

Math 1337 Sample Placement Exam
CALCULUS I EXAMINATION
Time - 2 hours

1. If $x^3 + y^3 + x^2y^2 = 3$, then the value of $\frac{dy}{dx}$ when $x = y = 1$ is
- a) 0 b) 1 c) -1 d) $\frac{2}{3}$ e) not defined
2. $\int_0^{19} \frac{dx}{(8+x)^{1/3}} =$
- a) $\frac{15}{2}$ b) 2 c) 0 d) $\frac{2}{15}$ e) $\frac{1}{2}$
3. The equation of the tangent line to the curve $y = f(x) = (x^2 - 5)^{3/2}$ at $x = 3$ is
- a) $y = 18x - 46$ b) $y = 18x + 8$ c) $y = 3x - 1$ d) $y = 3x + 8$
- e) none of a) - d)
4. The function $y = \frac{x^2 - 9}{2x^2 - 4}$ has vertical and horizontal asymptotes at
- a) $x = \pm 3$ and $y = \frac{1}{2}$ b) $x = \pm\sqrt{2}$ and $y = \frac{9}{4}$ c) $x = \pm 3$ and $y = \frac{9}{4}$
- d) $x = \pm\sqrt{2}$ and $y = \frac{1}{2}$ e) none of a) - d)
5. $\int \cos(3x + 7)dx =$
- a) $-\frac{1}{3} \sin(3x + 7) + C$ b) $\sin(3x + 7) + C$ c) $-\sin(3x + 7) + C$
- d) $\frac{1}{3} \sin(3x + 7) + C$ e) $\frac{1}{7} \cos(3x + 7) + C$

6. If $y = x^3 + 3$ and $u = 4x + 1$, then $\frac{dy}{du}$ is
- a) x b) $\frac{1}{x}$ c) $\frac{3}{4}x^2$ d) $u^3 + 3$ e) 0
7. $\int_0^{1/2} 24x\sqrt{1-x^2} dx =$
- a) $8 - 3\sqrt{3}$ b) $\frac{1}{2}$ c) $\frac{\pi}{2}$ d) $24 - 9\sqrt{3}$ e) none of a) - d)
8. If $f(x) = x + \frac{100}{x}$, $1 \leq x \leq 20$, then $f(x)$ has
- a) an absolute maximum of 101 and an absolute minimum of 20
b) an absolute maximum of 101 and an absolute minimum of 25
c) an absolute maximum of 20 and an absolute minimum of 1
d) an absolute minimum of 20 but no absolute maximum
e) no absolute maximum and no absolute minimum
9. $\int_{-2}^4 \sqrt{x^2} dx$ is
- a) 0 b) 6 c) ± 6 d) 10 e) nonexistent
10. $\frac{d}{dx} \left(\int_0^x \sqrt{1 + \tan t} dt \right)$ is
- a) $\sqrt{1 + \tan x} - 1$ b) $\sqrt{1 + \tan t}$ c) $\sqrt{1 + \tan x}$ d) $\frac{1}{2}(1 + \tan t)^{-1/2} \sec^2 t$
e) $\sqrt{1 + \sec^2 x}$
11. If $f(x) = x + \frac{4}{x}$, then $f(x)$ decreases on
- a) $(-2, 0) \cup (0, 2)$ b) $(-\infty, \infty)$ c) $(2, \infty)$ d) $(0, \infty)$ e) $(-1, 1)$

12. The point on the curve $y = \sqrt{x}$ nearest to $\left(\frac{7}{2}, 0\right)$ is
- a) $(0, 0)$ b) $(3, \sqrt{3})$ c) $(\sqrt{3}, 3)$ d) $(2, \sqrt{2})$ e) $(1, 1)$
13. The average value of $y = \cos x$ over the interval $0 \leq x \leq \frac{\pi}{2}$ is
- a) $\frac{\pi}{2}$ b) $\frac{2}{\pi}$ c) 1 d) 0 e) π
14. $\int_0^2 (x^2 - 4x + 4) dx$ is
- a) -2 b) 1 c) 2 d) -1 e) none of a) - d)
15. The graph of $y = 5x^4 - 3x^5$ has an inflection point at
- a) $(0, 0)$ and $(1, 2)$ b) $(0, 0)$ only c) $(1, 2)$ only d) $(0, 0)$ and $(4, 0)$
e) none of a) - d)
16. If $f(x) = x \cos x$, then $f'(x) =$
- a) $\sin x$ b) $-\sin x$ c) $-x \sin x$ d) $-x \sin x + \cos x$ e) $x \cos x - \sin x$
17. $\int (x^5 - 4x^2) dx =$
- a) $5x^4 - 8x + C$ b) $6x^6 - 12x^3 + C$ c) $\frac{x^6}{6} - \frac{4x^3}{3} + C$
d) $x^6 - 4x^3 + C$ e) $\frac{x^6}{6} - \frac{x^3}{3} + C$
18. If $y = \sin^3 2x$, then $\frac{dy}{dx} =$
- a) $-6 \sin^2 3x \cos 3x$ b) $3 \sin^2 2x$ c) $-3 \sin^2 2x$ d) $6 \sin^2 2x \cos 2x$
e) $3 \sin^2 3x \cos^2 3x$

