

Multipole Spatial Correlation Assessment
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Research Report in Partial Fulfillment of Task I:
Warm/Cold Multipole Correlation

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Multipole Spatial Correlation Assessment

Measurement-error regression modeling of the relationship between cold and warm multipole values necessitates the estimation of the measurement-error variability of warm and cold measurements and an assessment of whether the multipole values can be considered statistically independent across positions on a magnet. A previous report on this project titled "Measurement-Error Model Multipole Variance Estimation" determined that the multipole variability across positions on prototype dipole magnets could be considered constant for each of the cold and warm sets of multipole values. In addition, estimates of variability for each of the prototype dipoles were calculated from several rotations at each position for four skew (a_1, a_2, a_3, a_4) and four normal (b_1, b_2, b_4, b_6) multipoles. In this report, an assessment of the assumption that the multipole values are statistically independent across positions on a magnet is conducted.

Cold and warm multipole values are available at each of the 24 (nonoverlapping) mole positions along the lengths of prototype dipole magnets DCA 311-319. There are several ways in which the multipole correlations can be investigated. Two methods are detailed in this report: (a) an examination of spatial autocorrelations and (b) an examination of spatial semivariogram values.

1. Spatial Autocorrelations

The classical approach to an investigation of the correlations of temporally or spatially indexed data is through the calculation of the autocorrelations for each of several lags, where a lag is the difference between two index values. Denoting a multipole value at position k on a magnet by m_k and the average across all n positions by $\bar{m}$, the lag t sample autocorrelation is calculated from the formula $r(t) = c(t)/c(0)$, where

$$c(t) = \sum_{k=1}^{n-t} (m_{k+t} - \bar{m})(m_k - \bar{m}) / (n - t).$$

Appendix A contains plots of the autocorrelations for the four skew and normal multipoles on each of the prototype magnets. Since there are only $n = 24$ positions along the magnets, there are a limited number of lags for which sample autocorrelations can be calculated with adequate statistical precision. For the purposes of this report, only the first 10 autocorrelations are calculated and plotted. Superimposed on the plots are control limits. If any of the autocorrelations for lags 1 through 10 exceeds the control limits, the autocorrelation is deviating from zero by an amount that cannot be attributed to chance variation; i.e., the sample autocorrelation is significantly different from zero.

Scanning the plots in Appendix A, one sees that there are several lag-1 autocorrelations that exceed the control limits. This occurs on most of the magnets for all four cold skew multipoles and the warm skew a_2 multipoles. None of the other cold or warm multipoles show consistent significant autocorrelations on the magnets. These plots,

therefore, provide limited evidence of spatial autocorrelation, suggesting that it may be possible to ignore spatial autocorrelations for the purpose of calculating multipole averages across positions on a magnet. Further support for this suggestion is given in Section 3.

It is important to note that these autocorrelations do not take into account the use of warm multipoles to predict cold multipole values in a regression model. One interpretation of these significant autocorrelations is that there are changes in the mean multipole values at different positions. Another is that there is an autoregressive structure to the multipoles. If the former holds, the positional effects of the cold multipoles may be predictable from the warm values. The extent to which the autocorrelations of the cold multipole values are explained by the warm multipole values can be investigated by a variogram analysis of the residuals from a regression fit. The next section presents an analysis of semivariogram values for the cold and warm multipole values as well as for residuals from a regression fit of the cold to the warm multipoles.

2. Sample Semivariograms

Sample semivariograms are measures of spatial variability from which information on spatial correlation can be obtained. For two multipole measurements, one taken at position k and the other taken at position $k + t$, the variogram function is $\text{var}(m_{k+t} - m_k)$. The sample semivariogram estimates half this quantity. If multipoles have a common variance and the spatial correlation $\rho(t)$ is only a function of the lag t , one can write the semivariogram as

$$\text{var}(m_{k+t} - m_k) / 2 = \sigma^2 \{1 - \rho(t)\}.$$

A key feature of the sample variogram values is that they will not change as a function of the lag t if the multipoles are statistically independent. A substantive change in the semivariogram as a function of t indicates that the spatial correlations are changing as a function of the distance between positions on a magnet.

Sample semivariograms are calculated by taking half of the average of the squared differences of all multipoles that differ by t positions; i.e.,

$$s(t) = \sum_{n(t)} (m_{k+t} - m_k)^2 / \{2n(t)\}$$

In this equation, $n(t)$ indicates the number of pairs of multipoles that differ by t positions and the summation is over all such pairs. Note that in these calculations, a specific multipole may be used 1 or 2 times for a particular lag. Appendix B contains plots of the cold and the warm multipole semivariograms for all the prototype dipole magnets and the first two skew and normal multipoles.

There are four plots for each multipole-magnet combination. The upper left and right plots are semivariogram plots for the cold and the warm multipole measurements. The lower left plot is a semivariogram plot for the residuals from a least squares fit of the cold to the warm multipoles. The lower right plot contains the actual residuals from the least squares fit plotted against the position number.

When examining these plots, it is important to notice that the vertical scale on the residual semivariogram plots (lower left) is approximately an order of magnitude smaller than those of the cold and warm semivariogram plots shown above them. The reduction in spatial variability occurs because in most of the fits the warm multipole measurements do account for an appreciable amount of the variability observed in the cold multipole measurements. Thus, many of the trends seen in the residual semivariogram plots describe spatial variability patterns that are small in comparison to the overall variability of the cold multipoles.

Another overall feature that appears throughout the plots is that there are some departures from the desired horizontal band of points in the plots of residuals versus position (lower right). Hence, the warm multipole measurements do not in general capture all of the effects due to position; i.e., the systematic trends in the warm multipoles across positions do not completely account for the systematic trends observable in the cold multipole measurements. Thus the residual semivariogram plots do not necessarily reflect only spatial variability.

Few general comments can be made about the residual semivariogram plots. In some instances, notably the a_2 residual semivariogram plots, the plotted points appear to be randomly scattered about a horizontal line. This is the pattern expected if the residuals are formed by a model that has constant variance and no spatial correlation, a white noise model. On the other hand, the a_1 residual semivariogram plots sometimes appear consistent with a white noise model (e.g., magnets 311, 316, 317, 319) and sometimes exhibit linear (e.g., magnets 312, 313, 315, 318) or cyclic trends (e.g., magnet 314). The strong trends, linear or cyclic, occur when the residual versus position plot also shows a strong trend of the cold multipole residuals with position.

The trends in the residual plots and the residual semivariogram plots indicate that better modeling of the position effects is needed before a determination can be made about the importance of spatial correlations in the multipole values. The plots of the a_2 multipoles suggests that the spatial correlations may be minor. The trends in the plots of the other multipoles leaves such a general claim uncertain. The inclusion of coil size, collar, and other measurements may account for the remaining position effects. These modeling efforts are yet to be conducted.

A final concern which highlights the need for further consideration of the modeling of position effects on the cold multipoles is apparent from the b_2 plots of residuals versus position. There is a large negative residual at position 10 for all the magnets and one at position 12 for all except DCA311. These large residuals occur because the cold b_2

multipole values at these positions are the smallest of all the b_2 multipoles on each magnet.

3. Effect of Spatial Correlation on Magnet Estimates

If the cold multipole measurements are spatially autocorrelated by position, the mean multipole value across positions may not be the optimal estimator of the magnet mean multipole value. In Section 1 of this report, there was evidence of first-order spatial autocorrelation; i.e., multipole values one position apart from one another were significantly correlated. This type of autocorrelation can be modeled by a first-order autoregressive model:

$$(m_k - \mu) = \alpha(m_{k-1} - \mu) + w_k ,$$

where m_k is again a multipole value at position k , μ is the magnet multipole mean, α is a coefficient that determines the strength of the autocorrelation ($\rho = -\alpha$), and w_k is a white noise error process. One can show that the optimal (minimum variance unbiased) estimator of the magnet mean is not the overall magnet average, $\bar{m}$. Rather the optimal estimator is related to the magnet average by the following formula

$$\hat{\mu} = \bar{m} + \frac{m_1 + m_n}{n} \times \frac{\rho}{\{n/(n-2) - \rho\}}$$

If the autocorrelation is small, the magnet mean is close to being optimal. Estimated autocorrelations for the significant lag-1 autocorrelations in Appendix A are approximately 0.4-0.5. Using 0.45 as a typical value and $n = 24$ for the number of positions, the optimal estimate is approximately

$$\hat{\mu} = \bar{m} + 0.7 \frac{m_1 + m_n}{24}.$$

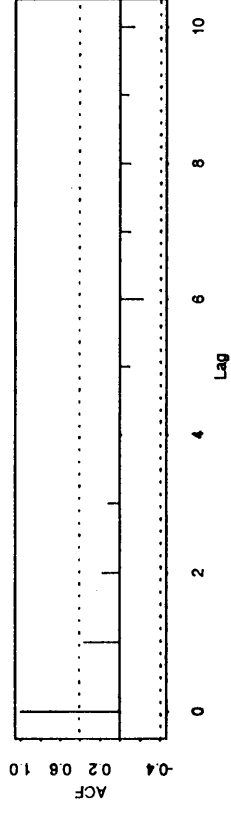
The difference between the optimal estimator and the sample mean is not trivial for these multipoles. Note, however, that this adjustment does not take into account the use of warm multipole values and physical measurements such as coil sizes in the modeling of the cold multipole values. Whether this type of an adjustment is actually necessary will depend on whether these additional variates satisfactorily account for the spatial autocorrelations seen in the cold multipoles.

Appendix A

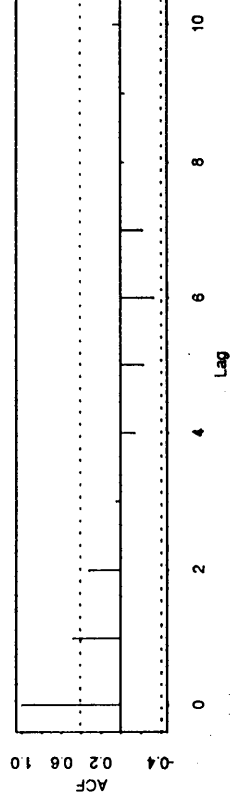
Multipole Spatial Autocorrelations

Sample Autocorrelations for Magnet DCA311 Skew Multipoles

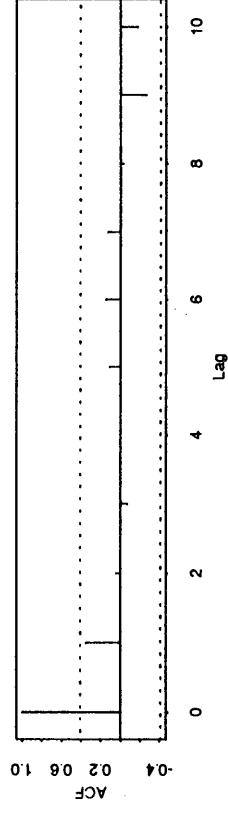
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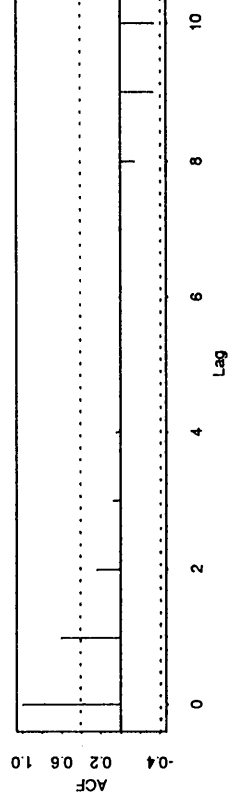
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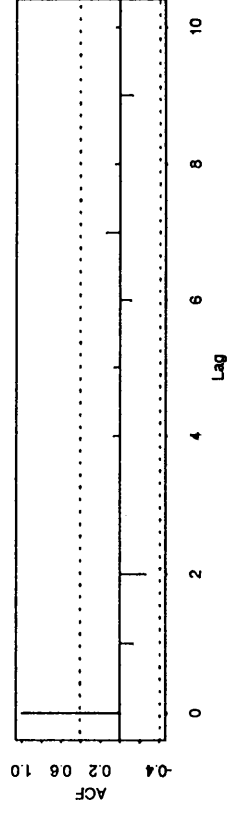
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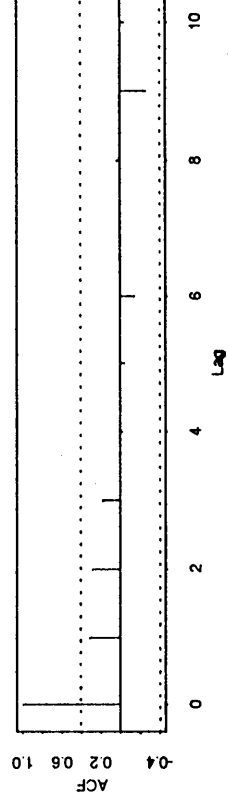
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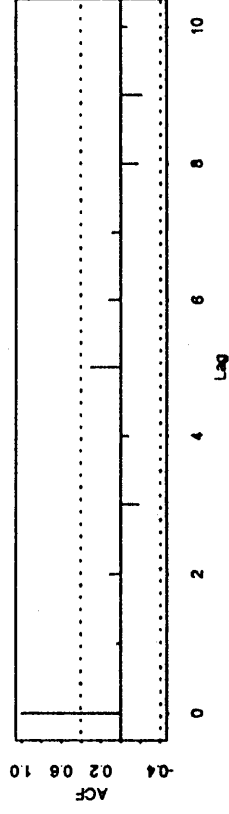
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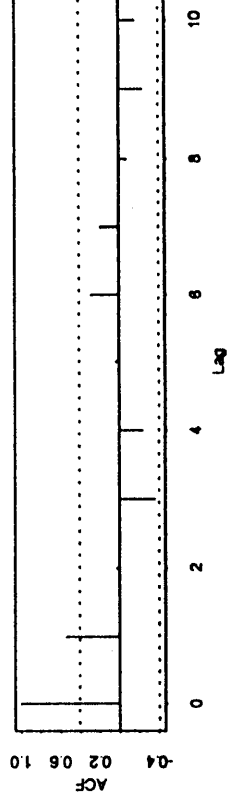
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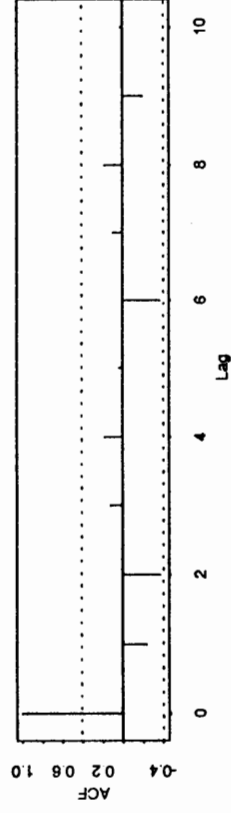


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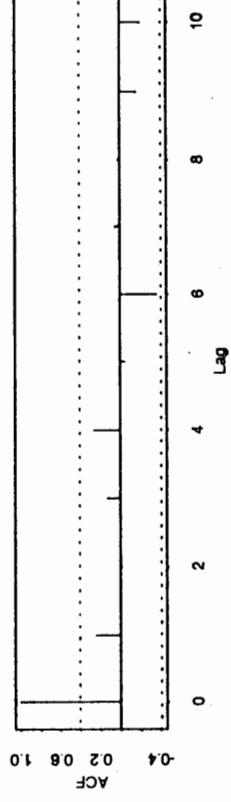


Sample Autocorrelations for Magnet DCA312 Skew Multipoles

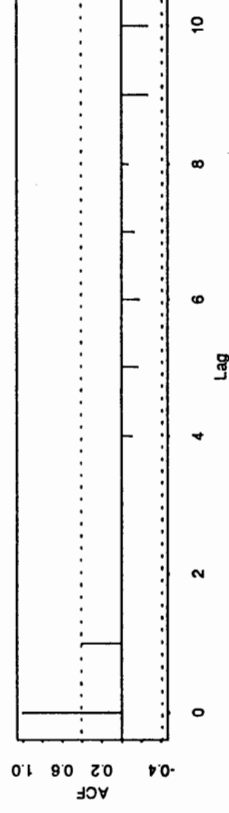
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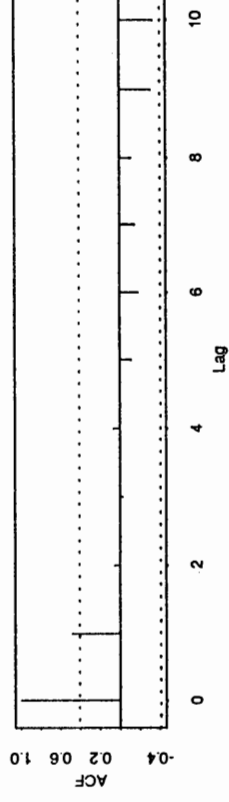
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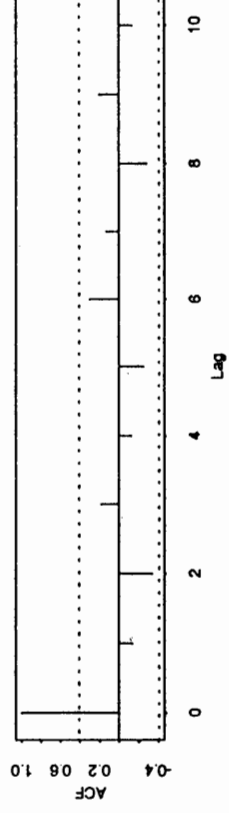
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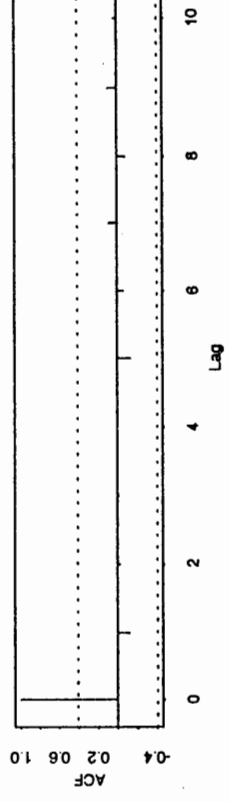
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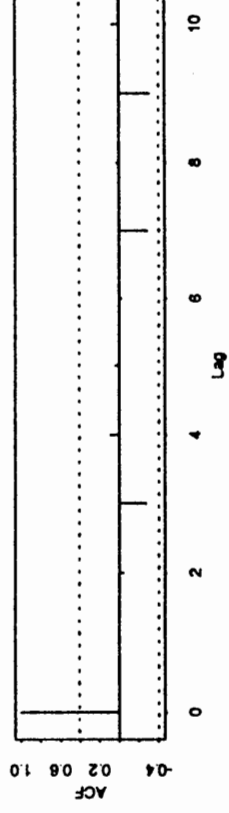
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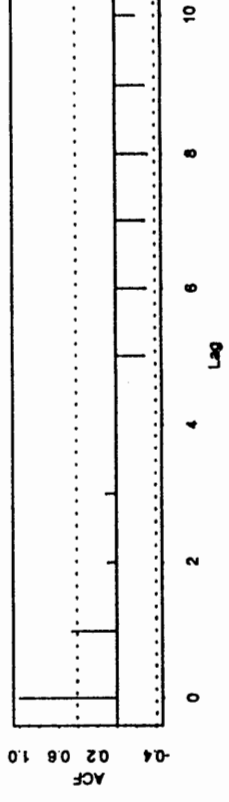
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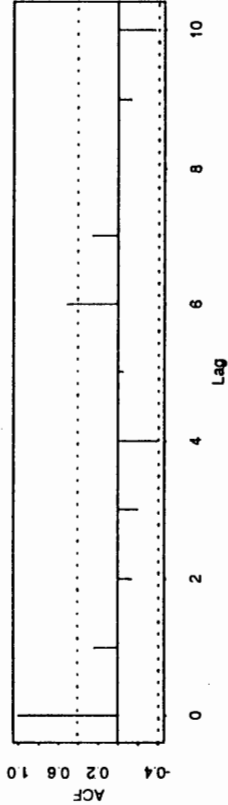


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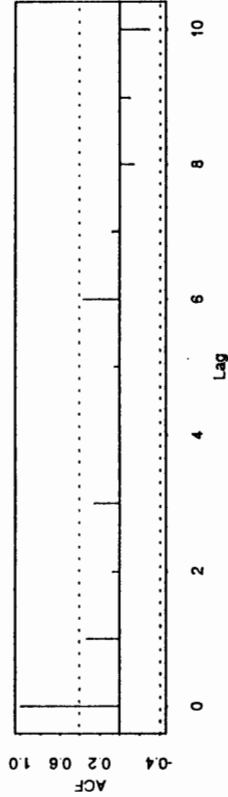


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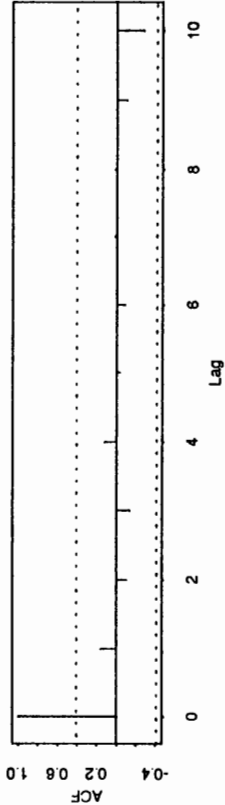
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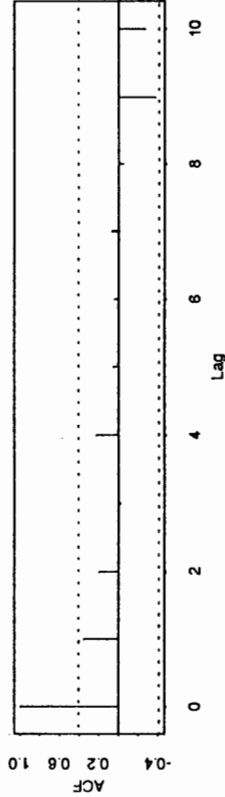
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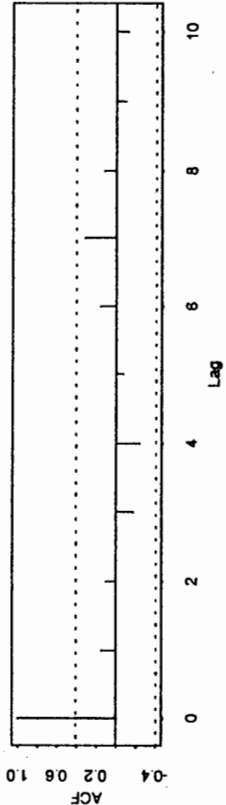
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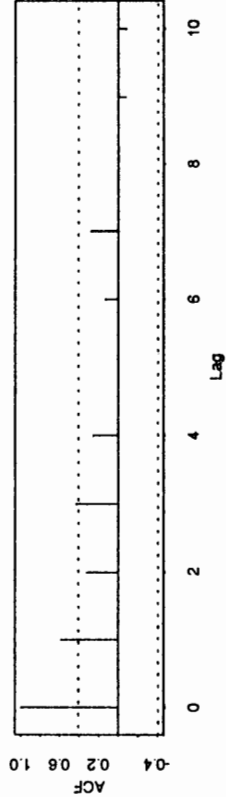
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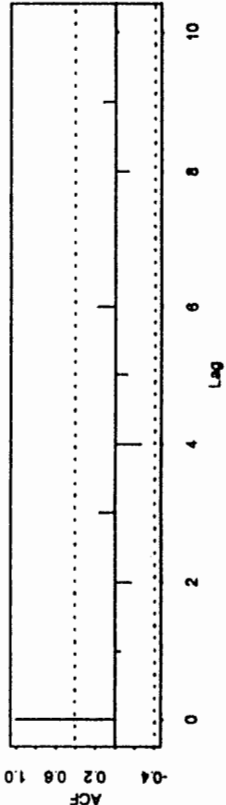
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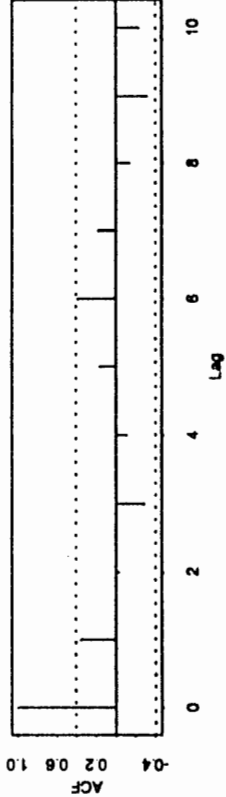
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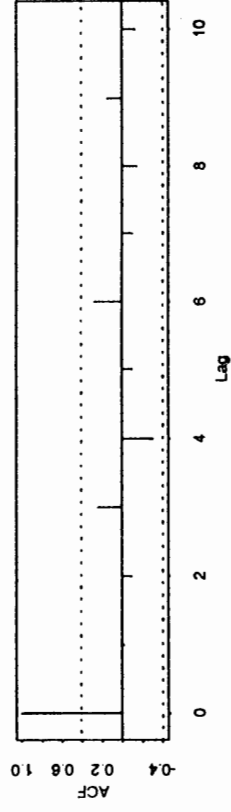


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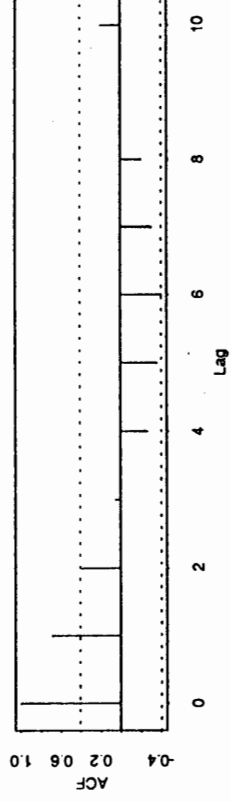


Sample Autocorrelations for Magnet DCA314 Skew Multipoles

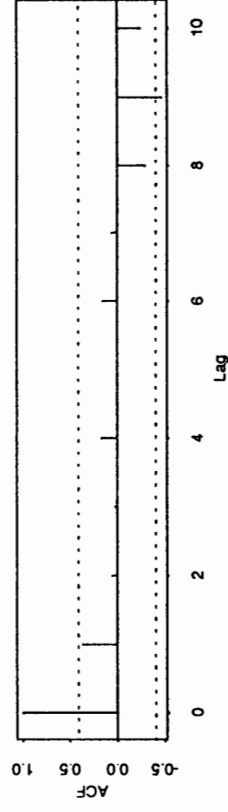
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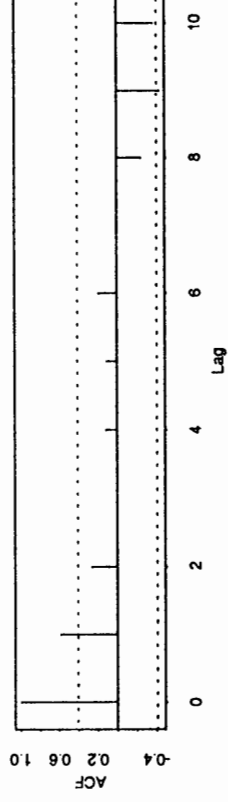
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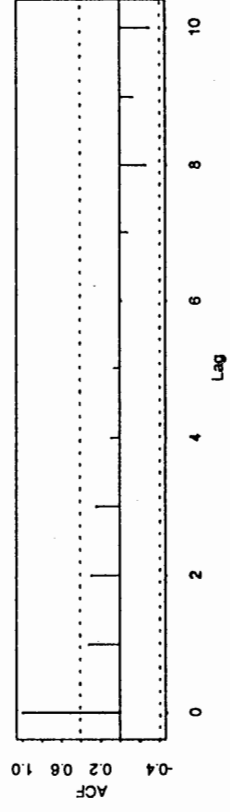
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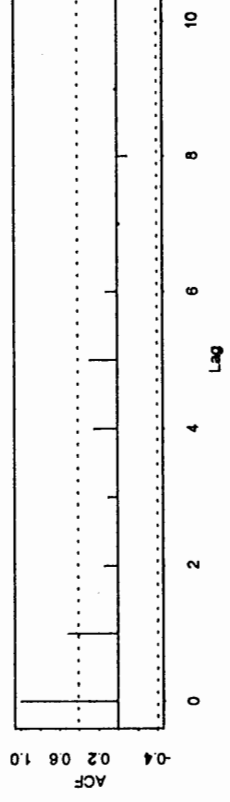
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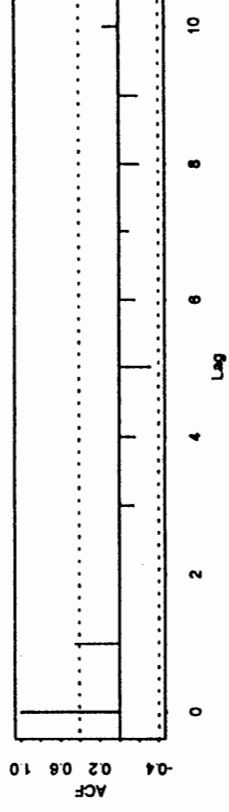
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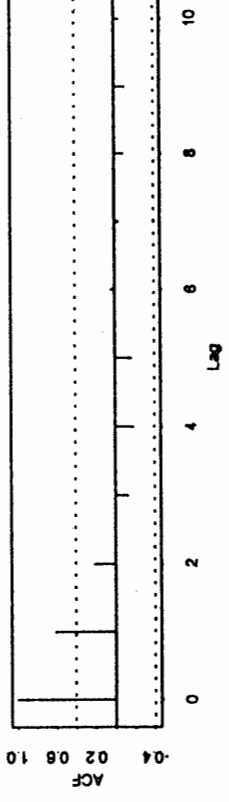
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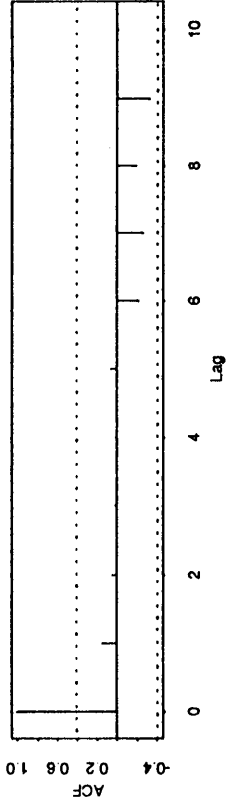


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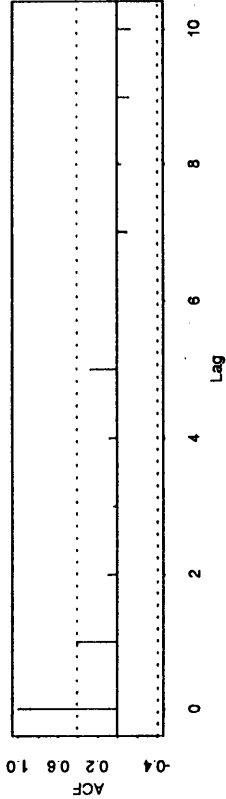


