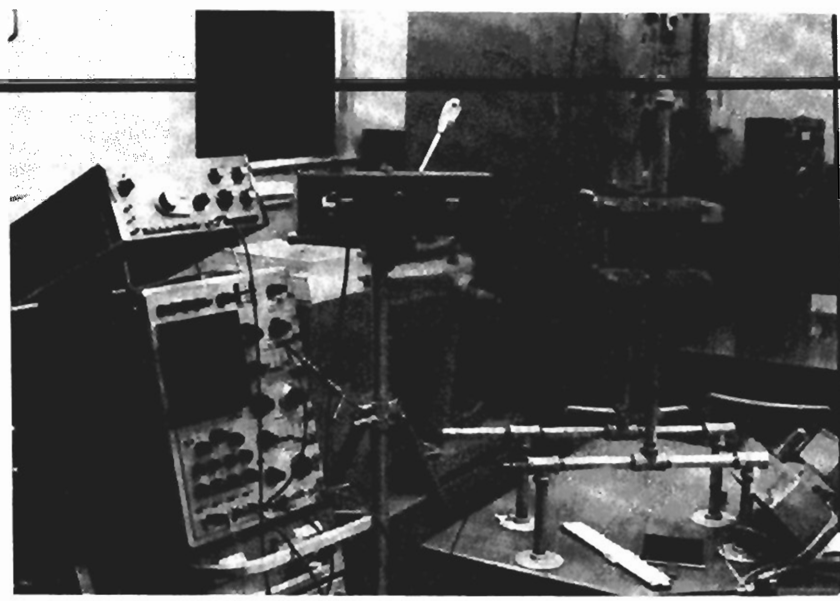


FIGURE 1b.

OVERALL EXPERIMENTAL SET-UP

One of the photomultiplier tubes is shown in the lower right.

