ONE-SIDED PREDICTION INTERVALS FOR THREE-STAGE SAMPLING FROM A NORMAL POPULATION

by

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ABSTRACT

One-sided prediction intervals for a normal population are extended to a third sampling stage. Procedures and tables are given for two situations. In the first situation, methods for obtaining such intervals are presented, and tables for calculating such prediction intervals are provided. In the second situation, a two-stage prediction interval has been applied, and a third stage is now required. Sample sizes are given for the third stage.

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1. INTRODUCTION

A random sample of n observations from a normal population is available. We wish to use its sample mean and sample standard deviation to compute a prediction interval for future samples. A survey article by Hahn and Nelson (1973) discusses prediction intervals for various populations. Mann, Schafer and Singpurwalla (1974) give an interval which contains with probability γ all m observations of a future sample from the same population. Fertig and Mann (1977) construct prediction intervals to contain at least $m-k+1$ out of m future observations from a normal distribution with probability $1-\beta$. They consider life-test data and the variate of interest is the failure time of an item. Their lower prediction limit constitutes a "warranty period."

In this paper, we will extend these results to a third stage. The second stage production (the future production of Fertig and Mann (1977)) met the warranty criterion and we are now required to go into a tertiary stage where at least $\ell-t+1$ out of ℓ additional items are needed to meet the same warranty criterion given that at least $m-k+1$ out of m items met the criterion at the second stage. That is, we know that at least $m-k+1$ out of m items produced at the second stage were larger than $\bar{X}-rS$ (where $\bar{X}$ and S were based on a sample of size n at the first stage) and we are required to have $\ell-t+1$ out of ℓ items in the third stage larger than $\bar{X}-rS$.

We will solve this three-stage prediction problem for two situations. First, in Section 3 we consider the case where we know in advance that we are going to have three stages and in this case we solve for the value of r in the criterion such that

$$P\{Z_{(t)} > \bar{X}-rS \mid Y_{(k)} > \bar{X}-rS\} = 1-\beta$$

where $Z_{(t)}$ and $Y_{(k)}$ are order statistics as defined in Section 2 below. Secondly, we assume that the criterion of Fertig and Mann (1977) was met by the second stage sample, i.e. $Y_{(k)} > \bar{X}-rS$. Then it is necessary for additional production to meet the criterion that $\ell-t+1$ out of ℓ items on a third stage are larger than $\bar{X}-rS$. Here we solve for the largest possible sample size, ℓ, given the values of β, n, m, k and t, and that the value of r from Fertig and Mann (1977) was used at the second stage and will be used again at the third stage.

2. FORMULA DEVELOPMENT FOR THE THREE-STAGE PROBABILITY

Let $X_1,\ldots,X_n$ be a random sample from the $N(\mu,\sigma^2)$ distribution. At the first stage of our three-stage procedure the sample mean $\bar{X}$ and the sample standard deviation S are computed from this sample. Let $Y_1,\ldots,Y_m$ and $Z_1,\ldots,Z_\ell$ be independent random samples for the second and third stages, respectively, from the same $N(\mu,\sigma^2)$ distribution. All $n+m+\ell$ observations are independent of one another. In both of the situations under consideration we have to investigate the conditional probability $P\{Z_{(t)} > \bar{X}-rS \mid Y_{(k)} > \bar{X}-rS\}$, where $Z_{(t)}$ is the t^{th} smallest observation from the third stage sample of size ℓ ; $Y_{(k)}$ is the k^{th} smallest observation from the second stage sample of size m; and $\bar{X} = \sum_{i=1}^{n} X_i/n$ and $S^2 = \sum_{i=1}^{n}(X_i-\bar{X})^2/(n-1)$ are the sample mean and sample variance of the first stage of size n.

