THEMIS SIGNAL ANALYSIS STATISTICS RESEARCH PROGRAM

OPTIMUM SEQUENTIAL SEARCH WITH DISCRETE LOCATIONS AND RANDOM ACCEPTANCE ERRORS

by

Mark W. Smith and John E. Walsh

Technical Report No. 44
Department of Statistics THEMIS Contract

August 28, 1969

Research sponsored by the Office of Naval Research
Contract N00014-68-A-0515
Project NR 042-260

DEPARTMENT OF STATISTICS
Southern Methodist University

OPTIMUM SEQUENTIAL SEARCH WITH DISCRETE LOCATIONS AND RANDOM ACCEPTANCE ERRORS

Mark W. Smith
Texas Instruments Incorporated

and

John E. Walsh
Southern Methodist University

Abstract

Much work has been done in search theory. However, very little effort has occurred where an object's presence at a location can be accepted when no object is present there. The case analyzed is of this type. The number of locations is finite, a single object is stationary at one location, and only one location is observed each step of the search. The object's location has a known prior probability distribution. Also known are the conditional probability of acceptance given the object's absence (small) and the conditional probability of rejection given the object's presence (not too large); these probabilities remain fixed for all searching and locations. The optimum sequential search policy specifies that the next location observed is one with the largest posterior probability of the object's presence (evaluated after each step from Bayes Rule) and that the object is at the first location where acceptance occurs. Placement at the first acceptance seems appropriate when the conditional probability of acceptance given the object's absence is sufficiently small. The policy is optimum in that, for any number of steps, it minimizes the probability of no acceptances and, simultaneously, maximizes the probability that an acceptance occurs and the object is accurately located. Search always terminates (with probability one). Optimum truncated sequential policies are also considered. Methods are given for evaluating some pertinent properties and for investigating the possibility that no object occurs at any location.

INTRODUCTION AND DISCUSSION

Development of efficient search policies has received much attention (Ref. 1 and 2). Virtually without exception, however, these results are for situations where observation (for detection of an object) is such that there is zero probability of accepting an object's presence at a place where no object is located. In practice, many important cases occur where this probability is nonzero (although it often may be small).

This paper considers a case where the conditional probabilities of acceptance and rejection (given presence, or absence, of an object) are neither zero nor unity. The search space is discrete, with a finite number N of possible locations. There is only one object and it remains stationary at one of the locations. The problem is to optimally identify a location for this object subject to observing one location at each step of the search (the same location could be observed at successive steps). Given is a valid prior probability distribution for the true location of the object. Also, the conditional probabilities of acceptance and rejection have known values that remain the same for all locations and all steps of the search.

A sequential search policy would seem satisfactory if, for any number of steps, it minimizes the probability of not having identified a location and, simultaneously, maximizes the probability that a location is identified and is the true location. The optimum policies considered have this property (subject to a criterion).

The principal interest is in situations where the conditional probability of acceptance given the object is absent (denoted by α) is small. Then, if the conditional probability of rejection given that the object is present (denoted by β) is not too large, there is a good chance that the first location accepted is the true location. Such situations are found to occur when $(1-\beta)/(N-1)\alpha$ is much larger than unity. A criterion which identifies the first location accepted as that of the object seems appropriate under these circumstances (and corresponds to previous results for the limiting situation of $\alpha=0$). Given this criterion, and any number of steps, the optimum policy minimizes the probability of no acceptances and simultaneously maximizes the probability that a location is identified and also that the object is at the location. A policy with this property is developed. The properties obtained for this policy hold when $\alpha,\beta < \frac{1}{2}$, but may not be very desirable unless $(1-\beta)/(N-1)\alpha$ is sufficiently large.