Sample Autocorrelations for Magnet DCA315 Skew Multipoles

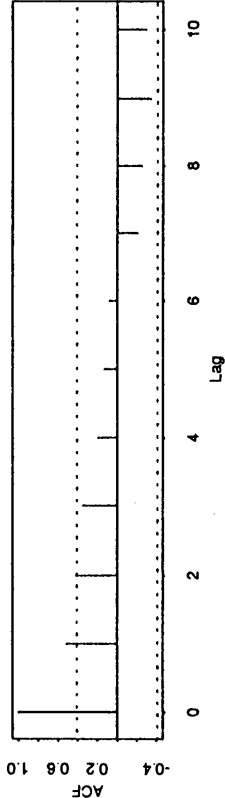
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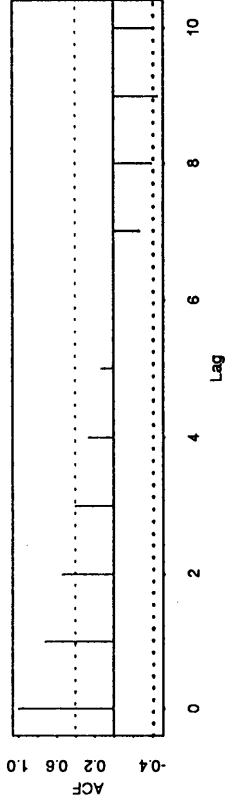
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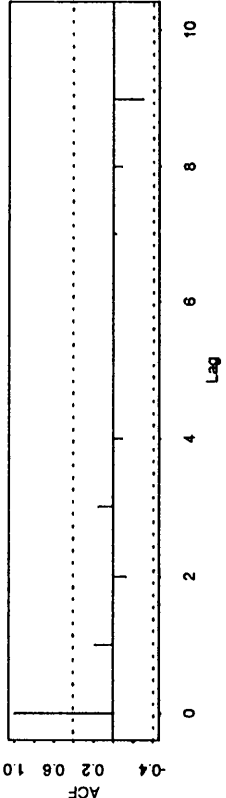
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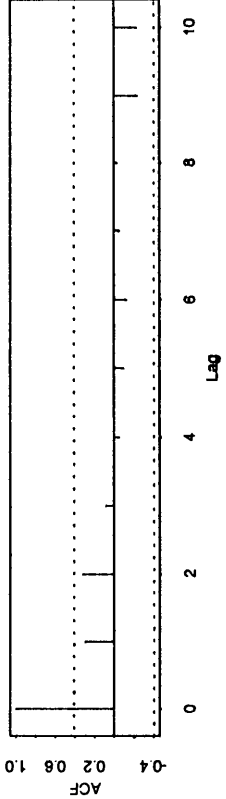
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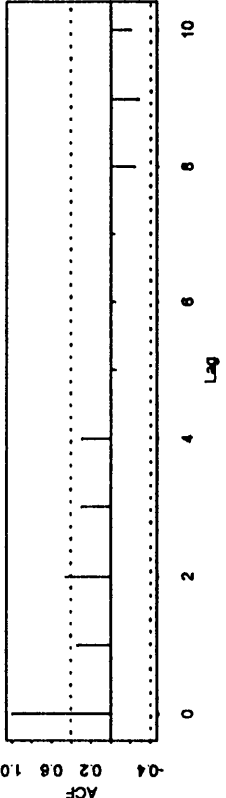
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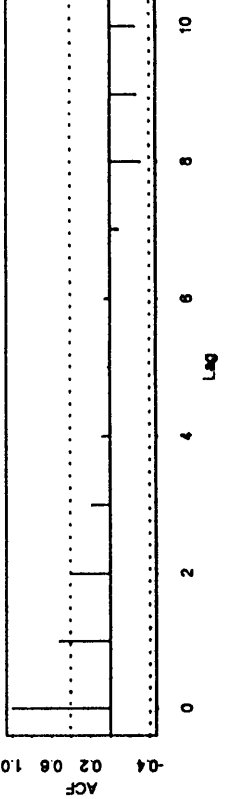
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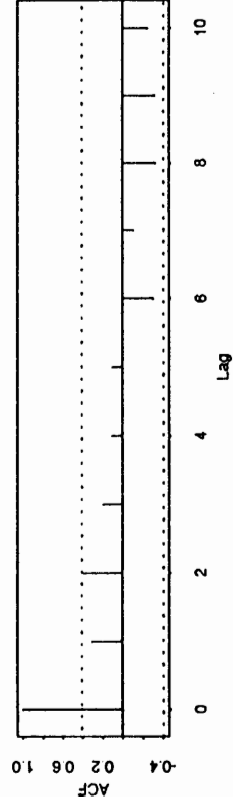


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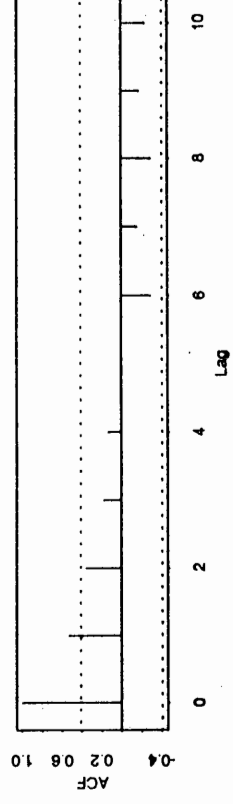


Sample Autocorrelations for Magnet DCA316 Skew Multipoles

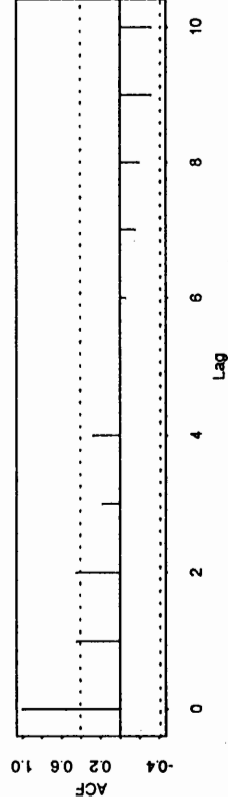
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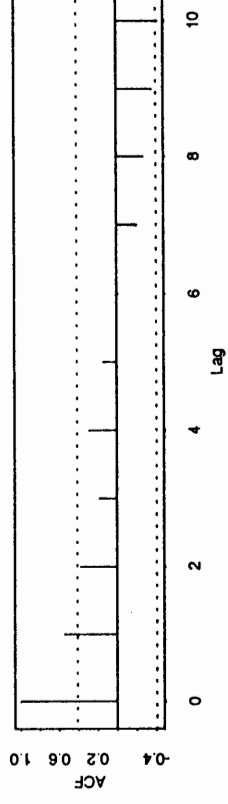
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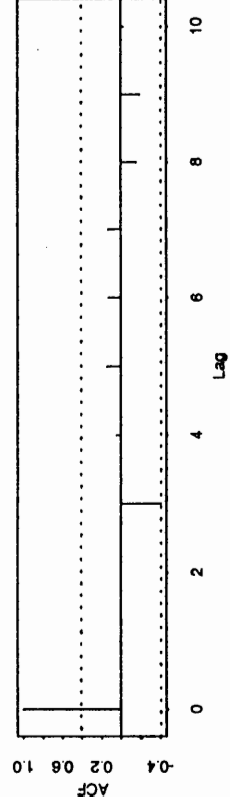
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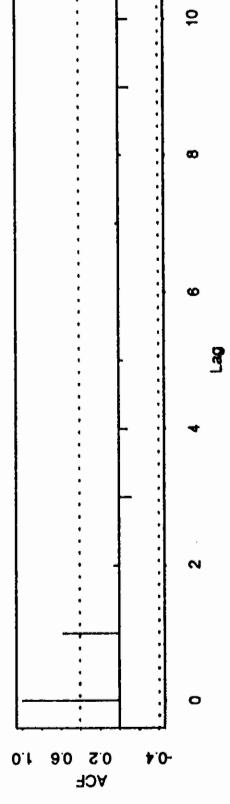
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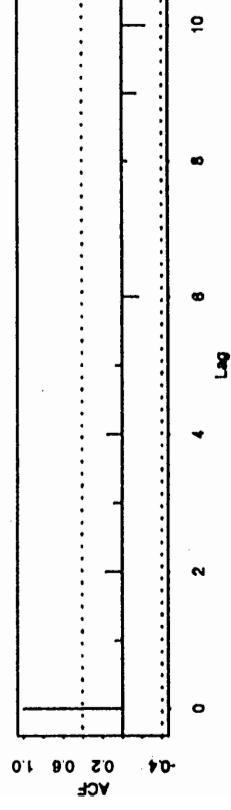
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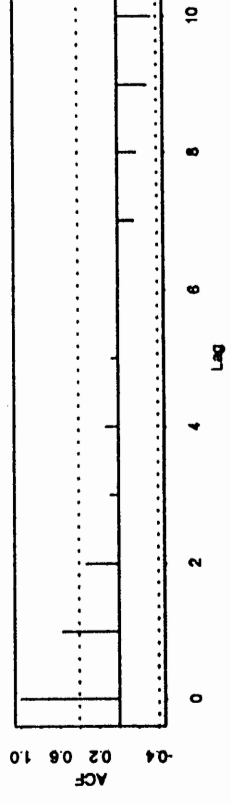
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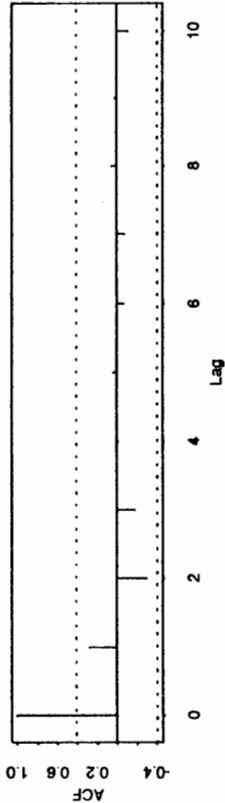


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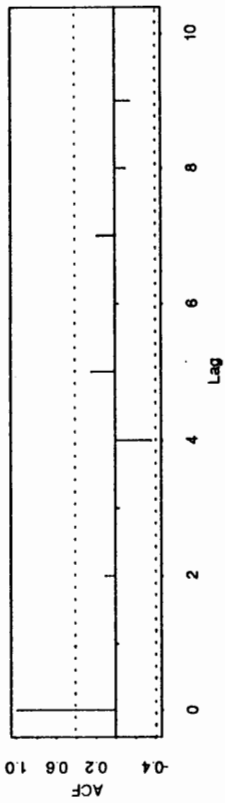


Sample Autocorrelations for Magnet DCA317 Skew Multipoles

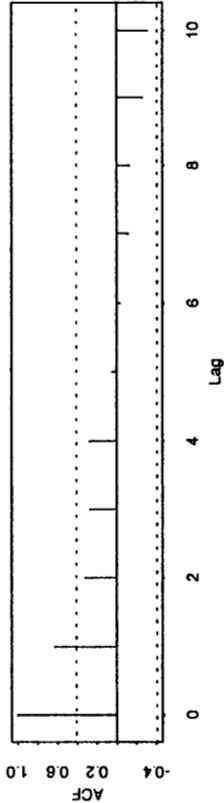
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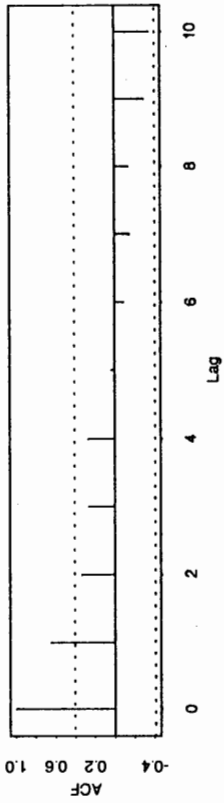
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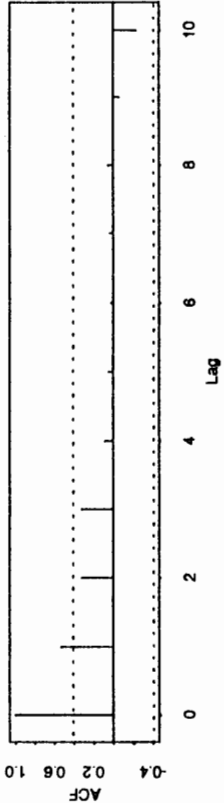
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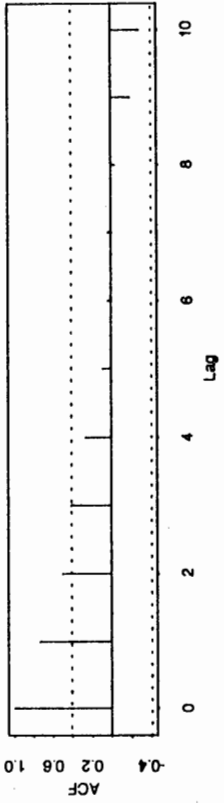
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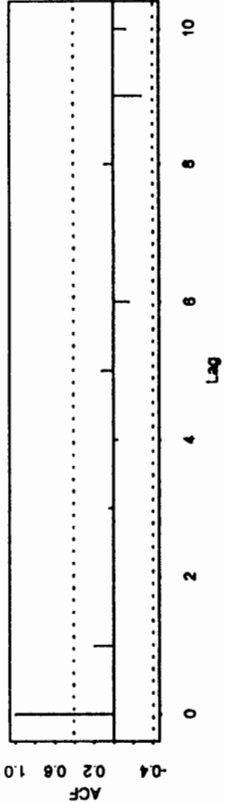
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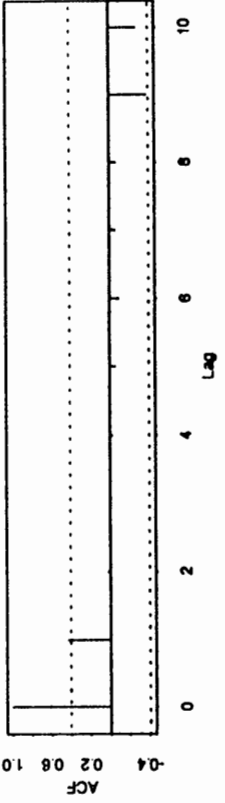
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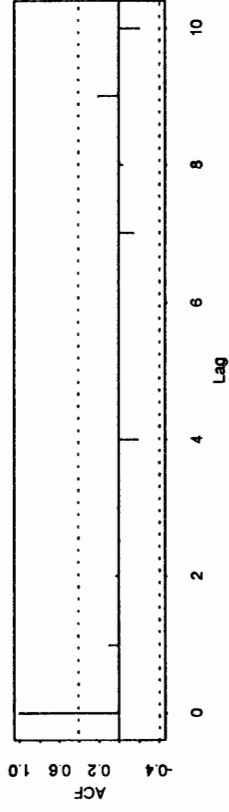


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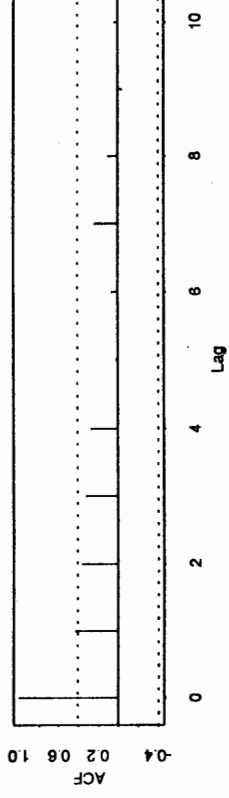


Sample Autocorrelations for Magnet DCA318 Skew Multipoles

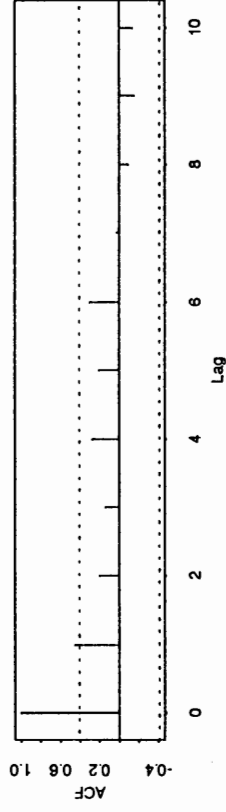
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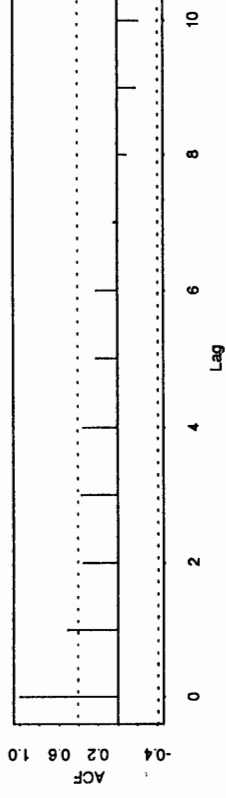
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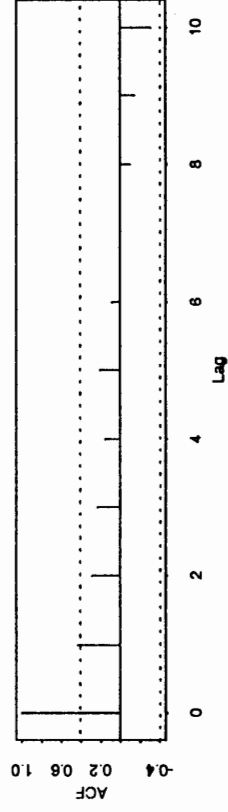
Series : Warma2



