A BIBLIOGRAPHY FOR THE ABLUE

by

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Summary. A bibliography is provided for publications pertaining to the asymptotically best linear unbiased estimators (ABLUE's) of location and scale parameters based on a subset of the sample quantiles and the associated problem of optimal quantile, or spacing, selection. Applications of the estimators to problems of hypothesis testing, percentile estimation and robust statistical inference are also included. A selected set of references is provided which discuss the relationship between the problem of optimal spacing selection and other statistical problems of optimal grouping, regression design for continuous time regression and optimal stratification.

Key Words: Asymptotically best linear unbiased estimator, optimal grouping, optimal spacings, optimal stratification, quantiles.

The ABLUE: a Bibliography

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The asymptotically best linear unbiased estimators, or ABLUE's, are easily computed location and scale parameter estimates constructed from a subset of the sample quantiles. General formulae for these estimators and their asymptotic relative Fisher efficiencies (ARE's) were developed by Ogawa (1951) motivated, in part, by the work of Mosteller (1946) and Yamanouchi (1949). These estimators are frequently quite efficient, even when based on quantile subsets as small as 5 or 7, provided the quantiles, or equivalently their spacing (i.e., their corresponding percentile points), is chosen correctly. The problem of selecting spacings that provide estimators with maximum ARE's is often called the optimal spacing problem. This problem has developed an extensive literature which includes articles that furnish tabulations of optimal spacings for most of the standard probability laws. Early work on optimal spacing selection (although not explicitly discussed as such) dates back to papers by Sheppard (1899) and Pearson (1920). However, as the computation of optimal spacings requires the solution of a system of nonlinear equations, it is of no surprise that the majority of work on this problem has occurred since the 1960's when high speed computers first began to be widely available.

Provided the user has available tables of optimal spacings for a probability distribution of interest, ABLUE's with high ARE's are readily computed. However, if such information is not available it is necessary to either i) compute the optimal spacings, which may negate the computational advantage of the ABLUE over other possibly more efficient estimators,

or ii) use nonoptimal spacings that may result in a considerable loss of estimator efficiency. Consequently, one objective of this bibliography is to provide the practitioner with a collection of references concerning spacing selection for various standard probability distributions. For this purpose, a listing of references that provide, or discuss how to obtain, optimal or nearly spacings is given, by the distributions to which they pertain, in the appendix. It is also hoped that this bibliography will aid researchers in this field in the identification of open as well as solved problems.

The references include only those relating, or being closely related, to the ABLUE and optimal spacing selection. Thus, material on the best linear unbiased estimator (BLUE) constructed from sample order statistics is excluded except for certain papers which compare properties of BLUE's and ABLUE's. Discussions regarding the BLUE's as well as some of the early work on the ABLUE can be found in Sarhan and Greenberg (1962), Johnson and Kotz (1970a,b) and Harter (1971). A selected set of references pertaining to problems of optimal grouping, stratification and regression design for time series, that are closely related and in some cases equivalent to the optimal spacing problem, are also provided. The emphasis is on papers that either discuss spacing selection or discuss the relationship between two or more of these problems.

A subject classification is added, in brackets, following each reference. This is intended to provide a rough differentiation between the various uses for the ABLUE such as estimation in censored samples, estimation of percentiles, hypothesis testing and robust inference. The following abbreviations are used:

C Provides optimal (or nearly optimal) spacings for censored samples from a particular distribution

D Optimal design for continuous time regression

E Comparison of the properties of BLUE's and ABLUE's

G General theory of the ABLUE and/or optimal spacing selection

GR Optimal grouping

O Other related problems

P Considers optimal (or nearly optimal) spacing selection for the estimation of a percentile using the ABLUE

R Robust procedure(s) involving the ABLUE

S Provides optimal (or nearly optimal) spacings for uncensored samples from a particular distribution

ST Optimal stratification

T Considers spacing selection for hypothesis testing.

It should be noted that the classification 'C' implies 'S' but not conversely.

Undoubtedly omissions and errors exist in this bibliography. Information regarding these would be greatly appreciated by the author.

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Classification of References by Distribution

Cauchy distribution

Balmer, Boulton and Sack (1974)
Bloch (1966)
Cane (1974)
Chan (1970)
Chan, Chan and Mead (1973)
Chan and Cheng (1971, 1974)
Chan, Cheng, Mead and Panjer (1973)
Dalcher (1955)
Eubank (1979)
Govindarajulu and Rao (1968)

Double exponential distribution

Ali, Umbach and Hassanein (1981)
Cheng (1978)
Saleh, Ali and Umbach (1981a)

Exponential distribution

Ali, Umbach and Hassanein (1981)
Chan and Cheng (1974)
Cheng (1975)
Dalcher (1955)
Eubank (1979, 1981a)
Kulldorff (1962, 1963a)
Ogawa (1960)
Saleh (1966, 1981)
Saleh and Ali (1966)
Saleh, Ali and Umbach (1981a)
Sarhan and Greenberg (1962)
Sarhan, Greenberg and Ogawa (1963)
Särndal (1961, 1962)

Extreme value distribution

Chan and Cheng (1974)
Chan, Cheng and Mead (1972)
Chan and Kabir (1969)
Chan and Mead (1971)
Cheng (1975)
Eubank (1979, 1981a)
Hassanein (1968, 1969a, 1972)
Kulldorff (1973)
Särndal (1962)

Gamma distribution

Dalcher (1955)
Hassanein (1977)
Rhodin (1975, 1976b)
Särndal (1964)

(Tukey's) Lambda distribution

Chan and Rhodin (1980)
Rhodin (1976c)

Logistic distribution

Chan (1969)
Chan, Chan and Mead (1971)
Chan and Cheng (1971, 1972, 1974)
Chan, Cheng, Mead and Panjer (1973)
Cheng (1975)
Eubank (1979, 1981a,b)
Gupta and Gnanadesikan (1966)
Hassanein (1969b, 1974)
Kulldorff (1964b)
Särndal (1962)

Lognormal distribution

Eubank (1979)
Särndal (1962)

Normal distribution

Benson (1949)
Chan and Chan (1973)
Chan and Cheng (1971, 1974)
Chan, Cheng, Mead and Panjer (1973)
Cheng (1980b)
Chernoff (1971)
Dalcher (1955)
Eisenberger and Posner (1965)
Eubank (1979, 1981a)
Govindarajulu and Rao (1968)
Higuchi (1954)
Kulldorff (1963b, 1964a)
Mosteller (1946)
Ogawa (1951, 1977a,b)
Pearson (1920)
Sarhan and Greenberg (1962)
Särndal (1961, 1962)
Sheppard (1899)
Yamanouchi (1949)

Pareto distribution

Chan and Cheng (1973, 1974)
Cheng (1975)
Eubank (1979, 1981a)
Kulldorff and Vännman (1973)
Umbach, Ali and Hassanein (1981)

Rayleigh distribution

Cheng (1975, 1980a)

Weibull distribution

Chan and Cheng (1974)
Chan, Cheng and Mead (1974)
Cheng (1975)
Dubey (1967)
Eubank (1979)
Hassanein (1971)
Särndal (1962)