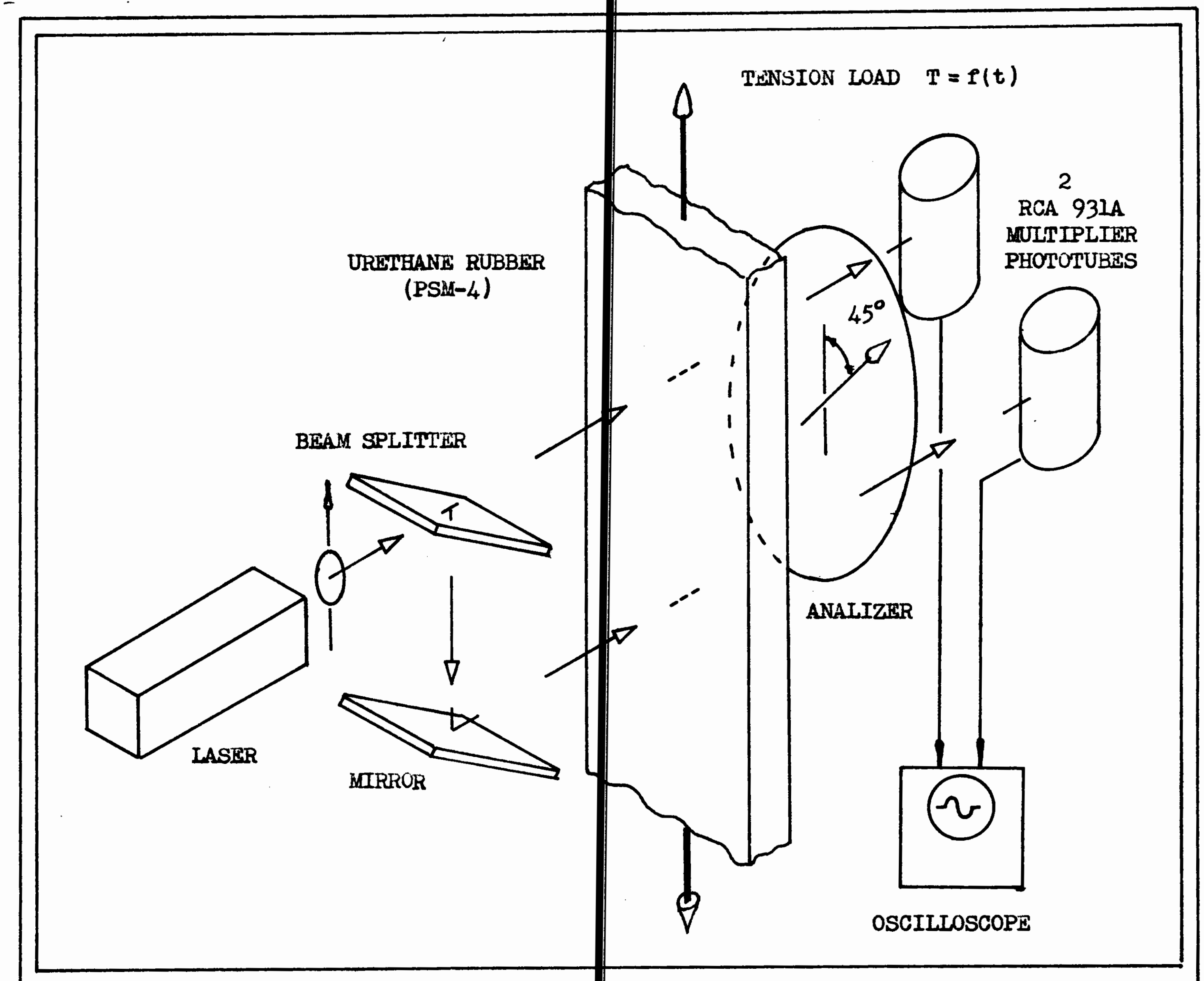

TENSION LOAD T = f(t)
2
RCA 931A
MULTIPLIER
PHOTOTUBES
URETHANE RUBBER
(PSM-4)
45°
BEAM SPLITTER
ANALIZER
LASER
MIRROR
OSCILLOSCOPE

FIGURE 2

is polarized light. This beam was split along two seperate paths which intercepted the speciman, then passes into the analyzer. The resulting light vectors are displayed on a multiplier phototube. The photomultipliers, being sensitive to light intensity, delivers an electrical signal of corresponding magnitude. These signals are displayed on an oscilloscope.

The analyzer's axis of polarization is rotated 45° with respect to the axis of the laser . This placed the light intensity in the so called grey field (2) as apposed to the standard light or dark field. Hence, the oscilloscope display will be similar to the more familiar pulse shape displayed by a wire-type strain gage.

The overall experimental set-up is shown in Figure 1 and in Figure 2 a sketch of the preceding discription is given.

RESULTS

The figure below (Figure 3) is photographic record of a typical oscilloscope record of the stress wave event. Data collected on this speciman yields a value of 750 pounds per inch squared for Young's modulus. (Peak-to-peak time lapse is 1.54 milliseconds in figure 3 for a distance of 4.25 inches; density being 0.99×10^{-4} pounds mass per inch cubed.) The published value of Young's modulus (3) for PSM-4 is 1000 psi. This is a twenty-five percent reduction from the published value.

In order to confirm either the published value or the experimental results, a static stress-strain curve was run on the speciman. As shown in figure 4, three test yields a mean slope of 710 psi. Thus, the value determined by the dynamic method yields satisfactery results.

CONCLUSIONS

The results from this experiment indicates that the method outlined here will produce a satisfactory value of Young's modulus for this class of materials.

Even though this particular experiment was conducted at room conditions, there is the possibility that this technique could be extended to testing in enviromentally controlled studies. However, one should be concerned as to the viscoelastic effects (4) that may be present in photoelastic materials at elevated temperatures.

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13. ABSTRACT

A technique of determining the modulus of elasticity of a birefringent material by means of sound wave transmission is discussed.

In the experiment, the sound wave velocity is obtained by measuring the change in the transmitted polarized light field intensity at two separate spatial locations along the length of the specimen by means of photomultiplier tubes and a cathode ray oscilloscope and camera. The relationship between the sound velocity, density, and modulus of elasticity of the material is used to calculate the latter quantity.

The technique was applied to a typical low modulus (750 psi) photoelastic material, urethane rubber (PSM-4) and the measured value of the modulus of elasticity is compared to values obtained from cumbersome and slow static stress-strain measurements and to the value published by the manufacturer. The technique lends itself very well to application in either a high or low temperature environment.

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