Consider the details of the optimum policy. At the first step, a location with largest value for the prior probability of the object's presence is observed. If the location is accepted, the object is said to be there and the search is completed. When rejection occurs, the posterior probabilities of the object's location are computed using Bayes Rule (from the location observed, occurrence of rejection, the prior distribution, α, and β). The next location observed is one with the largest of these posterior probabilities. If acceptance occurs, this location is identified as that with the object and search ends. When there is rejection, new posterior probabilities are computed by Bayes Rule (from the location last observed, occurrence of rejection, the previous posterior distribution, α, and β). This procedure is continued until an acceptance, which ultimately happens (with probability one). Then search is terminated and the identified location is that where acceptance occurred. When more than one location has the largest probability value, one of them is randomly chosen on an equally-likely basis.

The probability of correctly selecting the object's location can be made as near unity as desired. For fixed β, it is found that this can be accomplished by having α sufficiently small. However, the largest usable value for α increases as β decreases, so that having β small is desirable.

Optimum truncated sequential policies are also obtained. Here, a maximum is specified for the number of steps. The search policy is the same as that already given through this number of steps. When rejection occurs for all the steps, search ends and the identified location is one with largest posterior probability (computed after the last step). The probability of correct decision is less than for the policy without truncation but has very nearly the same value when the maximum for the number of steps is sufficiently large. Computations can be made to determine a suitable maximum number of steps.

The prior distribution for the object's location is assumed to be completely known and valid. No consideration is given to situations where the prior distribution is unknown in any respect or is inaccurate.

Exact expressions are developed for some properties such as the probability of correct location, the expected number of steps to search termination, and the probability of search termination within a stated number of steps. These expressions can be evaluated as closely as desired (exactly when the number of steps is limited) by direct utilization of the optimum policy being considered. Upper and lower bounds are developed for the properties considered that involve an unlimited number of steps. These bounds tend to be the same value with an increasing number of steps and are useful in deciding when an approximation to an evaluation is sufficiently accurate.

Part of the approach to providing bounds consists in using the uniform distribution (probability 1/N for each location) as the prior distribution. That is, part of one of the two bounds is evaluated on the assumption of a uniform prior distribution. In particular, a probability of correct decision evaluated for the situation of a uniform prior distribution is a lower bound for values of this probability. Also, an expected number of steps to search termination obtained on this basis provides an upper bound, etc. A desirable feature of bounds determined from the uniform distribution is that they often have reasonably uncomplicated expressions of a closed form. Then, sufficient conditions on α and β for obtaining desired values of properties can be determined from examination of an uncomplicated function of α and β.

Sometimes an object is believed to be at some one of the locations but its occurrence is not certain. That is, the possibility exists that no object is present at any location. Then a form of truncated policy should be used, since the policy with an unlimited number of steps is certain to ultimately identify a location even when no object occurs. For such a truncated policy, the conclusion that there is no object is adopted after the maximum number of steps is reached without an acceptance. Acceptance at any of the steps implies that there is an object and identifies its location (search also stops). Here the prior distribution used for a search policy is conditional (it is given that the object occurs). Ideally, the policy used should provide a good chance of claiming that no object occurs when this is the case and also of finding it (including location) when there is an object. Such an ideal situation is found to occur when α is sufficiently small and the maximum for the number of steps is suitably chosen.

Sometimes α and β are related so that decreasing the value of either one increases the value of the other. Then a "best" combination of values can often be determined for α and β. A specific example involving communications search is given in Ref. 3. Also, some potential fields of practical application for the optimum sequential search policies, with emphasis on electronic problems, are mentioned in Ref. 3. The material of Ref. 3 (unpublished) provides the basis for much of this paper and will be referred to again when derivations are considered.

Statement of notation and of expressions concerning the properties considered occurs in the next section. Justification of these results, and of the optimum properties stated for the sequential policies, is outlined in the following section. The final section is concerned with the situation where the possibility exists that no object is present at any location.