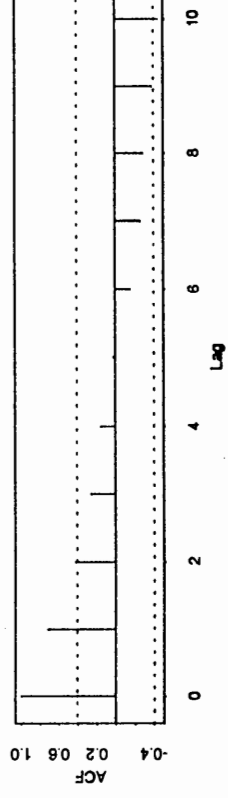
Series : Colda2



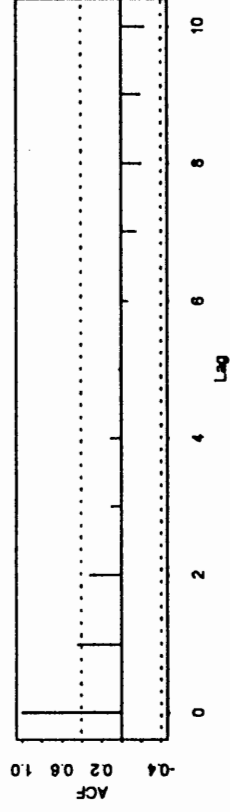
Series : Warma3



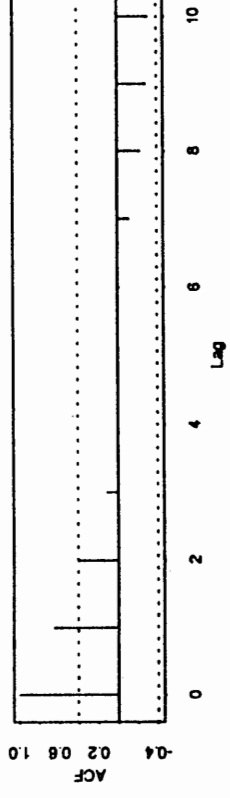
Series : Colda3



Series : Warma4

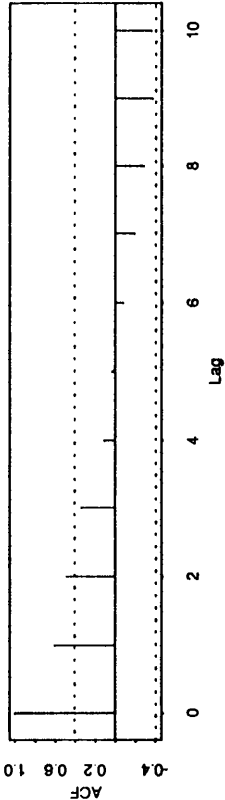


Series : Colda4

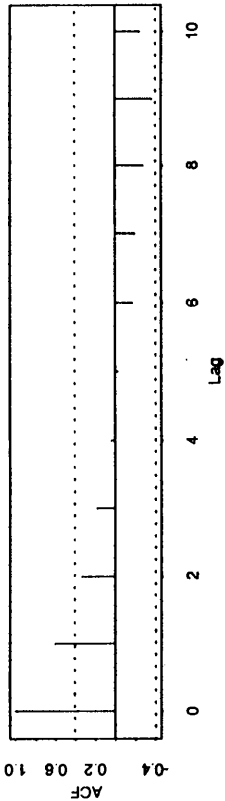


Sample Autocorrelations for Magnet DCA319 Skew Multipoles

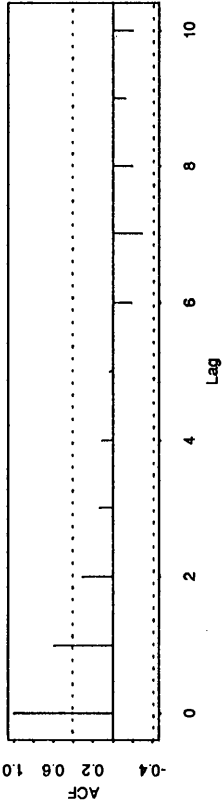
Series : Warma1



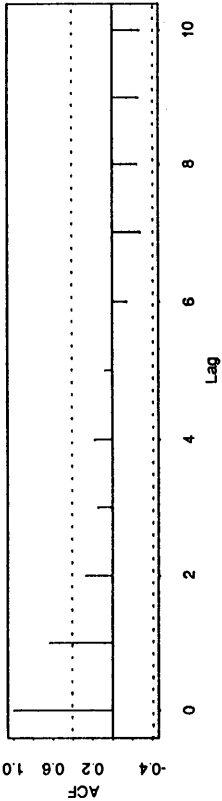
Series : Colda1



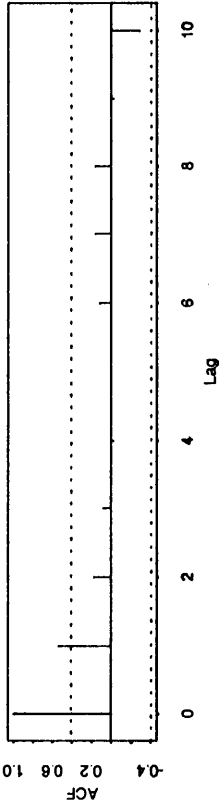
Series : Warma2



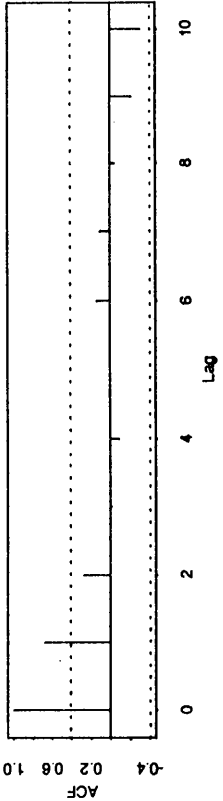
Series : Colda2



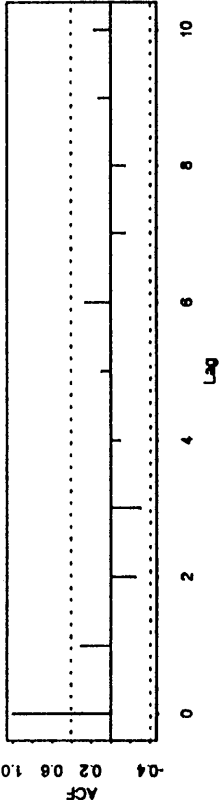
Series : Warma3



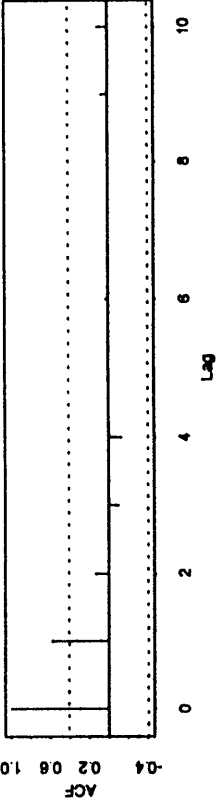
Series : Colda3



Series : Warma4

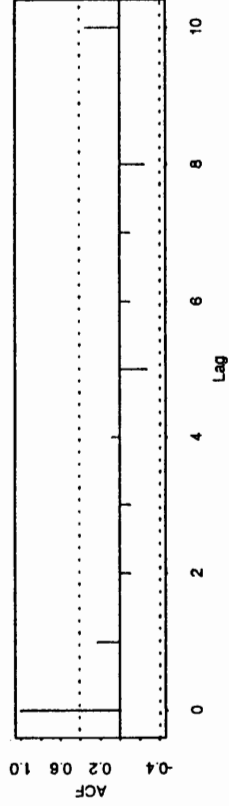


Series : Colda4

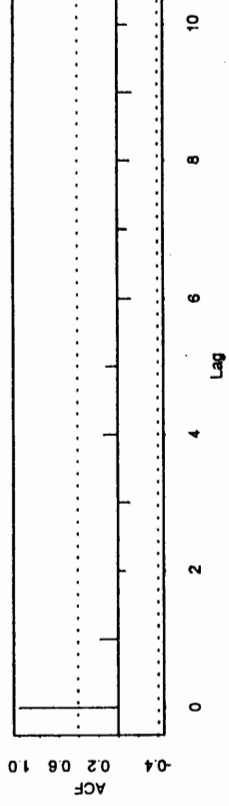


Sample Autocorrelations for Magnet DCA311 Normal Multipoles

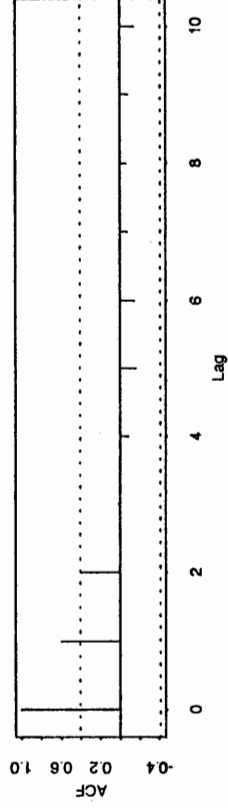
Series : Warmb1



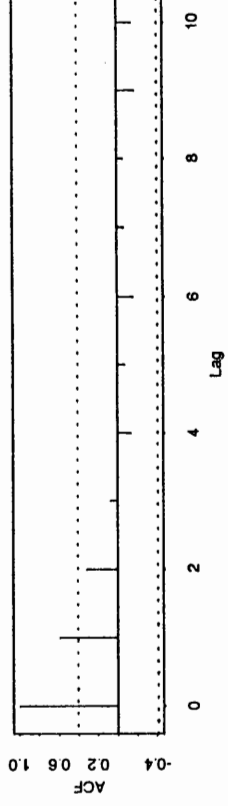
Series : Coldb1



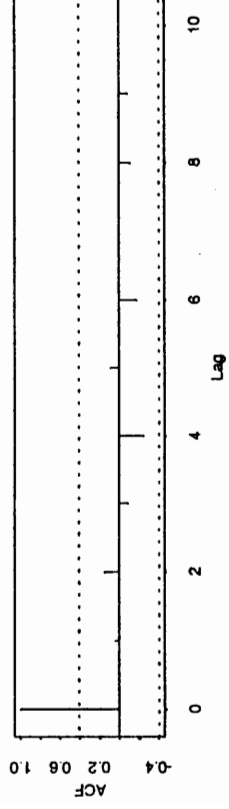
Series : Warmb2



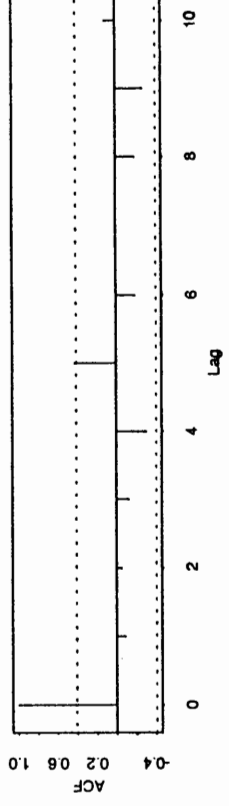
Series : Coldb2



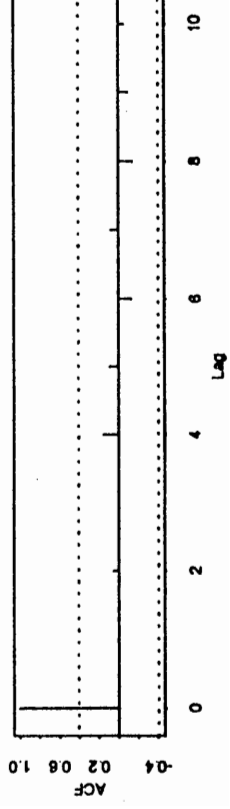
Series : Warmb4



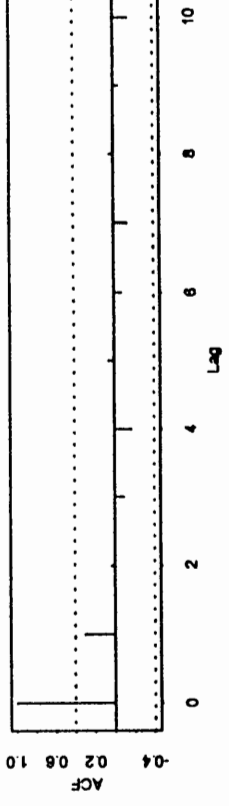
Series : Coldb4



Series : Warmb6

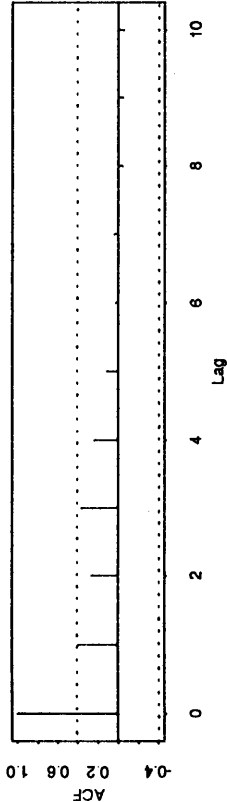


Series : Coldb6

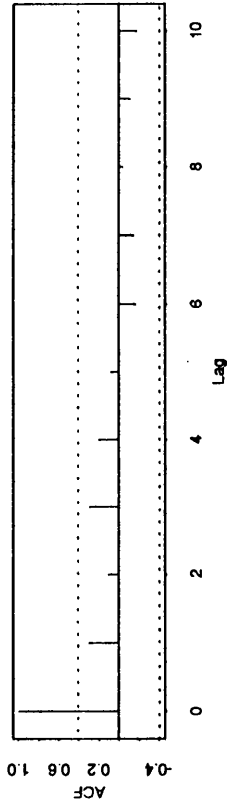


Sample Autocorrelations for Magnet DCA312 Normal Multipoles

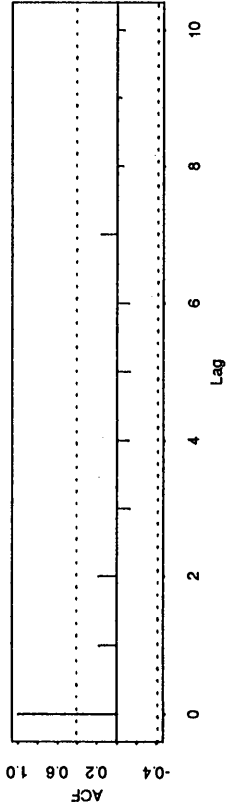
Series : Warmb1



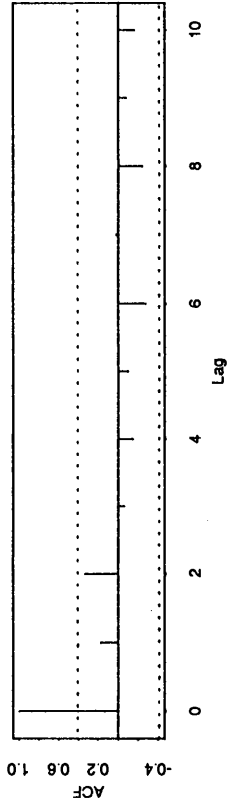
Series : Coldb1



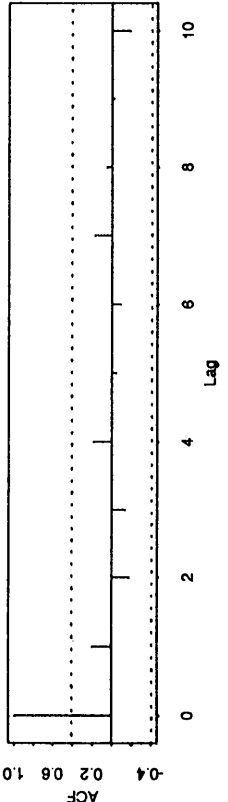
Series : Warmb2



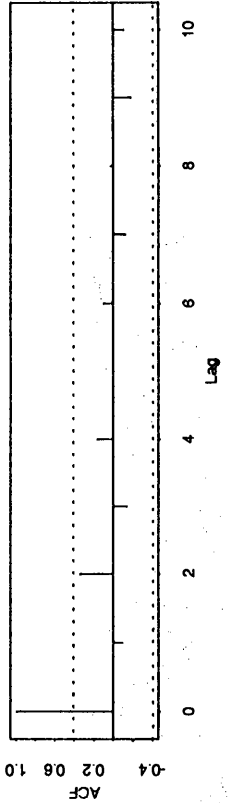
Series : Coldb2



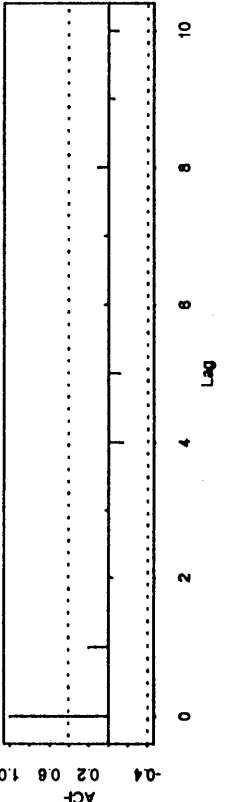
Series : Warmb4



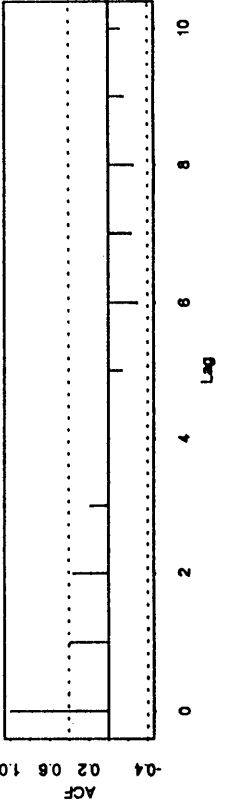
Series : Coldb4



Series : Warmb6

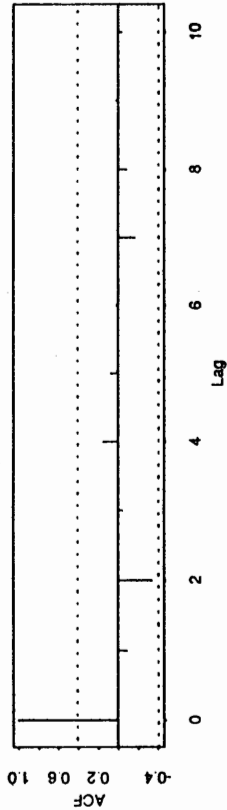


Series : Coldb6

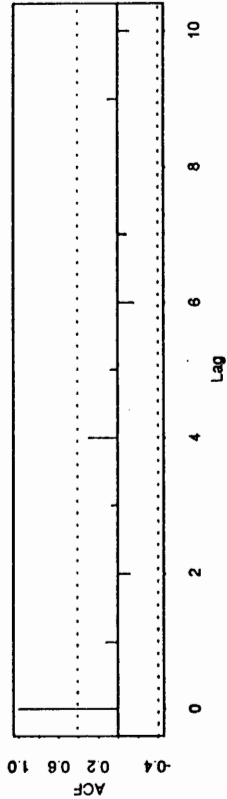


Sample Autocorrelations for Magnet DCA313 Normal Multipoles

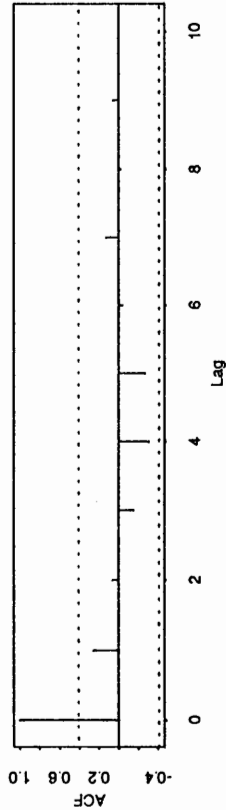
Series : Warmb1



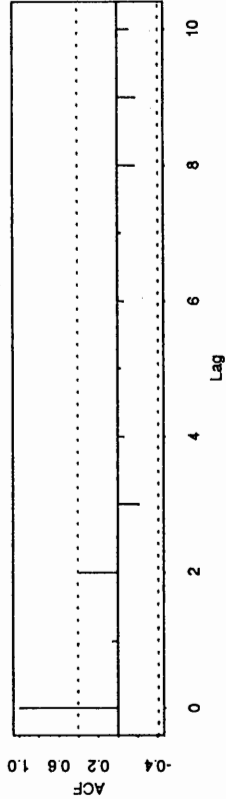
Series : Coldb1



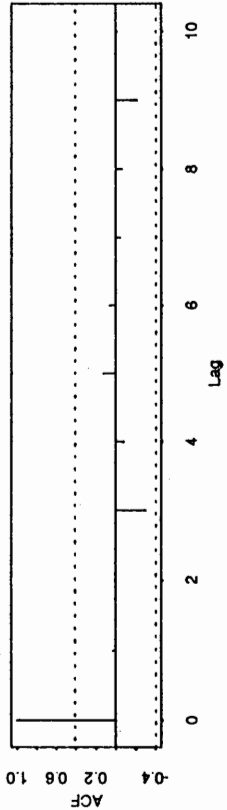
Series : Warmb2



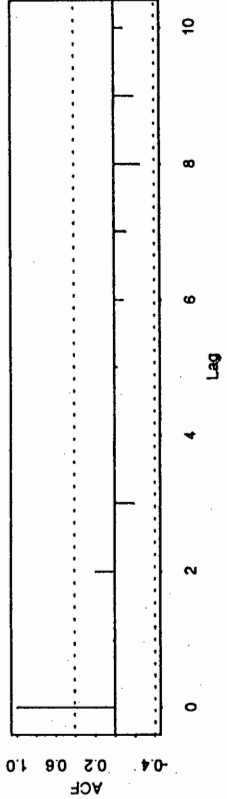
Series : Coldb2



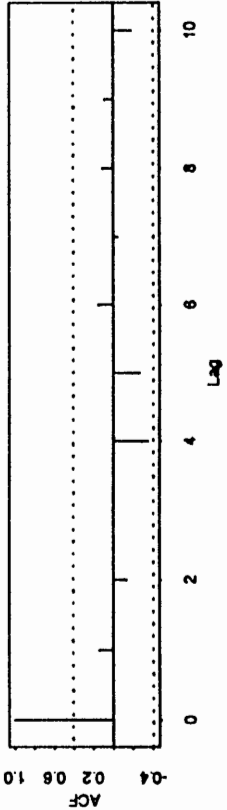
Series : Warmb4



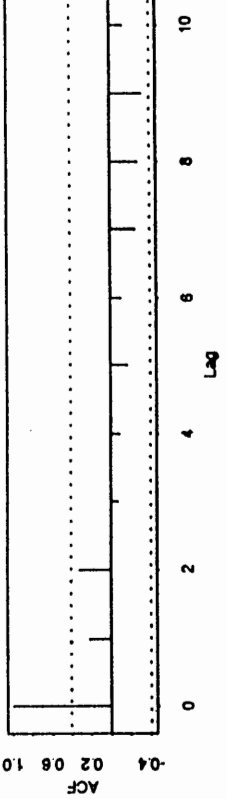
Series : Coldb4



Series : Warmb6

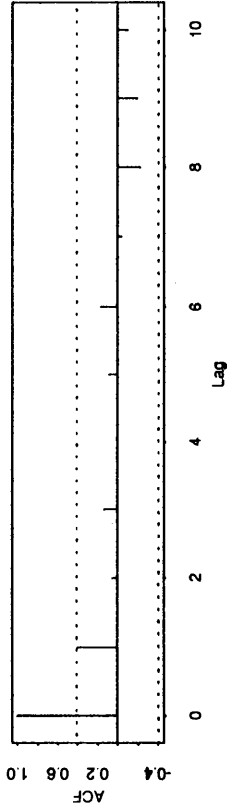


Series : Coldb6

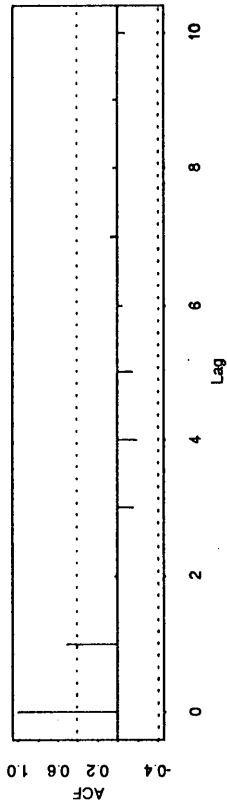


Sample Autocorrelations for Magnet DCA314 Normal Multipoles

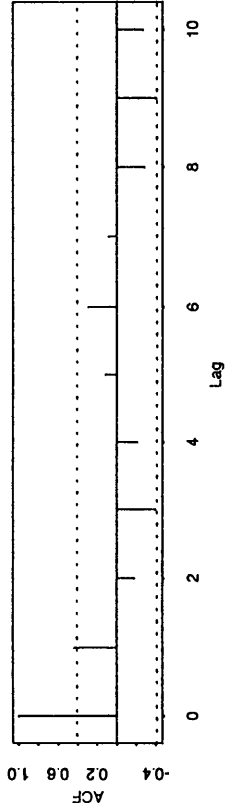
Series : Warmb1



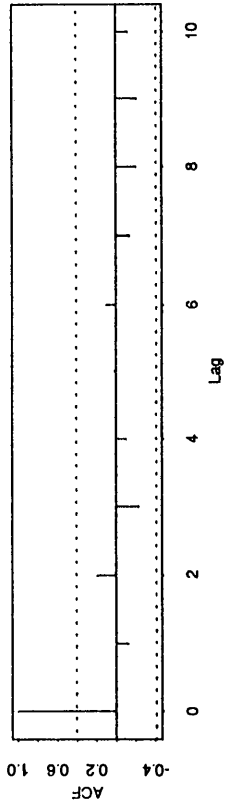
Series : Coldb1



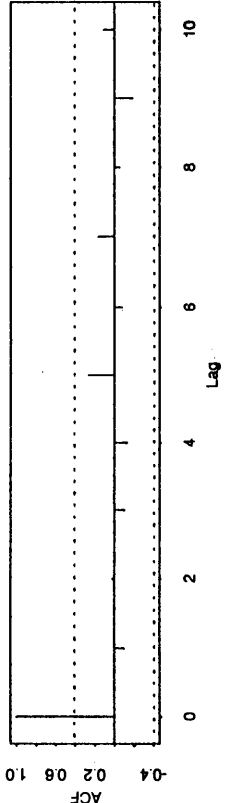
Series : Warmb2



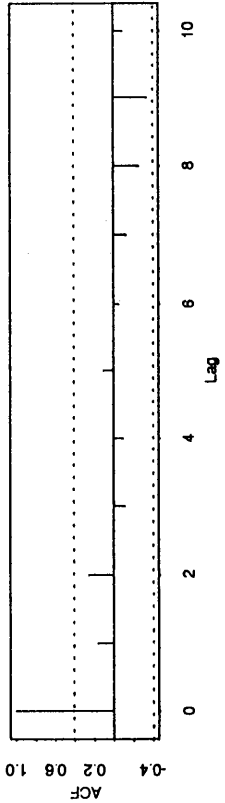
Series : Coldb2



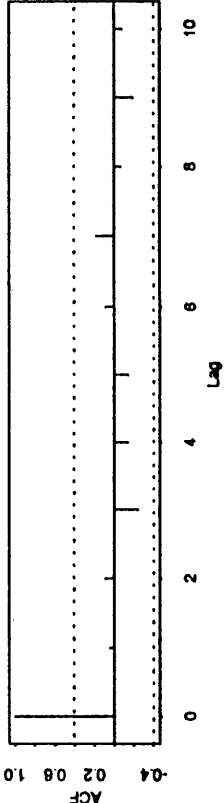
Series : Warmb4



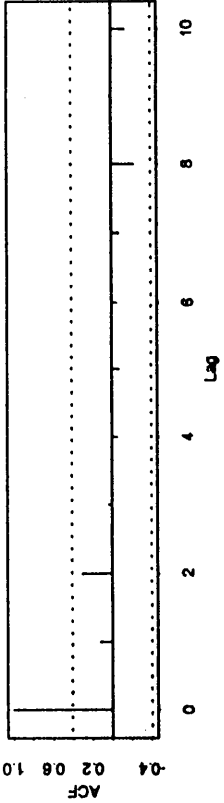
Series : Coldb4



Series : Warmb6

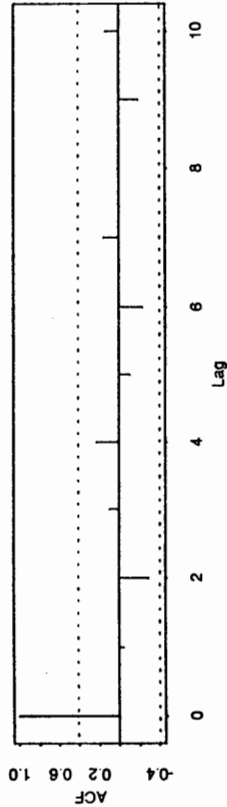


Series : Coldb6

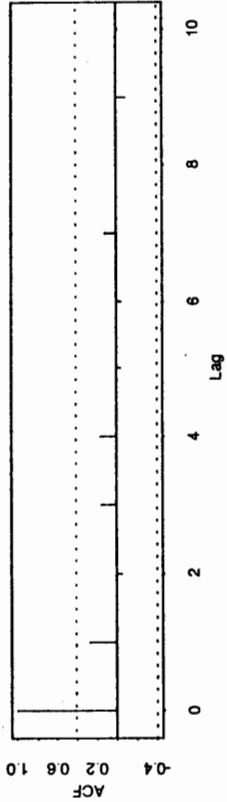


Sample Autocorrelations for Magnet DCA315 Normal Multipoles

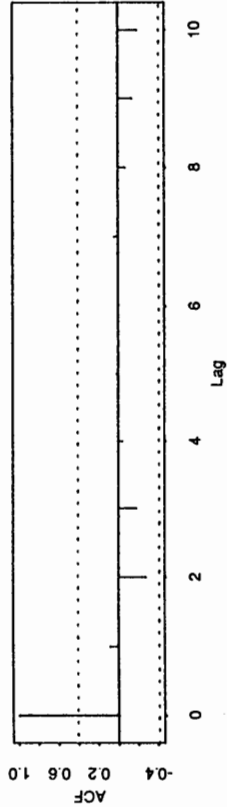
Series : Warmb1



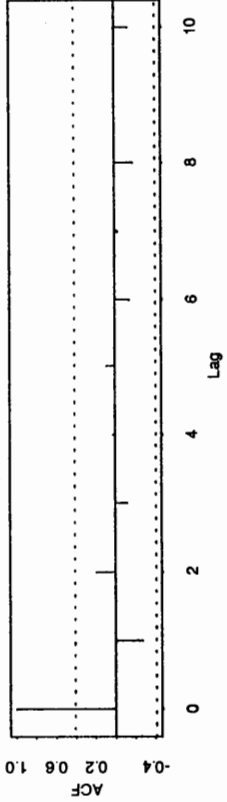
Series : Coldb1



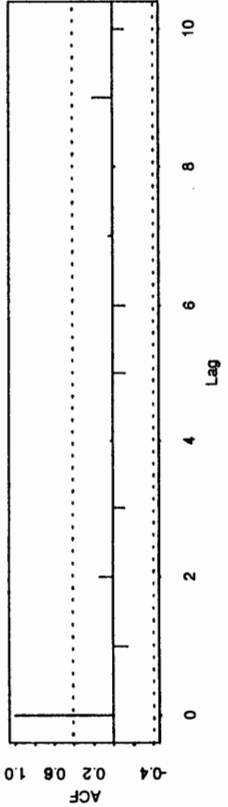
Series : Warmb2



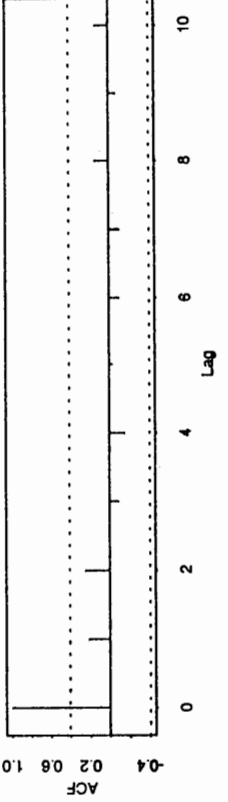
Series : Coldb2



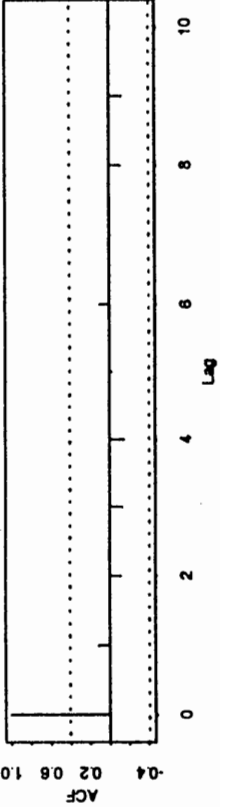
Series : Warmb4



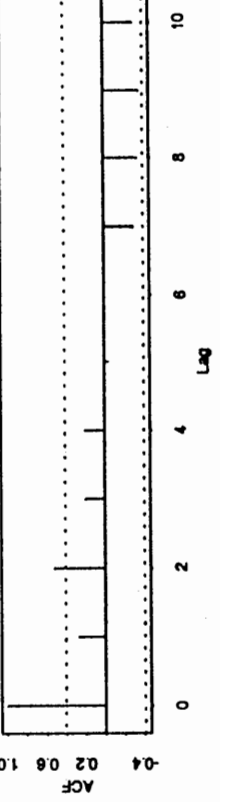