The probability that at least m-k+1 out of m observations in the second sample are greater than $\bar{X}$-rS is given by

$$\Pr(Y_{(k)} > \bar{X}-rS) = \int_{-\infty}^{\infty} \Pr(y > \bar{X}-rS) f_{Y_{(k)}}(y)dy$$

$$= \int_{-\infty}^{\infty} F_{\bar{X}-rS}(y) f_{Y_{(k)}}(y)dy \tag{2.1}$$

where $f_{Y_{(k)}}(\cdot)$ is the pdf of $Y_{(k)}$ and $F_{\bar{X}-rS}(\cdot)$ is the cdf of $\bar{X}$-rS. The computational form for this two-stage probability $P(Y_{(k)} > \bar{X}-rS)$ was given in Fertig and Mann (1977). The joint probability that at least (m-k+1) out of m observations in the second sample are greater than $\bar{X}$-rS and at least (ℓ-t+1) out of ℓ observations in the third sample are greater than $\bar{X}$-rS is given by

$$\Pr(\bar{Z}_{(t)} > \bar{X}-rS, Y_{(k)} > \bar{X}-rS)$$

$$= \int_{-\infty}^{\infty} \Pr\{Z_{(t)} > x, Y_{(k)} > x\} f_{\bar{X}-rS}(x)dx$$

$$= \int_{-\infty}^{\infty} \Pr\{Z_{(t)} > x\} \Pr\{Y_{(k)} > x\} F'_{\bar{X}-rS}(x)dx$$

$$= \sum_{j=0}^{t-1} \binom{\ell}{j} \sum_{i=0}^{k-1} \binom{m}{i} \int_{-\infty}^{\infty} [G(\tfrac{x-\mu}{\sigma})]^{i+j} [1-G(\tfrac{x-\mu}{\sigma})]^{m+\ell-i-j} F'_{\bar{X}-rS}(x)dx \tag{2.2}$$

where G(·) is the cumulative distribution for N(0,1). Equation (2.2) follows from the equations

$$\Pr\{Z_{(t)} > x\} = \sum_{j=0}^{t-1} \binom{\ell}{j} [G(\tfrac{x-\mu}{\sigma})]^{j} [1-G(\tfrac{x-\mu}{\sigma})]^{\ell-j}$$

and

$$\Pr\{Y_{(k)} > x\} = \sum_{i=0}^{k-1} \binom{m}{i} [G(\tfrac{x-\mu}{\sigma})]^{i} [1-G(\tfrac{x-\mu}{\sigma})]^{m-i} .$$

Carrying out the integration in equation (2.2) by parts, we obtain

$$\Pr\{Z_{(t)} > \bar{X}-rS \,, Y_{(k)} > \bar{X}-rS\}$$

$$= (m+\ell) \int_{-\infty}^{\infty} [1-G(\tfrac{x-\mu}{\sigma})]^{m+\ell-1} G'(\tfrac{x-\mu}{\sigma}) F_{\bar{X}-rS}(x)\,dx$$

$$+ \sum_{j=0}^{t-1} \binom{\ell}{j} \sum_{\substack{i=0 \\ i+j\neq 0}}^{k-1} \binom{m}{i} \Bigg\{ (m+\ell) \int_{-\infty}^{\infty} G'(\tfrac{x-\mu}{\sigma}) [G(\tfrac{x-\mu}{\sigma})]^{i+j} [1-G(\tfrac{x-\mu}{\sigma})]^{m+\ell-i-j-1} F_{\bar{X}-rS}(x)\,dx$$

$$- (i+j) \int_{-\infty}^{\infty} G'(\tfrac{x-\mu}{\sigma}) [G(\tfrac{x-\mu}{\sigma})]^{i+j-1} [1-G(\tfrac{x-\mu}{\sigma})]^{m+\ell-i-j-1} F_{\bar{X}-rS}(x)\,dx \Bigg\}$$