NOTATION AND RESULTS

The locations are identified by ℓ (ℓ=1,...,N). The location numbering is such that

$$p_o(1) \geq p_o(2) \geq \dots \geq p_o(N),$$

where $p_o(\ell)$, nonzero, is the prior probability that the object is at location ℓ and

$$p_o(1) + \dots + p_o(N) = 1.$$

The steps of the search are i=1,2,... and one location is observed, for possible detection of whether the object is there, at each step. $m_i(\ell)$ equals the number of times that location ℓ has been observed during steps $1,\ldots,i-1$. The location e_i is such that, for use of an optimum policy,

$$K_i p_o(e_i) = [\beta/(1-\alpha)]^{m_i(e_i)} p_o(e_i) = \max_{\ell}[\beta/(1-\alpha)]^{m_i(\ell)} p_o(\ell)$$

and is the location observed at the i^{th} step of the optimum policy. If the maximum occurs for more than one location, the value of e_i is chosen randomly so that all possibilities are equally likely (to avoid bias). For evaluation of properties, however, the value used for e_i is the smallest number among the possibilities.

Knowledge of α, β, N, and the $p_o(\ell)$ combined with methods of evaluating e_i and K_i over all i is sufficient both for application of an optimum policy and for evaluating properties of this policy. For a truncated optimum policy, the maximum number M of steps is also given.

Now consider statements of properties. In all cases optimum policies are used. Exact results for the general situation with no truncation are presented first. These are followed by exact results for the general situation with truncation. Next some results for a uniform prior distribution are given. Then, ways of developing bounds for some of the exact properties are stated. Finally, some conditions are given on α, β, N for assuring sufficiently high probabilities of correct decision.

The probability that the first acceptance is at the object location is

$$P_\infty = (1-\beta)\sum_{i=1}^{\infty}(1-\alpha)^{i-1}K_i p_o(e_i).$$

The probability that the first acceptance occurs on or before step n and also correctly locates the object is

$$P_n = (1-\beta)\sum_{i=1}^{n}(1-\alpha)^{i-1} K_i p_o(e_i).$$

The probability that search will terminate on or before the n^{th} step is

$$P'_n = 1-(1-\alpha)^n + (1-\alpha-\beta)(1-\alpha)^{n-1}\sum_{i=1}^{n} K_i p_o(e_i).$$

Of course, the probability that there are rejections at all of the first n steps is $1-P'_n$.

Since the last two terms of P'_n tend to zero as $n\to\infty$, there is unit probability that search terminates. In fact, search terminates with probability one for any search policy with the property that, ultimately, every location is observed an unlimited number of times.

The expected number of steps to termination of search is

$$\underline{L} = \frac{1}{\alpha}\left[1-(1-\alpha-\beta)\sum_{i=1}^{\infty}(1-\alpha)^{i-1} K_i p_o(e_i)\right].$$

The median number of steps to search termination can (conservatively) be evaluated as the smallest value of n satisfying $P'_n \geq 1/2$. For a perfect detection device ($\alpha=\beta=0$), the expected number of steps to search termination is easily seen to be

$$\sum_{\ell=1}^{N} \ell\ p_o(\ell).$$

Now, consider the general situation with truncation. For $n \leq M$, the probability that the first acceptance occurs on or before the n^{th} step is P_n, as given for no truncation. The probability of correctly locating the object is

$$P_M + (1-\alpha)^M K_{M+1} p_o(e_{M+1}).$$

For $n < M$, the probability that search will end on or before step n is P'_n. The expected number of steps to termination is

$$M - \sum_{i=1}^{M-1} P'_i .$$

The median number of steps to termination can (conservatively) be determined as the smallest n satisfying $P'_n \geq 1/2$, and equals M if no such n exists.

Next, consider the special situation of a uniform distribution. Examination of the results for the general situation shows that these are determined when expressions of the forms $K_{rN+s}\, p_o(e_{rN+s})$,

$$\sum_{i=1}^{rN+s} (1-\alpha)^i K_i p_o(e_i), \quad \text{and} \quad \sum_{i=1}^{rN+s} K_i p_o(e_i)$$

are evaluated for the uniform distribution, where $0 \leq r \leq \infty$, $1 \leq s < N$. It is found that