Series : Coldb4



Series : Warmb6

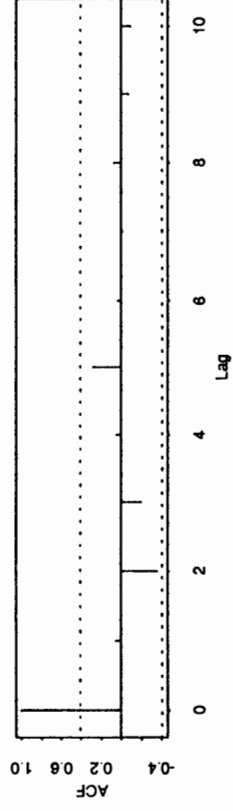


Series : Coldb6

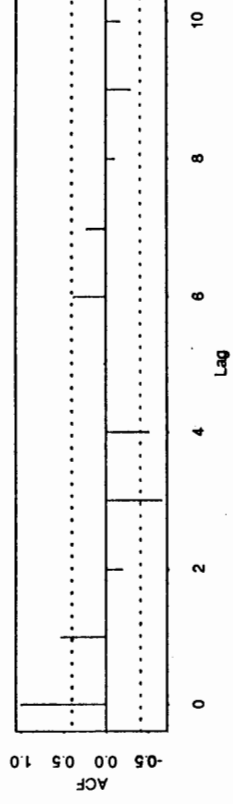


Sample Autocorrelations for Magnet DCA316 Normal Multipoles

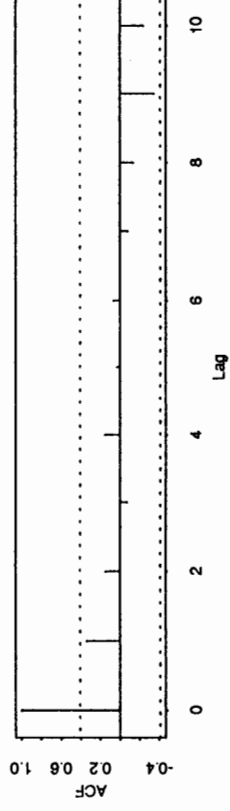
Series : Warmb1



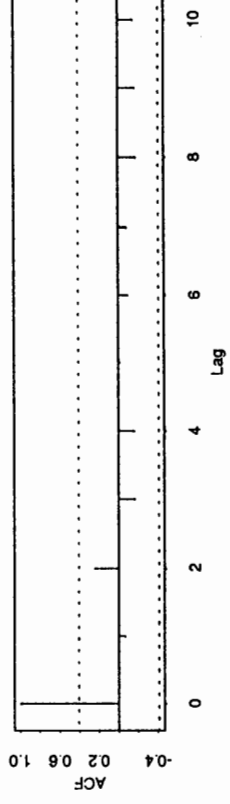
Series : Coldb1



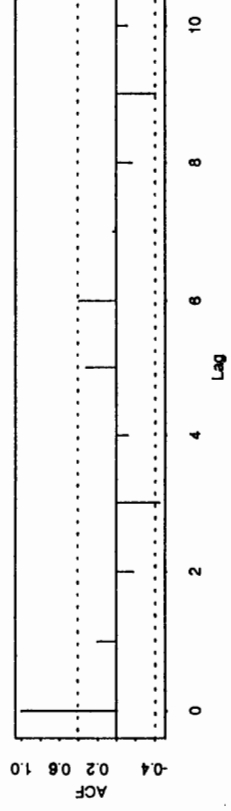
Series : Warmb2



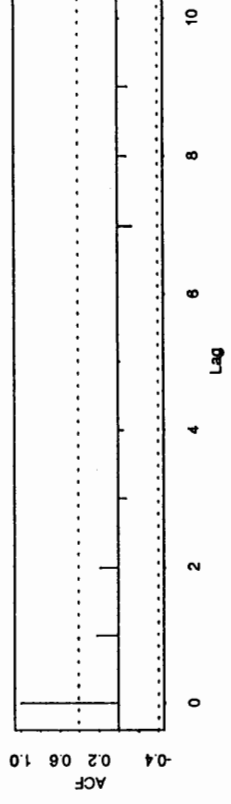
Series : Coldb2



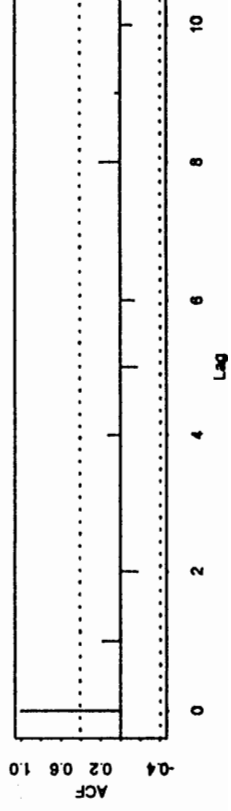
Series : Warmb4



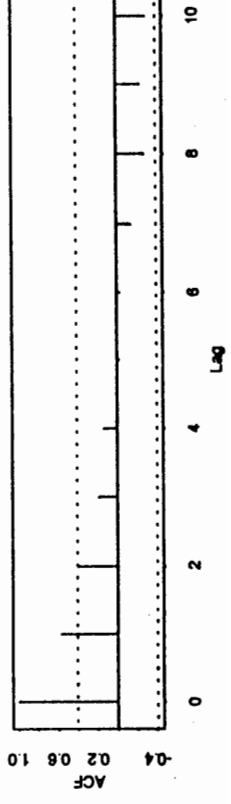
Series : Coldb4



Series : Warmb6

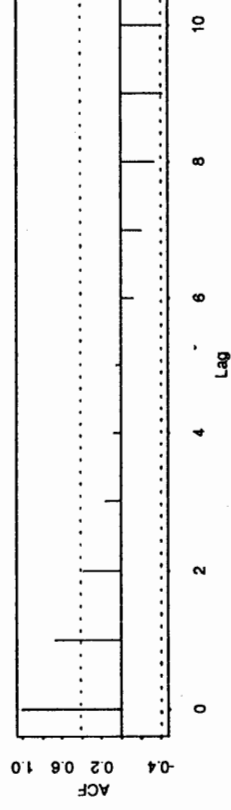


Series : Coldb6

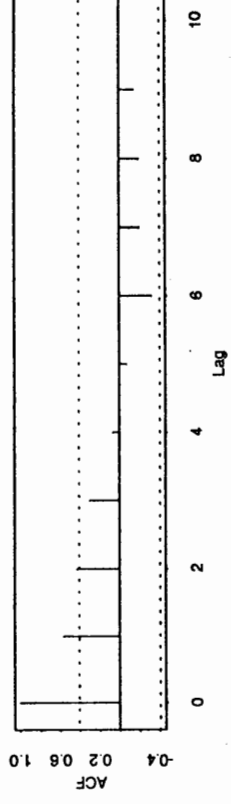


Sample Autocorrelations for Magnet DCA317 Normal Multipoles

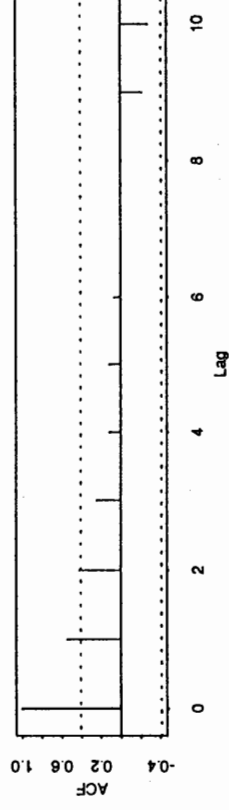
Series : Warmb1



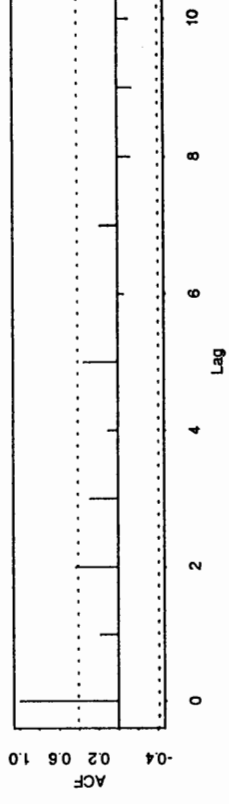
Series : Coldb1



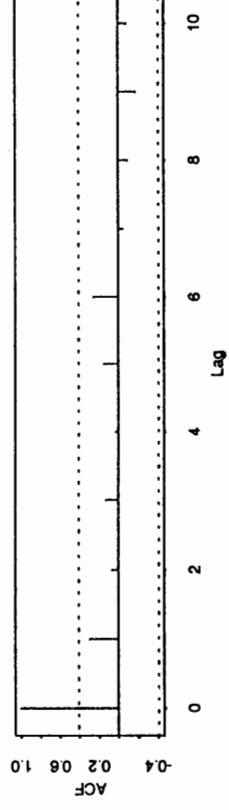
Series : Warmb2



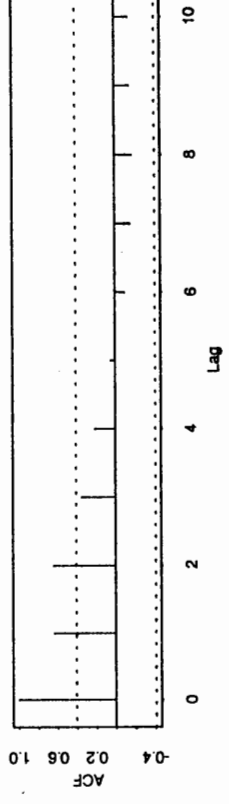
Series : Coldb2



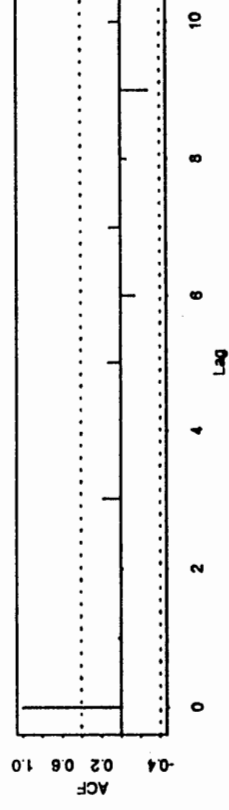
Series : Warmb4



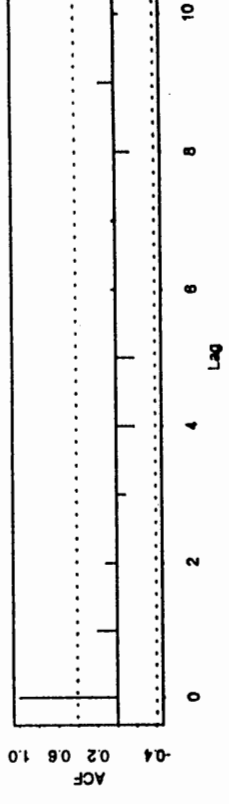
Series : Coldb4



Series : Warmb6

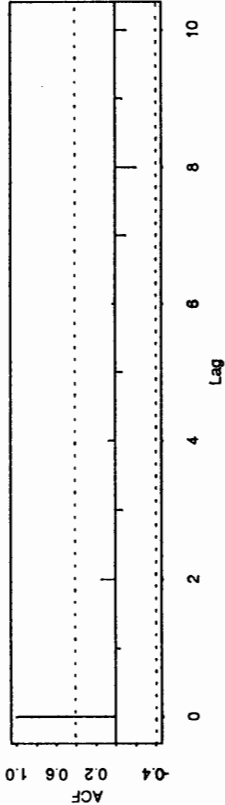


Series : Coldb6

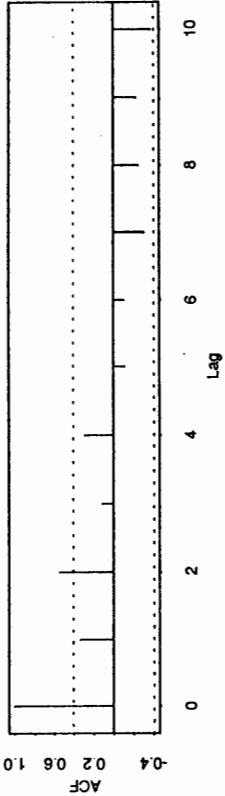


Sample Autocorrelations for Magnet DCA318 Normal Multipoles

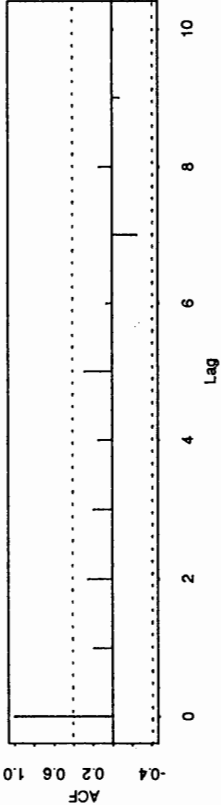
Series : Warmb1



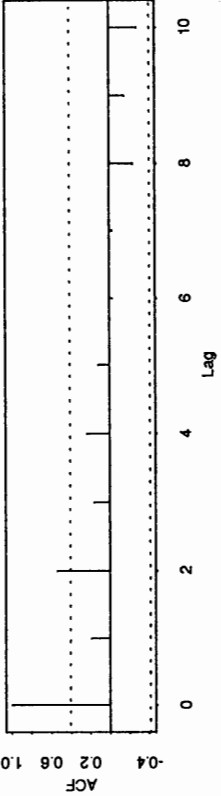
Series : Coldb1



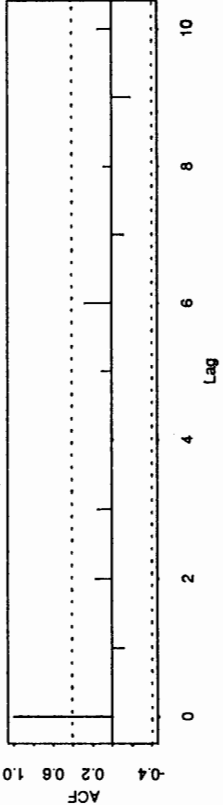
Series : Warmb2



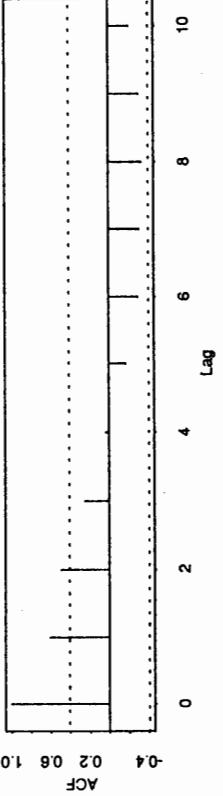
Series : Coldb2



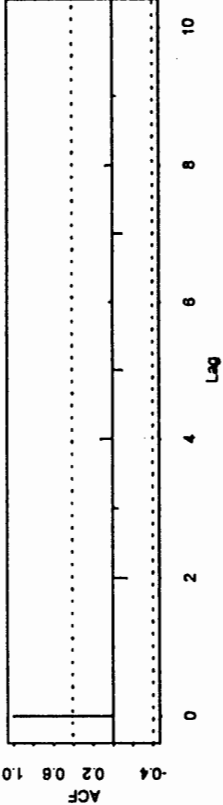
Series : Warmb4



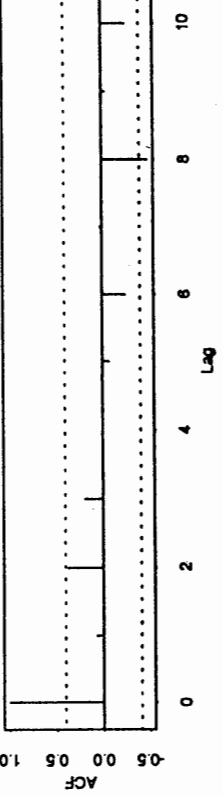
Series : Coldb4



Series : Warmb6

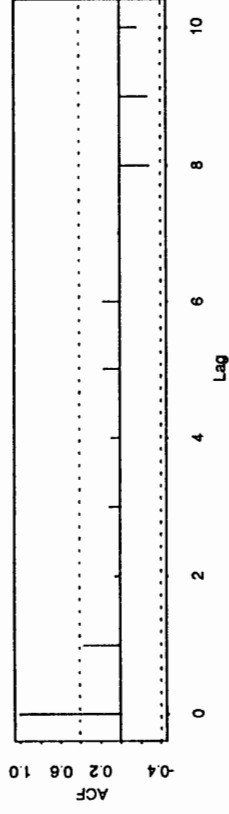


Series : Coldb6

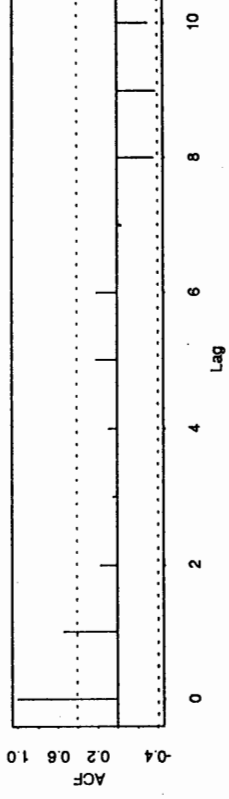


Sample Autocorrelations for Magnet DCA319 Normal Multipoles

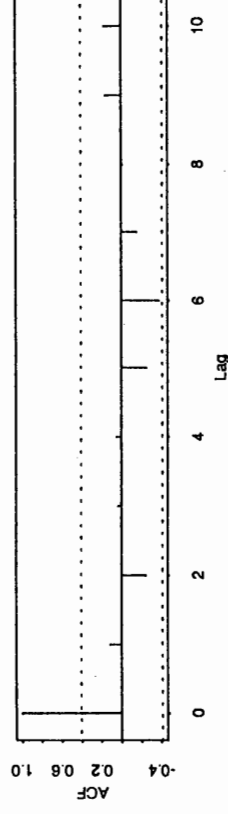
Series : Warmb1



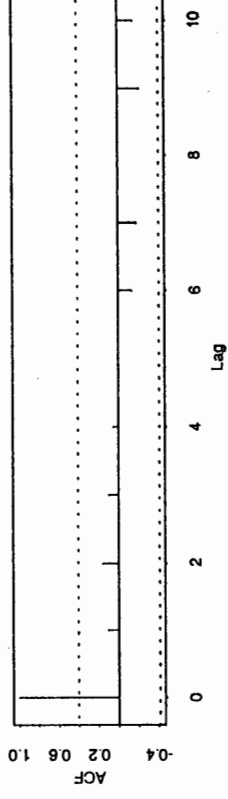
Series : Coldb1



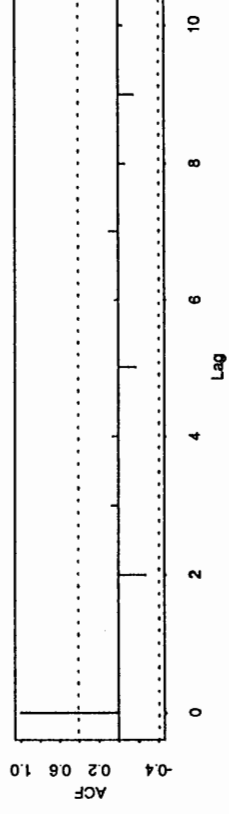
Series : Warmb2



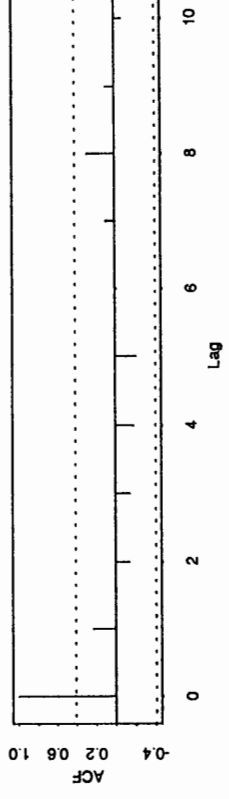
Series : Coldb2



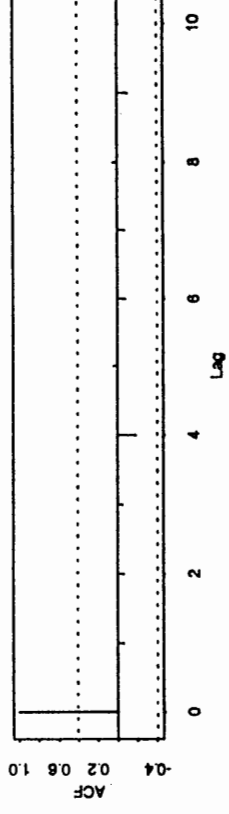
Series : Warmb4



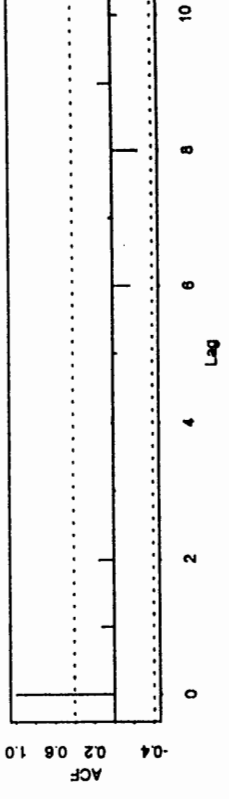
Series : Coldb4



Series : Warmb6



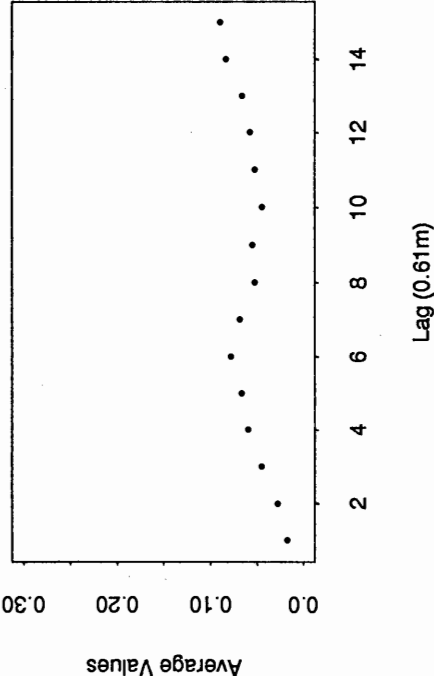
Series : Coldb6



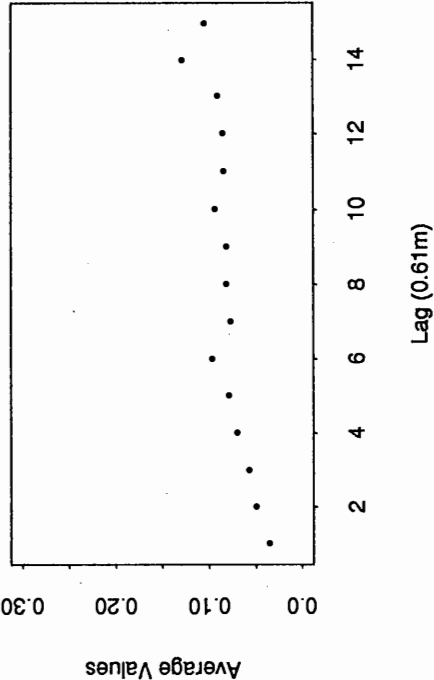
Appendix B
Multipole Semivariograms

Magnet DCA311 Skew Multipoles: a1

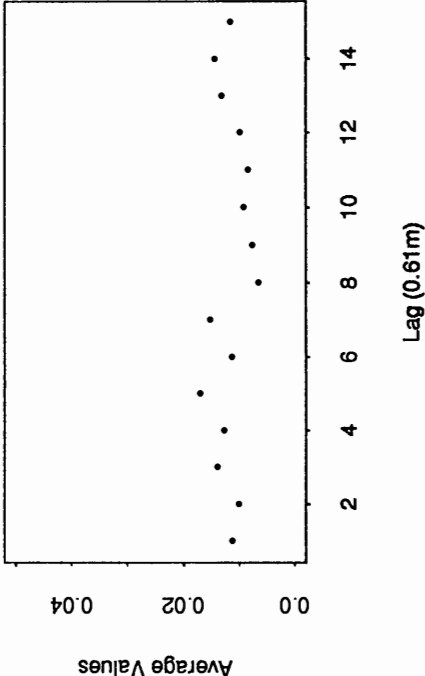
Cold a1 Semivariogram



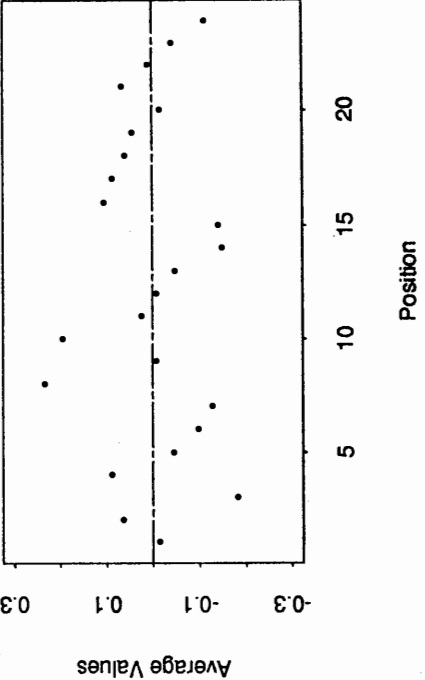
Warm a1 Semivariogram



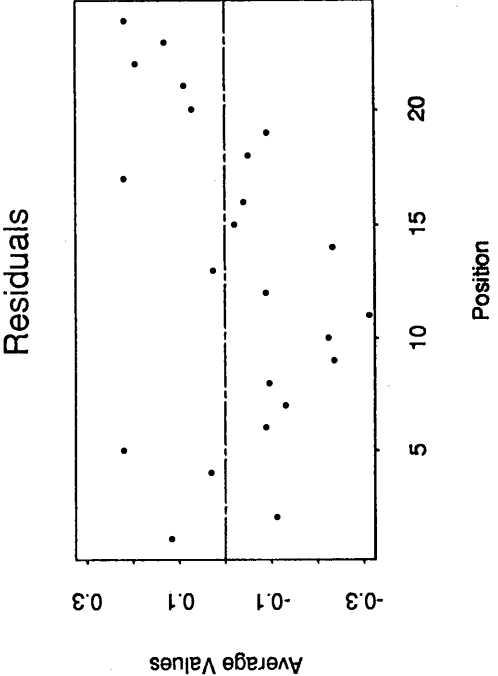
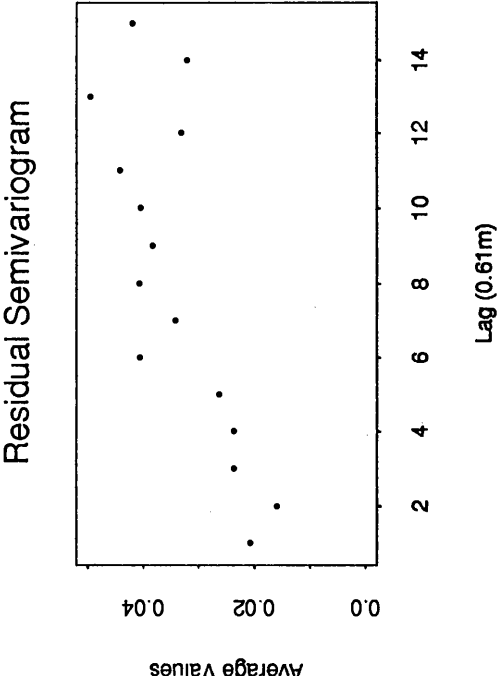
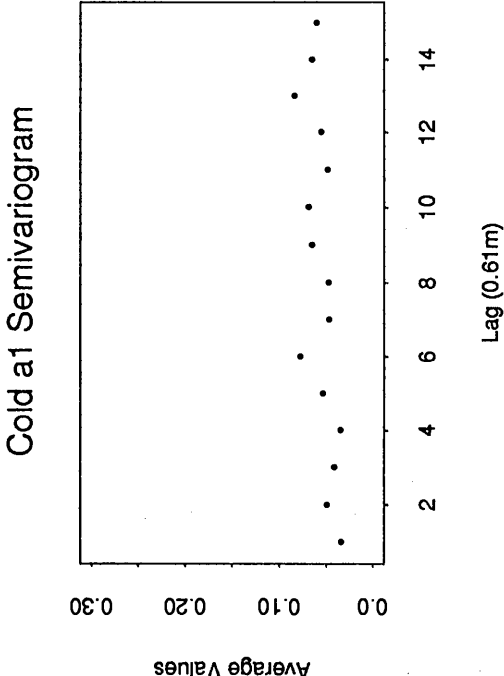
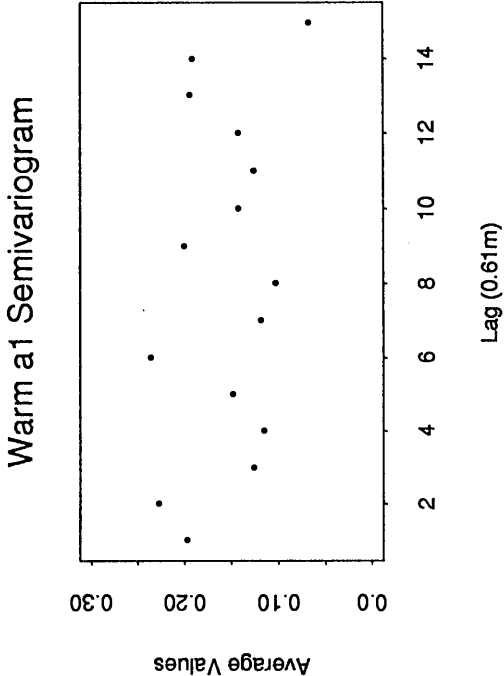
Residual Semivariogram



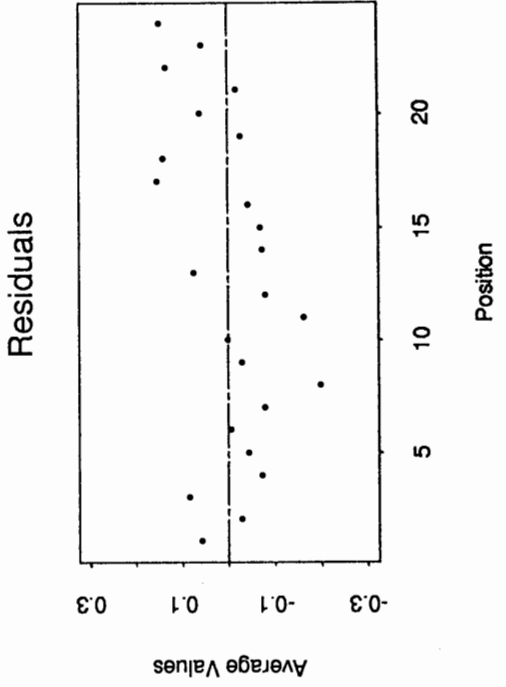
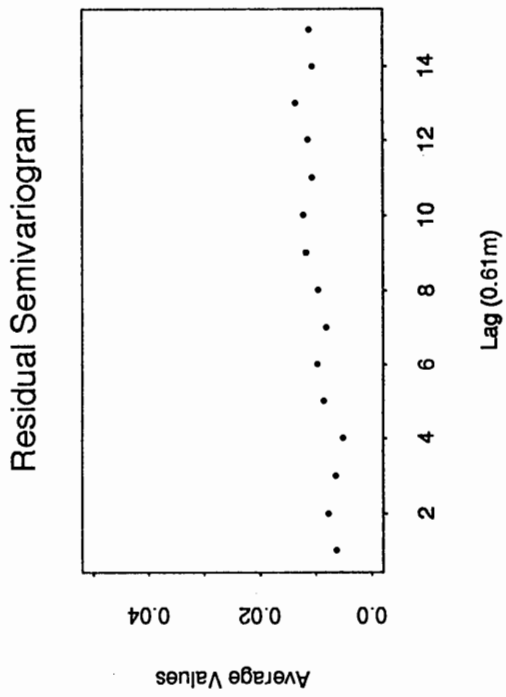
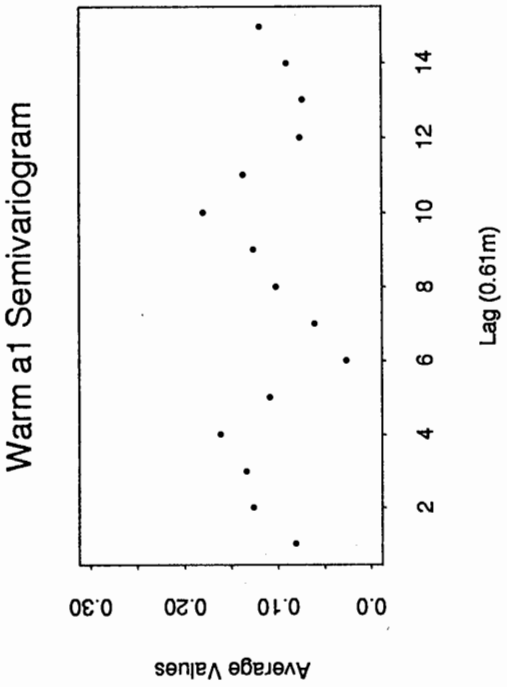
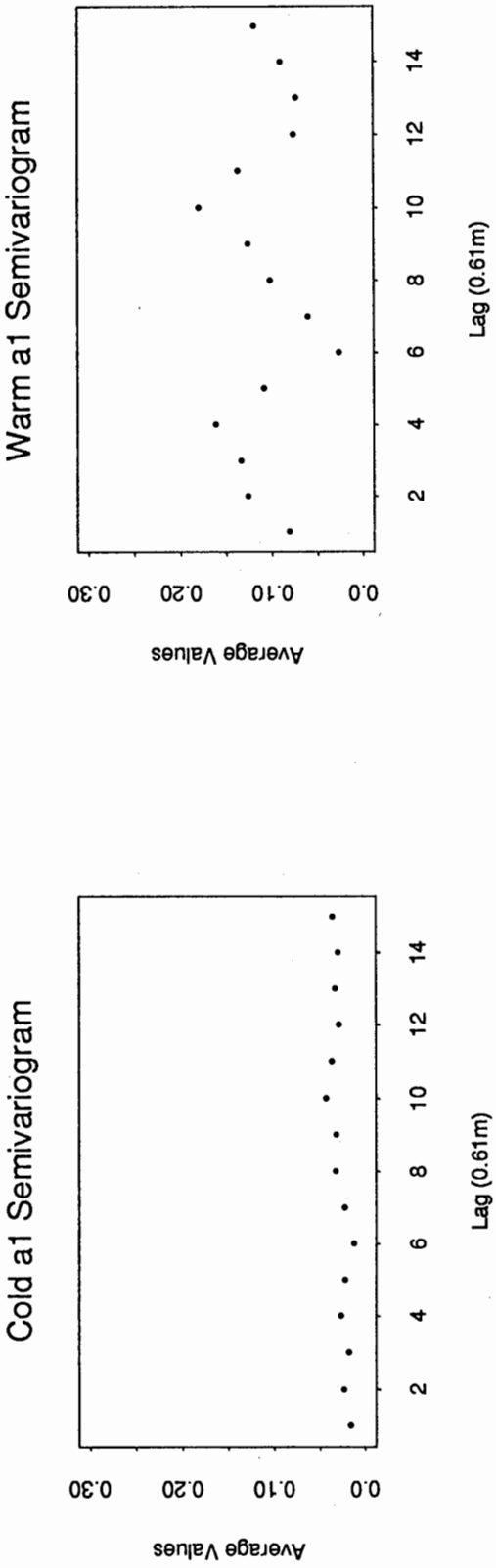
Residuals



Magnet DCA312 Skew Multipoles: a1

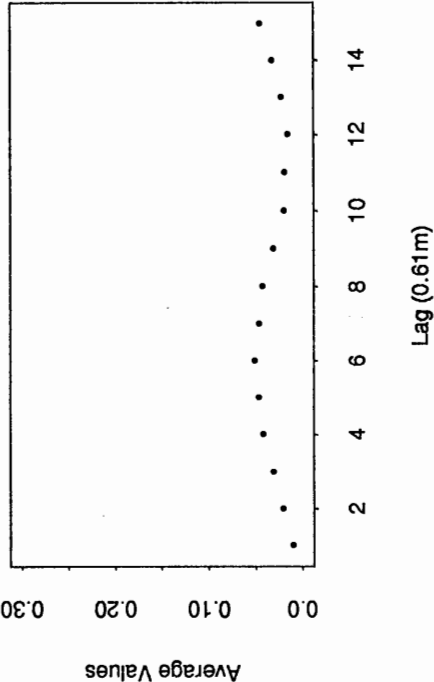


Magnet DCA313 Skew Multipoles: a1

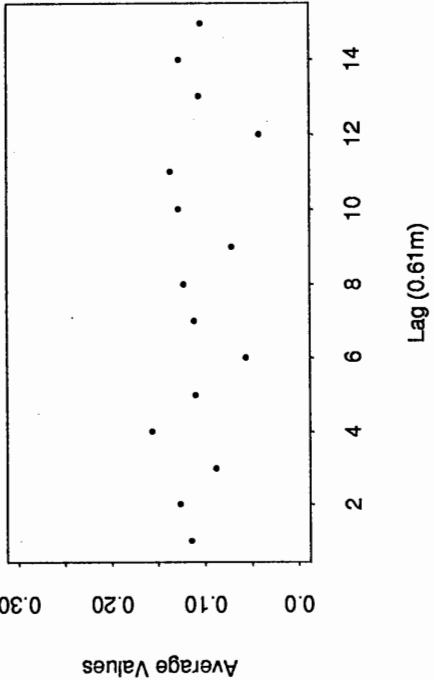


Magnet DCA314 Skew Multipoles: a1

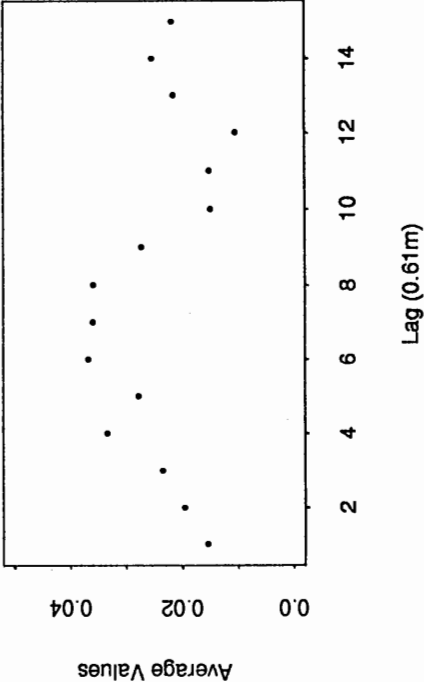
Cold a1 Semivariogram



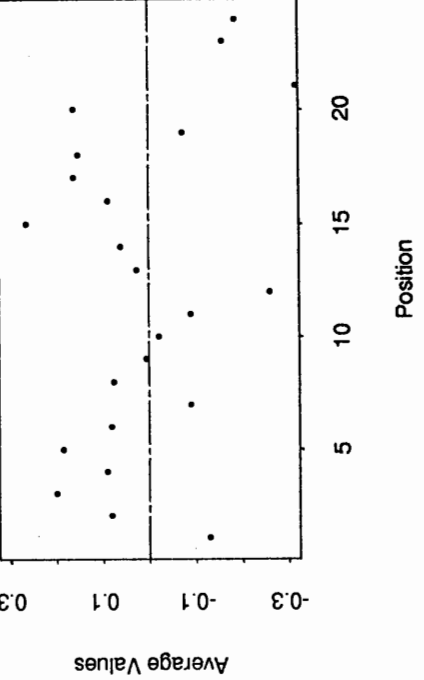
Warm a1 Semivariogram



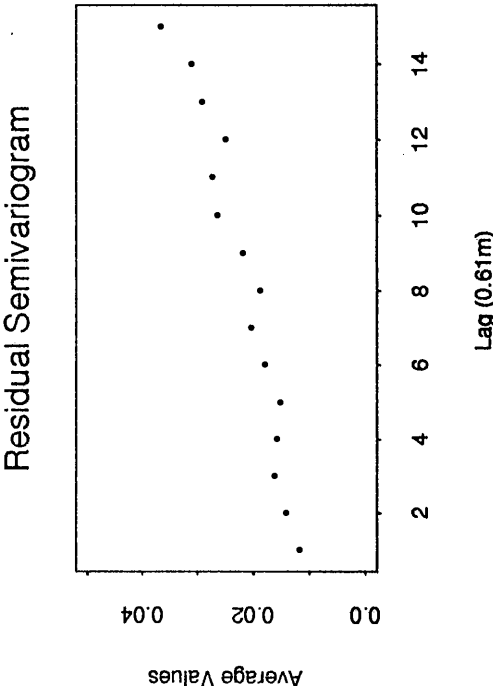
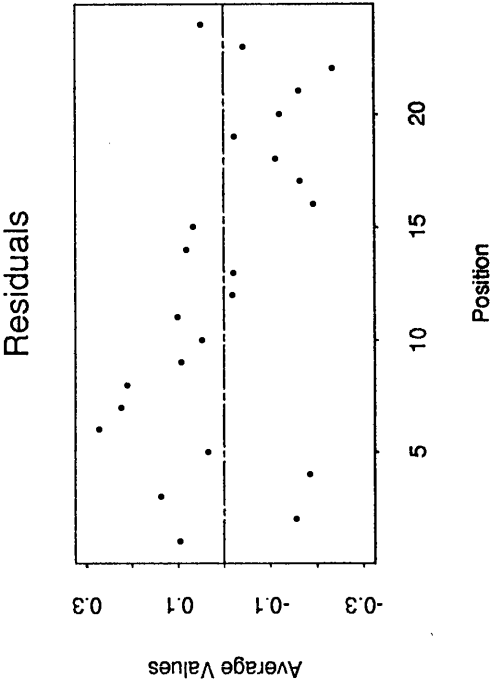
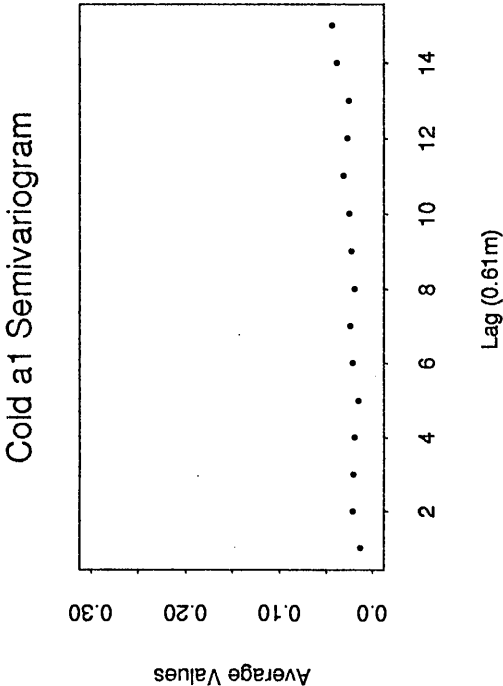
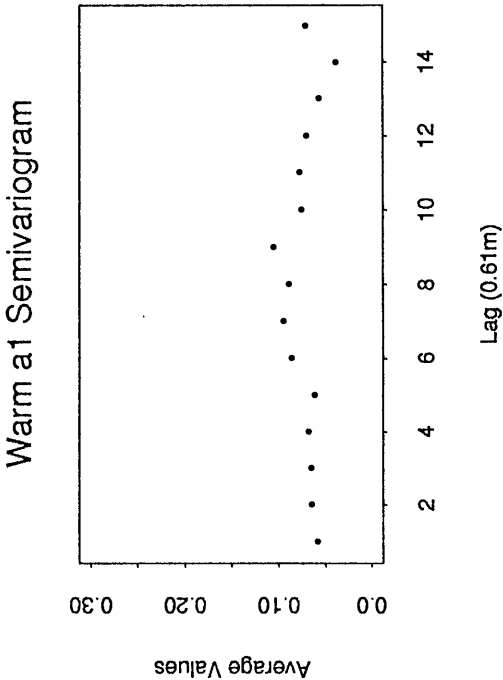
Residual Semivariogram