19. If $f(x) = \frac{x+2}{x-3}$ for all $x \neq 3$, then $f'(2) =$

- a) 0 b) -1 c) 5 d) -5 e) -4

20. $\int \cos^3 2t \sin 2t dt =$

- a) $\frac{1}{8} \cos^4 2t + C$ b) $\frac{1}{4} \sin^4 2t \cos^2 2t + C$ c) $-\frac{1}{4} \cos^4 2t + C$
d) $-\frac{1}{8} \cos^4 2t + C$ e) $-\frac{1}{4} \cos^4 2t \sin^2 2t + C$

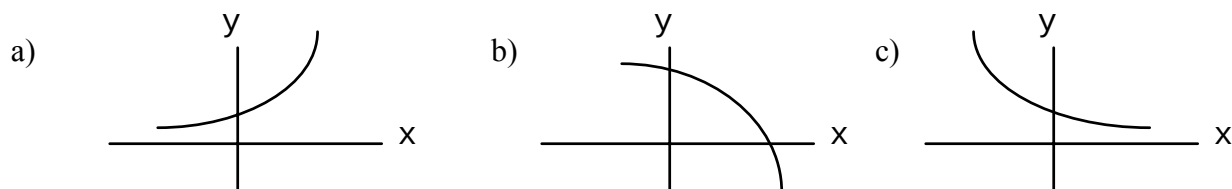
21. If $f(x) = x^2 - x$, find the slope of the secant line connecting the points $(3, f(3))$ and $(3+h, f(3+h))$. A simplified answer is

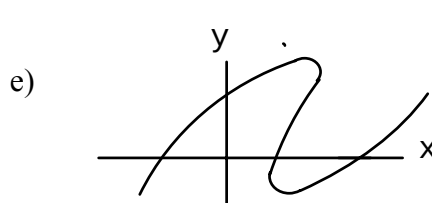
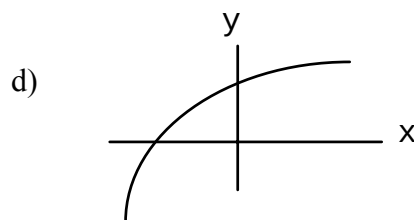
- a) $7+h$ b) $5h+h^2$ c) $\frac{12+7h+h^2}{h}$ d) $5+h$ e) $\frac{12+5h+h^2}{h}$

22. If $f(x)$ is continuous on $[a,b]$, which of the following is necessarily true?

- a) $f'(x)$ exists on (a,b) .
b) $f(x)$ has an absolute maximum and an absolute minimum on $[a,b]$.
c) $f(x)$ is either increasing on (a,b) , or decreasing on (a,b) .
d) all of the above.
e) none of the above.

23. If y is a function of x such that $y' < 0$ and $y'' > 0$ for all x , which of the following could be a part of the graph of $y = f(x)$:





24. If the area (in square units) of an expanding circle is increasing three times as fast as its radius (in linear units), then radius must be

a) $\frac{\pi}{2}$ b) $\frac{3}{2\pi}$ c) 1 d) $\frac{2\pi}{3}$ e) 2π

25. If $f(x) = x^{2/5}(x-4)^{1/5}$, then the domain of f' is

a) $\{x \mid x \neq 0\}$ b) $\{x \mid x > 0\}$ c) $\{x \mid 0 < x < 4\}$ d) $\{x \mid x \neq 0 \text{ and } x \neq 4\}$
 e) $\{x \mid x \text{ is a real number}\}$

26. The acceleration a of a body moving in a straight line is given in terms of time t by $a = 10 - 6t$. If the velocity of the body is 50 at $t = 2$ and if $s(t)$ is the position function of the body at time t , what is $s(5) - s(3)$?

a) 2 b) 33 c) 66 d) 0 e) none of a) - d)

27. If the line $27x - 16y = 0$ is tangent in the third quadrant to the curve $y = x^3 + k$, then k is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 0 d) $\frac{27}{32}$ e) $-\frac{27}{32}$

28. If $f(x) = 4x^5 - 10x^4 + 2$, then $f(x)$ has

a) a local (same meaning as “relative”) maximum of 2 and a local minimum of -30
 b) a local minimum of -30 but no local maximum
 c) no local extrema
 d) a local minimum of 2 but no local maximum
 e) a local minimum of 0 and a local maximum of 2

29. Given the function $f(x) = x^5 - 5x^4$, find all values of x for which its graph is concave upward.

- a) $x > 3$ b) $x < 3$ c) $0 < x < 3$ d) $x < 0$ e) $x < 0$ or $x > 3$.