$$K_{rN+s}\, p_o(e_{rN+s}) = \frac{1}{N}\,[\beta/(1-\alpha)]^r,$$

$$\sum_{i=1}^{rN+s} (1-\alpha)^{i-1} K_i p_o(e_i) = U_1(rN+s) =$$

$$\frac{1}{N\alpha}\Big\{[1-(1-\alpha)^N][1-\beta^r(1-\alpha)^{r(N-1)}]/[1-\beta(1-\alpha)^{N-1}] + \beta^r(1-\alpha)^{r(N-1)}[1-(1-\alpha)^s]\Big\},$$

$$\sum_{i=1}^{rN+s} K_i\, p_o(e_i) = U_2(rN+s) =$$

$$\{1-[\beta/(1-\alpha)]^r\}/[1-\beta/(1-\alpha)] + (s/N)[\beta/(1-\alpha)]^r,$$

and closed-form expressions for P_∞, P_n, P'_n, etc. can be directly stated in terms of α, β, N, and perhaps n or M.

Consideration of the uniform distribution is useful because of some inequalities that occur. Specifically, consider

$$\sum_{i=r_1N+1}^{r_2N+s} (1-\alpha)^{i-1} K_i p_o(e_i) \quad \text{and} \quad \sum_{i=r_1N+1}^{r_2N+s} K_i p_o(e_i),$$

where $0 \leq r_1 \leq r_2 \leq \infty$, $1 \leq s < N$, and the $p_o(\ell)$ have an arbitrary set of values. The inequalities

$$\begin{aligned} &\sum_{i=r_1N+1}^{r_2N+s} (1-\alpha)^{i-1} K_i p_o(e_i) \geq U_1(r_2N+s) - U_1(r_1N), \\ &\sum_{i=r_1N+1}^{r_2N+s} K_i p_o(e_i) \geq U_2(r_2N+s) - U_2(r_1N) \end{aligned} \tag{1}$$

are satisfied for all possible sets of values for the $p_o(\ell)$. Thus,

$$\sum_{i=1}^{r_2N+s} (1-\alpha)^{i-1} K_i p_o(e_i) \geq \sum_{i=1}^{r_1N} (1-\alpha)^{i-1} K_i p_o(e_i) + U_1(r_2N+s) - U_1(r_1N)$$

and

$$\sum_{i=1}^{r_2N+s} K_i p_o(e_i) \geq \sum_{i=1}^{r_1N} K_i p_o(e_i) + U_2(r_2N+s) - U_2(r_1N).$$

Suppose that r_2N+s is exceedingly large so that evaluation of $K_i p_o(e_i)$ would require much effort for that many steps. Often, a value occurs for r_1 such that r_1N is not excessive but large enough for the stated lower bound to be near the true value. The lower bound involving $U_1(r_2N+s)$ is especially useful for r_2 infinite.

Upper bounds are also needed in determining sufficiently accurate approximate values. The bounds developed are

$$\sum_{i=1}^{r_2N+s} (1-\alpha)^{i-1} K_i p_o(e_i)$$

$$\leq \sum_{i=1}^{r_1N} (1-\alpha)^{i-1} K_i p_o(e_i) + (1/\alpha) K_{r_1N}\, p_o(e_{r_1N}) \left[(1-\alpha)^{r_1N} - (1-\alpha)^{r_2N+s}\right],$$

$$\sum_{i=1}^{r_2N+s} K_i p_o(e_i)$$

$$\leq \sum_{i=1}^{r_1N} K_i p_o(e_i) + \left[(r_2-r_1)N+s \quad K_{r_1N}\, p_o(e_{r_1N})\right],$$

and follow from the inequality

$$K_i p_o(e_i) \geq K_j p_o(e_j), \qquad (i \leq j).$$

These upper bounds can be used, in combination with the lower bounds, to determine when r_1N is large enough to assure that a good approximation is obtained. The approximation could, for example, be the arithmetic average of the upper and lower bounds. The upper bounds are especially useful when

$$\sum_{i=1}^{r_2N+s} (1-\alpha)^{i-1} K_i p_o(e_i)$$

is to be evaluated (maybe with r_2 infinite). The forms of upper bounds are also usable when r_1N is replaced by any integer in the range 1 to r_2N+s.