$$= \sum_{j=0}^{t-1} \binom{\ell}{j} \sum_{i=0}^{k-1} \binom{m}{i} \cdot \frac{1}{\binom{m+\ell-1}{i+j}} \Bigg\{ \int_{-\infty}^{\infty} f_{V_{i+j+1,m+\ell}}(x) F_{\bar{X}-rS}(x)\,dx$$

$$- \int_{-\infty}^{\infty} f_{V_{i+j,m+\ell-1}}(x) F_{\bar{X}-rS}(x)\,dx \Bigg\} \qquad (2.3)$$

where $V_{a,b}$ is the a^{th} smallest observation in a random sample of b observations from the $N(\mu,\sigma^2)$ distribution and $f_{V_{a,b}}(\cdot)$ is the pdf of $V_{a,b}$. Using equation (2.1), we obtain

$$\Pr\{Z_{(t)} > \bar{X}-rS \,, Y_{(k)} > \bar{X}-rS\}$$

$$= \sum_{j=0}^{t-1} \binom{\ell}{j} \sum_{i=0}^{k-1} \binom{m}{i} \cdot \frac{1}{\binom{m+\ell-1}{i+j}} \{\Pr(V_{i+j+1,m+\ell} > \bar{X}-rS) - \Pr(V_{i+j,m+\ell-1} > \bar{X}-rS)\}$$

That is, the three-stage joint probability here can be expressed in terms of the two-stage probability.

3. DETERMINATION OF THE FIRST STAGE FACTOR r WHEN THREE STAGES ARE TO BE USED

In this section we assume that we know we have three stages before the first stage is completed. The lower prediction limit $\bar{X}$-rS can be obtained by determination of r such that

$$\Pr\{Z_{(t)} > \bar{X}-rS \mid Y_{(k)} > \bar{X}-rS\} = 1-\beta.$$

The intervals are constructed such that they will contain with probability $(1-\beta)$ at least $(\ell-t+1)$ out of ℓ observations from a third stage sample given that they contained at least $(m-k+1)$ out of m observations from a second stage sample.

For $n=2(2)\ 10(5)\ 30(10)50, \infty$ and $m=\ell=20(10)80$, and the following values of k, t and β we found the value of r as given in Tables 1 through 9:

Table	k	t	β
1	1	2	.10
2	1	2	.05
3	1	3	.05
4	1	3	.10
5	1	4	.10
6	2	2	.10
7	1	4	.05
8	2	3	.10
9	2	3	.05

4. SAMPLE SIZE REQUIRED FOR THE THIRD STAGE GIVEN THAT THE SECOND STAGE SAMPLE SATISFIES FERTIG AND MANN'S CRITERION

We assume that the user has conducted the two stage process as given by Fertig and Mann (1977) and that the second stage sample has met the criterion that at least m-k+1 out of m observations are greater than $\bar{X}$-rS where r is obtained from Fertig and Mann(1977). Now we wish to find the maximum third stage sample size, ℓ, allowed so that with probability 1-β at least (ℓ-t+1) out of ℓ observations are also greater than $\bar{X}$-rS.

That is, $\Pr\{Z_{(t)} > \bar{X}-rS \mid Y_{(k)} > \bar{X}-rS\} = 1-\beta$.

Since it seems logical that the same risk be applied to the third stage as was used on the second stage, we also assume that $\Pr\{Y_{(k)} > \bar{X}-rS\} = 1-\beta$. Then we wish to find the maximum possible sample size ℓ such that $\Pr\{Z_{(t)} > \bar{X}-rS\ ,\ Y_{(k)} > \bar{X}-rS\} \geq (1-\beta)^2$.