30. If F and f are continuous functions such that $F''(x) = f'(x)$ for all x , then $\int_a^b f'(x) dx =$

- a) $F(b) - F(a)$ b) $F'(b) - F'(a)$ c) $F(a) - F(b)$ d) $F'(a) - F'(b)$
e) none of a) - d)

31. $\int x\sqrt{9-x^2} dx =$

- a) $\frac{1}{3}(9-x^2)^{3/2} + C$ b) $-\frac{1}{3}(9-x^2)^{3/2} + C$ c) $\frac{x^2(9-x^2)^{3/2}}{3} + C$
d) $-\frac{x^2(9-x^2)^{3/2}}{3} + C$ e) $(9-x^2)^{3/2} + C$

32. If n is a positive integer, for what value of k is $\int_1^k x^{n-1} dx = \frac{2}{n}$?

- a) $3^{1/n}$ b) $2^{1/n}$ c) 2^n d) 0 e) 3^n

33. Let $y = x \sin x$. When $x = \pi$ and $dx = \pi$, the value of dy is

- a) $-\pi^2$ b) 0 c) 1 d) $\frac{\pi}{2}$ e) π^2

34. For what value of k will $x + \frac{k}{x}$ have a relative minimum at $x = 6$.

- a) 36 b) -36 c) 6 d) 0 e) none of the above

35. A farmer wishes to enclose 20,000 sq. ft. in a rectangular plot adjacent to a straight river. The fence for the side opposite the river costs \$4.00 per ft. and the fence for the other two sides costs \$1.00 per ft. In order to complete this project the farmer must spend at least

a) \$400 b) \$800 c) $200\sqrt{3}$ d) \$20,000 e) none of the above

36. Suppose that $f(x) = \frac{x}{x^2 + 1}$. Find $\int_0^2 f'(x) dx$.

a) $\frac{2}{5}$ b) $-\frac{28}{25}$ c) $\frac{28}{25}$ d) $\tan^{-1}(2)$ e) $\frac{1}{2}\ln(2)$

37. Let $f(x) = \cos(\ln x)$. $f'(x) =$

a) $-\sin(\ln x) + \cos\left(\frac{1}{x}\right)$ b) $-\sin(\ln x)$ c) $-\sin(\ln x) \cdot \frac{1}{x}$
d) $\frac{1}{\cos x} \cdot (-\sin x)$ e) $-\sin\left(\frac{1}{x}\right)$

38. Let $y = e^{\sin x}$. Find $\frac{d^2 y}{dx^2} =$

a) $e^{\sin x} (\cos^2 x - \sin x)$ b) $e^{\sin x} \cdot \cos^2 x \cdot \sin x$ c) $e^{\sin x} (\cos x + \sin x)$
d) $e^{\sin x} \cdot (-\cos x)$ e) $e^{-\sin x}$

39. Which of the following statements are correct?

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & x \neq 1 \\ 5 & x = 1 \end{cases}$$

I. $\lim_{x \rightarrow 1} f(x) = 2$ II. $\lim_{x \rightarrow 1} f(x) = 5$ III. $\lim_{x \rightarrow 1} f(x)$ does not exist

IV. $\lim_{x \rightarrow -1} f(x) = 0$ V. $\lim_{x \rightarrow -1} f(x)$ does not exist

- a) only I and IV are true b) only I and V are true c) only II and IV are true
d) only II and V are true e) only III and V are true

40. Find $\int x e^{x^2} dx$.

- a) $e^{x^2} + 2x^2 e^{x^2} + C$ b) $e^{x^2} + x e^{x^2} + C$ c) $e^{x^2} + C$
d) $\frac{x^2}{2} e^{x^2} + C$ e) $\frac{1}{2} e^{x^2} + C$

Answer Key

1. C	2. A	3. A	4. D	5. D
6. C	7. A	8. A	9. D	10. C
11. A	12. B	13. B	14. E	15. C
16. D	17. C	18. D	19. D	20. D
21. D	22. B	23. C	24. B	25. D
26. C	27. E	28. A	29. A	30. B
31. B	32. A	33. A	34. A	35. B
36. A	37. C	38. A	39. A	40. E

Passing score: 30 correct answers.