Residuals

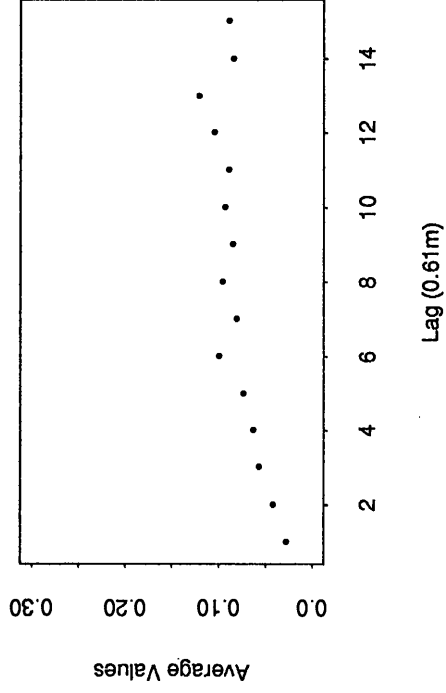


Magnet DCA315 Skew Multipoles: a1

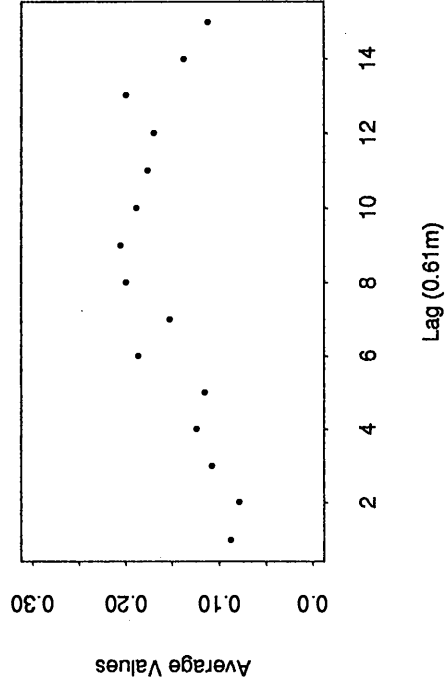


Magnet DCA316 Skew Multipoles: a1

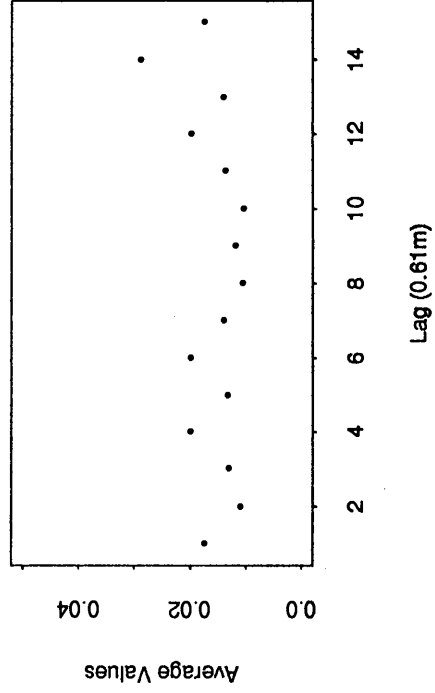
Cold a1 Semivariogram



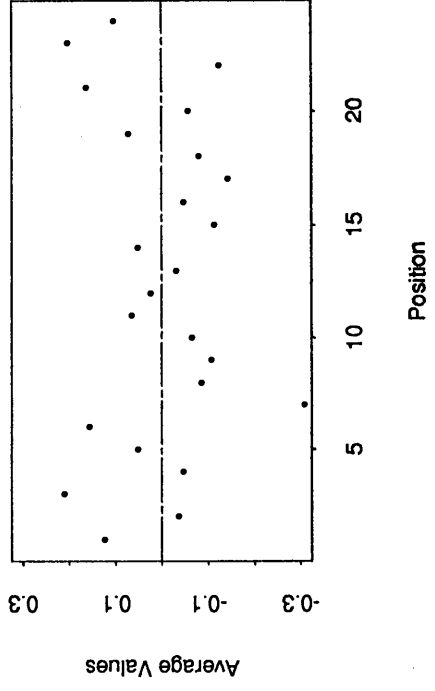
Warm a1 Semivariogram



Residual Semivariogram

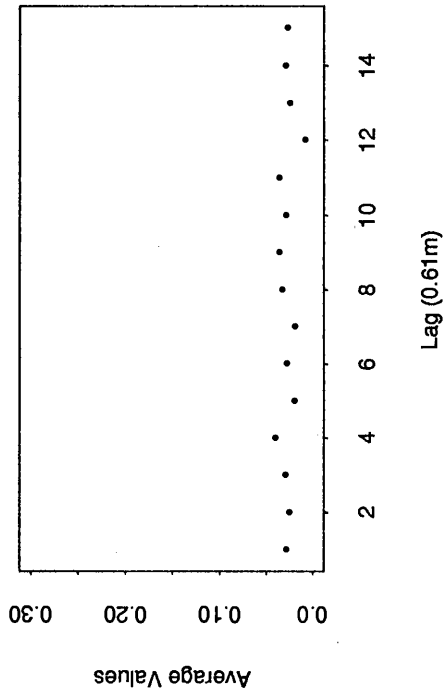


Residuals

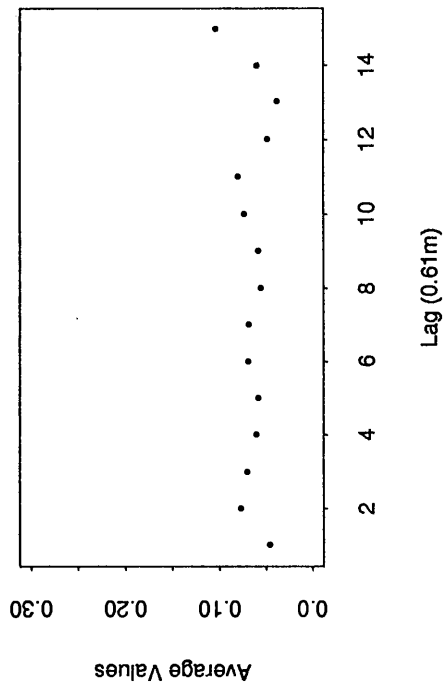


Magnet DCA317 Skew Multipoles: a1

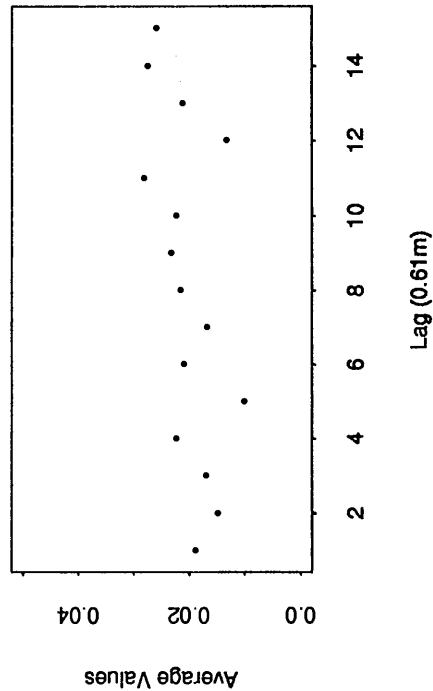
Cold a1 Semivariogram



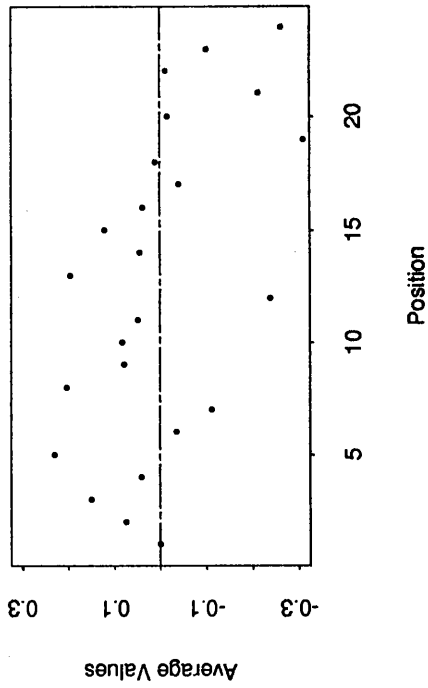
Warm a1 Semivariogram



Residual Semivariogram

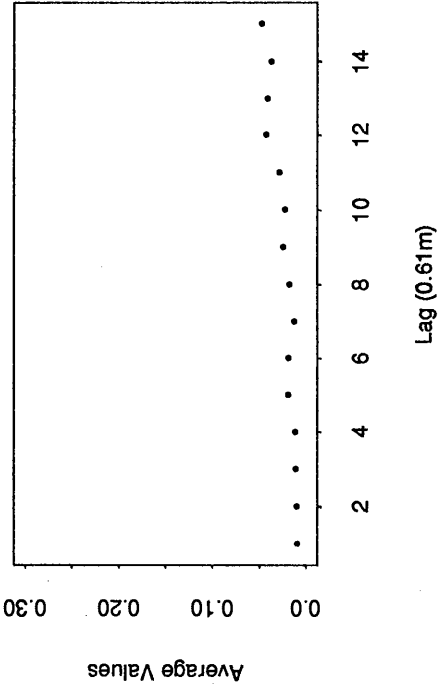


Residuals

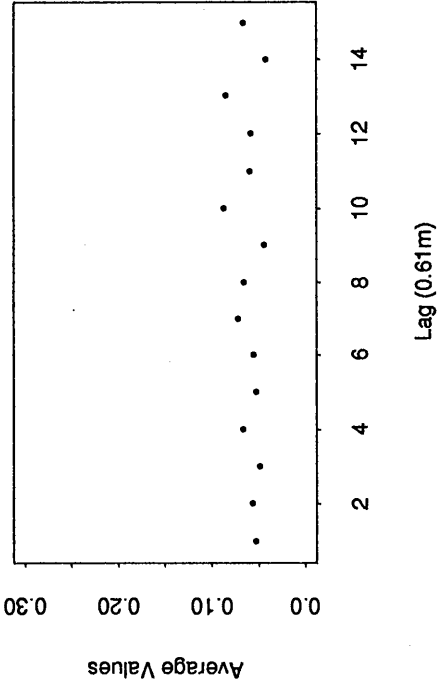


Magnet DCA318 Skew Multipoles: a1

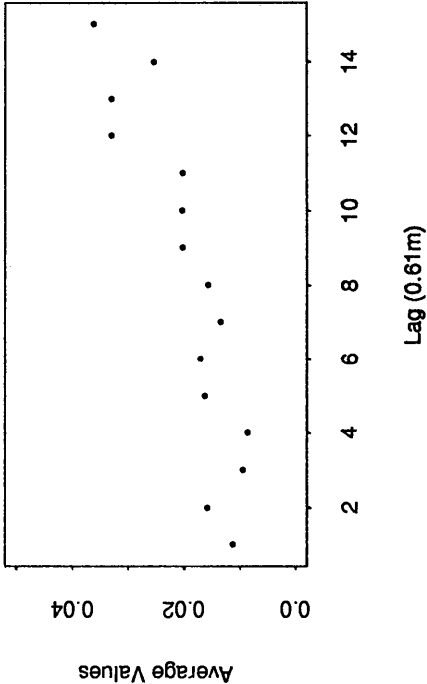
Cold a1 Semivariogram



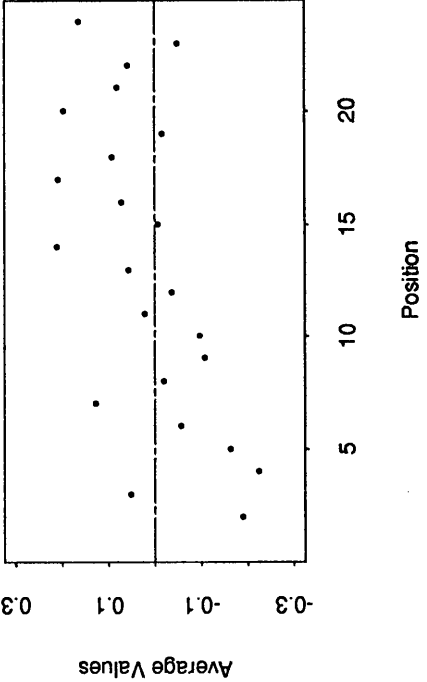
Warm a1 Semivariogram



Residual Semivariogram

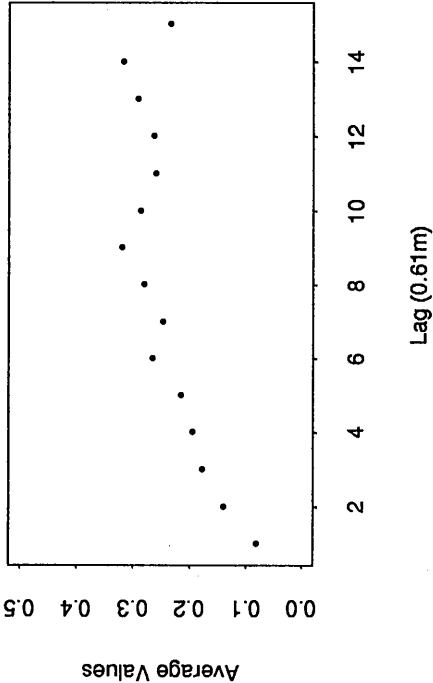


Residuals

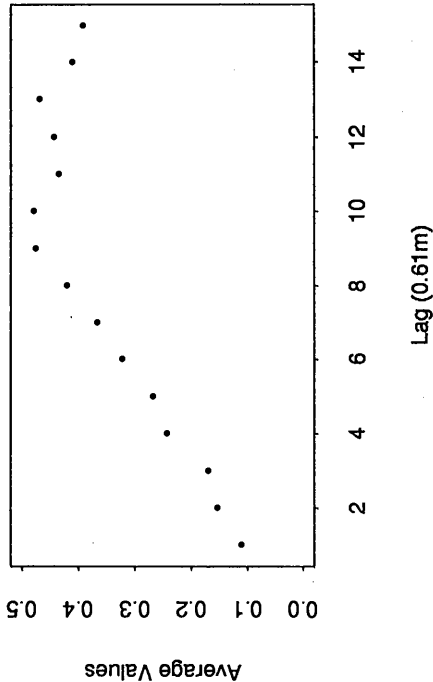


Magnet DCA319 Skew Multipoles: a1

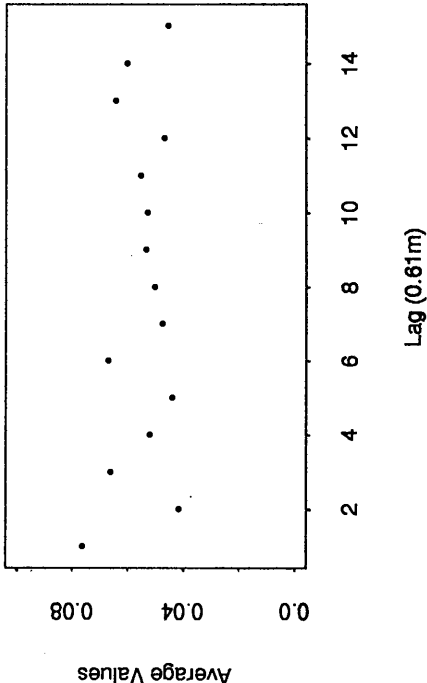
Cold a1 Semivariogram



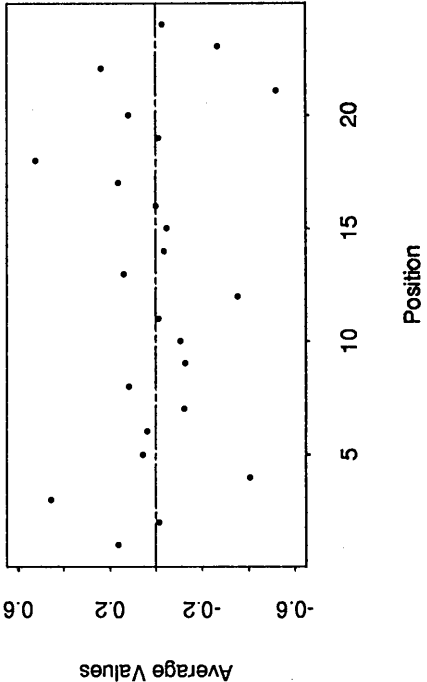
Warm a1 Semivariogram



Residual Semivariogram

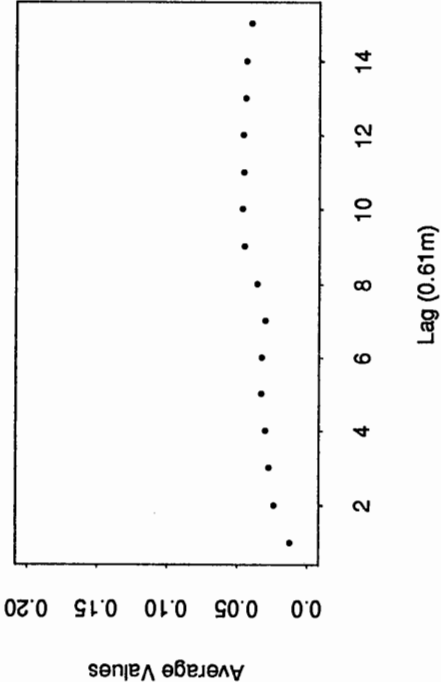


Residuals

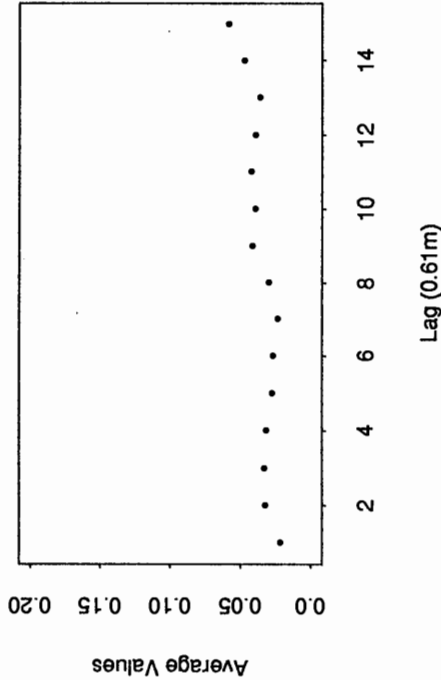


Magnet DCA311 Skew Multipoles: a2

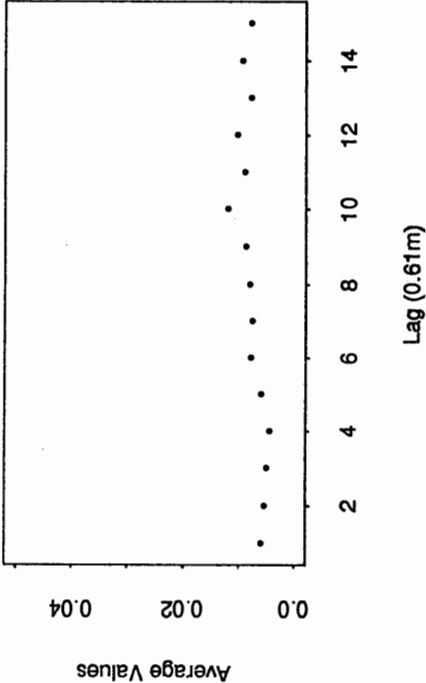
Cold a2 Semivariogram



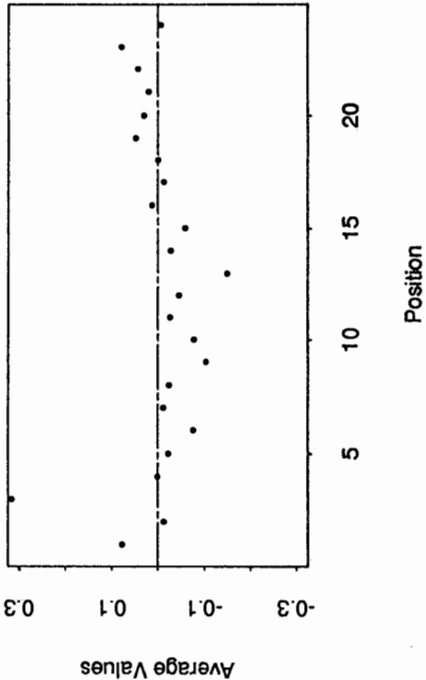
Warm a2 Semivariogram



Residual Semivariogram

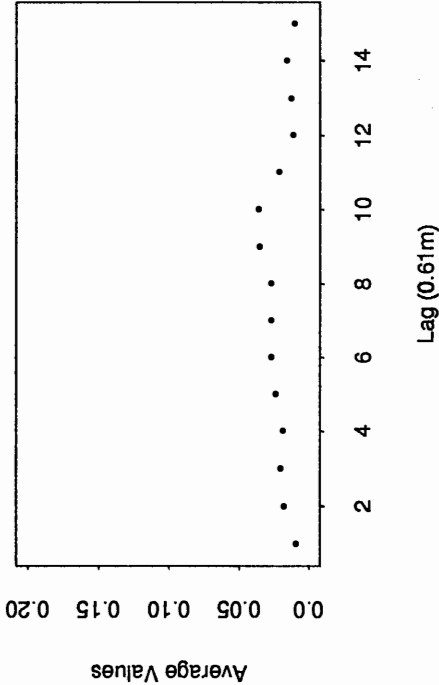


Residuals

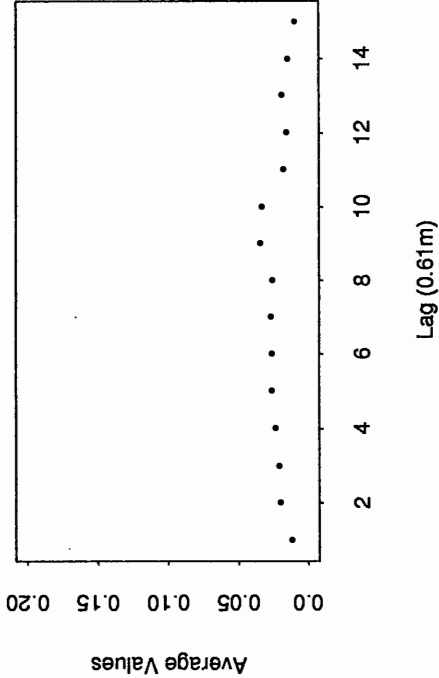


Magnet DCA312 Skew Multipoles: a2

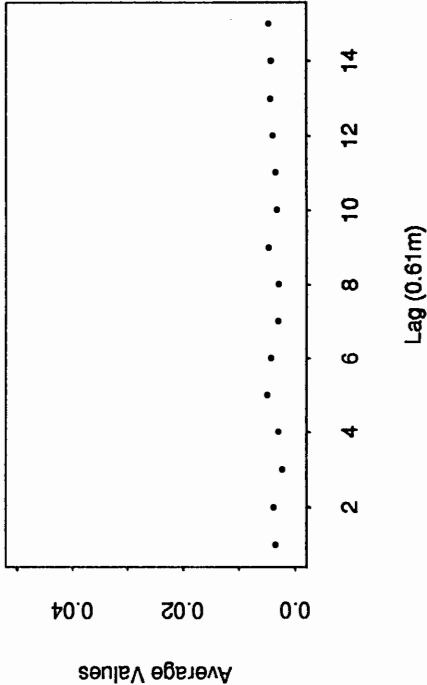
Cold a2 Semivariogram



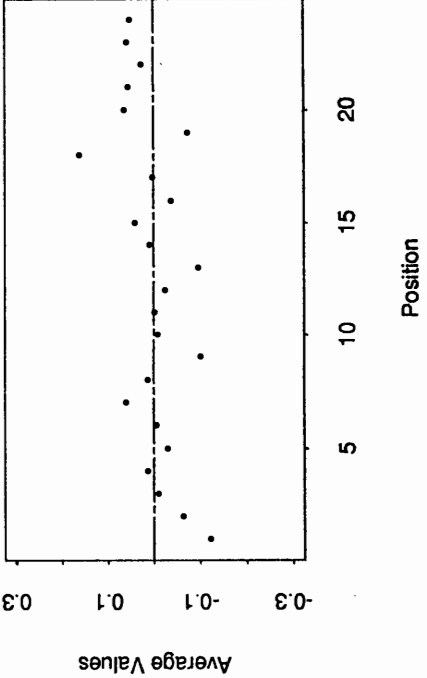
Warm a2 Semivariogram



Residual Semivariogram

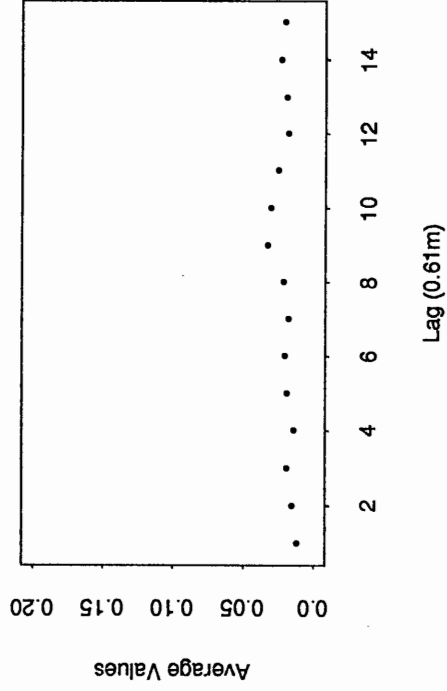


Residuals

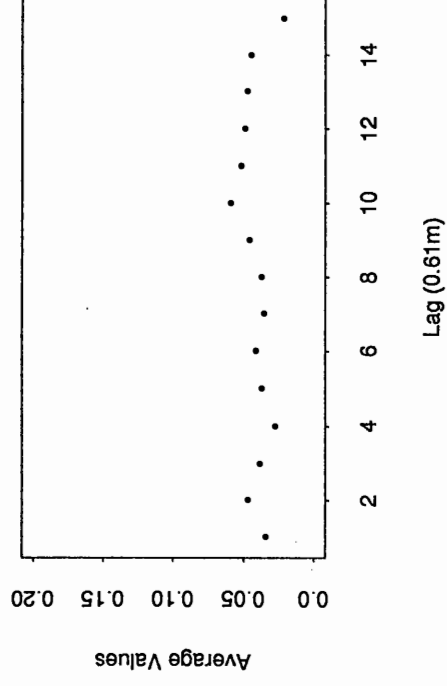


Magnet DCA313 Skew Multipoles: a2