For n=10(5) 30(10) 50, ∞ and m=20(10)80, and the following values of k, t and β we found the values of the third stage sample size ℓ as given in Table 10 through 12:

Table	k	t	β
10	2	2	.10
11	2	3	.10
12	2	2	.05

5. EXAMPLE

The breakdown voltage is available based on a random sample of 10 capacitors from a population of capacitors. The sample mean, $\bar{X}$, and sample standard deviation, S, of the breakdown voltages in kilovolts (KV) for these 10 capacitors are 9.53 and 1.00, respectively. A normal distribution for the breakdown voltage is assumed. We wish to use these 10 observations to obtain a 95 percent prediction interval for the 2nd smallest breakdown voltage of a future sample of 40 capacitors from the same population. For n=10, m=40, k=2 and β=.05, Fertig and Mann (1977) found that r=3.28. Hence a 95 percent lower prediction limit for the 2nd smallest breakdown voltage of the 40 capacitors is given by $\bar{X}-rS=9.53-(3.28)(1.00)=6.25$ KV.

For the problem discussed in Section 3 of this paper where we assume that we are going to have a third stage sample, we are given that the third stage sample size ℓ is 40, $n=10$, $m=40$, $k=2$, $t=3$ and $\beta=.05$. From Table 9, we get $r=2.3949$. Hence our breakdown voltage of the capacitors is at least $9.53-(2.3949)(1.00)=7.14$KV. Now we can be 95 percent sure that at least 38 of these 40 capacitors at the third stage will have breakdown voltages which exceed 7.14 KV given that at least 39 out of 40 capacitors at the second stage had breakdown voltages which exceeded 7.14 KV.

Next consider the problem discussed in Section 4 of this paper where we find that additional capacitors are needed given that with probability .95 at least 39 of these 40 capacitors at the second stage had breakdown voltages which exceeded 6.25 KV. We use the Tables 10 through 12 to answer the following question: What is the largest third stage sample size ℓ we can take and still be 95 percent sure that at least $\ell-1$ capacitors in the third sample will be greater than 6.25 KV? Hence, $m=40$, $n=10$, $k=2=t$, $\beta=.05$ and ℓ is to be determined. From Table 12, we find $\ell=64$ to be the largest third sample size which gives a 95 percent confidence that at least 63 out of 64 of the third stage sample will have breakdown voltages which exceed 6.25 KV. Of course, any sample size smaller than 64, say ℓ, will have at least $\ell-1$ voltages which exceed 6.25 KV with probability at least 0.95.

6. REFERENCES

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Factors for obtaining 100(1-β)% one-sided prediction intervals for containing at least ℓ-t+1 out of ℓ observations at a third stage given a first stage sample of size n and a second stage sample of size m(=ℓ) for which at least m-k+1 of m observations were in the prediction interval.

TABLE 1

$k = 1 \quad t = 2 \quad \beta = .10$

n \ ℓ	20	30	40	50	60	70	80
2	1.4800	1.4985	1.5040	1.5055	1.5054	1.5047	1.5037
4	1.7803	1.8728	1.9265	1.9626	1.9891	2.0097	2.0263
6	1.8744	1.9971	2.0722	2.1246	2.1642	2.1956	2.2213
8	1.9145	2.0534	2.1405	2.2024	2.2497	2.2876	2.3190
10	1.9345	2.0832	2.1778	2.2458	2.2981	2.3403	2.3754
15	1.9536	2.1146	2.2192	2.2954	2.3548	2.4031	2.4436
20	1.9584	2.1248	2.2341	2.3143	2.3771	2.4285	2.4717
25	1.9573	2.1284	2.2401	2.3226	2.3874	2.4405	2.4854
30	1.9588	2.1295	2.2427	2.3264	2.3925	2.4467	2.4925
40	1.9568	2.1290	2.2437	2.3289	2.3962	2.4517	2.4986
50	1.9547	2.1275	2.2429	2.3288	2.3968	2.4528	2.5004
∞	1.9282	2.0999	2.2152	2.3013	2.3698	2.4265	2.4747