These results for the two types of summations are directly usable in approximate evaluation of P_∞, P_n, P'_n, and $\underline{L}$. The exact results for the uniform distribution furnish lower bounds on the possible values of P_∞, P_n, and P'_n. They provide upper bounds on the possible values of $\underline{L}$, $M - \sum_{i=1}^{M-1} P'_i$, and the median number of steps to termination. These closed form expressions are reasonably uncomplicated and can be used to establish conditions on α and β that are sufficient to assure desired values of properties (at least a stated value for P_∞, P_n, or P'_n; at most a stated value for the expected or median number of steps to termination).

As an example, consider that at least a value P is desired for P_∞. Use of P_∞ for the uniform distribution provides the relation

$$\frac{1-\beta}{N\alpha}\left[1-(1-\alpha)^N\right]/\left[1-\beta(1-\alpha)^{N-1}\right] \geq P.$$

For $N\alpha<<1$ and $\beta<1$, this becomes, approximately,

$$(1-\beta)/(N-1)\alpha \geq (1+\beta)/2(1-P),$$

so that $(1-\beta)/(N-1)\alpha$ should be much greater than unity for P large. A somewhat related problem, non involving P_∞, would be to choose α and β so that the posterior probability (of correctness) for the accepted location is at least P irrespective of the step at which acceptance occurs. The minimizing posterior distribution at the step just before acceptance is the uniform distribution. Then the posterior probability of correct location after acceptance on the next step is seen to be

$$\alpha(1-\beta)\left[(1-\alpha-\beta)N\alpha(\alpha/(1-\alpha-\beta) + 1/N)\right]^{-1},$$

since the situation is the same as that of a uniform prior distribution and acceptance at the first step. The requirement that this value is at least P provides the (equivalent) inequality

$$(1-\beta)/(N-1)\alpha \geq P/(1-P).$$

Here too, $(1-\beta)/(N-1)\alpha$ should be much greater than unity for P large.

OUTLINE OF JUSTIFICATIONS

Detailed derivations of the various expressions used in proofs for nearly all of the results have been given in Ref. 3. Thus, nearly all of the justification given here is only sketched, and often uses the needed expressions without consideration of their derivations.

First, consider verification that for any number of steps n an optimum policy minimizes the probability that rejections occur at all of steps 1, ..., n. Mathematical induction is used to establish the form of the probability that rejections occur at the first n steps for any n. For the first step (n=1) and location ℓ_1 observed, the probability of rejection is easily seen to be

$$(1-\alpha-\beta)\left[(1-\alpha)/(1-\alpha-\beta) - p_0(\ell_1)\right].$$

Now consider $n \geq 2$ and assume that the probability of having rejections at the first n-1 steps, with location ℓ_i observed for the i^{th} step, is

$$(1-\alpha-\beta)(1-\alpha)^{n-2}\left[\frac{1-\alpha}{1-\alpha-\beta} - \sum_{i=1}^{n-1}\left(\frac{\beta}{1-\alpha}\right)^{m_i(\ell_i)} p_o(\ell_i)\right] . \qquad (2)$$

The induction is completed by showing that (2) implies the expression

$$(1-\alpha-\beta)(1-\alpha)^{n-1}\left[\frac{1-\alpha}{1-\alpha-\beta} - \sum_{i=1}^{n}\left(\frac{\beta}{1-\alpha}\right)^{m_i(\ell_i)} p_o(\ell_i)\right] \qquad (3)$$

for the probability of rejection on the first n steps, where location ℓ_n is observed at the n^{th} step.

For verification of (3) from (2), the probability of rejection on steps 1,...,n is expressed as the probability of rejection on the first n-1 steps multiplied by the conditional probability of rejection on the n^{th} step given that the first n-1 steps yielded rejections. The conditional probability is found to be

$$(1-\alpha)\left[\frac{1-\alpha}{1-\alpha-\beta} - \sum_{i=1}^{n}\left(\frac{\beta}{1-\alpha}\right)^{m_i(\ell_i)} p_o(\ell_i)\right]\left[\frac{1-\alpha}{1-\alpha-\beta} - \sum_{i=1}^{n-1}\left(\frac{\beta}{1-\alpha}\right)^{m_i(\ell_i)} p_o(\ell_i)\right]^{-1} .$$

Multiplication of this probability by the assumed expression (2) yields (3).