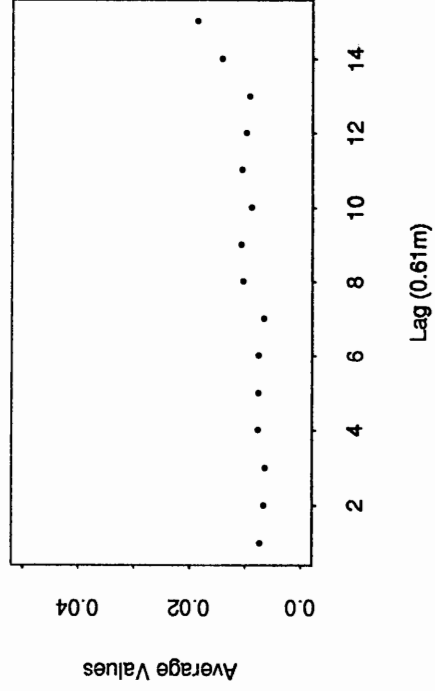
Cold a2 Semivariogram



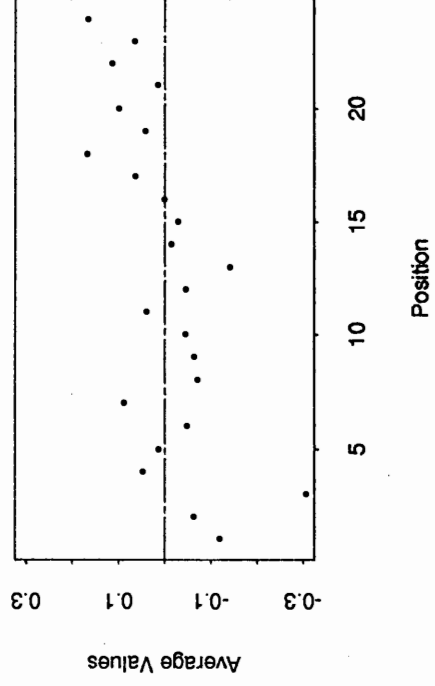
Warm a2 Semivariogram



Residual Semivariogram

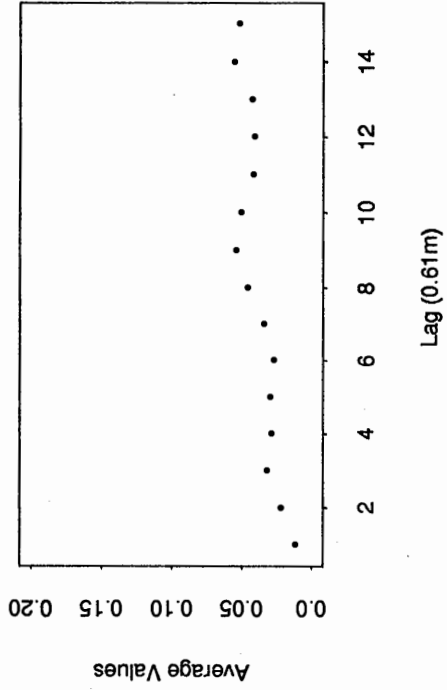


Residuals

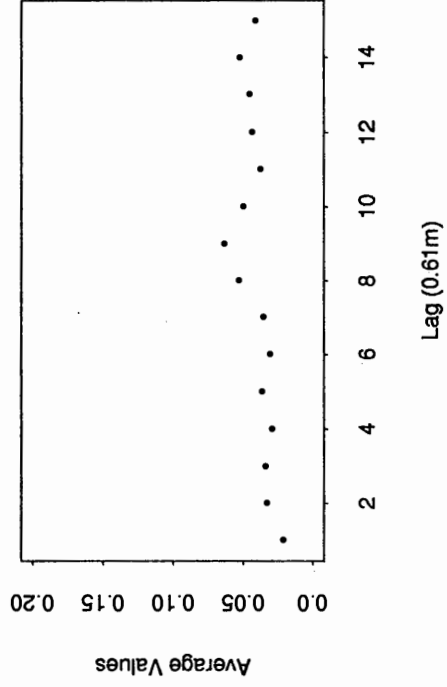


Magnet DCA314 Skew Multipoles: a2

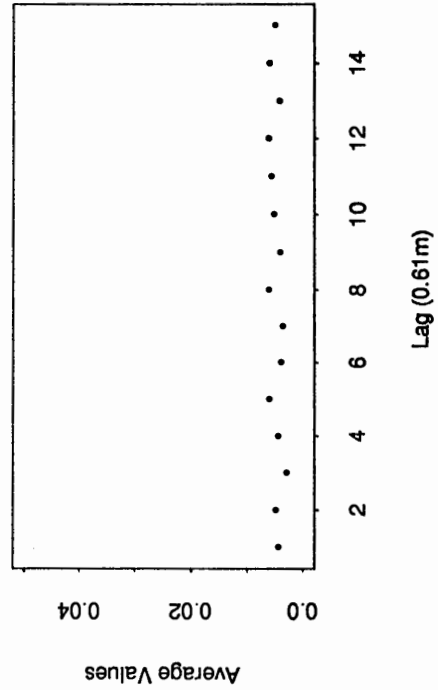
Cold a2 Semivariogram



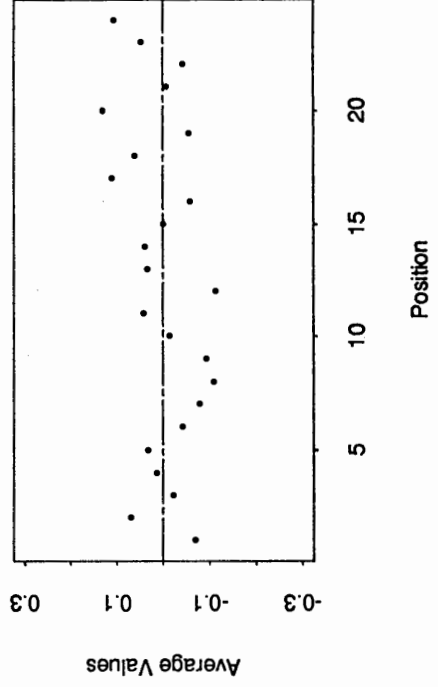
Warm a2 Semivariogram



Residual Semivariogram

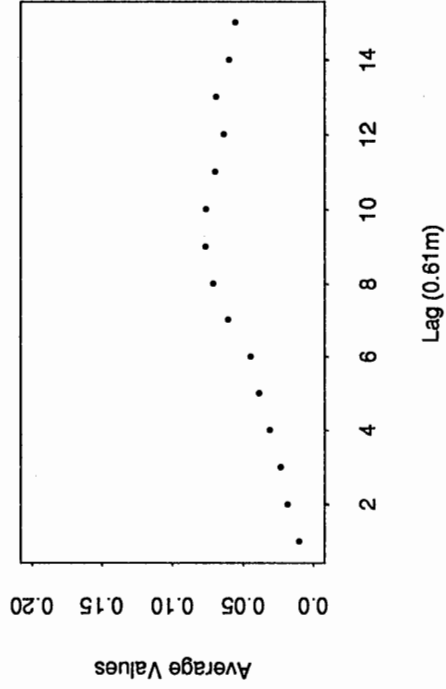


Residuals

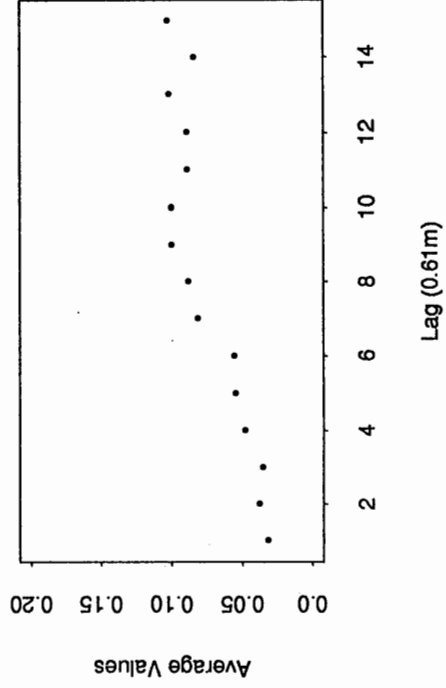


Magnet DCA315 Skew Multipoles: a2

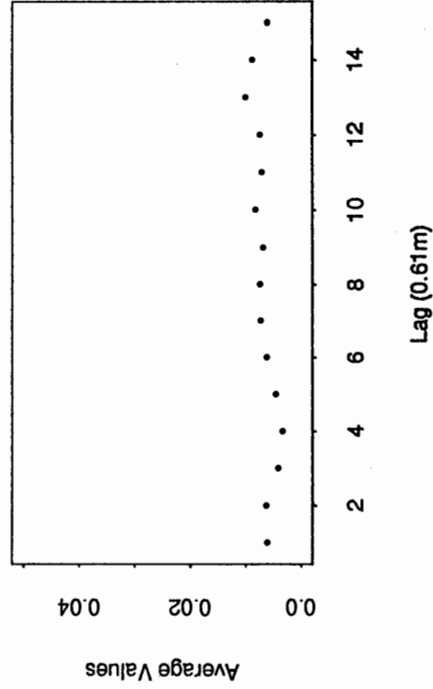
Cold a2 Semivariogram



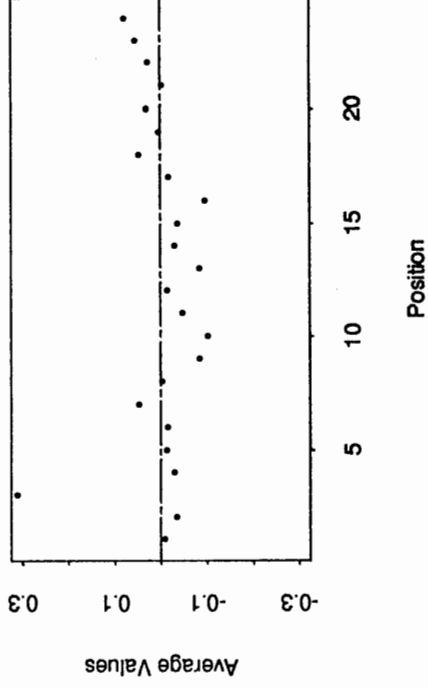
Warm a2 Semivariogram



Residual Semivariogram

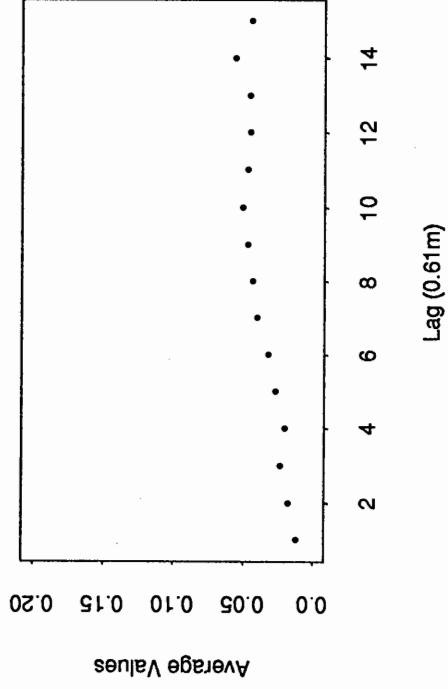


Residuals

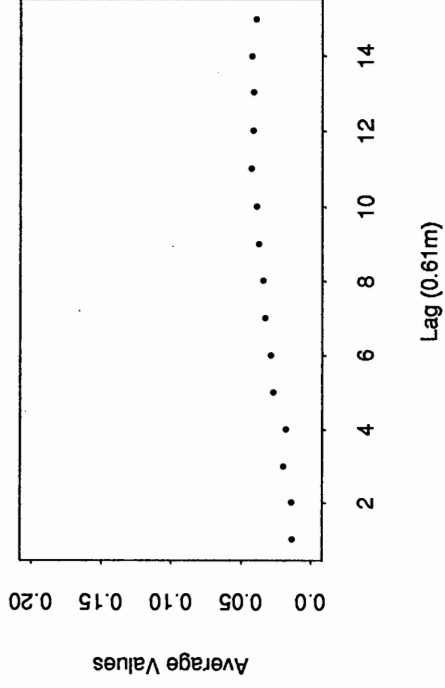


Magnet DCA316 Skew Multipoles: a2

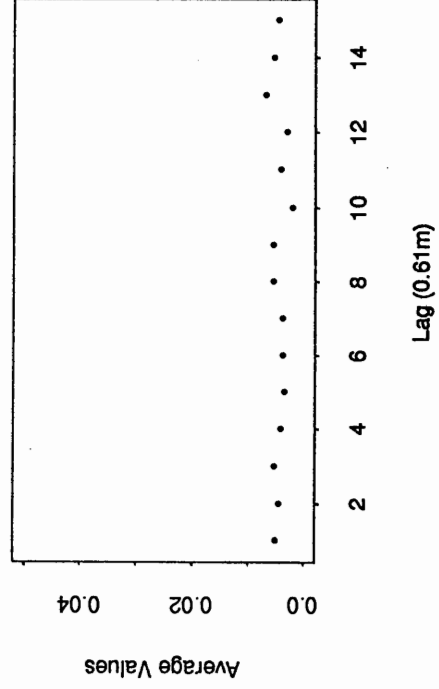
Cold a2 Semivariogram



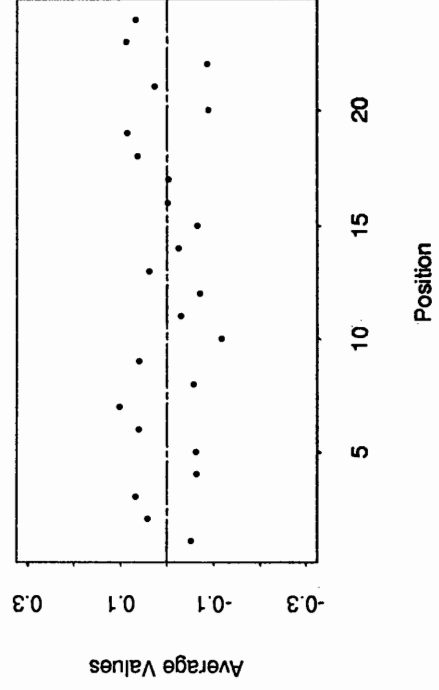
Warm a2 Semivariogram



Residual Semivariogram

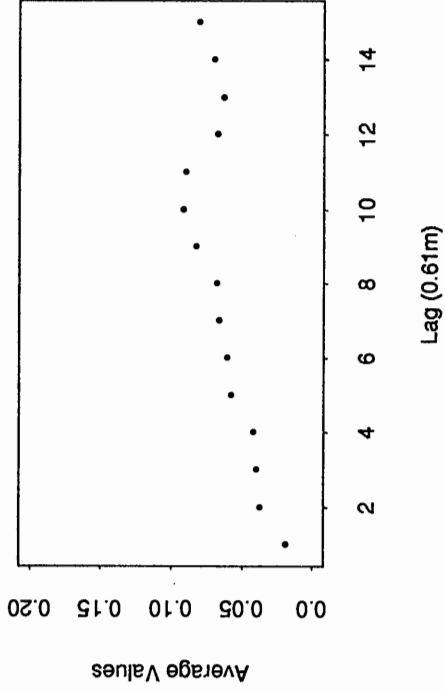


Residuals

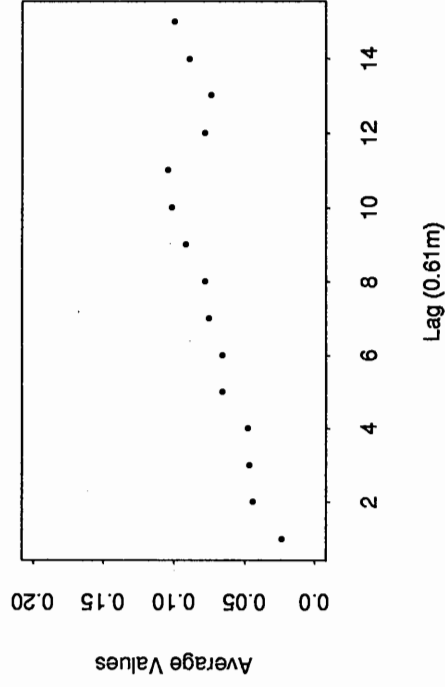


Magnet DCA317 Skew Multipoles: a2

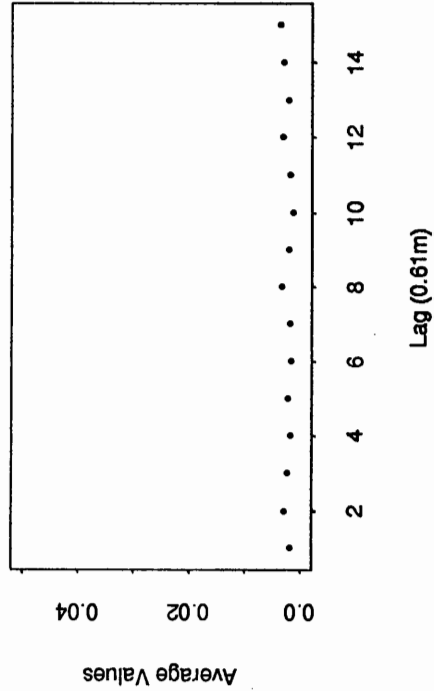
Cold a2 Semivariogram



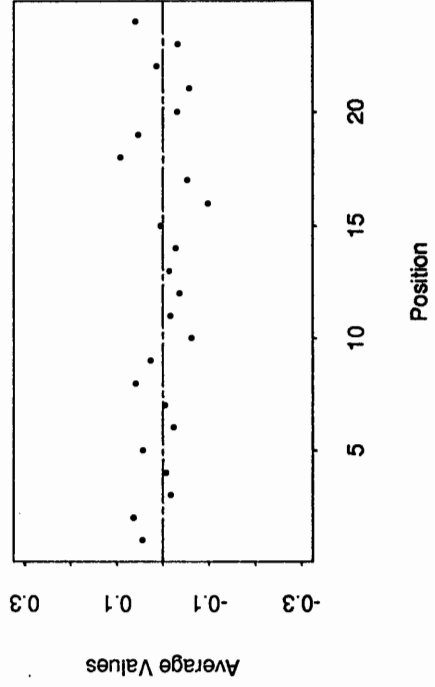
Warm a2 Semivariogram



Residual Semivariogram

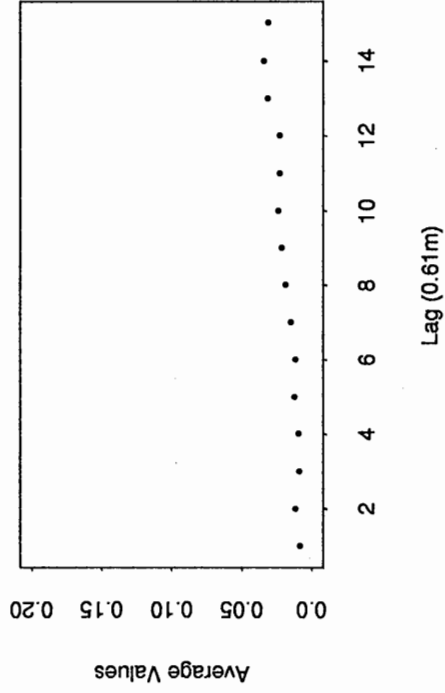


Residuals

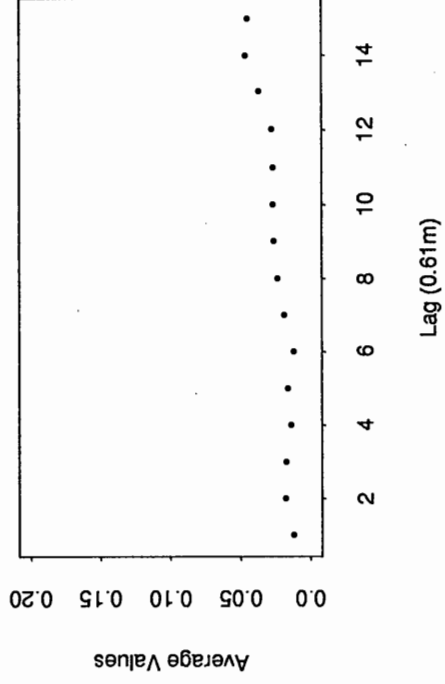


Magnet DCA318 Skew Multipoles: a2

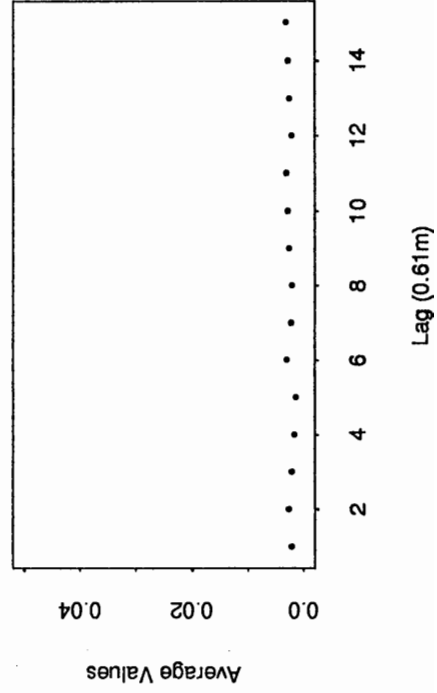
Cold a2 Semivariogram



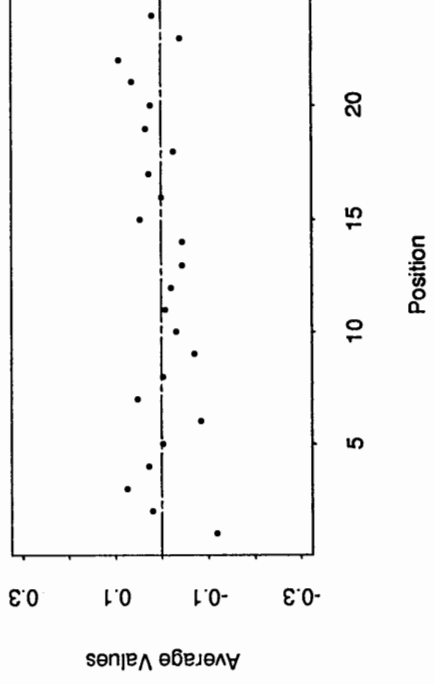
Warm a2 Semivariogram



Residual Semivariogram

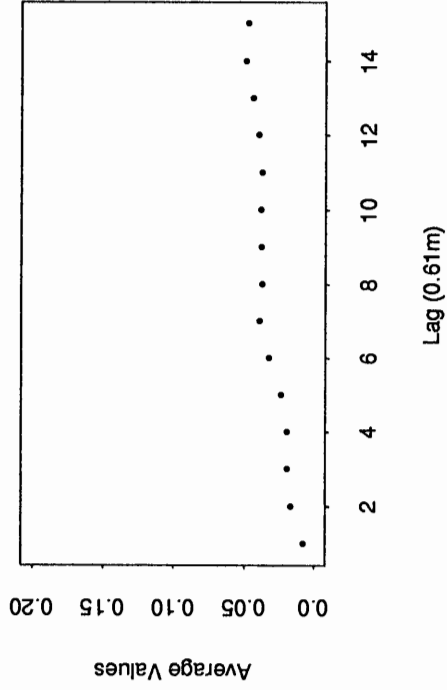


Residuals

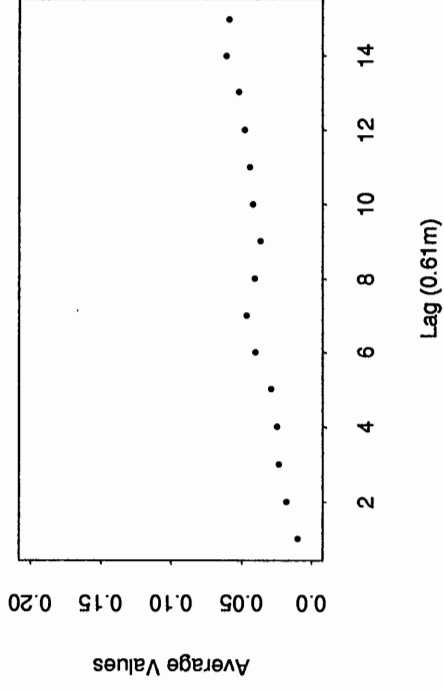


Magnet DCA319 Skew Multipoles: a2

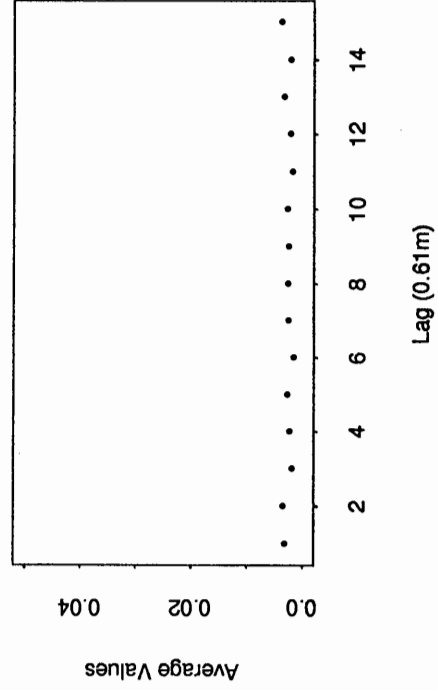
Cold a2 Semivariogram



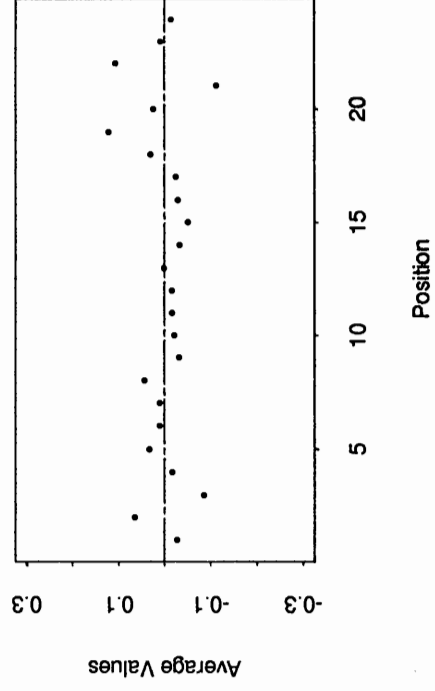
Warm a2 Semivariogram



Residual Semivariogram

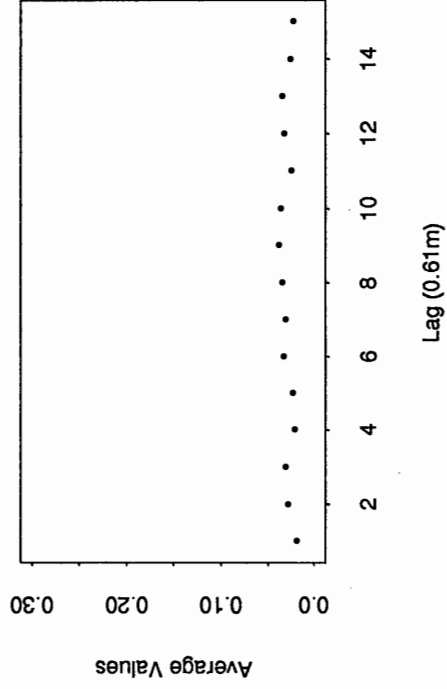