TABLE 2

$k = 1 \quad t = 2 \quad \beta = .05$

n \ ℓ	20	30	40	50	60	70	80
2	2.4762	2.4751	2.4652	2.4546	2.4450	2.4363	2.4287
4	2.4426	2.5622	2.6328	2.6810	2.7167	2.7446	2.7673
6	2.3981	2.5479	2.6413	2.7075	2.7580	2.7983	2.8317
8	2.3604	2.5227	2.6263	2.7009	2.7585	2.8049	2.8437
10	2.3306	2.4990	2.6079	2.6870	2.7486	2.7986	2.8405
15	2.2802	2.4533	2.5677	2.6519	2.7180	2.7721	2.8178
20	2.2483	2.4224	2.5384	2.6243	2.6921	2.7478	2.7949
25	2.2264	2.4004	2.5168	2.6034	2.6718	2.7282	2.7760
30	2.2105	2.3840	2.5004	2.5871	2.6559	2.7126	2.7607
40	2.1885	2.3611	2.4770	2.5636	2.6325	2.6893	2.7377
50	2.1745	2.3459	2.4612	2.5475	2.6162	2.6730	2.7213
∞	2.0955	2.2541	2.3674	2.4495	2.5148	2.5689	2.6151

TABLE 3

$k=1 \quad t=3 \quad \beta=.05$

n \ ℓ	20	30	40	50	60	70	80
2	1.3484	1.3941	1.4124	1.4213	1.4260	1.4287	1.4302
4	1.6214	1.7382	1.8046	1.8486	1.8805	1.9050	1.9246
6	1.7069	1.8525	1.9399	2.0001	2.0451	2.0804	2.1093
8	1.7425	1.9035	2.0026	2.0722	2.1249	2.1668	2.2013
10	1.7595	1.9297	2.0361	2.1117	2.1695	2.2157	2.2540
15	1.7739	1.9556	2.0718	2.1556	2.2204	2.2728	2.3165
20	1.7762	1.9626	2.0832	2.1709	2.2392	2.2947	2.3412
25	1.7755	1.9641	2.0869	2.1768	2.2470	2.3042	2.3524
30	1.7887	1.9592	2.0877	2.1788	2.2502	2.3085	2.3577
40	1.7711	1.9613	2.0865	2.1788	2.2514	2.3109	2.3611
50	1.7685	1.9588	2.0844	2.1772	2.2503	2.3103	2.3611
∞	1.7261	1.9139	2.0384	2.1308	2.2039	2.2642	2.3153

TABLE 4

$k=1 \quad t=3 \quad \beta=.10$

n \ ℓ	20	30	40	50	60	70	80
2	.7955	.8447	.8673	.8800	.8880	.8935	.8974
4	1.1670	1.2635	1.3173	1.3526	1.3779	1.3972	1.4125
6	1.3184	1.4421	1.5146	1.5638	1.6001	1.6284	1.6513
8	1.3979	1.5388	1.6236	1.6823	1.7261	1.7607	1.7890
10	1.4456	1.5982	1.6917	1.7569	1.8064	1.8456	1.8778
15	1.5069	1.6767	1.7832	1.8591	1.9171	1.9637	2.0023
20	1.5353	1.7138	1.8275	1.9092	1.9722	2.0231	2.0654
25	1.5510	1.7347	1.8526	1.9379	2.0041	2.0577	2.1025
30	1.5609	1.7477	1.8684	1.9561	2.0244	2.0798	2.1263
40	1.5726	1.7628	1.8867	1.9773	2.0481	2.1058	2.1544
50	1.5730	1.7715	1.8968	1.9889	2.0611	2.1202	2.1699
∞	1.5863	1.7828	1.9131	2.0095	2.0857	2.1483	2.2014