General verification of (3) implies that, for any number of steps, an optimum policy minimizes the probability of all rejections. That is, (3) is minimum for $\ell_i = e_i$, $(i=1,\ldots,n)$ since

$$K_i p_o(e_i) = \max_{\ell_i}[\beta/(1-\alpha)]^{m_i(\ell_i)} p_o(\ell_i)$$

for all possible choices of ℓ_i and for all i.

Next, consider an outline of the proof that, for any number n of steps, an optimum policy maximizes the probability that the first acceptance occurs within the first n steps and is at the object's location. Suppose that location ℓ_i is observed at the i^{th} step $(i=1,\ldots,n)$. The probability that the first acceptance occurs at step i and is at the object's location equals the probability of rejection on the first i-1 steps multiplied by two conditional probabilities. One is the conditional probability of acceptance at step i given that the object is at ℓ_i and that steps $1,\ldots,i-1$ yielded rejections. This probability equals $1-\beta$. The other is the conditional probability that the object is at ℓ_i given that steps $1,\ldots,i-1$ yielded rejections. This probability is found to be

$$\left(\frac{1-\alpha}{1-\alpha-\beta}\right)\left(\frac{\beta}{1-\alpha}\right)^{m_i(\ell_i)} p_0(\ell_i)\left[\frac{1-\alpha}{1-\alpha-\beta} - \sum_{j=1}^{i-1}\left(\frac{\beta}{1-\alpha}\right)^{m_j(\ell_j)} p_0(\ell_j)\right]^{-1}.$$

The product of the three probabilities simplifies to

$$(1-\beta)(1-\alpha)^{i-1}\,[\beta/(1-\alpha)]^{m_i(\ell_i)}\,p_0(\ell_i),$$

so that

$$(1-\beta)\sum_{i=1}^{n}(1-\alpha)^{i-1}[\beta/(1-\alpha)]^{m_i(\ell_i)}\,p_0(\ell_i) \tag{4}$$

is a general expression for the probability that the first acceptance occurs within the first n steps and is at the object's location.

Use of an optimum policy maximizes (4) for every value of n, since [illegible] of $\ell_i = e_i$ maximizes $[\beta/(1-\alpha)]^{m_i(\ell_i)}\,p_0(\ell_i)$ for all i. When the number of steps is unlimited, this result shows that an optimum policy results in the maximum probability that the first acceptance occurs at the object's location. For truncation at M steps, the probability of a correct decision is

$$(1-\alpha)^M \left[\beta/(1-\alpha)\right]^{m_{M+1}(\ell_{M+1})} p_0(\ell_{M+1})$$

$$+ (1-\beta) \sum_{i=1}^{M} (1-\alpha)^{i-1} \left[\beta/(1-\alpha)\right]^{m_i(\ell_i)} p_0(\ell_i)$$

for the general case. Use of an optimum policy, so that $\ell_i = e_i$, $(i=1,\ldots,M+1)$, evidently maximizes this probability.

For an optimum strategy, expression (3) equals $1-P_n'$ and expression (4) equals P_n. Letting $n \to \infty$ in P_n yields P_∞. Expressions for the truncated case are obtained directly from P_n, P_n', and an expression for the conditional probability of a location after M steps given that these steps yielded rejections.

A general expression for the expected number of steps to the first acceptance, with location i observed at step i and an unlimited number of steps, is

$$\frac{1}{\alpha}\{1-(1-\alpha-\beta) \sum_{i=1}^{\infty} (1-\alpha)^{i-1} \left[\beta/(1-\alpha)\right]^{m_i(\ell_i)} p_0(\ell_i)\} .$$

This equals $\underline{L}$ for use of an optimum policy, which evidently minimizes its value.