Residuals

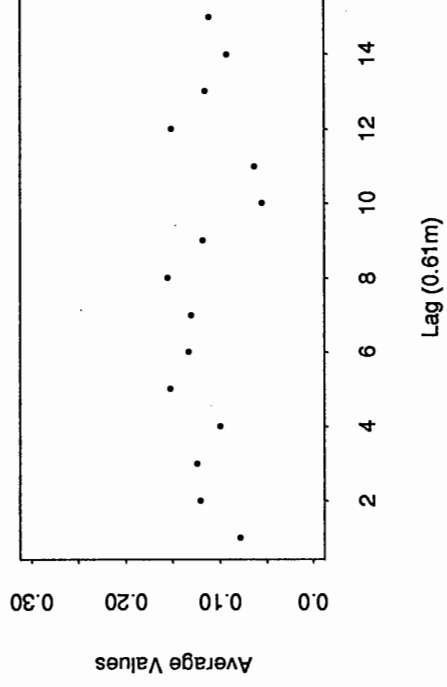


Magnet DCA311 Normal Multipoles: b1

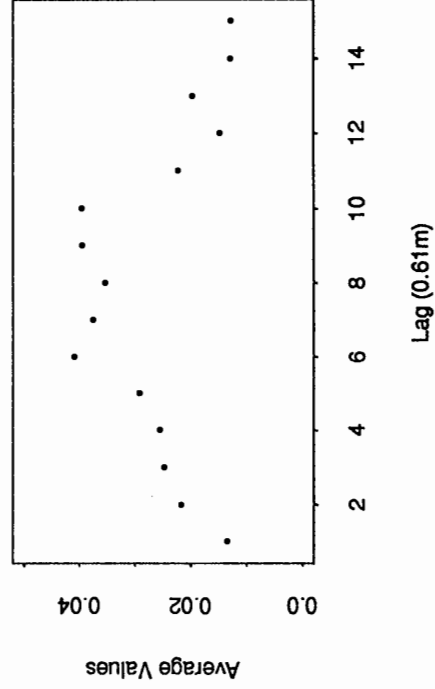
Cold b1 Semivariogram



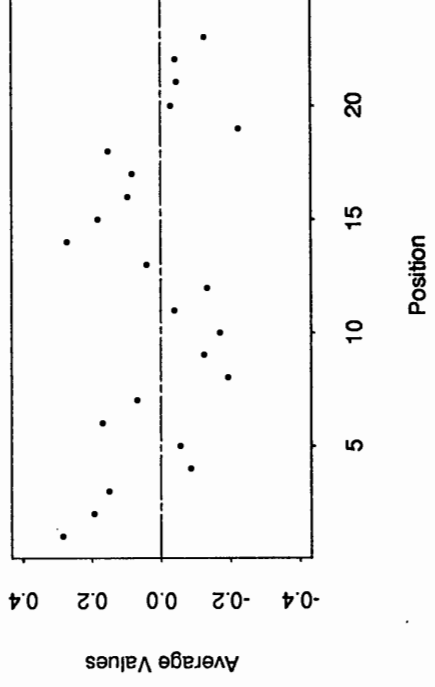
Warm b1 Semivariogram



Residual Semivariogram

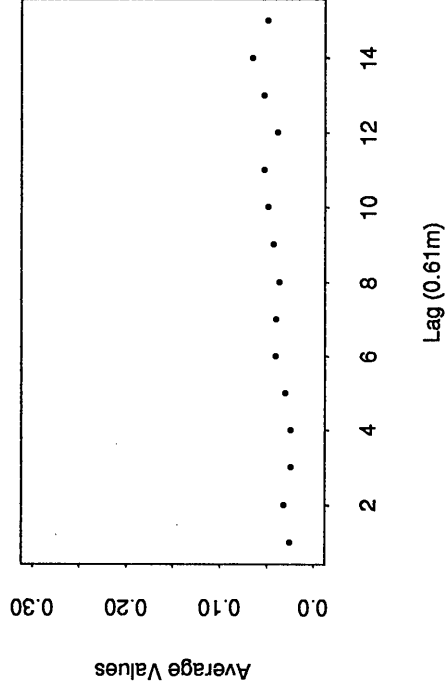


Residuals

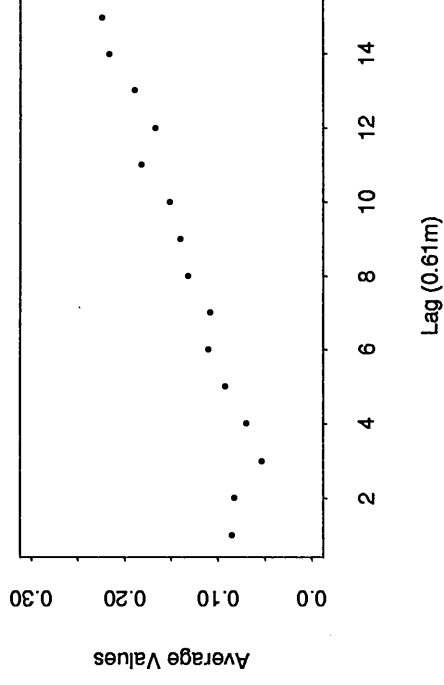


Magnet DCA312 Normal Multipoles: b1

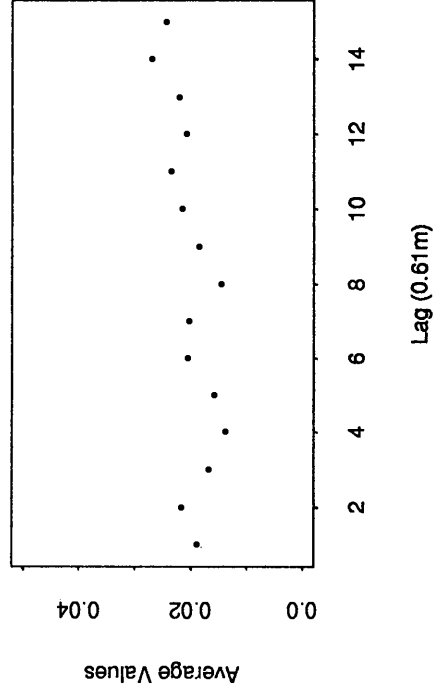
Cold b1 Semivariogram



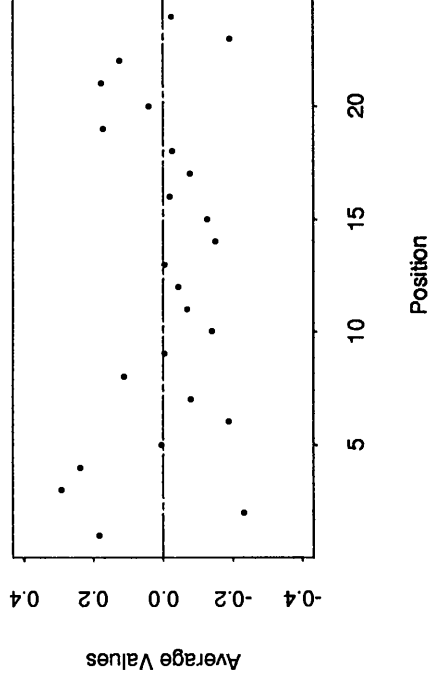
Warm b1 Semivariogram



Residual Semivariogram

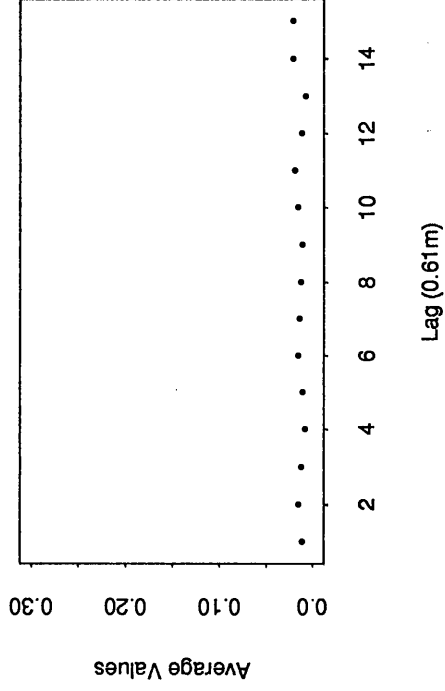


Residuals

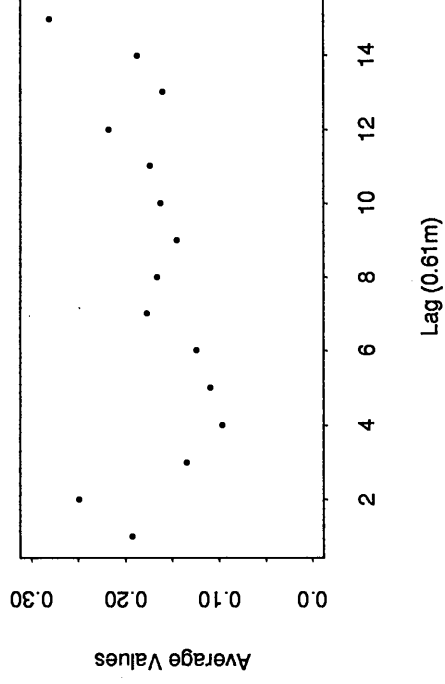


Magnet DCA313 Normal Multipoles: b1

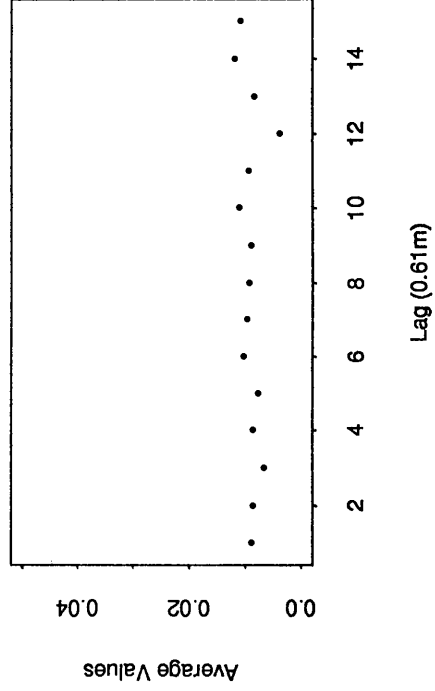
Cold b1 Semivariogram



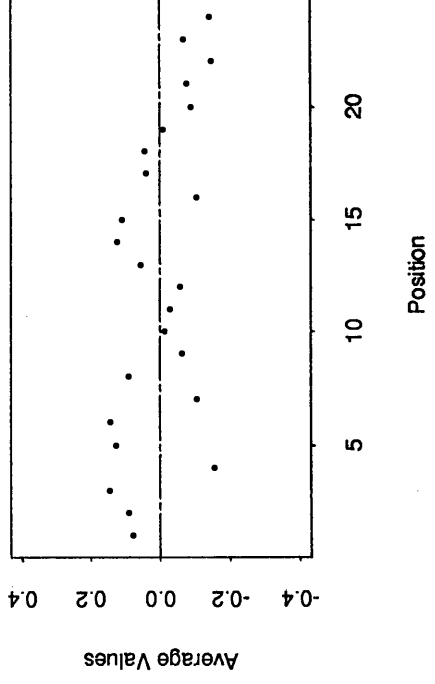
Warm b1 Semivariogram



Residual Semivariogram

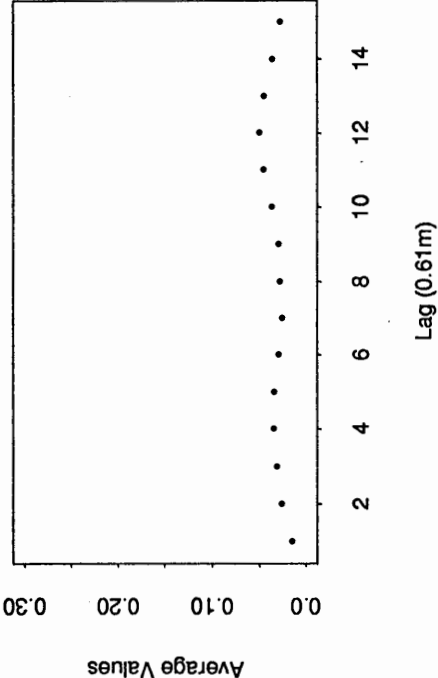


Residuals

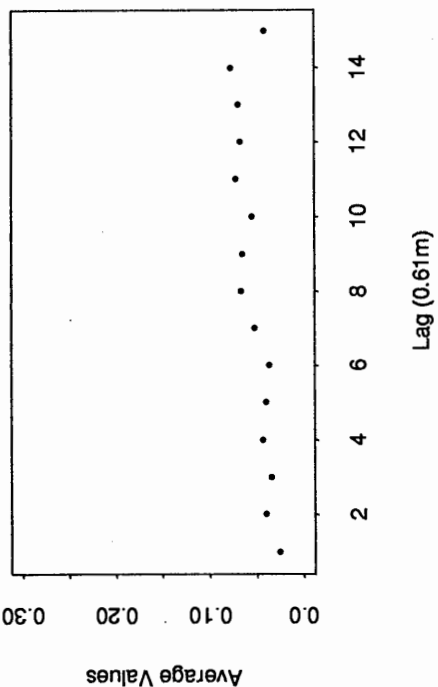


Magnet DCA314 Normal Multipoles: b1

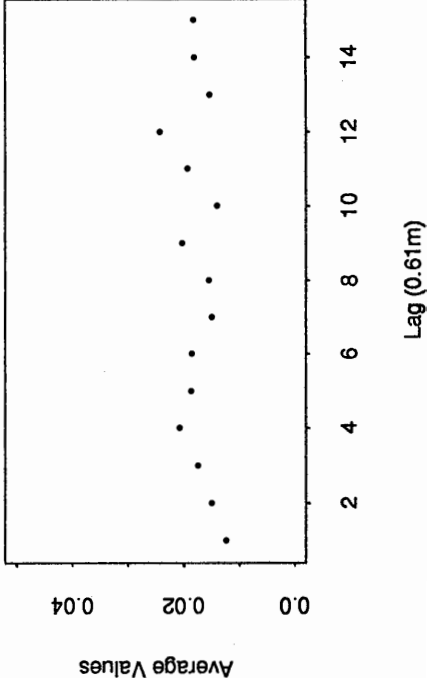
Cold b1 Semivariogram



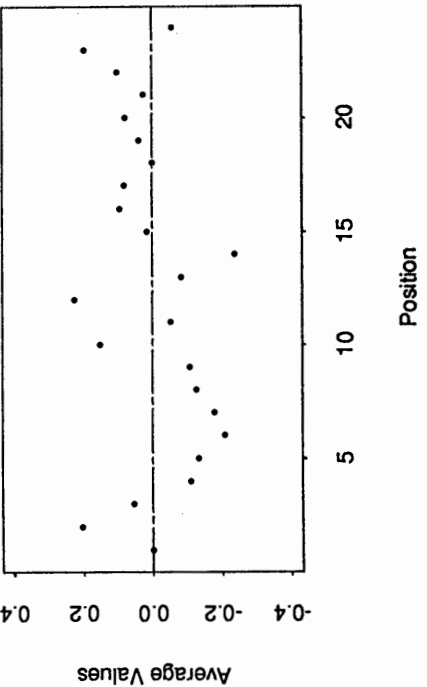
Warm b1 Semivariogram



Residual Semivariogram

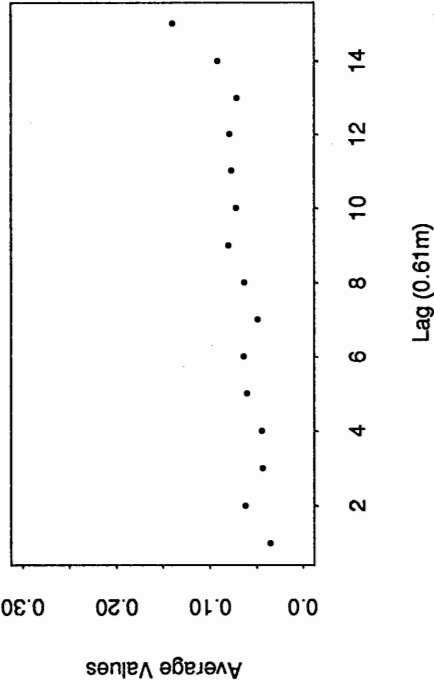


Residuals

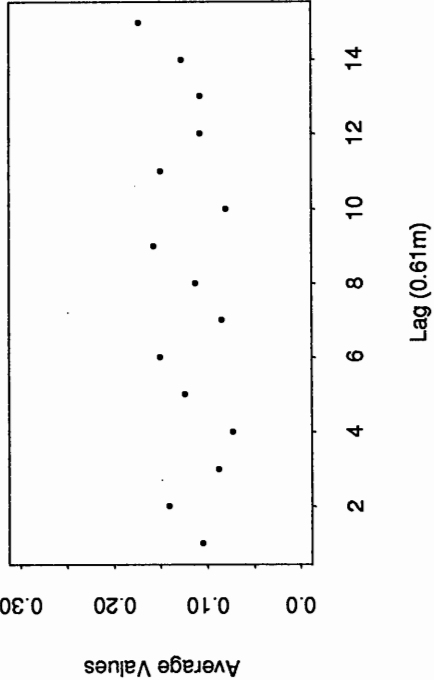


Magnet DCA315 Normal Multipoles: b1

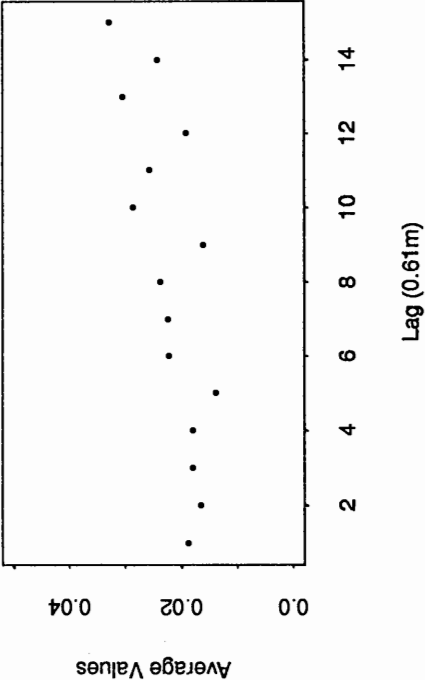
Cold b1 Semivariogram



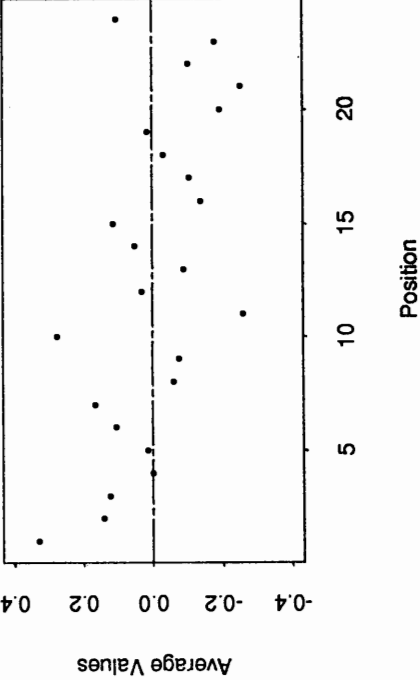
Warm b1 Semivariogram



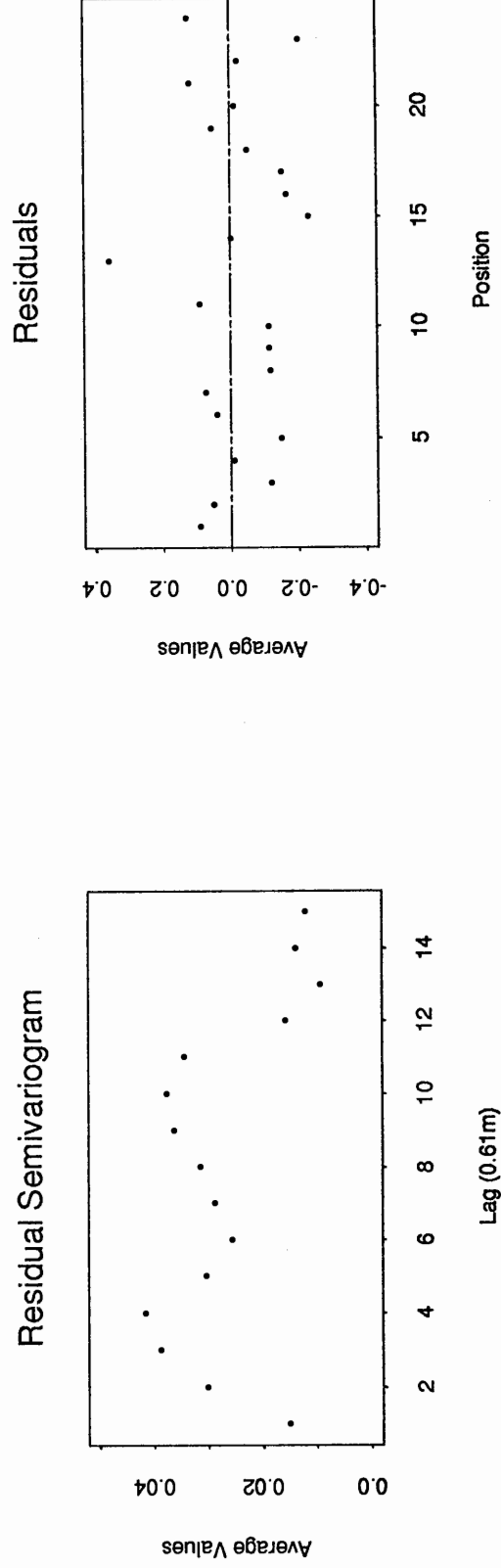
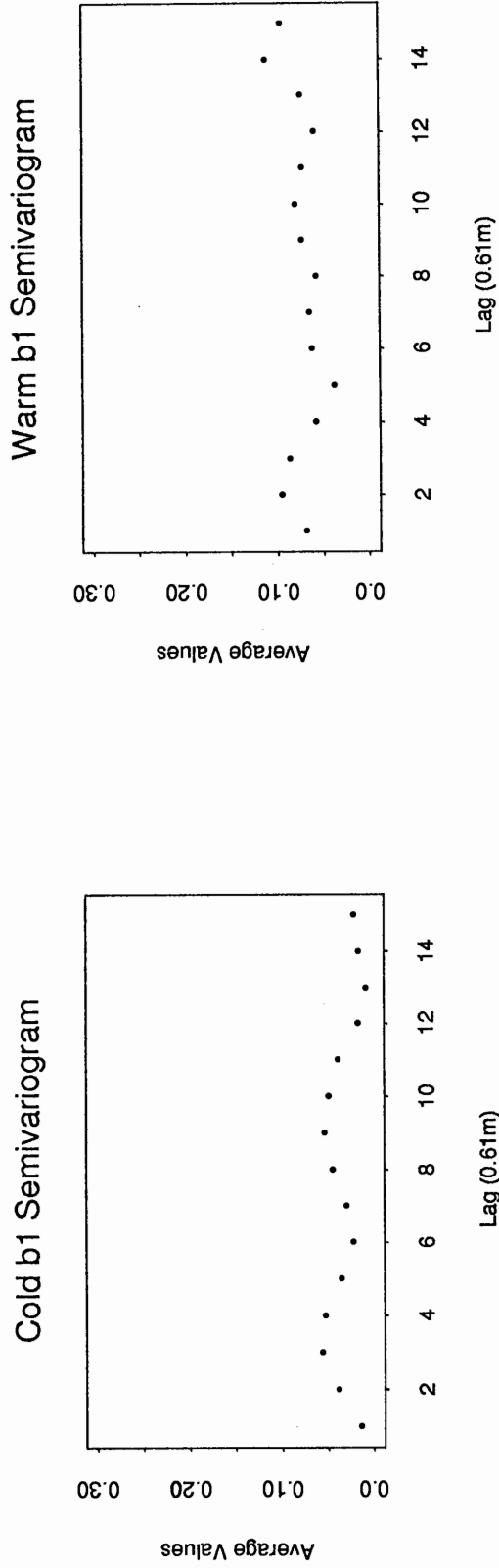
Residual Semivariogram



Residuals

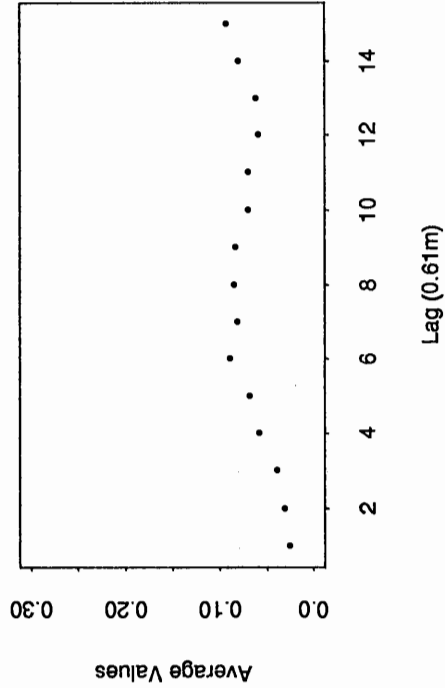


Magnet DCA316 Normal Multipoles: b1

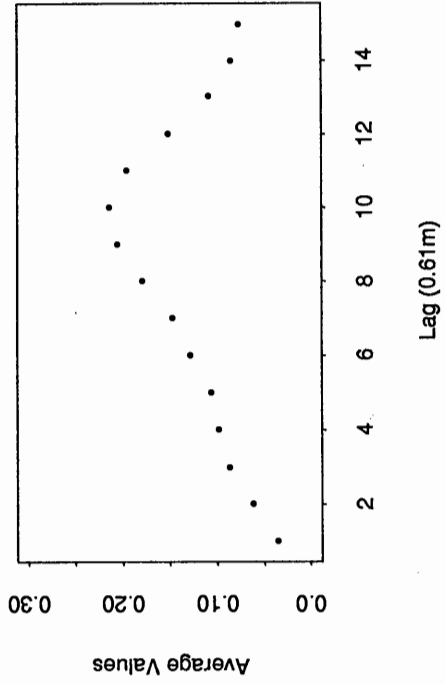


Magnet DCA317 Normal Multipoles: b1

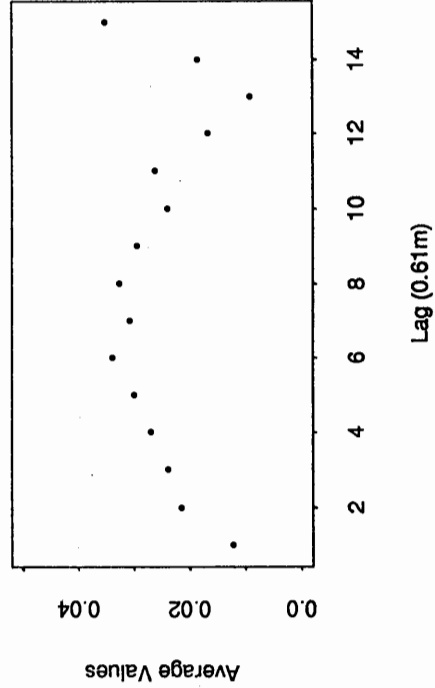
Cold b1 Semivariogram



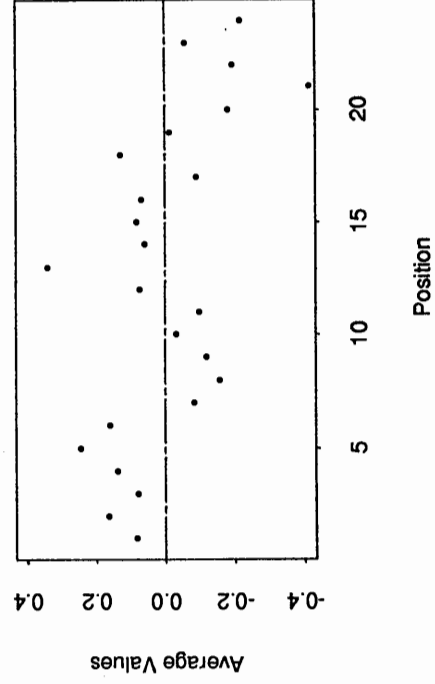
Warm b1 Semivariogram



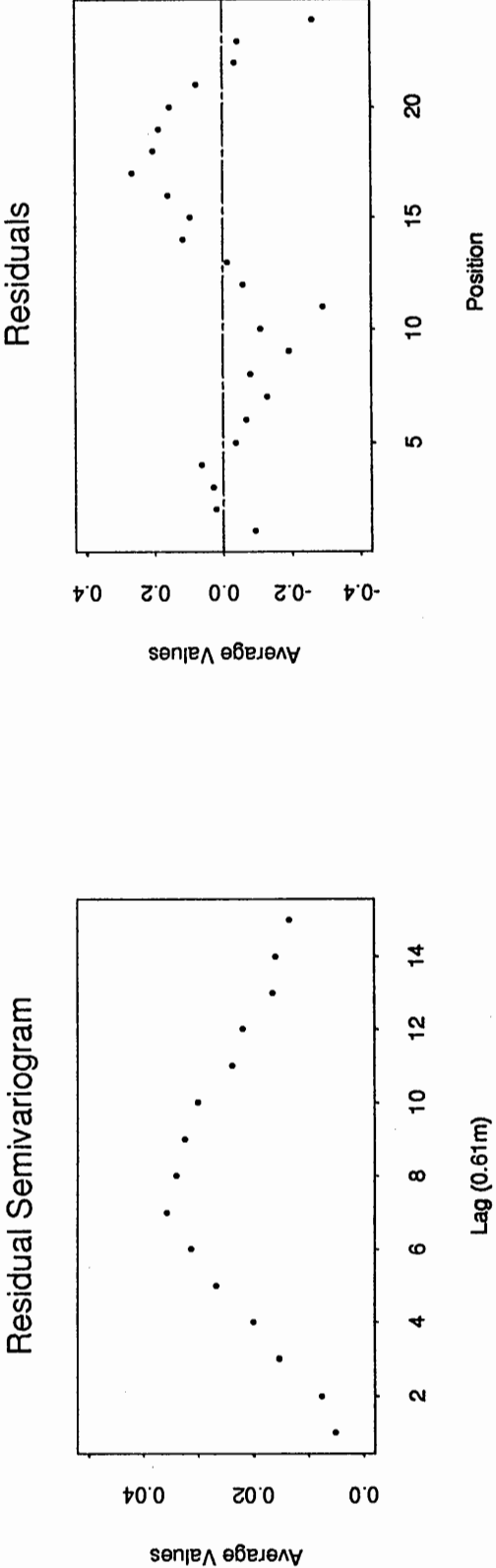
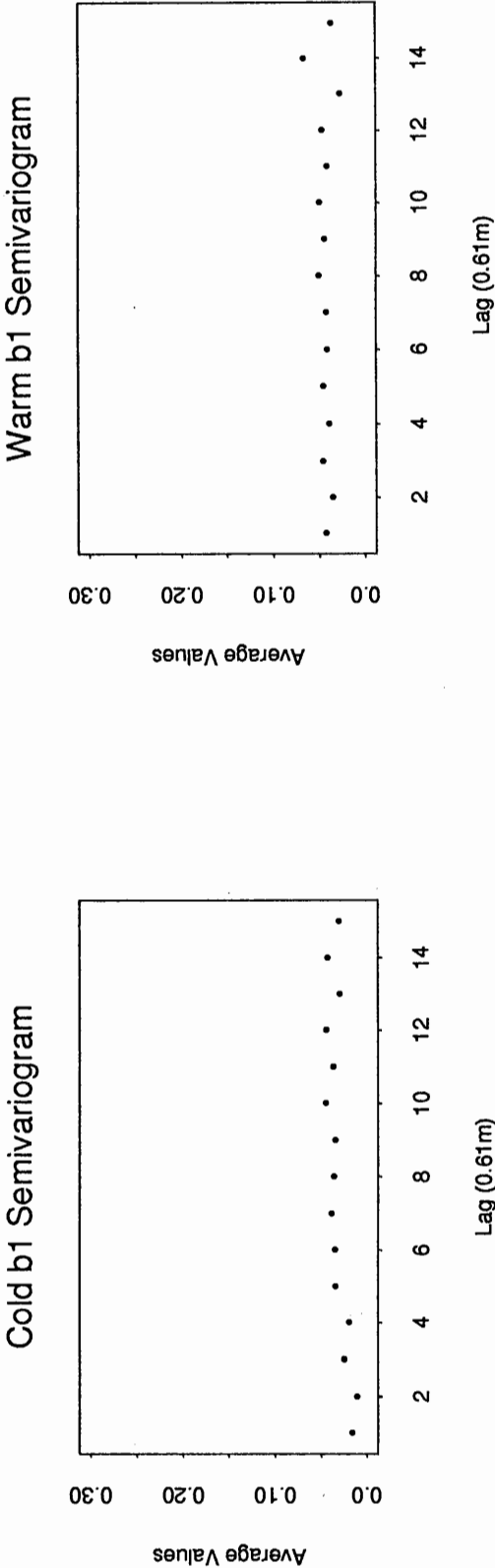
Residual Semivariogram