TABLE 5

$k = 1 \quad t = 4 \quad \beta = .10$

n \ ℓ	20	30	40	50	60	70	80
2	.3749	.4560	.4942	.5165	.5312	.5415	.5492
4	.8011	.9051	.9613	.9974	1.0230	1.0423	1.0574
6	.9808	1.1076	1.1797	1.2276	1.2623	1.2891	1.3106
8	1.0807	1.2241	1.3078	1.3645	1.4064	1.4389	1.4652
10	1.1435	1.2992	1.3918	1.4553	1.5025	1.5396	1.5698
15	1.2287	1.4042	1.5114	1.5864	1.6430	1.6879	1.7248
20	1.2700	1.4572	1.5733	1.6553	1.7177	1.7676	1.8089
25	1.2934	1.4881	1.6100	1.6967	1.7631	1.8164	1.8607
30	1.3079	1.5078	1.6337	1.7238	1.7931	1.8489	1.8953
40	1.3241	1.5306	1.6618	1.7563	1.8293	1.8884	1.9377
50	1.3323	1.5428	1.6772	1.7743	1.8497	1.9108	1.9619
∞	1.3394	1.5595	1.7025	1.8074	1.8898	1.9573	2.0144

TABLE 6

$k = 2 \quad t = 2 \quad \beta = .10$

n \ ℓ	20	30	40	50	60	70	80
2	2.5687	2.5560	2.5382	2.5220	2.5081	2.4962	2.4859
4	2.3773	2.5088	2.5865	2.6394	2.6785	2.7091	2.7340
6	2.2926	2.4560	2.5578	2.6298	2.6847	2.7286	2.7648
8	2.2364	2.4123	2.5244	2.6051	2.6672	2.7174	2.7591
10	2.1960	2.3778	2.4952	2.5804	2.6465	2.7002	2.7451
15	2.1313	2.3181	2.4406	2.5307	2.6013	2.6590	2.7076
20	2.0929	2.2803	2.4041	2.4957	2.5679	2.6271	2.6771
25	2.0698	2.2543	2.3783	2.4703	2.5430	2.6029	2.6535
30	2.0493	2.2354	2.3591	2.4511	2.5240	2.5840	2.6349
40	2.0266	2.2097	2.3325	2.4241	2.4968	2.5569	2.6079
50	2.0115	2.1930	2.3150	2.4061	2.4785	2.5383	2.5892
∞	1.9282	2.0999	2.2152	2.3013	2.3698	2.4265	2.4747

TABLE 7

$k = 1 \quad t = 4 \quad \beta = .05$

n \ ℓ	20	30	40	50	60	70	80
2	.8115	.8796	.9108	.9283	.9394	.9469	.9523
4	1.1592	1.2766	1.3415	1.3837	1.4138	1.4367	1.4547
6	1.2953	1.4397	1.5236	1.5802	1.6218	1.6541	1.6802
8	1.3641	1.5252	1.6213	1.6875	1.7368	1.7755	1.8071
10	1.4039	1.5762	1.6808	1.7536	1.8083	1.8517	1.8873
15	1.4517	1.6403	1.7575	1.8405	1.9038	1.9545	1.9964
20	1.4713	1.6682	1.7921	1.8808	1.9489	2.0037	2.0493
25	1.4807	1.6824	1.8104	1.9025	1.9736	2.0311	2.0791
30	1.4858	1.6903	1.8210	1.9154	1.9885	2.0478	2.0973
40	1.4907	1.6982	1.8317	1.9288	2.0044	2.0658	2.1174
50	1.4929	1.7016	1.8366	1.9350	2.0119	2.0746	2.1273
∞	1.4658	1.6761	1.8134	1.9144	1.9937	2.0589	2.1140