The exact results for a uniform prior distribution and an optimum policy follow directly from the fact that the locations are observed in the cyclic order 1,2,...,N,1,2,...,N,1,2,... . This determined order of search is also the basis for the inequalities (1). Examination of the way that $m_n(e_i)$ is selected from among the possibilities shows that

$$K_i p_o(e_i) \geq K_{i+1} p_o(e_{i+1}) \quad \text{and} \quad K_{rN+s} p_o(e_{rN+s}) \geq [\beta/(1-\alpha)]^r p_o(s).$$

That is, the quantity on the right side of one of these inequalities is at most equal to a value in a set over which the quantity on the left side is the maximum value. This, combined with the properties of the $p_o(\ell)$, shows that

$$\sum_{i=r_1N+1}^{r_2N} K_i p_o(e_i) \geq \sum_{j=r_1}^{r_2} [\beta/(1-\alpha)]^j = U_2(r_2N) - U_2(r_1N)$$

and that

$$\sum_{i=r_2N+1}^{r_2N+s} K_i p_o(e_i) \geq \frac{s}{N}[\beta/(1-\alpha)]^{r_2} ,$$

so that

$$\sum_{i=r_1N+1}^{r_2N+s} K_i p_o(e_i) \geq U_2(r_2N+s) - U_2(r_1N) .$$

In a similar fashion,

$$\sum_{i=r_1N+1}^{r_2N+s} (1-\alpha)^i K_i p_o(e_i) \geq U_1(r_2N+s) - U_2(r_1N) ,$$

is found to hold.

UNCERTAIN PRESENCE OF ANY OBJECT

The policies considered here have a maximum of m steps. When an acceptance occurs on or before the m^{th} step, the presence of an object is declared at the location of the acceptance and search stops. If rejections occur for all of the first m steps, the decision is that no object is present at any location.

The probability of deciding that no object is present equals $(1-\alpha)^m$ when this is the situation. That is, the locations are equivalent, so that the same probability is obtained for all search policies. The probability of a correct decision for this situation is large when α and m are such that $(1-\alpha)^m$ is large. For fixed m, this occurs as α decreases. For fixed α, the value of $(1-\alpha)^m$ increases as m decreases but never exceeds $1-\alpha$.

When an object is present, the probability of deciding on object occurs and also correctly locating it equals P_m. The value of P_m increases as m increases and, for fixed m, increases as α decreases. However, P_m also depends on β and decreases as β increases. For m large and $\beta<1$, a sufficiently small α will make P_m as near unity as desired. However, for P_m to be at least a stated value, the largest allowable α increases as β decreases, so that having β small is desirable.

Suppose that β is given and that a combination of α and m such that $(1-\alpha)^m \geq p_1$ and $P_m \geq p_2$ is desired, where p_1 and p_2 are specified values. This can be determined on an iteration basis by selecting a small value for α and trying to choose m so that both inequalities hold. If more than one value of m accomplishes this, increase α and try again. If no value of m suffices, decrease α and try again. The iteration stops when an α value is reached such that the inequalities are satisfied for exactly one value of m.

ACKNOWLEDGEMENT

Thanks are due to Texas Instruments Incorporated for support of Mark W. Smith, and to ONR Contract N00014-68-A-0515 and Mobil Research and Development Corporation for partial support of John E. Walsh.

REFERENCES

1. P.H. Enslow, Jr., "A Bibliography of Search Theory and Reconnaissance Theory Literature," Naval Res. Log. Quart. 13, 177-202 (1966).

2. James M. Dobbie, "A Survey of Search Literature," Operations Research 14, 525-537 (1968).

3. Mark William Smith, An Optimum Discrete Space Sequential Search Procedure Which Considers False Alarm and False Dismissal Instrument Errors, Ph.D. Thesis, Statistics Dept., Southern Methodist University, May 1969. Available as Tech. Report 35, THEMIS, Statistics Dept., Southern Methodist University, Dallas, Texas.