Residuals

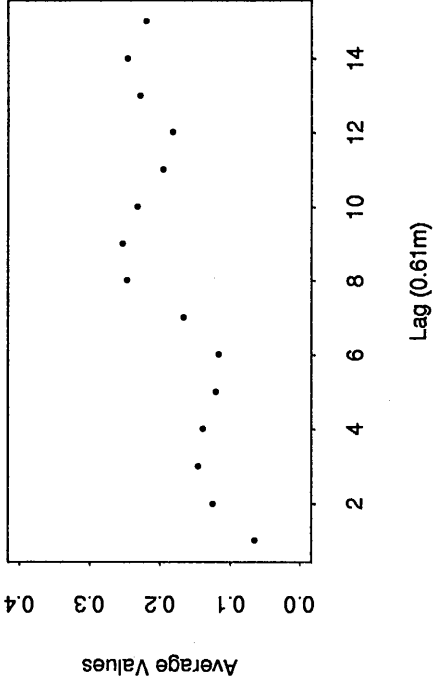


Magnet DCA318 Normal Multipoles: b1

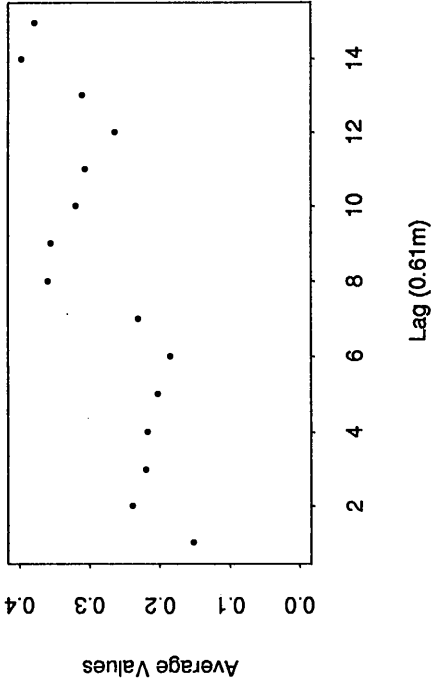


Magnet DCA319 Normal Multipoles: b1

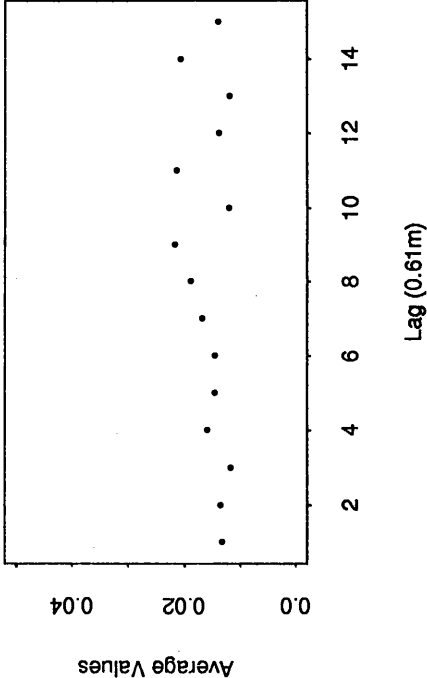
Cold b1 Semivariogram



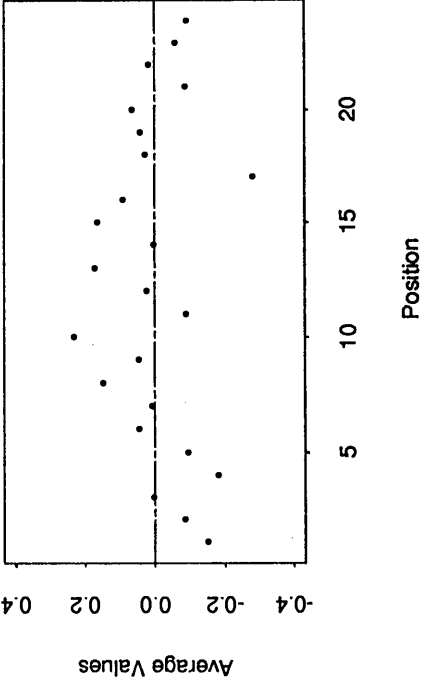
Warm b1 Semivariogram



Residual Semivariogram

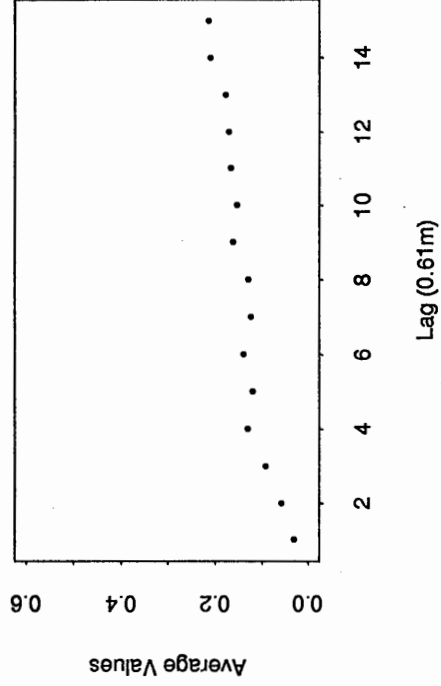


Residuals

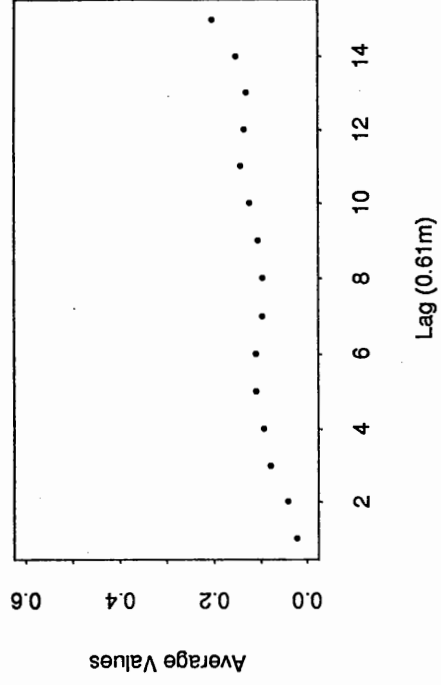


Magnet DCA311 Normal Multipoles: b2

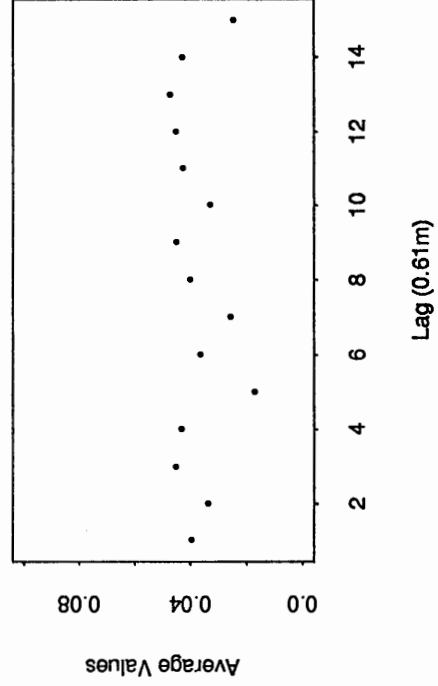
Cold b2 Semivariogram



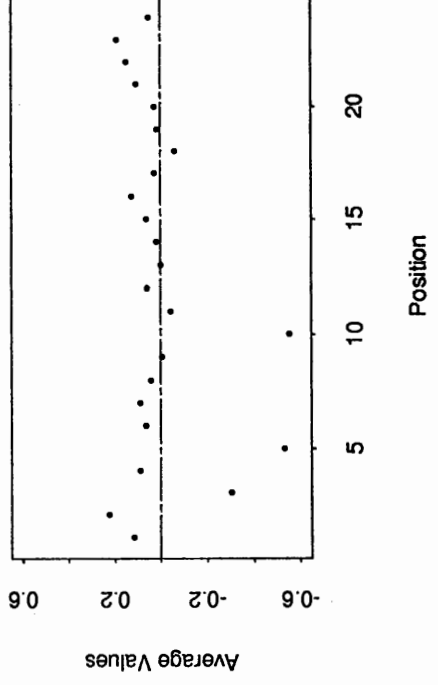
Warm b2 Semivariogram



Residual Semivariogram

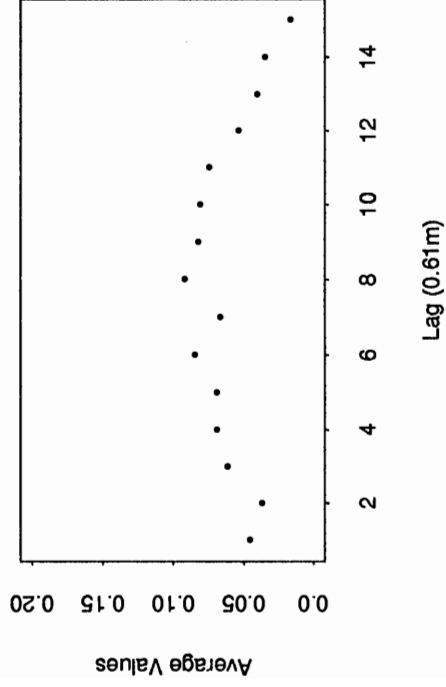


Residuals

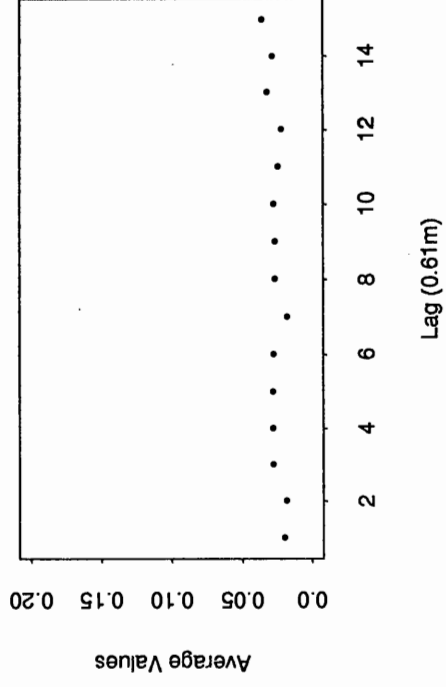


Magnet DCA312 Normal Multipoles: b2

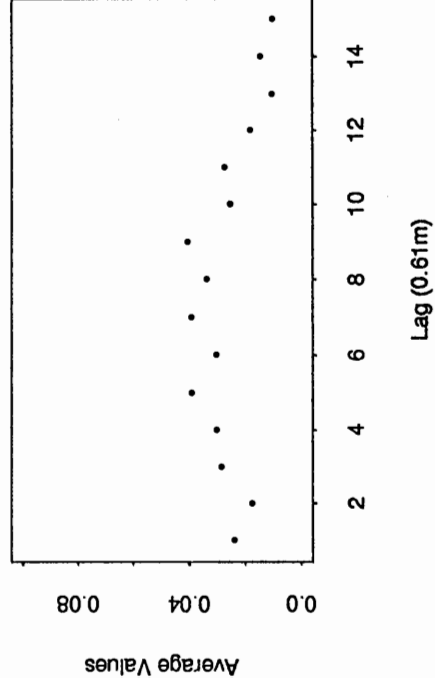
Cold b2 Semivariogram



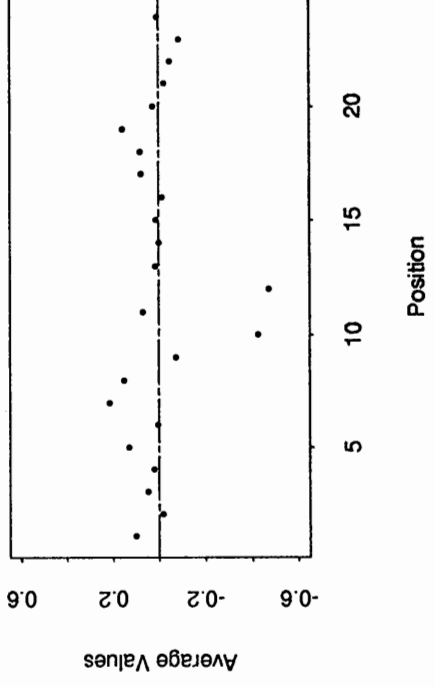
Warm b2 Semivariogram



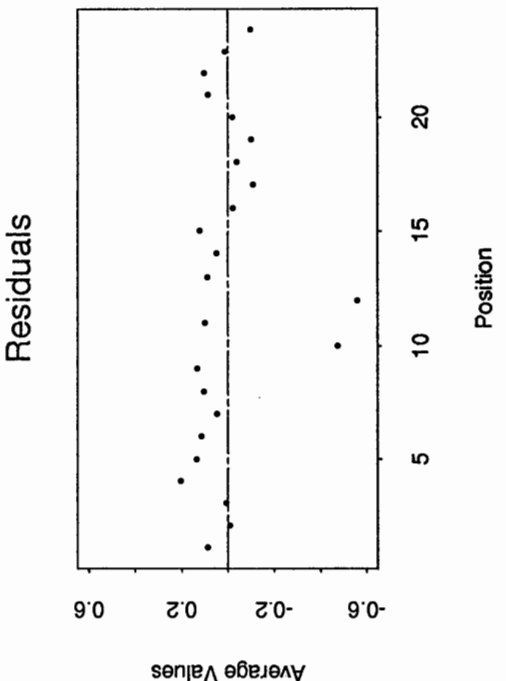
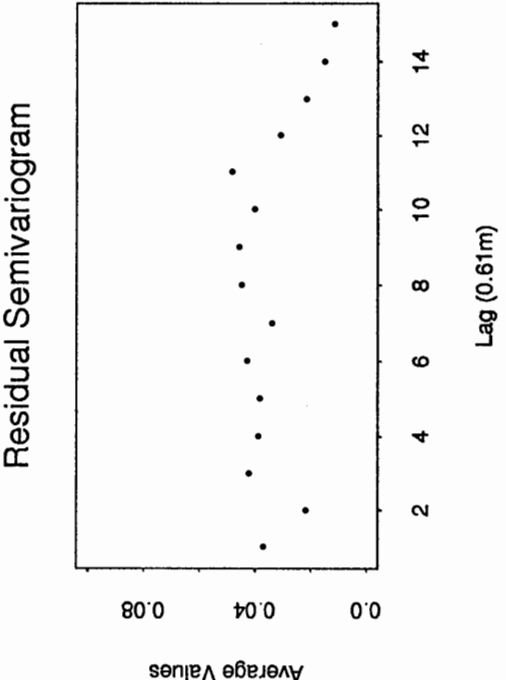
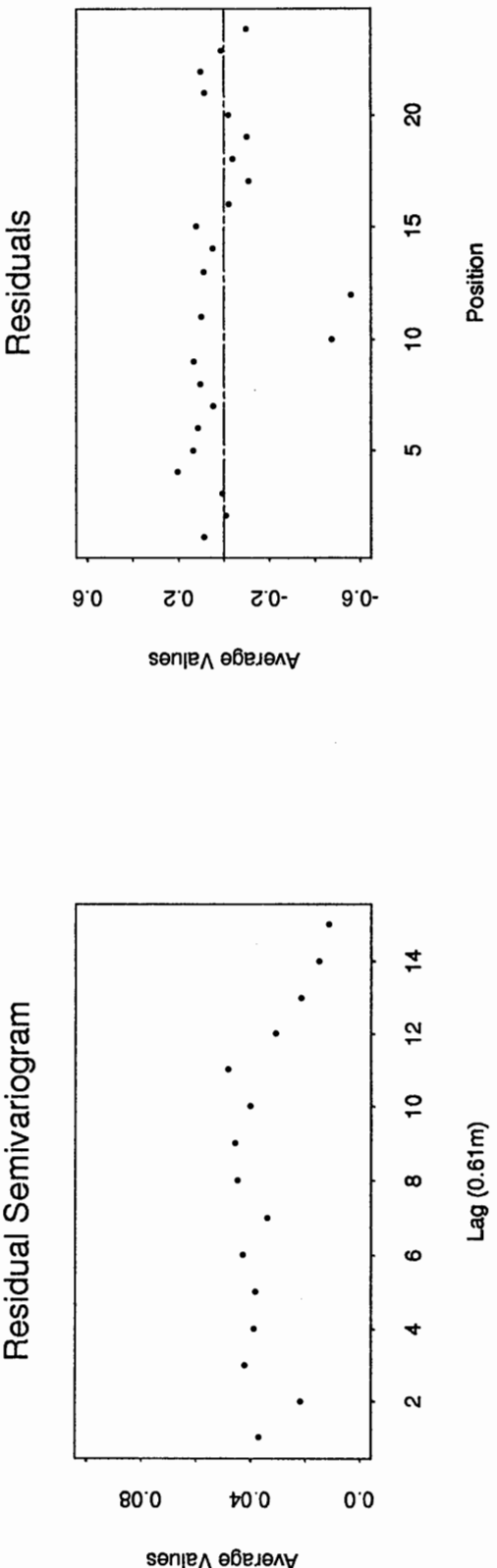
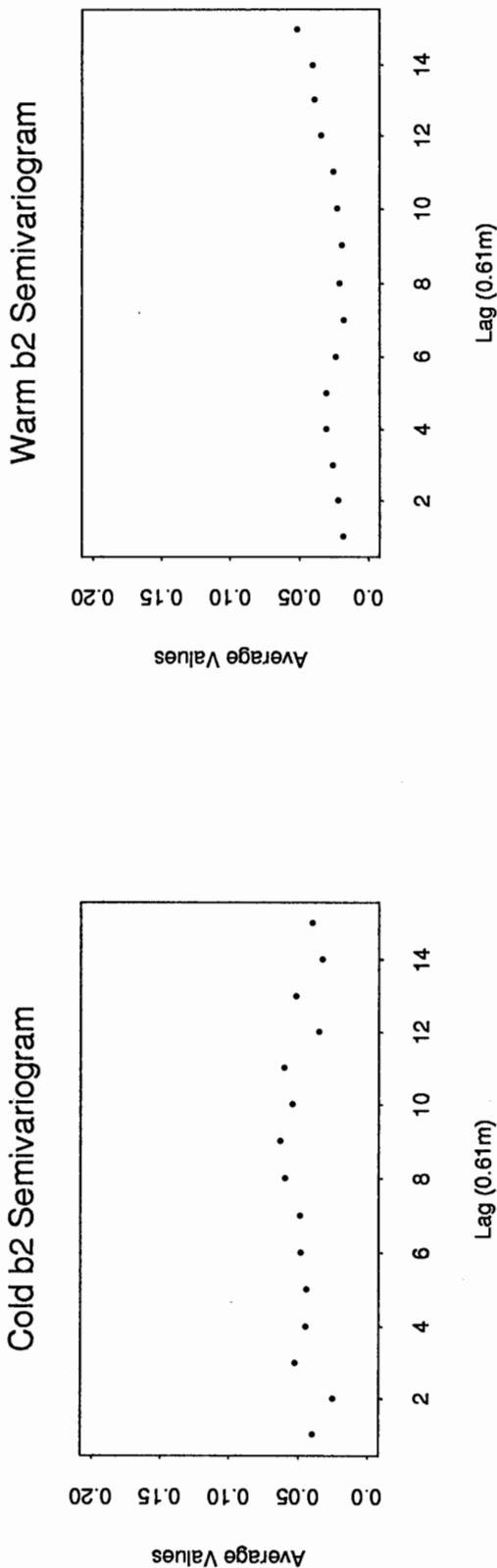
Residual Semivariogram



Residuals

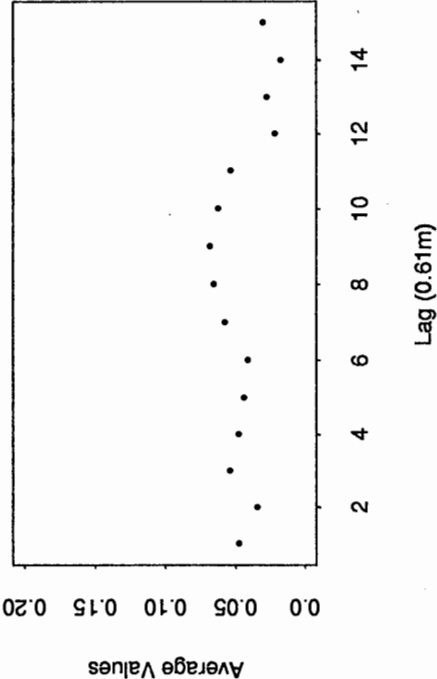


Magnet DCA313 Normal Multipoles: b2

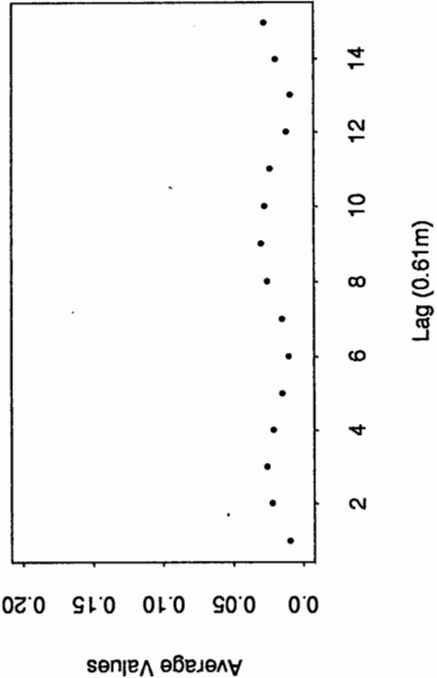


Magnet DCA314 Normal Multipoles: b2

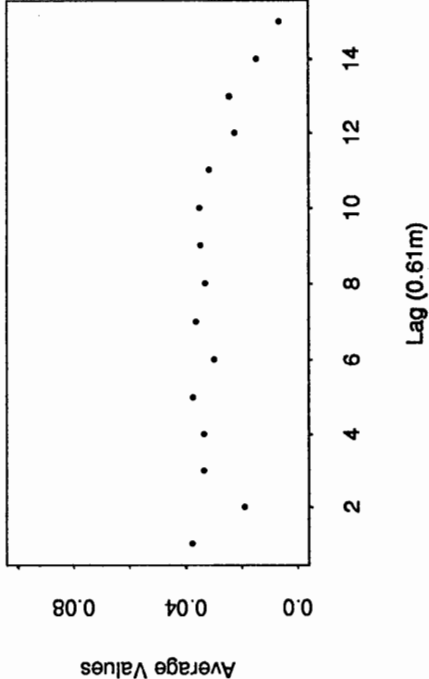
Cold b2 Semivariogram



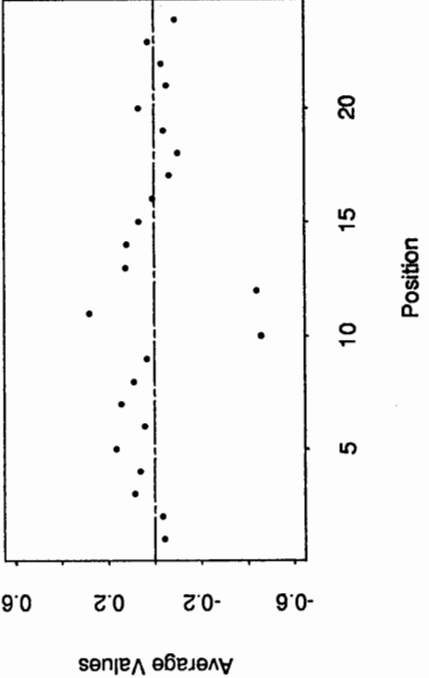
Warm b2 Semivariogram



Residual Semivariogram

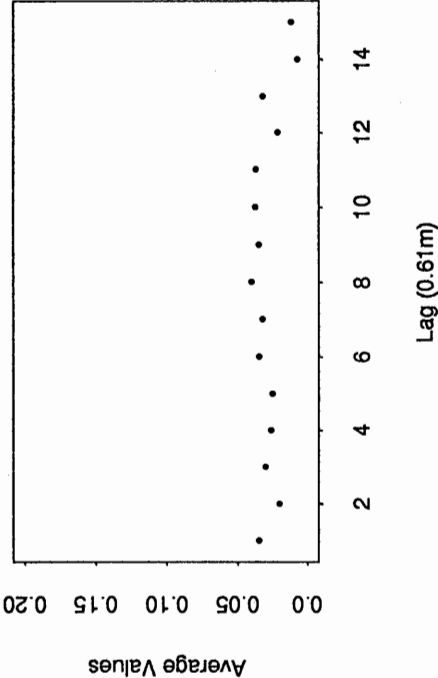


Residuals

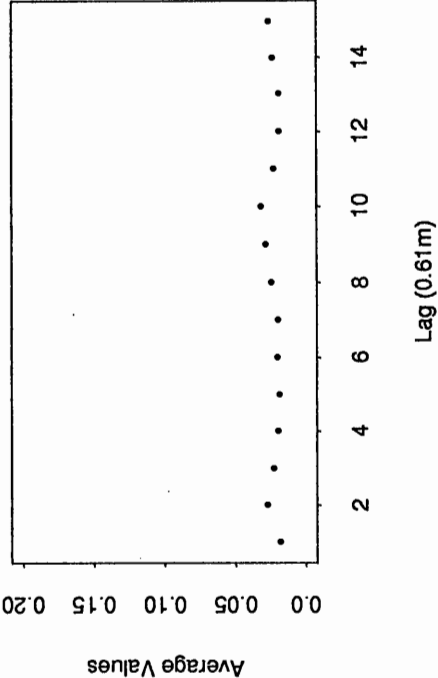


Magnet DCA315 Normal Multipoles: b2

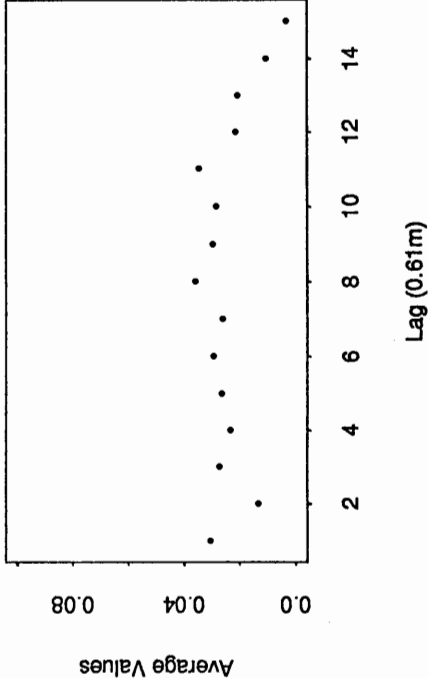
Cold b2 Semivariogram



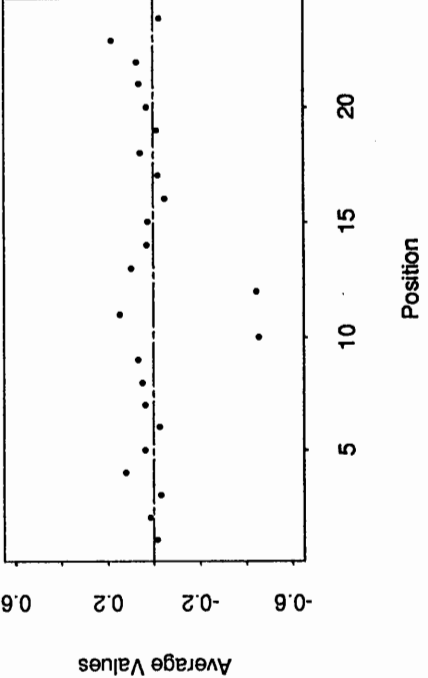
Warm b2 Semivariogram



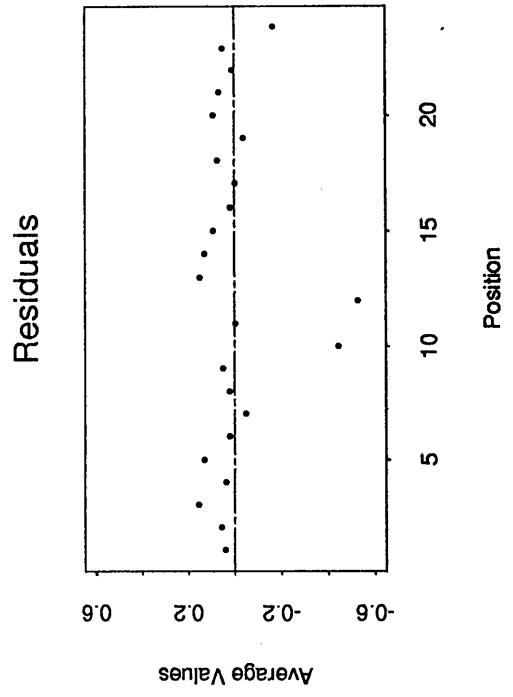
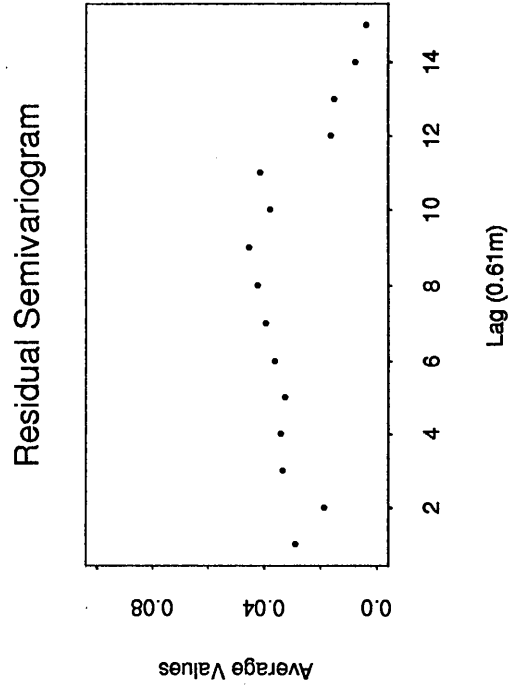
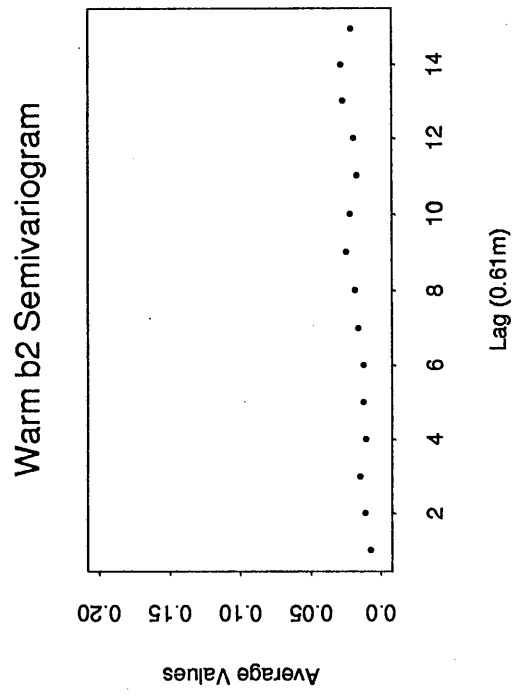
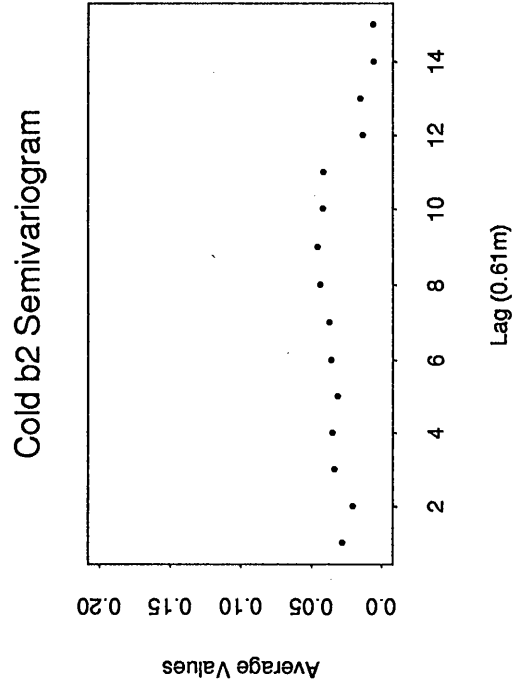
Residual Semivariogram



Residuals

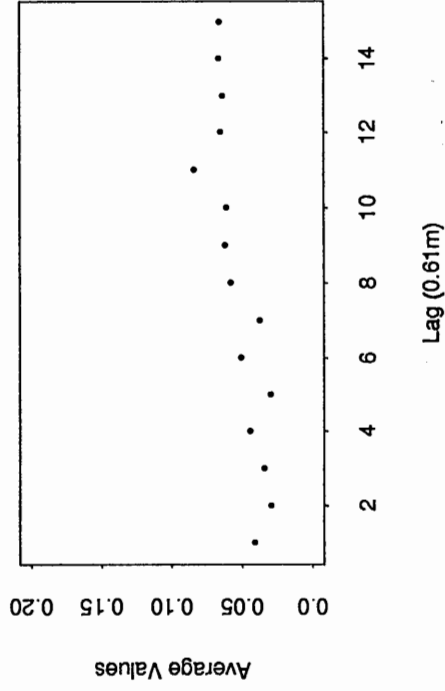


Magnet DCA316 Normal Multipoles: b2

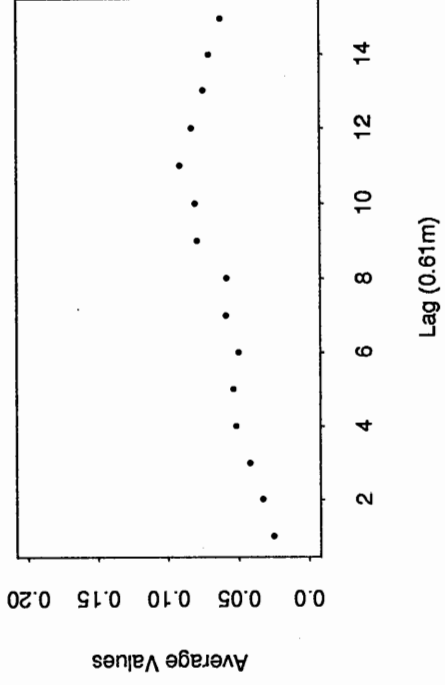


Magnet DCA317 Normal Multipoles: b2

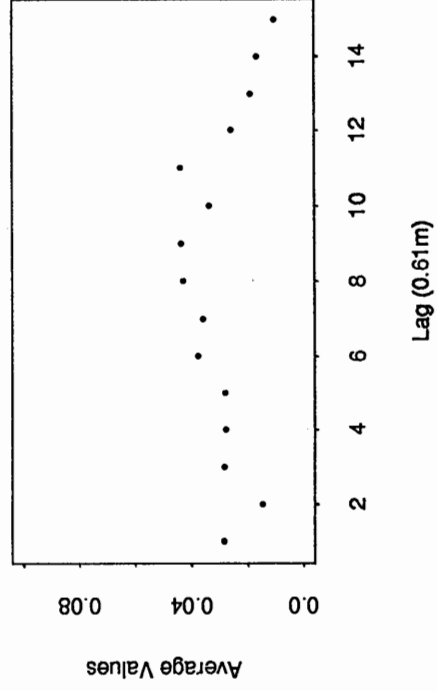
Cold b2 Semivariogram



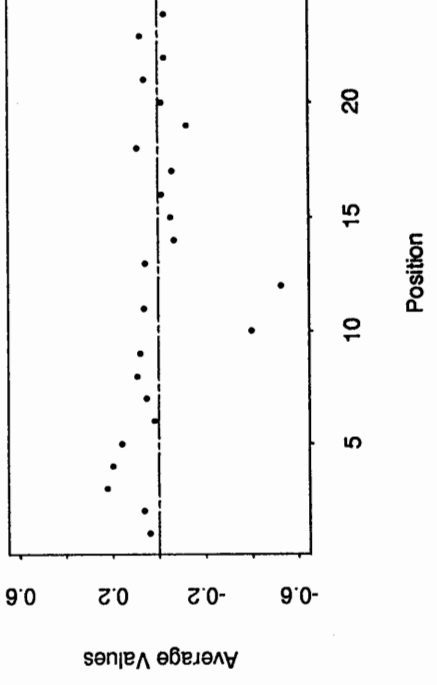
Warm b2 Semivariogram



Residual Semivariogram

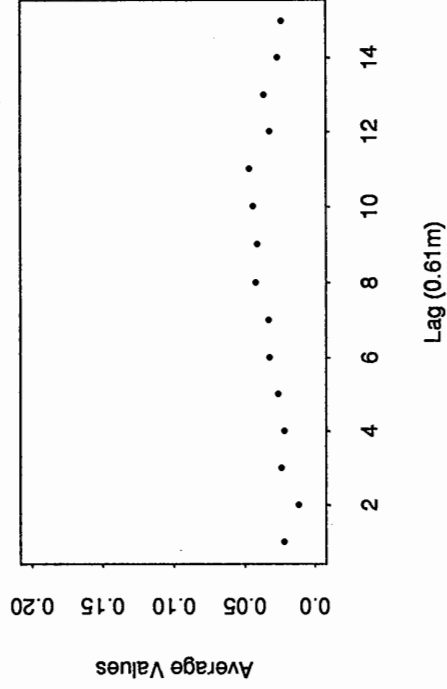


Residuals

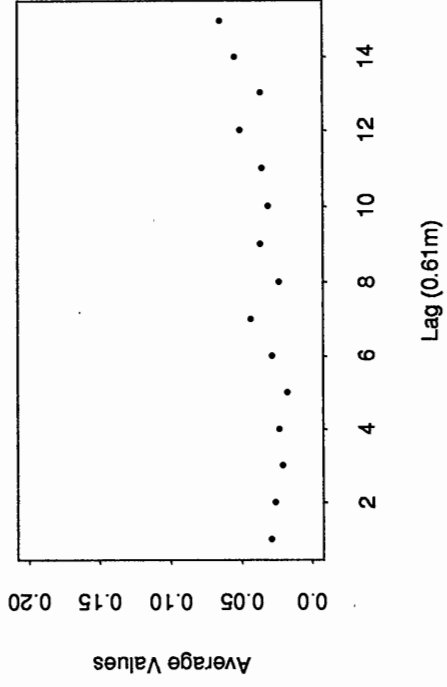


Magnet DCA318 Normal Multipoles: b2

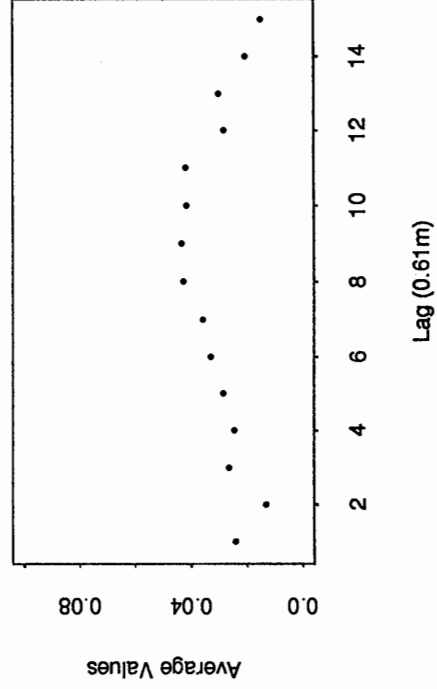
Cold b2 Semivariogram



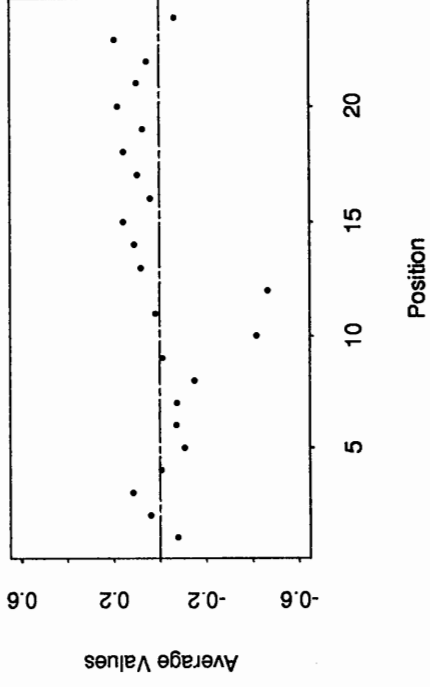
Warm b2 Semivariogram



Residual Semivariogram

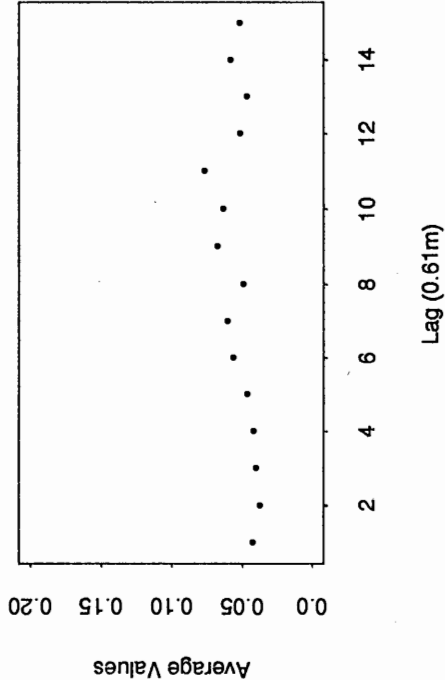


Residuals

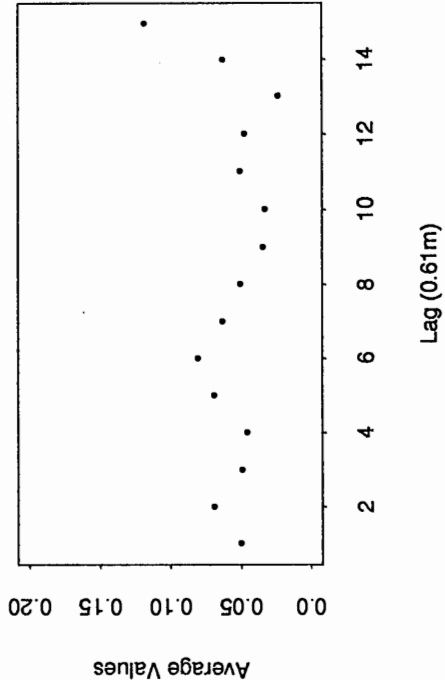


Magnet DCA319 Normal Multipoles: b2

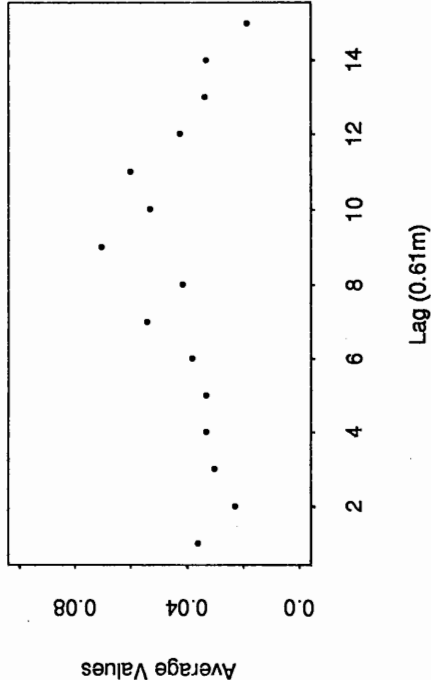
Cold b2 Semivariogram



Warm b2 Semivariogram



Residual Semivariogram



Residuals