TABLE 8

$k = 2 \quad t = 3 \quad \beta = .10$

n \ ℓ	20	30	40	50	60	70	80
2	1.4278	1.4746	1.4918	1.4993	1.5028	1.5043	1.5049
4	1.6044	1.7354	1.8097	1.8590	1.8948	1.9223	1.9442
6	1.6505	1.8118	1.9087	1.9755	2.0254	2.0647	2.0967
8	1.6646	1.8411	1.9498	2.0262	2.0841	2.1302	2.1681
10	1.6683	1.8534	1.9693	2.0516	2.1145	2.1649	2.2066
15	1.6649	1.8601	1.9850	2.0751	2.1447	2.2010	2.2480
20	1.6582	1.8573	1.9860	2.0797	2.1525	2.2117	2.2613
25	1.6521	1.8529	1.9835	2.0790	2.1535	2.2143	2.2653
30	1.6469	1.8485	1.9801	2.0766	2.1522	2.2139	2.2659
40	1.6336	1.8412	1.9735	2.0710	2.1476	2.2103	2.2633
50	1.6340	1.8355	1.9681	2.0660	2.1430	2.2062	2.2595
∞	1.5856	1.7828	1.9131	2.0095	2.0857	2.1483	2.2014

TABLE 9

$k = 2 \quad t = 3 \quad \beta = .05$

n \ ℓ	20	30	40	50	60	70	80
2	2.6220	2.6572	2.6584	2.6516	2.6428	2.6339	2.6254
4	2.2901	2.4645	2.5654	2.6333	2.6830	2.7215	2.7525
6	2.1704	2.3708	2.4935	2.5794	2.6443	2.6959	2.7383
8	2.0974	2.3067	2.4380	2.5315	2.6031	2.6605	2.7081
10	2.0467	2.2597	2.3949	2.4923	2.5673	2.6280	2.6785
15	1.9675	2.1827	2.3211	2.4221	2.5007	2.5648	2.6185
20	1.9213	2.1361	2.2745	2.3761	2.4558	2.5209	2.5758
25	1.8910	2.1049	2.2424	2.3439	2.4237	2.4891	2.5443
30	1.8761	2.0825	2.2191	2.3201	2.3997	2.4651	2.5203
40	1.8494	2.0527	2.1874	2.2873	2.3663	2.4312	2.4863
50	1.8327	2.0336	2.1669	2.2659	2.3442	2.4087	2.4634
∞	1.7261	1.9139	2.0384	2.1308	2.2039	2.2642	2.3153

TABLE 10

Maximum Sample Sizes Allowed for Third Stage when $k = 2$, $t = 2$ and $\beta = .10$

n \ m	20	30	40				
10	33	46	68	90	113	138	163
15	25	40	57	74	92	111	130
20	23	37	52	67	82	99	115
25	22	35	49	63	77	92	107
30	22	34	47	60	74	88	102
40	21	33	45	57	70	83	95
50	21	32	44	56	68	80	92
∞	20	30	39	49	59	69	79

TABLE 11

Maximum Sample Sizes Allowed for Third Stage when $k = 2$, $t = 3$ and $\beta = .10$

n \ m	20	30	40	50	60	70	80
10	59	82	121	160	200	242	286
15	47	74	104	135	166	199	233
20	45	70	96	124	152	181	211
25	43	67	92	118	144	171	198
30	43	66	90	114	139	165	190
40	42	64	87	110	134	157	182
50	42	63	86	108	131	154	177
∞	41	62	82	103	124	145	165

TABLE 12

Maximum Sample Sizes Allowed for Third Stage when $k = 2$, $t = 2$ and $\beta = .05$

n \ m	20	30	40	50	60	70	80
10	27	45	64	84	105	128	151
15	24	38	53	69	86	103	120
20	22	35	49	63	77	92	107
25	22	34	46	59	73	86	100
30	21	33	45	57	70	83	96
40	21	32	43	55	67	79	91
50	21	31	42	54	65	77	88
∞	20	29	39	49	60	70	79

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One-sided prediction intervals for a normal population are extended to a third sampling stage. Procedures and tables are given for two situations. In the first situation, methods for obtaining such intervals are presented, and tables for calculating such prediction intervals are provided. In the second situation, a two-stage prediction interval has been applied, and a third stage is now required. Sample sizes are given for the third stage.

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